

A MATHEMATICAL MODEL OF ADDICTION DYNAMICS

by

IRWIN J. SHIFFER and H. L. LUCAS

Institute of Statistics
Mimeograph Series 1113
Raleigh, NC

National Institute of General Medical Sciences

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INTRODUCTION

The General Problem and Goal

In the past two decades, narcotics addiction has grown from primarily a ghetto phenomenon to a problem affecting wide segments of most United States cities. Massive campaigns to prevent, control, or eradicate drug habits have had only limited success. A formidable obstacle to careful study and treatment of addiction has been the enactment of stringent laws which have created a criminal class among narcotics users, restricted physicians' efforts to provide effective rehabilitation, and spawned a lucrative black market for drugs.

Addiction research has continued throughout this century, but much of it has been in such specialized areas as physiological reactions and chemotherapy, psychological components of addict behavior and constituencies of addict families, or reports from particular treatment programs. While there have been elaborations of the pathways leading individuals to become full-scale addicts, these have tended to be descriptive in nature and vague with regard to the actual mechanisms.

If the referral of addicts to treatment is going to be made on a rational, rather than fortuitous, basis, the salient features of an addict's existence ought to be represented within a unified framework and its principal elements given explicit expression. With that aim, a mathematical system is developed here which utilizes findings of the addiction literature and some general concepts about human behavior.

Addiction and Crime

It should be stated at the outset, however, that many fundamental questions regarding addiction behavior remain unanswered. For example, the issue of heroin's addictiveness is unresolved, as is the relationship between narcotics use and crime. Regarding the latter, there is opinion on this subject covering a wide range of possibilities. Among those who view criminal activity as unrelated to addiction level is Lukoff [19], who states:

If we assume that heroin addiction is, for some segment of the addict population only an aspect of a more coherent pattern of deviance, then there are very important consequences. First, it casts doubt on the assumption that the crime associated with addiction is necessarily a function of the addict's desperate need for funds to purchase drugs.
(pp. 5-6)

On the other hand, O'Donnell [26] has found that, among patients at the Public Health Service facility in Lexington, Kentucky, the arrest rate of male addicts increases after the age of 31--their average age of addiction--while the overall national arrest rate for males declines after this age. Although law enforcement officials sometimes estimate addicts' responsibility for crime as high as 50 percent, Singer [30] argues that addicts are responsible for much less crime than is commonly believed. Brown and Silverman [5] and Goldman and Coate [14] have, however, noted a correlation between drug use and crime of about .30.

Some authors ([14], [26]) have suggested that the level of criminality depends as much on the level of legitimate income as on either the level of drug usage or a predisposition to deviance. Yet

no study was found which took level of conventional income into account when evaluating the correlation between drug usage and crime.

Problems of Study and Model Development

In the prevailing socio-political climate, facts are elusive. Generally, the addicts on whom data are collected have sought voluntary admission to treatment or have been referred by law enforcement authorities. In either case, they represent those among the addict population whose habits have become unmanageable. Even the ratio of detected to undetected addicts has not been ascertained, although a usual estimate is 1:2.

As illustrations of the methodological obtuseness being employed in this field, the studied by Brown and Silverman [5] and Goldman and Coate [14] attempting to determine the amount of criminal activity associated with drug use can be cited. Brown and Silverman correlated fluctuations in New York City heroin prices with reported criminal offenses (of the revenue-raising type) during the same time period. They found a significant relationship, but virtually nothing on the level of individual behavior is accounted for. Perhaps all that is learned is that increased enforcement efforts result in more arrests, while the disruption of the narcotics distribution network causes increases in prices.

Goldman and Coate [14], though examining individual behavior, utilize data which offer weak support for their conclusions. Based upon interviews with addicts at a New York therapeutic community, they report a 30 percent increase in crime for each dollar increase

in the drug habit. But questions about drug use and drug sales were not made time-specific to the period of criminal involvement. Their own calculations show that

. . . if half of the average dollar size of habit is supported by income from drug sales, as has been suggested by several researchers, over \$300 would be left over in 'discretionary income' for the average addict in the Phoenix House sample. And this is not to mention income secured from other legal and illegal sources. (p. 69)

Such averages of retrospective responses to ill-suited questionnaires seem to be commonplace in addiction research. Under these circumstances, the best that can be done with the mathematical system to be developed in the pages to follow is to justify the equation forms as much as possible with relevant empirical studies found in the literature and with plausible exposition for the remainder. This work, then, must be viewed as a first-generation effort. It attempts to demonstrate a potentiality of its format, rather than to claim precision for its terms. The model may appear to be primitive, but the data which support our comprehension of addiction dynamics are, likewise, primitive.

