

MECHANICAL EQUATION OF STATE ANALYSIS OF ELEVATED TEMPERATURE CYCLIC DEFORMATION OF AUSTENITIC STAINLESS STEELS

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SUMMARY

A mechanical equation of state is a functional relationship among appropriate state variables such that mechanical response can be predicted solely on the basis of current values of the state variables. Recent consensus requires that state relations incorporate at least one state variable whose value is controlled by some aggregate characteristic of the state of the microstructure. This concept has great potential value since, if such a relationship exists and if the state variables can be determined, the mechanical response of a structure may be predicted without knowledge of its history.

The load-relaxation experiment is commonly utilized to characterize the rate-dependent plastic behavior of a metal and thereby evaluate the potential for description in terms of an equation of state. Relaxation is usually assumed to progress while the microstructure remains constant because the plastic strain accrued is small. However, at high temperatures the assumption may be fallacious since thermally induced microstructural recovery may occur.

The present work considers load relaxation data reported by Conway, Stentz and Berling, for 304 and 316 stainless steels during cyclic deformation with various hold times. The experiments were conducted at 650 and 538 °C, temperatures sufficiently high so that recovery is likely to occur during the experiment. The experimental data are analyzed to determine if a power law suggested as an equation of state can represent the observed behavior.

The relation proposed to describe stress relaxation concurrent with microstructural recovery is:

$$\frac{\dot{\epsilon}}{\dot{\epsilon}_1} = \left[(1-A) \left\{ \frac{\sigma - \sigma_i}{\sigma_1} \right\}^{m/n} + A \left\{ \frac{\sigma - \sigma_i}{\sigma_1} \right\}^n \right]$$

The state variable m relates dislocation velocity to stress; the state variable n denotes the recovery rate; the state parameter σ_i is an internal stress which becomes zero at high temperatures; A is a functional relationship between m , n , strain hardening and recovery. σ_1 and $\dot{\epsilon}_1$ are the stress and strain rate respectively at the beginning of load relaxation. If no recovery occurs $A=0$ and the relation reduces to the simple power law commonly observed at low temperatures.

In 304 at 650 °C short hold times and large cyclic strain amplitudes inhibit recovery. Conversely, long hold times and small cyclic strains promote recovery. When recovery occurs, relaxation can be described with $\sigma_i=0$, $m \approx 40$, and $n \approx 10$. If no recovery occurs $\sigma_i = \sigma_f(\sigma)$; $m \approx 3$.

At 538 °C for 304 no evidence of recovery was found and $\sigma_i = \sigma_f(\sigma)$, $m \approx 2.5$. Likewise in 316 at 650 °C no recovery was found and $\sigma_i = \sigma_f(\sigma)$, $m \approx 7$.

These observations can be explained on the basis of thermal and strain induced recovery of the microstructure developed by cyclic deformation. When recovery occurs, stress relaxation no longer prescribes the loci of rate dependent behavior at a constant state; rather the material passes through a continuous succession of *states*. The results of this analysis indicate that the power law may provide an equation of *states* for the stainless steels.

1. Introduction

The currently recommended rules that are used to calculate the time-dependent in-elastic (creep) response of IMFBR components are based upon an equation-of-state approach. Specifically, constant-stress creep data is fitted to a functional form that relates creep strain to stress level, temperature, and time. For calculations involving time-varying loads, the creep strain rate is formulated in terms of stress level, temperature, and accumulated creep strain (time being explicitly eliminated); special treatment is required for load reversals. Since the creep strain rate at any instant depends upon the accumulated creep strain, the current stress, and current temperature, any history dependence is based upon the assumption that strain is uniquely related to microstructure.

