Dynamic analysis of liquid storage tank including hydrodynamic interaction by boundary element method

I.-T. Hwang & K. Ting
Institute of Nuclear Energy Research, Taiwan

1 INTRODUCTION

Dynamic response of liquid storage tanks considering the hydrodynamic interactions due to earthquake ground motion has been extensively studied. Several finite element procedures, such as Balendra et. al. (1982) and Haroun (1983), have been devoted to investigate the dynamic interaction between the deformable wall of the tank and the liquid. Further, if the geometry of the storage tank can not be described by axi-symmetric case, the tank wall and the fluid domain must be discretized by three dimensional finite elements to investigate the fluid-structure-interactions. Thus, the need of large computer memory and expense of vast computer time usually make this analysis impractical. Hence, to compensate for the inadequacies of these analysis, a rigorous and efficient numerical technique using the boundary elements to model the hydrodynamic effects is successfully developed in this work.

Based on the ideas of Porter and Chopra (1973) for analysis of gravity dams including hydrodynamic interactions, the tank wall and fluid domain are treated as two substructures of the total system. The hydrodynamic pressures due to ground motion which are determined by the boundary element method to solve wave equation over the fluid domain can be expressed as the frequency-dependent terms in equations of motion for the tank. The dynamic response of the tank wall subjected to ground motion including hydrodynamic effects can then be analyzed by the linear combination of the first few natural modes of the tank alone. Thus, the hydrodynamic interactions between the elastic flexible and the fluid are then solved.

To demonstrate the accuracy and reliability of the solution technique developed herein, the dynamic behavior of ground-supported,deformed, cylindrical tank with incompressible fluid conducted by Haroun (1983) are analyzed. Good correlations of hydrodynamic pressure distribution between the computed results with the referenced solutions are noted. Based on experiences of one of authors (Hwang 1985), The fluid compressibility significantly affects the hydrodynamic pressures of the liquid-tank-interactions and the work which is done on this discussion is still little attention. Thus, the influences of the compressibility of the liquid on the response of the liquid storage due to ground motion are then drawn. By the way, the complex-valued frequency response functions for hydrodynamic forces of Haroun's problem are also displayed.

2 SOLUTION PROCEDURE

The analysis procedure for the liquid storage tank including the hydrodynamic interactions in this work is divided into two steps. Firstly, the effects of fluid express as frequency-dependent terms in the governing equations for the elastic tank. Secondly, these equations are transformed in terms of the first few modes of vibration of the tank, thus enabling drastic reduction in numbers of unknowns leading to highly efficient solutions. The hydrodynamic terms in the structural equations are determined by the boundary element method to solve the wave equation over the fluid domain. The solution of equations of motion subjected to ground motion including hydrodynamic interactions will be calculated by modal superposition method.

Equations of motion for the elastic tank, idealized as the finite element system, subjected to the ℓ-component(ℓ = x, y or z) earthquake ground motion including hydrodynamic effects in cartesian coordinate system, are

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = -[M]\{e^{\ell}\}\ddot{u}_g^{\ell}(t) - \{Q^{\ell}(t)\} \tag{1}$$

where $\{u\}$, $\{\dot{u}\}$ and $\{\ddot{u}\}$ are the modal displacement, velocity and acceleration vectors, respectively. [M], [C] and [K] denote the consistent mass, damping and stiffness matrices, respectively. $\{e^{\ell}\}$ is defined as the pseudo-static influence vector. $\ddot{u}_g^{\ell}(t)$ is the eqrthquake ground acceleration in the ℓ-direction. $\{Q(t)^{\ell}\}$ is the nodal force vector in the ℓ-direction associated with hydrodynamic pressure. Eq.(1) can be expressed as a linear combination of the natural vibration modes $\{\phi_j\}$ corresponding to the frequency ω_j of the tank alone. By introducing the generalized displacements $Y_j^{\ell}(t)$ for the jth vibration mode, eq.(1) becomes