It is highly questionable as to whether the reinforcing or changing of particular value systems and life styles, be they drug oriented or people oriented, can ever be derived from mimeographed questionnaires, static statistics, or programs based on expediency and nurtured by anxiety and hysteria.

Stanley Einstein and
Stephen Allen

Preface, Proceedings of the
First International Conference
on Student Drug Surveys, 1971

THE LITERATURE

Modeling in the Behavioral Sciences

Since the 1920's and 1930's, when Lotka [18] and Rashevsky [27] demonstrated the theoretical possibilities for mathematics in the behavioral sciences, a diverse array of mathematical structures has appeared as representations of behavioral phenomena. Unfortunately, the range of phenomena to which mathematics has been applied is, as yet, narrow; and the types of evolutionary sequences depicted by these models are inappropriate to the charting of individual life histories as envisioned here. No mathematical model directly relevant to studies of drug addiction could be found. Furthermore, Carr-Hill and Macdonald [6] report that "adequate tools (conceptual and computational) for the handling of sociological life-histories are unavailable" (p. 57).

There is in the literature, however, an assortment of stochastic and deterministic models of social mobility. If one subscribes to the definition of Tibbitt [31] for social mobility as "the changing locations of individuals in social space, as measured by changes over time in a convenient system of social coordinates" (p. 30), then this work, which characterizes addicts as participants in selected activities for which there are appropriate measures of involvement, can be considered a model of social mobility.

Most studies in this area focus on occupational mobility. Stochastic versions by Blumen et al. [3], McGinnis [23], McFarland [22],

and Mayer [20] seek to account for numerical shifts between job status categories during some time interval. As such, these models are designed to fit data and not to elucidate processes. An exception appears in Harrison White's analysis of "vacancy chains" [32]. The impetus to career advancement is assumed to come from opportunities within an organization to fill vacancies created by promotion, emigration, or death; but the use of the Episcopal Church hierarchy as a paradigm severely limits its applicability to addiction studies.

Path Analysis Approach

The American Occupational Structure by Blau and Duncan [2] set a new direction in the study of work careers. By means of "path analysis," a causal network relating the educational and occupational attainments of an individual to those of his father is established. Standard regression coefficients are used to calculate direct and indirect effects of a sequence of variables on the ultimate (i.e., equilibrium) job status level. This approach seeks to explain observed stratification in terms of a lifelong process, but its mechanism does not serve well as the basis for a dynamic model. Since path coefficients are set only after the course of a career is known, the model applies exclusively to the individual career under examination, and extrapolations are tenuous. There may be considerable error involved in assuming that measurements are taken under equilibrium conditions. And the requirements that the causal relationships of all variables be strictly ordered are a serious limitation.

Some Dynamic Formulations

In the works of Fararo [10] and of Dorein and Hummon [9], process assumes primary importance; fitting model behavior to data becomes secondary. Both models are deterministic. Fararo has used a system of differential equations to express the status equilibration mechanisms proposed by Galtung [12, p. 125]--namely, that "an individual will attempt to maximize his status" and that "an individual will try to equilibrate upward his various component statuses" (i.e., the status levels tend to rise together). For a two-dimensional status vector, the equations appear as follows:

$$\frac{dS_1}{dt} = a_1 S_1 - b_1 (S_2 - S_1) - C_1$$

$$\frac{dS_2}{dt} = a_2 S_2 - b_2 (S_1 - S_2) - C_2 ,$$

where a_i , b_i , and c_i ($i = 1, 2$) are positive parameters.

