THE METHOD OF SLICED SUBSTRUCTURES IN THE ANALYSIS OF BOILER-PODDED PCPV

S. KAWAMATA
Institute of Industrial Science, University of Tokyo, Minato-ku, Tokyo, Japan

O. ISOBATA
Shimizu Construction Co. Ltd., Research Laboratory, Koto-ku, Tokyo, Japan

T. KURODA
Nuclear Power Department, Electric Power Development Co. Ltd., Chiyoda-ku, Tokyo, Japan

SUMMARY

The geometry of the boiler-podded PCPV, having vertical cavities within the thickness of the vessel wall, leads to the indispensable practice of three-dimensional analysis for each state of design, i.e. elastic, time-dependent and failure analyses. However, the three-dimensional finite element analysis, having large degrees of freedom and band width of the system stiffness matrix, requires a huge amount of computer memory, computing time and labour for input data preparation, and consequently high cost of analysis.

This high cost of three-dimensional analyses is often a heavy burden to the designers, especially in the time-dependent and fracture analyses where numerous steps of elastic analysis are necessary to be repeated. Therefore, the methods of simplified three-dimensional analysis, which can provide acceptable accuracy within reasonable computation cost, are worthwhile developing for the purpose of design study of the PCPV.

In this paper, "the method of sliced substructures", a new method of simplified three-dimensional analysis of the boiler-podded PCPV is proposed and the efficiency and the accuracy obtainable by this method are shown by examples of actual analysis of a boiler-podded PCPV for HTGR.

The method of sliced substructures is based upon the assumption that the stress distribution of this type does not so largely deviate from the axisymmetric pattern for axial (σ_z) and shearing (σ_{rz}) components, though the horizontal components (σ_r, σ_θ and $\tau_{r\theta}$) are severely disturbed by the vertical cavities. The system stiffness matrix is formed as the summation of the two parts representing the axi-symmetric resistance and the asymmetric one in the horizontal plane, respectively. In the formation of the former part, the components of D matrix which are relating stresses σ_r and σ_θ to strains ϵ_r and ϵ_θ are suppressed. The suppressed components of the element stiffness are later supplemented in the stage of the system stiffness matrix.

The components of the system stiffness representing the horizontal resistance are evaluated for horizontally sliced layers separately, in which plane stress state is assumed. The matrix condensation technique is used for each horizontal substructures in order to reduce the two-dimensional degrees of freedom into ones for those nodes which are on the vertical plane of symmetry between cavities.

The results of elastic analysis of a PCPV for 770 MW HTGR by the new method are presented in comparison with the results of full three-dimensional analysis by the use of iso-parametric elements.

For the elastic analysis, very good coincidence in the stress distribution was obtained though there was some discrepancy in displacement. The CPU time for the new method was reduced to 1/8 of the full three-dimensional analysis.

1. Introduction

Prestressed Concrete Pressure Vessels (PCPV) of "podded boiler type" or "multi-cavity type" used for High Temperature Gas-cooled Reactors are characterized by vertical cylindrical cavities incorporated within the thickness of vessel wall in order to contain heat exchangers. For this type of PCPV three-dimensional stress analyses are inevitably required because of stress disturbance which is caused mainly by the boiler cavities.

Three-dimensional finite element analysis of PCPV, however, is often a heavy burden to designers because of laborious imput data preparation, large number of nodal freedoms, large band-width of system stiffness matrix and therefore long computation time required. For practice, it is uneconomical to carry out the full three-dimensional analysis for each configuration proposed in the process of design. Also, time-dependent creep analysis as well as failure analysis, in which numerous steps of stress computation and modification of stiffness matrix must be repeated, is extremely difficult for its computation cost.

From these reasons it is desirable to develop methods of approximate analysis of the boiler-podded PCPV, alternative to the full three-dimensional one, which allow to attain the accuracy tolerable for design purpose within reasonable computation cost.

Kawamata, one of the present authors proposed a new method of the approximate three-dimensional analysis of the boiler-podded PCPV, "the method of sliced substructures" [1].