$$M_j\ddot{Y}_j^{\ell} + C_j\dot{Y}_j^{\ell} + K_jY_j^{\ell} = -\{\phi_j\}^T[M]\{e^{\ell}\}\ddot{u}_g^{\ell} - \{\phi_j^f\}^T\{Q^{\ell}\} \tag{2}$$

where $M_j=\{\phi_j\}^T[M]\{\phi_j\}$; $K_j=\{\phi_j\}^T[K]\{\phi_j\}$ and $C_j=2\xi_j\omega_jM_j$; ξ_j is the modal damping ratio for the tank. $\{\phi_j^f\}$ is the subvector of $\{\phi_j\}$ which nodes contact with liquid. Transorming these equations into frequency domain, eq.(2) can be written as

$$[-\omega^2M_j + i\omega C_j + K_j]\overline{Y}(\omega) = -\{\phi_j\}^T[M]\{e^{\ell}\} - \{\phi_j^f\}^T\{\overline{Q}^{\ell}(\omega)\} \tag{3}$$

where $\overline{Y}(\omega)$ and $\overline{Q}^{\ell}(\omega)$ are frequency response functions of generalized displacement and hydrodynamic forces, respectively.

For the compressible inviscid fluid, the hydrodynamic pressure assoceated with small amplitude are governed by the wave equation

$$\nabla^2 P = \frac{1}{c^2}\frac{\partial^2 P}{\partial t^2} \tag{4}$$

where ∇^2 is Laplacian operator and c is the velocity of sound in liquid. P(t) is hydrodynamic pressure. Eq.(4) in frquency domain can be written as

$$\nabla^2\overline{P} + k^2\overline{P} = 0 \tag{5}$$

where $\overline{P}(\omega)$ is the frequency response function for hydrodynamic pressure; $k=\omega/c$ is the wave number.

The frequency response function of hydrodynamic pressure $\overline{P}(\omega)$ will be solved by boundary element method. Once $\overline{P}(\omega)$ is solved, the dynamic response of the tank can be obtained by eq.(3)

3 CALCULATION OF HYDRODYNAMIC PRESSURE BY BOUNDARY ELEMENT METHOD

The boundary element method is applied to the wave equation over the fluid domain to determine the hydrodynamic pressure expressed as the frequency-dependent terms $\overline{P}(\omega)$. By Green's second identity, eq.(5) becomes

$$(6) \qquad \int_S \{ G(\vec{r}|\vec{r}_0) \frac{\partial \overline{P}(\vec{r})}{\partial n} - \overline{P}(\vec{r}) \frac{\partial G(\vec{r}|\vec{r}_0)}{\partial n} \} dS = -\tfrac{1}{2}\overline{P}(\vec{r}_0)$$

where $S=S_1+S_2$ represents the surface of the fluid domain; S_1 is the wetted area to contact with the tank wall, S_2 is the free surface of fluid. $\vec{r}$ and $\vec{r}_0$ denote the position vector of observation point and source point, respectively. n is the unit outward normal vector at the surface. $G(\vec{r}|\vec{r}_0)$ is Green's function.

By introducing the half-space Green's function $\overline{G}(\vec{r}|\vec{r}_0)$;

$$(7) \qquad \overline{G}(\vec{r}|\vec{r}_0) = \frac{1}{4\pi} [\mathrm{Exp}(ikR_1)/R_1 - \mathrm{Exp}(ikR_2)/R_2]$$

where $R_1 = [(x-x_0)^2+(y-y_0)^2+(z-z_0)^2]^{\frac{1}{2}}$ and $R_2=[(x-x_0)^2+(y-y_0)^2+(z+z_0)^2]^{\frac{1}{2}}$ (x,y,z) and (x_0,y_0,z_0) are the coordinate at the observation and source point, respectively. In viewing of eq.(7), $\overline{G}(\vec{r}|\vec{r}_0)$ satisfy the Dirchlet boundary condition, i.e. the free surface surface of fluid (z=0) is traction free. Thus,the surface integral over the free surface S_2 will be vanished.