Dorein and Hummon [9] use a nonlinear differential system to produce an alternative to the Blau-Duncan model. Their equations are

$$\frac{dE}{dt} = KE(E_p - E) + U ;$$

$$E_p = a_1 E_f + a_2 S_f + a_3 ;$$

$$\frac{dS}{dt} = C_1(S_f - S) + C_2(S_0 - S)$$

$$+ C_3(\gamma E - S) + C_4 + N ;$$

$$S_0 = b_1 S_f + b_2 E + b_3 ,$$

in which E and S represent educational and occupational status, respectively; E_f and S_f refer to the father's equilibrium statuses; S_0 is the son's initial job level; E_p is son's educational potential; U and N are said by the authors to be noise terms, although they are treated as additional constants.

It is in the spirit of these last two works that the present project is undertaken.

PERSPECTIVE

General Approach

The primary task in developing a model for addiction behavior is to decide upon a conceptual framework, including a set of coordinates, in which to construct the quantification. Because of the multiplicity of factors involved, considerable simplification is required. To manage the problem in a logical way, one must resolve the complex issues of addiction dynamics into a few basic variables, driving forces and their interactions, and leave many of the factors which condition addict behavior implicit and avenues for further development.

The presentation of this work is organized in the following manner: A general discussion of the components within a community which relate to addiction phenomena prefaces the elucidation of explicit assumptions and the rationale for the framework chosen. The model is then presented with justification for and characteristics of the constituent equations. The time-course of system variables under a variety of conditions is next graphically displayed and interpreted. In the concluding section, possible alternative constructions and extensions, including treatment effects, are outlined. The appendices contain outputs from a stochastic version of the model and reference to the stability of the original system.

Factors to Be Considered

Figure 1 illustrates one way in which community addiction problems can be viewed. For our purpose, attention will be focused primarily on factors encompassed by the two boxes within the broken-bordered rectangle. Other factors will be taken into account in only a general way by parameters which can take different values under different circumstances and for which explicit mathematical expression must await future refinements.

By the time a person has become an addict, Family genetic and cultural factors have already been transmitted. More specifically, the family background parameters of race, ethnicity, education level and socio-economic status of the parents will partially determine the job advancement potential and earning capacity of the subject, his propensity to drug use and ability to self-sustain the cost of a drug habit, and his involvement with conventional community affairs.

Depending upon the mix of a subject's associates in the various activities, there will be reinforcing or extinguishing pressures on the path of each of the selected variables of interest. The magnitude of the influence will be determined by the intimacy of the relationships and the amount of contact time with Other People. Some of the associates will be suppliers of the demanded drugs.

The Community sector is important because it controls the opportunities in many of the other sectors of the system. It establishes (or licenses or closes) treatment facilities, determines the types of patients they will serve, and sets the probabilities on the