The practice of estimating "creep" damage fractions during elevated temperature strain hold periods assumes at least that creep and relaxation are different manifestations of the same phenomenon. However, the rate-dependent response of metals depends upon their entire thermomechanical history [1-3]. Calculation of creep damage from relaxation data may be meaningful for a fixed state of the microstructure, but totally misleading if strain hardening or recovery ensue. Some experiments have been conducted to evaluate the accuracy with which creep and stress relaxation are interpredictive; [4] however, no studies have been directed toward analysis of equation-of-state assumptions, taking cognizance of the possibly different evolution of the microstructure during monotonic deformation, cyclic deformation, creep, and stress relaxation.

When the mechanical state of a metal is uniquely defined by stress and plastic strain rate, the differential equation relating those variables is always integrable. Furthermore, the integrating factor is a microstructure dependent (internal) state variable, [5] such that the thermomechanical history of the material can be incorporated into the current value of the internal variable. In general, two or more independent variables seem to be required for incorporation of history-dependent response. This situation arises under conditions of nonsteady loading [1,3,6]. The question of whether an equation of state exists when the description requires more than two independent variables is not answerable solely from mathematical analysis [7-9]. Confirmation must come from experiments.

If the kinetics of microstructural evolution during elevated temperature deformation can be described with internal state variables, then a firm basis would be provided from which to calculate "damage", i.e., changes in microstructure leading toward failure. Of course, if relaxation proceeds at a constant microstructural state, no "damage" is accumulated during the relaxation period. This, however, is not supported by experiment [10,11]. In this report we do not attempt an in-depth evaluation of current elevated-temperature design practice in terms of equation-of-state concepts. Our limited goal consists of analyzing the stress relaxation in stainless steel during strain hold periods, utilizing a function which admits the possibility of damage through strain hardening and/or recovery. The function incorporates history dependent response into two macroscopically identifiable internal state variables. It is suggested that improved calculations of accumulated damage should follow from the concepts presented.

2. Theory

Much of the stress relaxation data obtained for stainless steels at elevated temperatures can be represented by the following equation [10,12]

$$\ln \frac{\sigma_0}{\sigma} = \frac{B}{1+P} t^{1+P} \quad (1)$$

where σ_0 is the initial stress, σ and t are stress and time, respectively, and B and P are constants. The relation requires that σ approach zero for long relaxation times. This requirement may restrict application of the equation to data taken at the highest temperatures only, as will be shown later. Furthermore, in attempting to fit eq. (1) to the relaxation during hold periods in tension, Jaske [10] found the "constants" B and P to be dependent upon strain amplitude.

In an effort to address the problem of amplitude dependence, Bui-Quoc [11] modified eq. (1) by substituting a strain-compensated time

$$t^* = \left(\frac{\Delta \epsilon}{2k} \right)^{2k} t \quad (2)$$

where $\Delta \epsilon$ is the cyclic strain amplitude and k is the monotonic strain hardening exponent. The resulting equation becomes

$$\ln \frac{\sigma_0}{\sigma} = B_1 t^{*P_1} \quad (3)$$

where B_1 and P_1 were found to be independent of hold time and strain amplitude. If eq. (3) can be shown to describe relaxation following arbitrary thermomechanical histories, its equation-of-state nature will be established. Use of the monotonic exponent k suggests that Bui-Quoc believes this to be the case.

No explanations have been offered as to why eq. (3) should work; therefore, the following analysis is proposed as a likely rationale. We first differentiate eq. (1) with respect to t . Because time is not a valid state variable, [13] we use eq. (1) to eliminate t from the resulting relation. During relaxation at fixed strain

$$S \frac{d\sigma}{dt} = - \frac{d\epsilon}{dt} \quad (4)$$

where S , ϵ are compliance of specimen plus load train, and inelastic strain respectively; from eqs. (1) and (4) we obtain

$$\frac{d\epsilon}{dt} = SB\sigma \left[\frac{1+P}{B} \right]^{\frac{P}{1+P}} \left[\log \frac{\sigma_0}{\sigma} \right]^{\frac{P}{1+P}} \quad (5)$$