The method of sliced substructures bases upon the assumption that the stress disturbance by the boiler cavities takes place mainly in the form of disturbance of two-dimensional stress components in the horizontal plane (σ_r, σ_θ, $\tau_{r\theta}$) while the distribution of other components (σ_z, τ_{rz}) remain in effect axi-symmetric. In this method the three-dimensional stress pattern is represented as a combination of the two-dimensional stress components of horizontally sliced layers with cavities and the axi-symmetric components of a solid of revolution. The nodal freedoms of the sliced layers, where plane stress state is assumed, are eliminated by the static condensation used in the substructure technique and the final degrees of freedom are equal to those of the corresponding axi-symmetric problems.

In the preceding paper [1], the efficiency and the accuracy of the method of sliced substructures were illustrated by simple examples. In the present paper the same method is applied to actual problems of the boiler-podded PCPV. The resuls of elastic analysis of PCPV by the new method with regard to inner pressure and thermal load are presented and compared with the same results of fully three-dimensional analysis by the use of the iso-parametric elements.

2. Method of Sliced Substructures

2.1 Basic Assumption

The method of sliced substructures is a method of approximate three-dimensional stress analysis which can be applied to boiler-podded PCPV under loads periodically distributed between boiler cavities such as dead load, prestress, thermal load, inner pressure, etc.

By the condition of symmetricity, a wedge shaped unit zone is to be analysed by the use of finite element displacement method. As shown in Fig.1, the unit to be analyzed is divided into layers of horizontal slices.

In the method of substructures, the following assumptions are adopted:

1) the perforated stand-pipe zone of the top slab is transformed into an equivalent transversely isotropic continium whose effective coefficients of elasticity are evaluated by

finite element analysis of unit area [1],

2) in the "main section," the vertical surface of symmetry inbetween the two boiler cavities, axi-symmetric nodal freedoms, $\widetilde{u}_i$ and $\widetilde{w}_i$, are assumed, from which axisymmetric strain components, ε_z and γ_{rz} , are evaluated as shown in Fig.2, and
3) for each slices, two-dimensional nodal freedoms in the horizontal plane, $\overline{u}$ and $\overline{w}$, are assumed which are constant in the thickness of the slice, as shown in Fig.3, from which strain components ε_x , ε_y , and τ_{xy} are evaluated.

2.2 Finite Element Idealization

Finite element subdivision of the main section, for which axi-symmetric nodal freedoms are allotted, is done in the same way as in the axi-symmetric problems. Here, rectangular ring elements being used, a mesh in lattice pattern is adopted as shown in Fig.4(a). It must be noted that some ring elements intersecting the boiler cavity has their arc length shorter than in the ordinary axi-symmetric problem without cavity.

The slicing of the main section is done in such a manner that a surface in the midhight of rectangular ring elements forms one boundary surface of a sliced layer as shown in Fig.4 (a). Then, each slice is subdivided into two-dimensional finite element mesh as shown in Fig.4(b).

As shown in Fig.4(b), the radial displacement $\overline{u}_i$ is the only nodal freedom for the nodes located on the vertical main section. Important point is that this radial displacement of the slice $\overline{u}_i$ is taken identical with the horizontal displacement $\widetilde{u}_i$ of the main section. As the result, the horizontal displacement of the slices becomes continuous in the vertical main section through the continuous displacement field for the ring elements, although it has some discontinuity between upper and lower layers of the slices as shown in Fig.3.

2.3 Stiffness of Ring Elements

In this paper, rectangular ring elements are used, for which the bilinear displacement field is assumed.

The stiffness matrix of the ring element is evaluated by the known formula

$$[\widetilde{K}_e] = \int_{vol} [B]^T [\widetilde{D}_1][B]\, dV \tag{1}$$

in the same manner as in the usual axi-symmetric problems except the following two points:

1) in order to suppress the contribution of the strains ε_r , ε_θ and $\gamma_{r\theta}$, the elasticity matrix $\widetilde{D}_1$ takes the following form:

$$\{\widetilde{\sigma}\} = [\widetilde{D}_1]\{\widetilde{\varepsilon}\} \tag{2}$$

in which

$$\begin{Bmatrix} \widetilde{\sigma}_r \\ \widetilde{\sigma}_\theta \\ \widetilde{\sigma}_z \\ \widetilde{\tau}_{rz} \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & d_{33} & 0 \\ 0 & 0 & 0 & d_{44} \end{bmatrix} \begin{Bmatrix} \widetilde{\varepsilon}_r \\ \widetilde{\varepsilon}_\theta \\ \widetilde{\varepsilon}_z \\ \widetilde{\gamma}_{rz} \end{Bmatrix} \tag{3}$$

where the coefficients d_{33} and d_{44} are the same to the corresponding ones of the matrix for the usual axi-symmetric problems.

2) in order to allow for the stiffness reduction due to the boiler cavity, the volume integral of Eq.(1) is carried out with regard to the real volume of the ring element, the volume of the element reduced by the cavity where necessary.

2.4 Stiffness of Sliced Substructures

The horizontal slices are dealt with as substructures where the freedoms of internal nodes are to be eliminated with only those on the main section maintained. In this paper, constant strain triangular plane stress elements are utilized.

For each substructures, the system equations for plane stress problem in the form of

$$[\bar{K}]\{\bar{d}\} = \{f_s\} \tag{4}$$

are obtained, where f_s is nodal external force in horizontal plane.

The nodal displacements of a sliced substructure, as shown in Fig.5, are divided into two groups, i.e. radial displacements on the main section, $\bar{u}_I$, and the remaining ones, $\bar{d}_{II}$. Correspondingly Eq.(4) takes a partitioned form of

$$\begin{bmatrix} \bar{K}_{11} & \bar{K}_{12} \\ \bar{K}_{21} & \bar{K}_{22} \end{bmatrix} \begin{Bmatrix} \bar{u}_I \\ \bar{d}_{II} \end{Bmatrix} = \begin{Bmatrix} f_{sI} \\ f_{sII} \end{Bmatrix} \tag{5}$$

where

f_{sI} : horizontal nodal loads acting on the nodes on the main section

f_{sII} : horizontal nodal loads acting on the other nodes

Elimination of $\bar{d}_{II}$ in Eq.(5) leads to the condensed equation of equilibrium as

$$[\bar{K}_I]\{\bar{u}_I\} = \{\bar{f}_I\} \tag{6}$$

where

$$[\bar{K}_I] = [\bar{K}_{11}] - [\bar{K}_{12}][\bar{K}_{22}]^{-1}[\bar{K}_{21}] \tag{7}$$

$$\{\bar{f}_I\} = \{f_I\} - [\bar{K}_{12}][\bar{K}_{22}]^{-1}\{f_{II}\} \tag{8}$$

When the number of the nodes located on the main section is m, the dimension of matrix in Eq.(6) becomes $(m \times m)$.

2.5 System Equations

As the nodal displacement $\bar{u}_I$ of each slice is identical with the radial components of the nodal displacements of the main section, $\tilde{u}$, the final system equations are formulated by the summation of the two of the system stiffness matrices, i.e.

$$\{[\tilde{K}] + [K]\}\{\tilde{d}\} = \{\tilde{f}\} + \{\bar{f}\} \tag{9}$$

where

$\{\tilde{d}\}$: two-dimensional nodal freedom allotted to the main section

$\{\tilde{f}\}$: vertical (Z) component of nodal load

$\{\bar{f}\}$: horizontal (r - θ) component of nodal load condensed according to Eq.(8)

$[K]$: system stiffness matrix assembled from axi-symmetric ring elements

$[\overline{K}]$: system stiffness matrix assembled from the condensed stiffness matrix $\overline{K}_I$ for each slice

The system stiffness matrix $\widetilde{K}$ in Eq.(9) for the main section has the degrees of freedom and the band structure identical with those in the usual axi-symmetric problems.

The matrix $\overline{K}$ in Eq.(9) is formed by putting $\overline{K}_I$ of Eq.(7) in the main diagonal of the system stiffness matrix with some modification. The following is a illustrative example of the arrangement of $\overline{K}$ matrix.