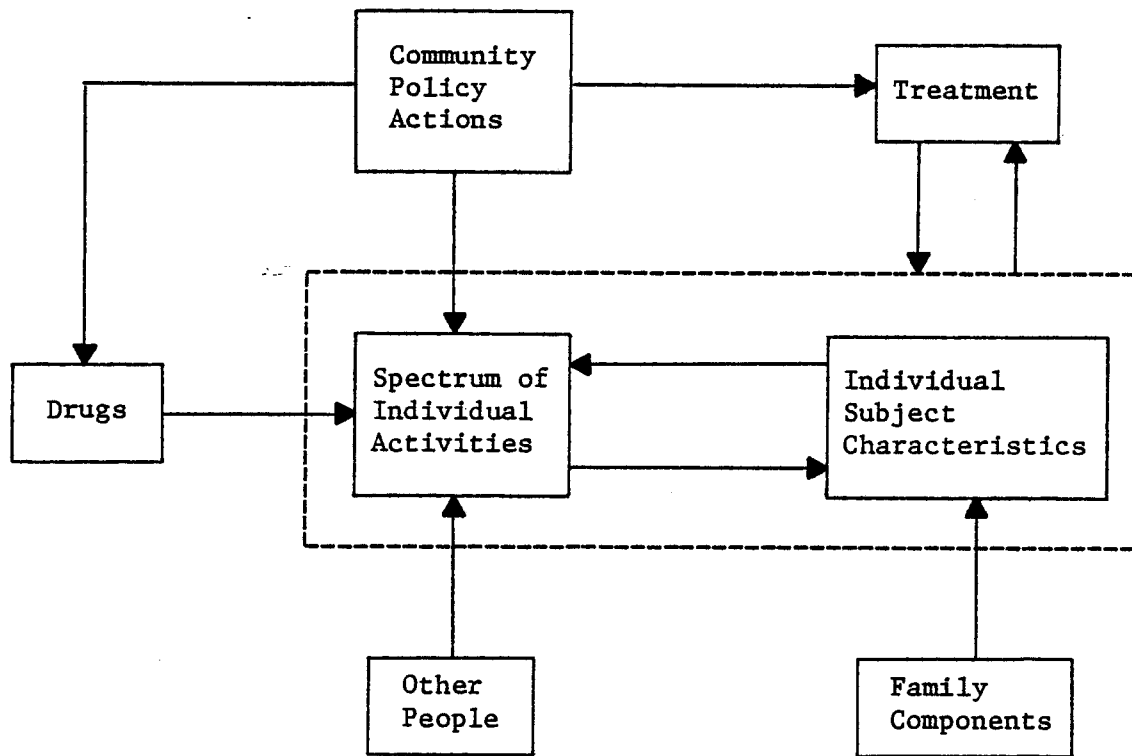


Figure 1. Chart of community addiction system

client-routing alternatives of the agency referral network (as described by Fréidson [11]). Law enforcement policy and agents regulate the flow of drug supplies to some degree and condition the risks of detection for those in the drug scene. Community policy shapes the Activity environment by encouraging location of industry, construction of housing, clearance of slums, building of new schools and job-training centers, etc. It is also responsible for the expected rates of flow through the system for community members with different backgrounds and qualifications--in a manner analogous to the "structurally enforced inertia" of Mayer's [20] absorbing state mobility model.

Choice of Coordinate System

Yet, having given some specification of the types of interactions to be found within the community system, there is still the problem of a common scale of measurement for the interchanges described above. One possibility is the study of satisfaction levels among each of the participants in the system relative to each of the activities. The operationalizing of satisfactions would be a major drawback to this approach, although the notion of goals is incorporated into the present model. Time spent by the subject in the various activities was considered. But the time dimension of activity participation does not necessarily reflect the quality of the activity experiences. Another alternative is the study of the intensity of involvement with the particular activities; but here is encountered the difficulty of equating the relevance of the various involvement levels.

A reasonable solution appears to be an economic perspective, with money as the common commodity. In this case there is a readily recognizable standard of exchange, fully operationalized, and applicable to the four activities with which we shall be primarily concerned--drug usage, job achievement, criminal activity, and maintenance of standard of living. This is in line with the recommendation by Tibbit [31] that attention be given to mobility indicators other than job status. Tibbit (p. 35) states: "There is a need to include consideration of dimensions of prestige concerned with consumption. . . how income is spent."

Simplifying Features

To clarify the picture of our object of study, the following comments are made: No account will be taken of the circumstances surrounding the subject's initiation to drug use. The drug of choice will be a narcotic--let us think of heroin as the representative of that class of drugs--and it will constitute the only controlled substance on which the individual relies for euphoria, relief of tension, and social identification. Modifications to accommodate for polysubstance use would not be especially difficult under the conceptualization to be employed here. In order to avoid discontinuities in the time-course of some of the variables resulting from child care and job incapacity complications, the subject is considered to be male. The behavior actions depicted by the model are assumed to occur in a single community. All variables are expressed in inflation-adjusted-dollar terms. Local variations in price and

purity of heroin will, of course, affect the model's parameters. Amplification of this point is made when the parameter values are assigned. While there is no assurance of uniformity in the relationship between cost and amount of heroin, we would do no better to record the number of bags purchased or the number of injections per unit time, since the addict is not likely to verify the dilutions. The economic approach has the advantage of highlighting the distribution of a limited resource to drugs rather than to alternative, more constructive activities.