Eq. (5) is then transformed to t^* -space by differentiating eq. (2) and substituting in (5) to obtain

$$\frac{d\epsilon}{dt^*} = \dot{\epsilon}_0 \left[\ln \frac{\sigma_0}{\sigma} \right]^{\frac{P}{1+P}} \quad (6)$$

where

$$\dot{\epsilon}_0 = SB\sigma \left(\frac{1+P}{B} \right)^{\frac{P}{1+P}} \left(\frac{2k}{\Delta \epsilon} \right)^{2k} \quad (7)$$

The magnitude of σ_0 increases with $\Delta \epsilon$, as shown by the equation for the cyclic stress-strain curve, i.e.,

$$\sigma_0 = K(\Delta \epsilon)^{k'} \quad (8)$$

where K , k' are material constants. Since σ increases with σ_0 , eq. (7) has stress-

dependent terms in both numerator and denominator, and there is a possibility that eq. (7) will depend only upon σ_0 . The dependence will be exact if $\sigma \propto \sigma_0^{2k/k'}$. When this is approximately the case, we can set $(1 + P)/P = -\lambda$ (P is always negative) [10] and write eq. (6) as

$$\ln \frac{\sigma_0}{\sigma} = \left(\frac{\dot{\epsilon}_0}{\dot{\epsilon}} \right)^\lambda \tag{9}$$

which is the equation of state form proposed by E. W. Hart [14]. Huang, et. al., [15] have successfully used eq. (9) to describe the stress relaxation of 316 stainless steel following monotonic deformation at elevated temperatures.

During stress relaxation at low homologous temperatures, the stress may decrease to some nonzero equilibrium value at long times [2,16]. In this event neither eqs. (1) nor (9) can describe the data. Furthermore, stress relaxation kinetics appear to be history dependent at low temperatures [1]. Recently we have considered the application of another function to description of stress relaxation data in a 50% Sn-50% In alloy over a range of homologous temperatures from 0.2 to 0.9 [16]. This function, which is derived from a power-law description of dislocation dynamics, modified to account for microstructural evolution, is

$$\dot{\epsilon}/\dot{\epsilon}_0 = \left[(1 - A) \left\{ \frac{\sigma - \sigma_1}{\sigma_0} \right\}^{m/N} + A \left\{ \frac{\sigma - \sigma_1}{\sigma_0} \right\} \right]^N \tag{10}$$

Here A is a function of m , N , strain hardening and recovery, m denotes the stress dependence of the dislocation glide velocity, N is the parameter denoting the stress dependence of the recovery rate, σ_1 is the internal stress, and $\dot{\epsilon}_0$ and σ_0 are the inelastic strain rate and stress, respectively, at the beginning of relaxation. The expression is analagous to the present creep equation if the latter is written in terms of stress and temperature-dependent constants. In eq. (10), however, σ_1 and σ_0 replace c as the microstructure-dependent variables.

Load relaxation in the Sn-In alloy [16] can be characterized over the entire range of temperatures by eq. (10). At low homologous temperatures a nonzero value for σ_1 was measured; this stress vanishes at higher temperatures. The analysis indicated that relaxation at intermediate and high temperatures may be interpreted in terms of an evolving microstructure. Under this description the material passes through a continuous succession of mechanical states during a relaxation event. A necessary consequence of the rationale is that the relaxation equation cannot be used to predict steady state creep damage under conditions of constant microstructure until the appropriate values of the internal state variables are determined.

In the next section we consider the application of eq. (10) to the relaxation behavior of 304 and 316 stainless steels during cyclic deformation with hold periods in tension. Because eq. (10) is descriptive of relaxation at low as well as high homologous temperatures, in contrast to eqs. (1) and (9), we feel that the former offers potential for more accurate calculations of the "damage" which accumulates during deformation. The equation can be integrated to obtain the functional "creep" relationship between c and t for a given σ ; damage fraction calculations may then be performed on a time basis in the usual way. The procedure has a more meaningful basis, however, because evolution of the microstructure is explicitly accounted for.