If the wetted area S_1 is divided into N segments and the constant elements are employed, $\partial\overline{P}/\partial n$ and $\overline{P}$ appeared in eq.(6) will be constant for every element. Eq.(6) can be written as

$$(8) \qquad \sum_{i=1}^{N} \overline{P}(\vec{r}_i) \int_{S_{1i}} \frac{\partial \overline{G}(\vec{r}_i|\vec{r}_j)}{\partial n} dS_i - \tfrac{1}{2}\overline{P}(\vec{r}_j) = \sum_{i=1}^{N} \frac{\partial \overline{P}(\vec{r}_i)}{\partial n} \int_{S_{1i}} \overline{G}(\vec{r}_i|\vec{r}_j)\, dS_i$$

Eq.(8) can be expressed as the matrix form;

$$(9) \qquad [A]\{\overline{P}\} = [B]\{\partial\overline{P}/\partial n\}$$

To calculate coefficients of [A],the singular integral as $R \rightarrow 0$ will be specially treated. For want of space, coefficients of matrices [A] and [B] are not discussed here and the detailed information will be referred to the subsequent paper.

As we know, $\{\partial\overline{P}/\partial n\}$ can be expressed as:

$$(10) \qquad \{\partial\overline{P}/\partial n\} = -\{\rho \overline{a}_n^{\ell}\}$$

where ρ is the mass density of liquid; $\overline{a}_n^{\ell}$ is the frequency response function of ground acceleration in the direction of the outward normal vector of each element and is expressed as

$$(11) \qquad \{\overline{a}_n^{\ell}\} = \{\overline{a}_R^{\ell}\} + \sum_{j=1}^{J}\{\phi_j^n\} \quad \ddot{\overline{Y}}(\omega)$$

where $\{\ddot{\bar{a}}_R^{\ell}\}$ is the frequency forcing function for acceleration of rigid tank due to ground motion in the ℓ-th direction. $\{\phi_j^n\}$ is the natural vibration modes corresponding to frequency ω_j in the normal direction of the tank wall. J is the number of vibration modes. By the linear combination of the first few vibration modes and known rigid body acceleration, $\{\ddot{\bar{a}}_n^{\ell}\}$ will be calculated. From eq.(9) and (10), $\{\bar{P}\}$ will be obtained.

4 DYNAMIC RESPONSE OF TANK

$\{Q^{\ell}(t)\}$ is the vector of loads at the nodal points on the inside wall of the tank associated with hydrodynamic pressure P(t). The complex frequency response function for this load vector corresponding to eq. (11) can be expressed as

$$\{\bar{Q}^{\ell}(\omega)\} = \{\bar{Q}_R^{\ell}(\omega)\} + \sum_{k=1}^{J} \ddot{\bar{Y}}_k^{\ell}(\omega)\{\bar{Q}_{Fk}^{\ell}(\omega)\} \tag{12}$$

where $\{\bar{Q}_R^{\ell}(\omega)\}$ and $\{\bar{Q}_{Fk}^{\ell}(\omega)\}$are the frequency response function for load vector corresponding to the hydrodynamic pressure due to acceleration of tank with the rigidity and flexibility. The hydrodynamic forces due to acceleration of deformed body have been expressed in terms of unknown generalized coordinate responses $\ddot{\bar{Y}}_k^{\ell}(\omega)$.

Substituting eq.(12) into eq.(3) and performing necessary manipulations ,eq.(3) becomes

$$[S^{\ell}(\omega)]\{\bar{Y}^{\ell}(\omega)\} = \{L^{\ell}(\omega)\} \tag{13}$$

where $S_{jk}^{\ell}(\omega) = -\omega^2\{\phi_j^f\}^T\{\bar{Q}_{Fk}^{\ell}(\omega)\}; \; j \neq k$

$$S_{jj}^{\ell}(\omega) = -\omega^2 M_j + i\omega C_j + K_j - \omega^2\{\phi_j^f\}^T\{\bar{Q}_{Fj}^{\ell}(\omega)\}$$

$$L_j^{\ell}(\omega) = -\{\phi_j\}^T[M]\{e^{\ell}\}-\{\phi_j^f\}^T\{\bar{Q}_{Fk}^{\ell}(\omega)\}$$