Referring to a simple pattern shown in Fig.6, let the nodal displacement vector of the main section, $\widetilde{d}$, be defined as

$$\{\widetilde{d}\} = \{\ \widetilde{u}_1\ \widetilde{w}_1\ \widetilde{u}_2\ \widetilde{w}_2\ \widetilde{u}_3\ \cdots\cdots\ \} \tag{10}$$

then $\overline{K}$ matrix takes the following form:

$$[\overline{K}] = \begin{array}{c} \begin{array}{ccc} \widetilde{d}_{1\sim4} & \widetilde{d}_{5\sim8} & \widetilde{d}_{9\sim12} \end{array} \\ \left[\begin{array}{c|c|c} \overline{K}^a & & \\ \hline & \overline{K}^b & \\ \hline & & \overline{K}^c \end{array}\right] \end{array} \tag{11}$$

where a,b and c are superfixes indicating the slice.

The partitioned diagonal submatrix $\overline{K}^a$, for example, takes the form of

$$[\overline{K}^a] = \left[\begin{array}{cc|cc|cc|cc} * & 0 & * & 0 & * & 0 & * & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline * & 0 & * & 0 & * & 0 & * & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline * & 0 & * & 0 & * & 0 & * & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline * & 0 & * & 0 & * & 0 & * & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \tag{12}$$

where * means non-zero element.

Also, load vectors $\widetilde{f}$ and $\overline{f}$ for this example have the following arrangement:

$$\{\widetilde{f}\} = \{\ 0\ \ f_{z1} \mid 0\ \ f_{z2} \mid 0\ \ f_{z3} \mid \cdots\cdots\cdots \mid 0\ \ f_{z12}\} \tag{13}$$

$$\begin{aligned}\{\overline{f}\} = \{\ & \overline{f}_1^a\ \ 0 \mid \overline{f}_2^a\ \ 0 \mid \overline{f}_3^a\ \ 0 \mid \overline{f}_4^a\ \ 0 \mid \\ & \overline{f}_5^b\ \ 0 \mid \overline{f}_6^b\ \ 0 \mid \cdots\cdots \mid \overline{f}_8^b\ \ 0 \mid \overline{f}_9^c\ \ 0 \mid \cdots\cdots \mid \overline{f}_{12}^c\ \ 0\ \}\end{aligned} \tag{14}$$

Solving Eq.(9), two-dimensional nodal displacements of the main section $\widetilde{d}$ are obtained. Then back substitution of $\widetilde{u}_I$ into Eq.(5) yields $\widetilde{d}_{II}$, the remaining displacement of each slice.

As regards to element stresses, the axi-symmetric components, σ_z and τ_{rz} , are evaluated by Eq.(3) and other components, (σ_r, σ_θ and $\tau_{r\theta}$) or (σ_x, σ_y and τ_{xy}), are obtained as plane stress of the slices.

3. Application of Method of Sliced Substructures to Elastic Problems of PCPV

To see the applicability of the new method, the elastic problems of a boiler-podded PCPV, designed for 770 MWe HTGR by General Atomic Co., was analyzed. The analyses for inner pressure and thermal load were carried out by both the new method and iso-parametric solid elements for comparison. The general view of the model for analysis is shown in Fig.7 indicating the iso-parametric element subdivision.

Applied inner pressure was 54.6 kg/cm^2. The assumed temperature was 62.8°C ~ 65.6° C in the surface of core cavity and 37.8° ~ 43.3° C in the outer surface of PCPV.

For analysis by the method of substructures, the main section of the PCPV was divided into ring elements as shown in Fig.8 and slices were divided into triangular elements, Fig.9 showing an example of the mid-height slice.

Fig.10 shows the comparison of displacements obtained by both methods. In Fig.10(a) for the case of inner pressure, displacement improved by modification, in which the contraction of the thickness of each slice by Poisson's ratio was considered and reflected to Z-component, is also shown.

Figs.11 and 12 show the comparison of principale stresses of the vertical main section and the horizontal section in the mid-height of core -cavity, respectively.