THE MODEL

Monetary Flow Diagram

The manner in which the principal elements of an addict's economic situation are seen to interact is depicted in Figure 2. That flow diagram also introduces the state variables of our mathematical system. The subject's money requirement, \underline{m} , is met by earnings from legitimate employment, \underline{l} , and (when necessary) from reserve resources, \underline{R} ; in the event of an additional financial shortfall, illicit, or "criminal," earnings, \underline{c} , will provide the difference. The money requirement is allocated either for the purchase of addictive drugs, \underline{a} , or to support the addict's standard of living, \underline{s} . The variable $\dot{R}^* = \underline{l} - \underline{m}$ (i.e., the difference between legitimate income and money requirement) controls the rate of accumulation or depletion of reserve resources as well as the necessity for illicit earnings. The \underline{R} appears in a circle rather than a rectangle because--unlike the other variables shown--it represents an amount of cash (\$), rather than a rate of cash flow (e.g., \$/week).

The thick arrows indicate money flow; the thinner arrows pointing to the shaded triangles represent the anticipated influences of the state variables on those rates of flow. For instance, the level of illicit earnings is expected to damp drug use if the addict is concerned about having his habit detected by police, employers, family, or friends. Also, the amount of money being used for standard of living is made to depend on the fraction of earnings being spent on drugs. Some of the variables are shown to influence