in the above equations, j=1,2,...,J and k=1,2,...,J

From eq.(13),the generalized displacement $\bar{Y}^{\ell}(\omega)$ will be obtained. The frequency responses for generalized accelerations may be obtained from

$$\ddot{\bar{Y}}_j(\omega) = -\omega^2\bar{Y}_j^{\ell}(\omega)$$

The complex frequency responses of accelerations to the ground motion at the nodal points of the are

$$\{\ddot{\bar{u}}^{\ell}(\omega)\} = -\omega^2\sum_{j=1}^{J} \bar{Y}_j^{\ell}(\omega)\{\phi_j\}$$

5 RESULTS AND DISCUSSIONS

To show the effectiveness of the proposed technique, the solution of Haroun's problem is presented. The dynamic response of a cylindrical tank with partially filled incompressible fluid is discussed. The tank wall is discretized by linear triangular thin shell elements as shown in Fig.1. Due to the symmetric excitation loading and geometry of the tank, the half of the tank is considered. Total numbers of shell element 240 are used in this analysis. For comparsion purposes,the hydrodynamic pressure of rigid tank with incompressible fluid obtained by analytic method (Haroun) and the boundary element method is drawn in Fig.2. Excellent agreement between the present results and referenced solution can be found.

To demonstrate the significant effects of the compressibility of fluid to the hydrodynamic pressure, Fig.3 and Fig.4 display the normalized absolute value of these frequency response functions, $p/\rho H_f$ versus the depth of liquid for five different noramlized excitation frequencyω/ω_1 . H_f is the height of liquid and $\omega_1 = c/2H_f$. The hydrodynamic pressure without considering the effect of compressibility of fluid can be calculated by setting$\omega = 0$. From Fig. 3 and 4, the compressibility of fluid is significantly influence on hydrodynamic pressure.

The responses of tank due to the excitation of horizontal and vertical ground motion are analyzed using the solution procedure presented in this paper. The modulus of complex-valued frequency response function of hydrodynamic force acting on the tank wall are displayed in Fig.5 and 6. The region near the first few natural frequencies of the tank, The hydrodynamic force considering tank-water interaction is quite different from the one neglecting this interaction for the horizontal excitation. However, effects of interaction are not important for the vertical exitation as shown in Fig.6.

The modulus of the complex frequency response function of the horizontal accelerations at the point A (Fig.1) on the tank wall due to the horizontal and vertical excitations are presented in Fig.7 and 8, respectively. Obviously, effects of compressibility on acceleration are significant.

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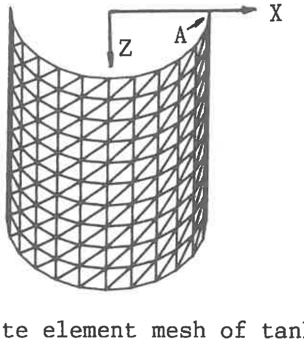