Comparison of the computing time by the use of IBM 370/158 machine for inner pressure is presented in Table I. The system equation for the substructure method was solved by Cholesky elimination, the one for iso-parametric solution, by Gauss-Seidel iteration starting with $\tilde{u}$ -values obtained by the substructure solution. The ratio of the computing times of 1 : 8 will be more favorable for the new method when the iteration for the other starts with zero values.

Table I. Comparison of Computing Time Unit: sec.

Substr. Stiffness Matrix incl. Condensation (6 zones)	623
Main Stiffness Matrix	55
Equations Solution (91 nodes)	90
Substr. Displacement and Stress	181
Total	949 sec

Stiffness Matrix	4378
Eqns. Solution (1390 nodes)	2924
Stress	384
Total	7686 sec

4. Conclusions

The method of substructures, an approximate method of three-dimensional stress analysis developed for the design of boiler-podded PCPV, was presented. The application of the new method to elstic problems of an PCPV indicated the accuracy obtainable by the method and, especially, a drastic reduction of computing time compared to the conventional full three-dimensional analysis.

This advantage of the method of sliced substructures will act more favourably to designers in the step analysis for creep and failure problems.

Acknowledgements

The authors are indebted to the collaboration of Dr.Y.Hangai, Univ.of Tokyo, Mr.S.Shioya, Nihon Univ, Mr.N.Tanaka and Mr.H.Akiyama, Shimiau Construction Co. and Mr.H.Murakami, Electric Power Development Co. Also appreciation is extended to General Atomic Co. for their offering the sample problems.

Reference

[1] KAWAMATA, S., "Analysis of Podded Boiler Type PCPV with Reference to the Analysis of Solid of Revolution," IABSE Seminar on CONCRETE STRUCTURES SUBJECTED TO TRIAXIAL STRESSES, May 1974, Bergamo, Italy

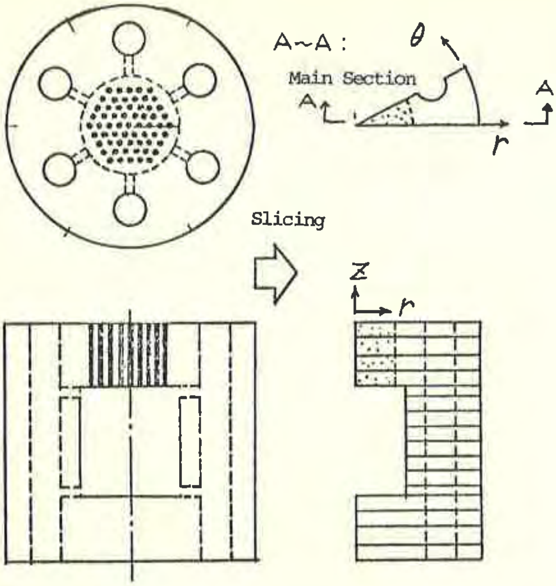


Fig.1 Analysis Model of Boiler-Podded PCPV

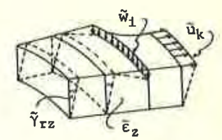


Fig.2 Axi-Symmetric Components of Strain and Displacement

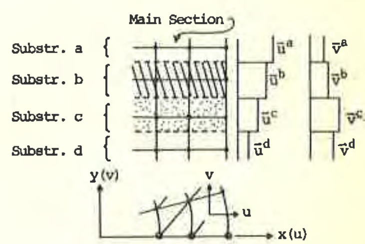
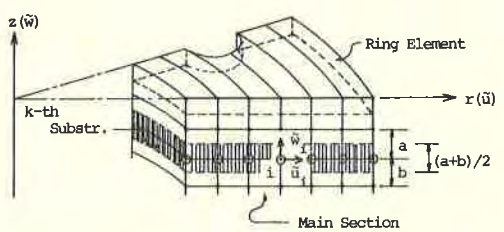
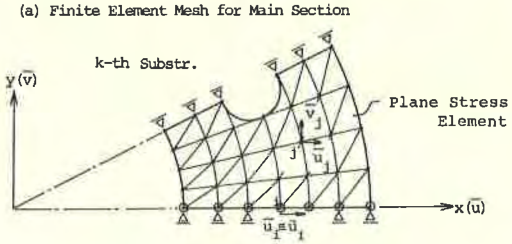