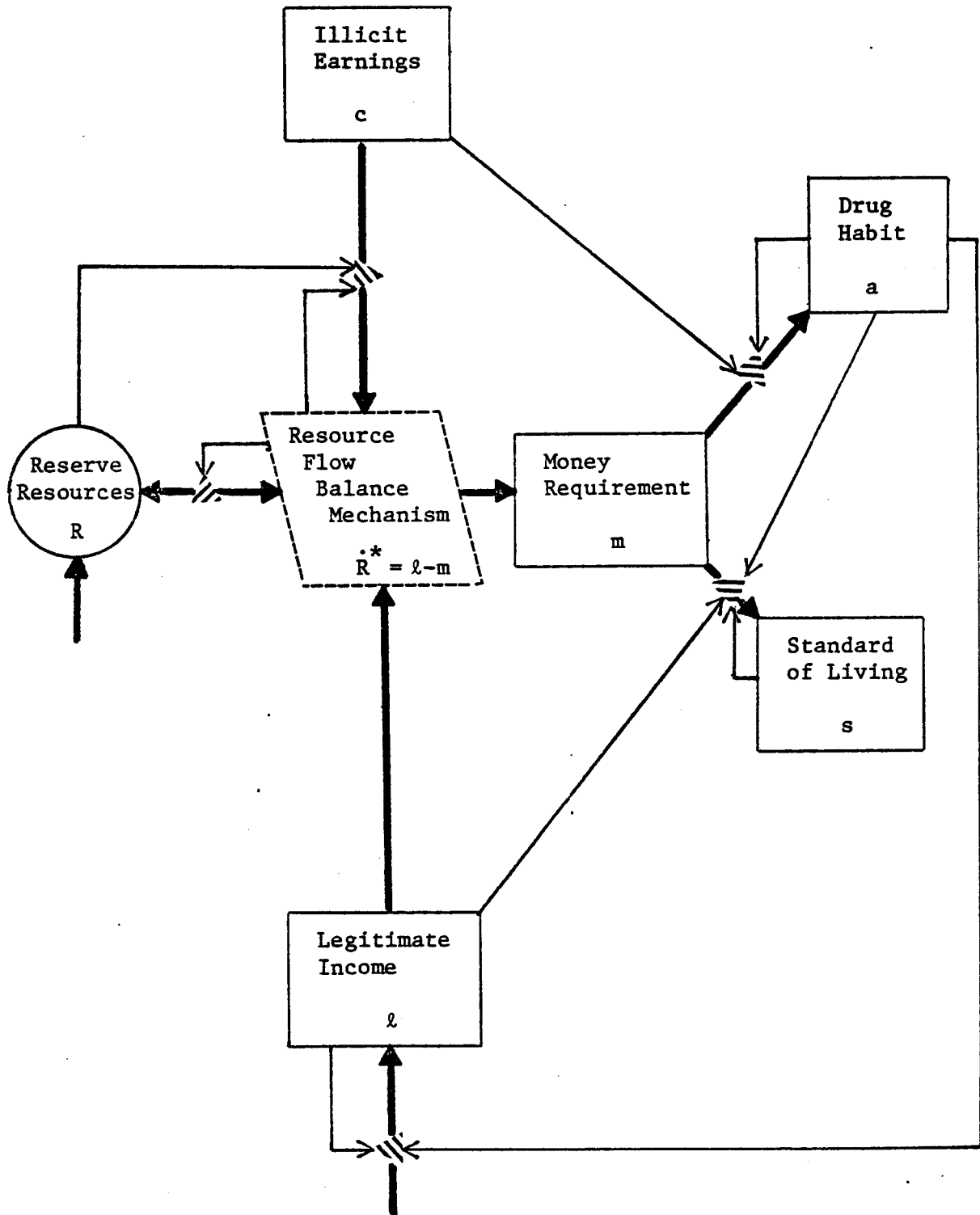


Figure 2. Flow diagram for model

their own rates of change. Increasing tolerance makes this effect plausible for drug use, as does current job status help determine future salary; and families become accustomed to their most immediate standard of living.

The System Equations

Tables 1, 2, and 3 explicitly introduce the mathematical model adopted for addiction behavior. Table 1 presents the time changes of the state variables as a system of ordinary, nonlinear differential equations. Table 2 gives expressions for some of the parameters appearing in the equations of Table 1. Table 3 identifies some relationships among variables which must be satisfied throughout.

The state variables, as already indicated, are designated by the Roman letters a, l, s, m, R, R^{*}, and c; the parameters of the model, by Greek letters. All parameters and variables are assumed to be non-negative. A dot appearing above a variable denotes the time derivative; for example, $\dot{a} \equiv \frac{da}{dt}$ represents the instantaneous rate of change of a with respect to time t.

General system performance--with particular regard to the attributes of the side conditions (Table 3)--is displayed in Figure 3. Table 4 gives a complete listing of the variables and parameters of the model along with brief definitions.

Table 1. Differential equations of model

$$(1) \quad \dot{a} = \kappa_a a (\alpha \langle c \rangle - a)$$

$$(2) \quad \dot{l} = \kappa_l l \left[\log_e \left(\frac{\lambda \langle a \rangle}{l} \right) \right] + \pi l$$

$$(3) \quad \dot{s} = \kappa_s s \left[\log_e \left(\frac{\sigma \langle a, m, c \rangle}{s} \right) \right] + \epsilon \pi l$$

$$(4) \quad \dot{m} = \dot{a} + \dot{s}$$

$$(5) \quad \dot{R}^* = l - m$$

$$(6) \quad \dot{R} = \begin{cases} \dot{R}^*, & \text{if } R^* \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

$$(7) \quad \dot{c} = \begin{cases} \dot{m} - \dot{l}, & \text{if } R^* < 0 \\ 0, & \text{otherwise} \end{cases}$$

Table 2. Parametric equations of model

$$(8) \quad \alpha_{\langle c \rangle} = \frac{\alpha_0}{1 + \xi_a \gamma_a c^2}$$

$$(9) \quad \lambda_{\langle a \rangle} = \lambda_0 \left[1 - \frac{a(a - \xi_\ell)}{\alpha^*(\alpha^* - \xi_\ell)} \right]$$

$$0 \leq \xi_\ell \leq (2\alpha^*\lambda^*/\lambda_0) \left(\frac{\lambda_0}{\lambda^*} - 1 + \sqrt{1 - \frac{\lambda_0}{\lambda^*}} \right) \quad \text{if } \lambda_0 > 0$$

$$0 \leq \xi_\ell < \alpha^* \quad \text{if } \lambda_0 = 0$$

$$(10) \quad \sigma_{\langle a, m, c \rangle} = \frac{\sigma_0 + \sigma^- \gamma_s x}{1 + \gamma_s x} ; \quad x = \frac{\xi_s a}{m - c}$$

Table 3. Side conditions of model

$$(11) \quad m = a + s$$

$$(12) \quad c = \begin{cases} m - \ell, & \text{if } R = 0 \\ 0, & \text{if } R > 0 \end{cases}$$

$$(13) \quad R^* = R, \quad \text{if } \dot{R}^* \geq 0 \quad \text{or} \quad R > 0$$

Table 4. Definitions of variables and parameters

<u>Variable</u>	<u>Definition</u>
t	Time--weeks
a	Level of addiction--dollar cost of heroin per week; $a \geq 0$
ℓ	Legitimate income--dollars per week by legitimate means; $\ell > 0$
s	Standard of living--dollars per week for conventional expenditures; $s > 0$
m	Money requirement--dollars per week; $m = a + s > 0$
c	Criminal income--dollars per week from illegitimate sources; $c \geq 0$
R	Reserve resources--total dollars available in savings and low detection-risk cash sources; $R \geq 0$
R*	Dummy variable--dollars; $R^* = R$, if $R \geq 0$
\dot{a} , etc.	Time derivatives; instantaneous rates of change of variables etc.