Fig.1Finite element mesh of tank

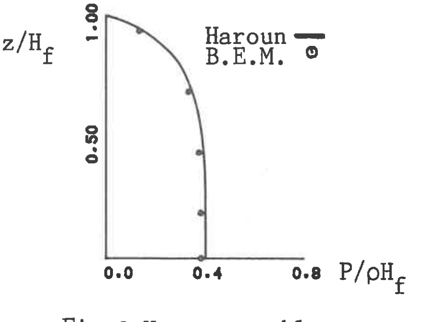

Fig.2 Haroun problem

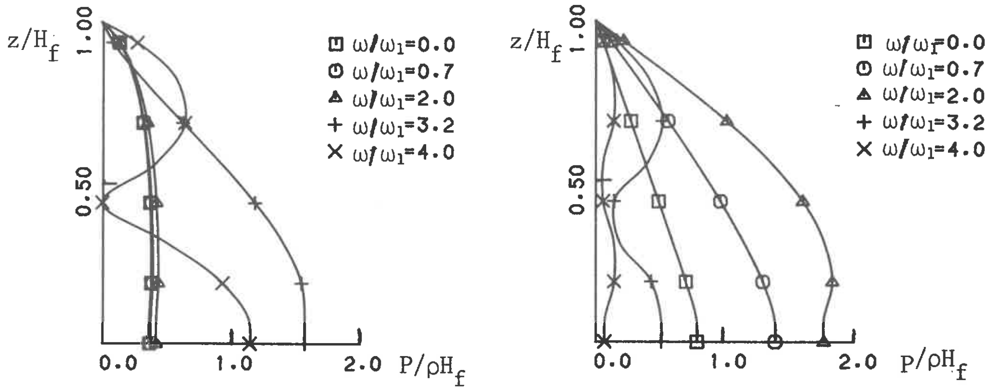

Fig. 3 & 4 Hydrodynamic pressure distribution

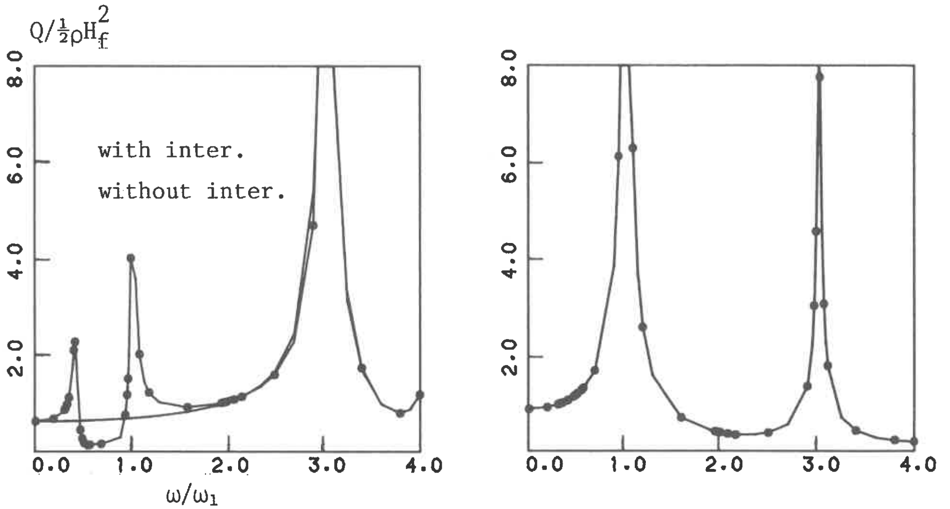

Fig. 5 & 6 Modulus of complex valued frequency response function

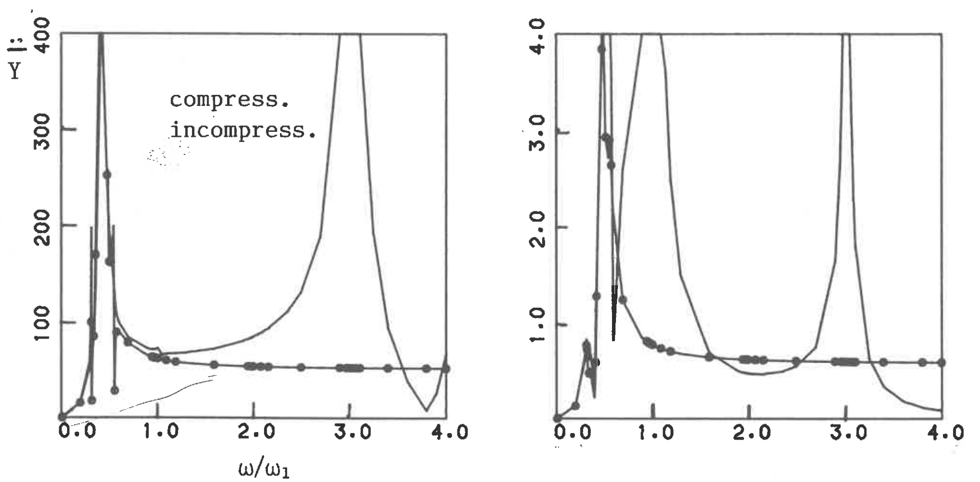

Fig. 7 & 8 Modulus of complex valued frequency response function