Fig.3 Assumed Displacement for Slices



(a) Finite Element Mesh for Main Section



(b) Finite Element Mesh for Sliced Substructure

Fig.4 Finite Element Subdivision

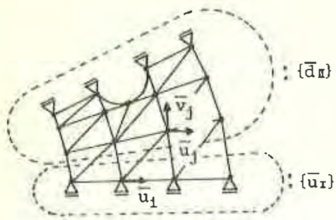


Fig.5 Patition of Nodal Freedoms in Slice

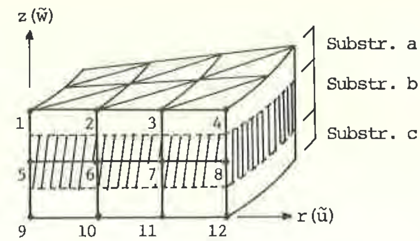


Fig.6 Example of Subdivision of Main Section

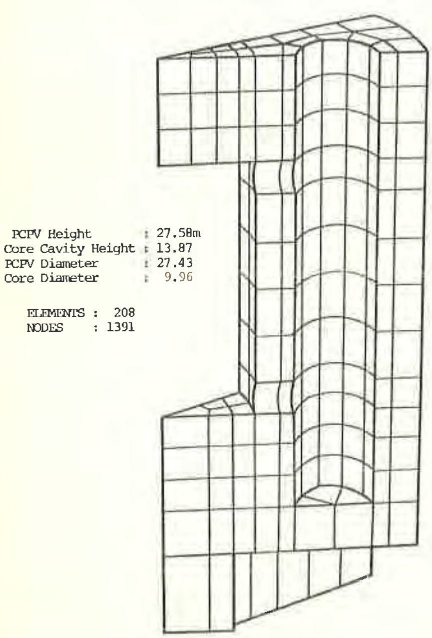


Fig.7 Subdivision of PCPV into Iso-Parametric Elements of 20 nodes

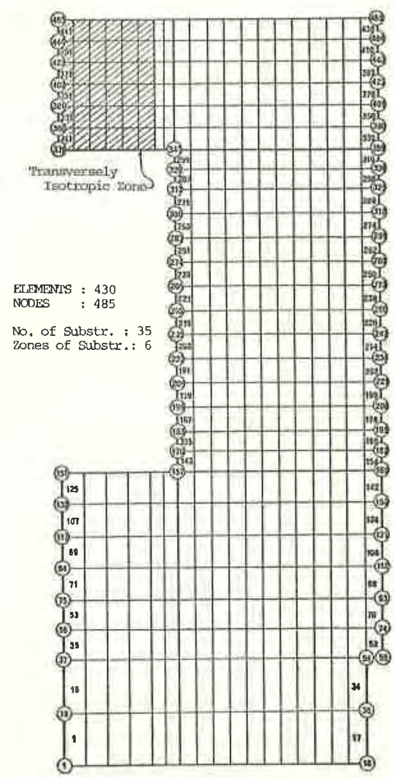


Fig.8 Subdivision of Main Section for Sliced Substructure Analysis