3. Analysis of Data

Conway [12] and Conway, Stentz and Berling [17] have published data from an extensive set of experiments on the relaxation behavior of 304 stainless steel. A sufficient number of variables were studied so that the equation-of-state nature of eq. (10) can be tested for the material. Relaxation was measured at constant strain after monotonic tensile loading and during various tensile hold times in reversed strain cyclic deformation. The bulk of the data was taken for 304 at 650°C ($T/T_m = 0.552$); however, some information was obtained for 304 at 538°C ($T/T_m = 0.485$). Limited data were presented for the relaxation of 316 stainless steel at 650°C ($T/T_m = 0.561$).

The following observations of Conway, *et. al.*, about their own data are pertinent to the present study: (1) For a given strain, the relaxation kinetics of 304 stainless steel at 650°C subsequent to cyclic or monotonic loading are similar; (2) Stress relaxation subsequent to cyclic deformation is independent of the number of cycles, at least after an initial transient encompassing approximately ten percent of the cyclic life; (3) At a constant cyclic strain range the relaxation kinetics are independent of hold time; (4) Relaxation behavior was significantly dependent upon strain range.

It is important to note that relaxations subsequent to cyclic and monotonic loading were similar. This observation indicates that the inelastic deformation kinetics may be history independent (at these temperatures) and provides some basis for use of eq. (10) as an equation of state.

With these observations in mind we tested the ability of eq. (10) to describe stress relaxation data of 304 stainless steel at 650°C. It will be shown that the description is satisfactory. The analysis of the data at 650°C for 304 is then compared to a similar treatment of relaxation data of 316 stainless steel at 650°C and of 304 stainless steel at 583°C.

The data of Conway, [12] plotted as stress versus time, were digitized and recorded on magnetic tape to provide for subsequent spline-fitting with a computerized routine to better than 0.1% at each datum. The calculated curves were differentiated to determine stress rates vs. time or stress.

Plots of the logarithm of stress versus the logarithm of stress rate were then prepared. This plotting format was chosen since the curvature of the data thus displayed indicates which homologous temperature regime characterizes the behavior [16]. Upward curvature indicates a "low temperature" response characterized by $A = 0$ and $\sigma_1 \neq 0$. A straight line (or nearly so) indicates "intermediate temperature" behavior with zero, or very small internal stress. Downward curvature indicates "high temperature" behavior. This regime is characterized by a non-zero value of A and a very small or vanishing internal stress σ_1 .

Eq. (10) was fit to the data by appropriate choice of the parameters m , N and A . Selection of m and N was facilitated by noting that m , the stress exponent for dislocation glide, is given by the limiting slope of the $\log \sigma$ vs. $\log \dot{\sigma}$ curve at high stress rates; and N , the exponent for the kinetics of microstructural recovery, is given by the limiting slope of the curve in the low stress rate regime. The variable A then is used as an adjustable parameter, affecting the point of transition between glide controlled and recovery controlled behavior.

Figures 1, 2 and 3 show fits to Conway's data on 304 at 650°C for 10 minute hold times as a function of cyclic strain range. These data are concave downward, indicating "high temperature" behavior. In this case σ_1 was zero. It was found that $m \approx 120$ and $N \approx 7$ for all the data analyzed. The value of A was variable; this is expected since A depends on the initial stress and stress rate as well as the recovery and strain hardening coefficients. As can be seen, quite satisfactory descriptions of the data were obtained. The consistency of m and N indicate that the kinetics of glide and recovery are independent of strain range at this temperature.

As delineated earlier, the shape of the relaxation curves varies with cyclic strain amplitude. The consistency of m and N indicate, however, that this change in shape is a consequence of the dependence of A upon the initial conditions. The mechanisms responsible for the kinetics of dislocation glide and microstructural recovery are not affected by strain range. Furthermore, since Conway *et. al.* [12] report that stress relaxation subsequent to monotonic and cyclic deformation are similar and that hold time does not affect relaxation kinetics, we see that at this temperature in 304 stainless steel eq. (10) functions as an equation of state as defined in section 2. Even though strain hardening and recovery are occurring during relaxation, the behavior may be described in terms of history independent material properties.

Analysis of the rather limited data of Conway on 316 stainless steel at 650°C indicates that the rate-controlling kinetics for dislocation glide and recovery differ from their counterparts in 304. Fig. 4 shows a curve fit to 316 relaxation data. A tendency for slight concave downward behavior is observed, but overall less curvature is displayed in the 316 data. The 316 appears to be operating in the "intermediate temperature" regime at 650°C. The difference from 304 must be associated with the mechanical stability of the microstructure. The best data fit was found for $m \approx 400$ and $N \approx 10$, values significantly different from their counterparts in 304. Because of the limited amount of data on 316 the state nature of eq. (10) a more precise determination of the state values of m and N must await more experimental study.

A striking change in rate controlling kinetics is seen when stress relaxation in 304 stainless at 538°C (Fig. 5) is compared with the 650°C data (Fig. 3). At the lower temperature a concave upward curvature, indicative of low temperature behavior, is seen. In this case σ_1 is non-zero and best data fit was obtained using eq. (10) with $m \approx 2$, $A = 0$ and $\sigma_1 = 3.48 \times 10^8 \text{ N/m}^2$. The fact that $A = 0$ suggests that microstructural recovery is negligible.

This rapid change in material response with temperature is not without precedent [16]. The observation of the change in material stress relaxation response with temperature demonstrates the need of a single equation-of-state formulation that can be used to describe material behavior in both temperature regimes. Eq. (10) satisfies this requirement but eq. (1) does not. In the preceding discussion of the 650°C data for 304 sufficient experiments existed to indicate that material response was independent of deformation history. At 538°C sufficient data do not exist. Indeed, there is some indication that in the low temperature regime [1,6] stress relaxation response is history dependent. Thus, application of eq. (10) as a state relation for 304 at 538°C must await further experiments designed to test history effects.

4. Summary

We have demonstrated that the proposed state relation given by eq. (10) can describe the stress relaxation behavior of 304 stainless steel in two different behavioral regimes; "low temperature" (Fig. 5) and "high temperature," (Fig. 5, 1-3). Under this description at 650°C the parameters m and N function as material constants as required by theory. Since Conway reports that relaxation is the same subsequent to either monotonic or cyclic histories, the relation at 650°C is also presumed to be history independent and satisfies the criteria for an equation of state. Analyses of a limited amount of data on 316 stainless steel indicates that the kinetics of dislocation glide and microstructural recovery differ from their counterparts in 304.

Analysis of 304 relaxation at 538°C showed that an internal stress exists and that microstructural recovery is insignificant during relaxation times. However, insufficient data exists to determine if eq. (10) will act as a state relation for the material at that temperature.

It was found in 304 and 316 at 650°C that microstructural recovery was responsible for a significant portion of the stress relaxation curve. This finding implies that the basic assumption in current high-temperature design practice is unsatisfactory and that microstructural changes must be considered when inverting creep and stress relaxation equations.

The generally good fits of the model to the experimental relaxation data suggest that it might be applied successfully to damage fraction calculations in the usual manner. However, more precise experiments over a larger temperature range are required to verify the conclusions of this work and to allow a more precise determination of the state parameters before such calculations could be made with confidence.