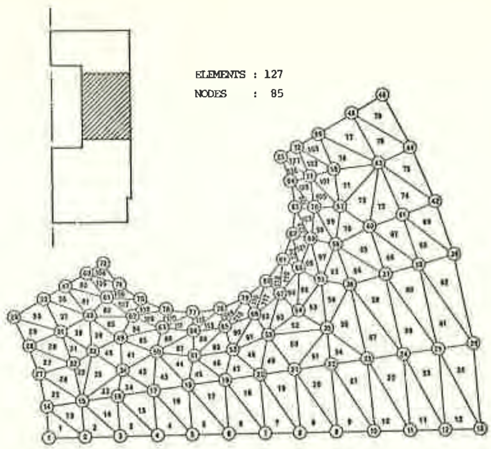


Fig.9 Subdivision of Mid-Height Slices

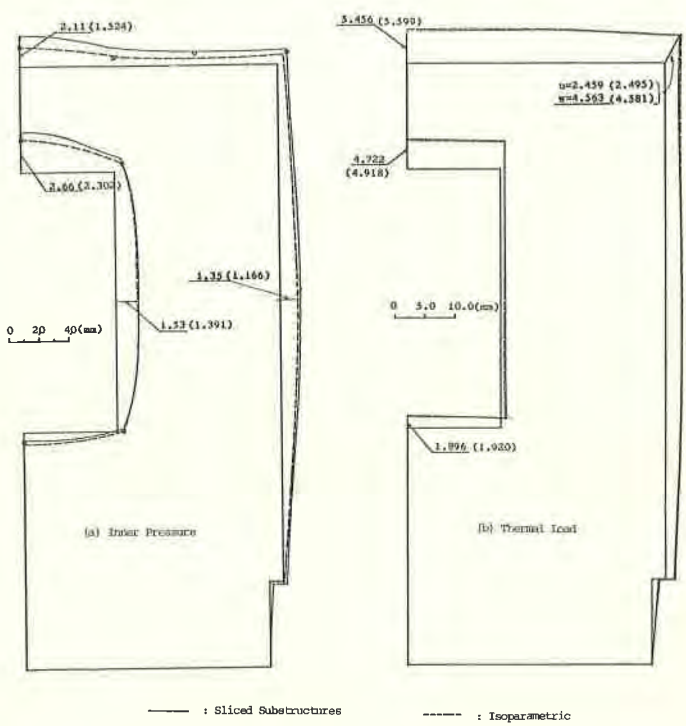
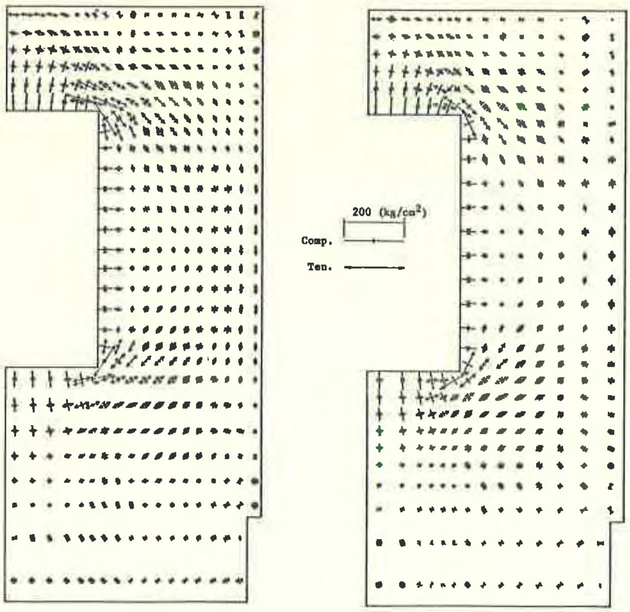


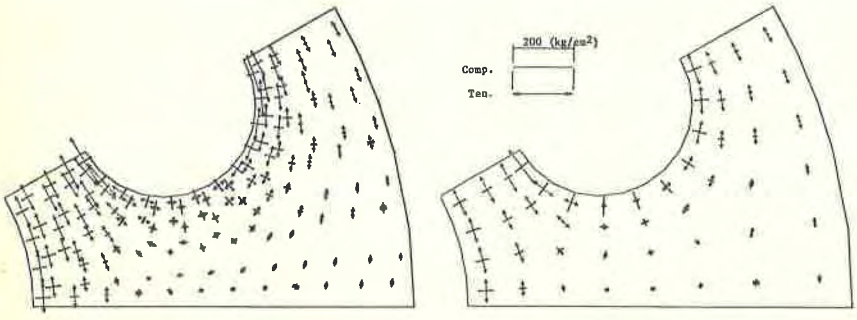
Fig.10 Comparison of Deformation of Main Section



(a) Sliced Substructures Solution

(b) Isoparametric Solution

Fig.11 Comparison of Principal Stress in Main Section for Inner Pressur



(a) Sliced Substructures Solution

(b) Isoparametric Solution

Fig.12 Comparison of Principal Stress in Mid-Height Horizontal Plane for Inner Pressure