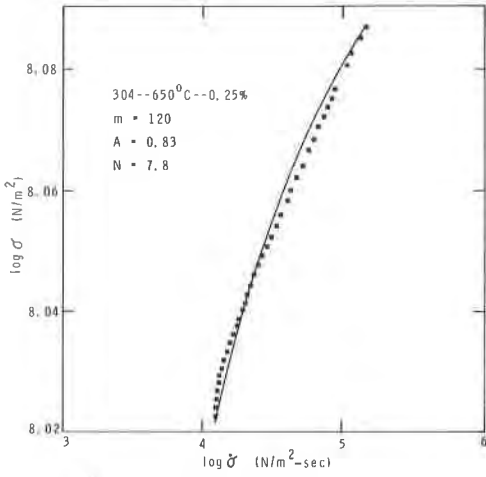


Fig. 1: Stress relaxation in 304 at 650°C. Ten minute hold time; reversed cyclic strain of 0.25%.

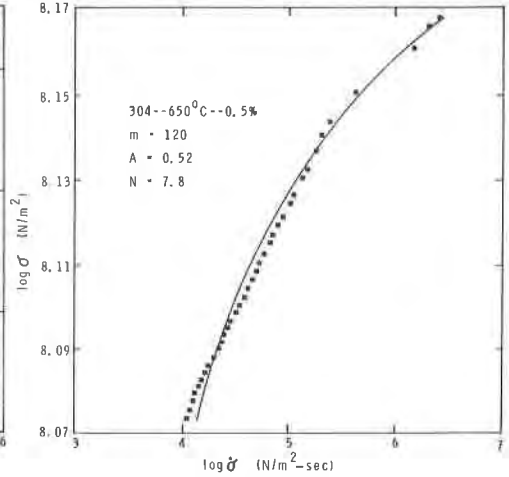


Fig. 2: Stress relaxation in 304 at 650°C. Ten minute hold time; reversed cyclic strain of 0.5%.

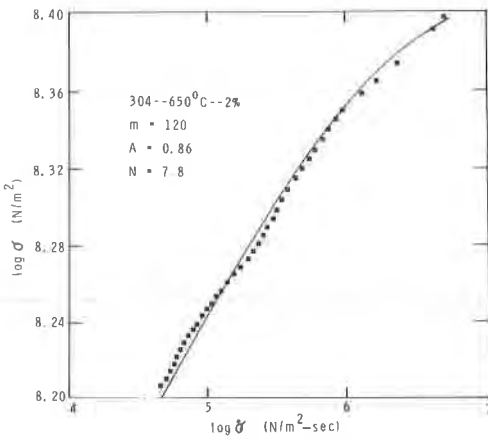


Fig. 3: Stress relaxation in 304 at 650°C. Ten minute hold time; reversed cyclic strain of 2%.

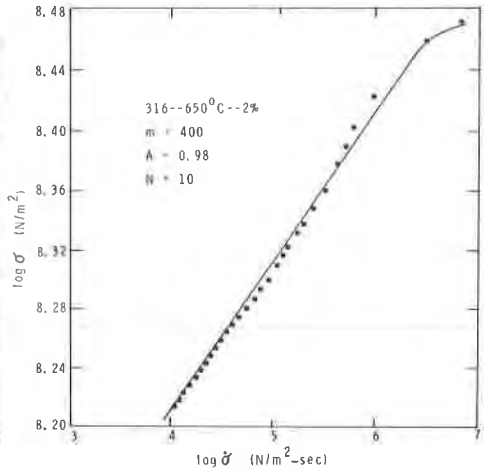


Fig. 4: Stress relaxation in 304 at 538°C. Ten minute hold time; reversed cyclic strain of 2%.

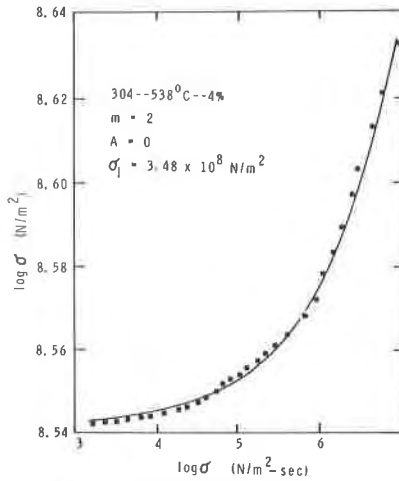


Fig. 5: Stress relaxation in 304 at 538°C. Ten minute hold time; reversed cyclic strain of 4%

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References

- [1] SWEARENGEN, J. C., ROHDE, R. W., "The History Dependence of Microdeformation Parameters," *Scripta Met.*, 8, 1409 (1974).
- [2] ROHDE, R. W., SWEARENGEN, J. C., "Mechanical State Relations for Inelastic Deformation of Iron: An Analytic Description," *J. Engr. Mat'ls. & Tech.*, 99, 59 (1977).
- [3] KREMPL, "Cyclic Creep - An Interpretive Literature Survey," *WRC Bulletin No. 195*, p. 63, Welding Research Council, 1974.
- [4] PUGH, C. E., "Constitutive Equations for Creep Analysis of LMFBR Components," *Adv. in Design for Elevated Temperature Environment*, S. Y. Zamrik & R. I. Jetter, Eds., p. 1, ASME, 1975.
- [5] HART, E. W., "A Phenomenological Theory for Plastic Deformation of Polycrystalline Metals," *Acta Met.*, 18, p. 599 (1970).
- [6] SWEARENGEN, J. C., ROHDE, R. W., HICKS, D. L., "Mechanical State Relations for Inelastic Deformation of Iron: The Choice of Variables," *Acta Met.*, 24, p. 969 (1976).
- [7] RICE, J. R., "Continuum Mechanics and Thermodynamics of Plasticity in Relation to Microscale Deformation Mechanisms," *Constitutive Equations in Plasticity*, A. S. Argon, Ed., MIT Press, 1975.
- [8] LI, J. C. M., "Stable Steady State and the Thermokinetic Potential," *J. Chem. Phys.*, 37, 1592 (1962).
- [9] LI, J. C. M., "Stable Steady States in Plasticity," *Symp. on Adv. in Metal Deformation: Phenomena and Mechanism*, Cornell University, Oct. 25-27, 1976.

- [10] JASKE, C. E., MINDLIN, H., PERRIN, J. S., "Combined Low-Cycle Fatigue and Stress Relaxation of Alloy 800 and Type 304 Stainless Steel at Elevated Temperatures," Fatigue at Elevated Temperatures, ASTM ATP 520, Am. Soc. for Testing and Materials, p. 365, 1975.
- [11] BUI-QUOC, T., SAKET, R. E., BIRON, A., "Evaluation of Hold-Time Effect in Fatigue at Elevated Temperature on a Stainless Steel," ASME Paper No. 75-WA/PVP-17, Am. Soc. for Mech. Engrs., 1975.
- [12] CONWAY, J. B., "An Analysis of the Relaxation Behavior of AISI 304 and 316 Stainless Steel at Elevated Temperature," Report No. GEMP-730, Gen. Electric Co., 1969.
- [13] MCCARTNEY, L. N., "No Time-Gentlemen Please,!" *Phil. Mag.*, 33, 689 (1976).
- [14] HART, E. W., "Constitutive Relations for the Nonelastic Deformation of Metals," *J. Engr. Mat'ls. & Tech.*, p. 193, July 1976.
- [15] HUANG, F. H., ELLIS, F. V., LI, CHE-YU, "Plastic Equation of State in Type 316 Austenitic Stainless Steel," Report No. COO-2172-8, Cornell University, October 1975.
- [16] SWEARENGEN, J. C., ROHDE, R. W., "Application of Mechanical State Relations at Low and High Homologous Temperatures," *Met. Trans.*, 1977 (in press).
- [17] CONWAY, J. B., STENTZ, R. H., BERLING, J. T., "Fatigue, Tensile, and Relaxation Behavior of Stainless Steels," NTIS Pub. No. TID-26135, 1975.