<u>Parameter</u>	<u>Definition</u>
κ_a	Rate of addiction change (1/dollar-week); implicitly function of anxiety, job progress, associates, etc.
$\alpha < c >$	Addiction "goal" (dollars per week); $\alpha^- \leq \alpha < c > \leq \alpha^*$
α_0	Addiction proclivity (dollars per week) in the absence of illicit-income detection risk; $\alpha^- \leq \alpha_0 < \alpha^*$
α^*	Physiological heroin tolerance limit (dollars per week)--minimum lethal dosage for experienced addicts
α^-	Physiological heroin demand (dollars per week)--minimum dosage to prevent withdrawal symptoms; $0 \leq \alpha^- < \alpha^*$
γ_a	Personal reactivity to risk of detection for illegitimate earnings (unit free); $0 \leq \gamma_a \leq 1$

(continued)

Table 4 (continued)

<u>Parameter</u>	<u>Definition</u>
ξ_a	Parameter for effect of illegitimate earnings on addiction goal when $\gamma_a = 1$ (weeks ² /dollars ²); $\xi_a \geq 0$
κ_s	Rate of standard-of-living changes (1/week); implicitly function of family pressures, personal wishes, availability of goods, etc.
π	Productivity increases in wages (unit free)
$\lambda\langle a \rangle$	Legitimate income goal (dollars per week); $0 \leq \lambda\langle a \rangle \leq \lambda^*$
λ_0	Legitimate earnings goal (dollars per week) in the absence of drug use; $0 \leq \lambda_0 \leq \lambda^*$
λ^*	Maximum legitimate earning capacity (dollars per week)
ξ_ℓ	Parameter for effect of drug use on earnings goal (dollars per week); $0 \leq \xi_\ell \leq \alpha^*$
κ_ℓ	Rate of legitimate income changes (1/week); implicitly function of education, skills, motivation, etc.
ϵ	Standard-of-living elasticity (unit free); response of standard of living to changes in legitimate earnings
$\sigma\langle a, m, c \rangle$	Standard-of-living "goal" (dollars per week); $\sigma^- \leq \sigma\langle a, m, c \rangle \leq \sigma_0$
σ_0	Standard-of-living goal (dollars per week) in the absence of drug usage
σ^-	"Subsistence" level (dollars per week) for standard of living; $0 \leq \sigma^- \leq \sigma_0$
γ_s	Personal reactivity to importance of financial position (unit free); $0 \leq \gamma_s \leq 1$
ξ_s	Parameter for effect of drug use on standard of living (unit free)

THE EQUATION FORMS

Let us examine now the rationale for and characteristics of the equations which comprise our mathematical system.

Addiction Equation

There exists scant hard data on the time course of drug use rate. Hence, the logistic form, which seems to characterize many "growth" phenomena reasonably well, is adopted for equation (1). A possibly more realistic expression can replace (1) in the future if evidence dictates. The purpose here is to provide an explicit form which is qualitatively appropriate. The "growth-rate" parameter, κ_a , and the "asymptote," α , are presumably both functions of various factors previously discussed under perspective. Cognizance of "real-world" complexity is here taken only for α , which is expressed in (8) as a function of illegitimate earnings, \underline{c} , and yet to be defined parameters. Graphs of (1) and (8) are given in Figures 4 and 5; evidence and reasoning in support of those equations will follow.

There are several sociological works which bear upon the choice of the logistic form. Alksne et al. [1] explain the enculturation of a neophyte narcotics user. A new user requires a period of time for adjustment of self-image, for learning the crafts of efficient drug usage, and for developing the skills which enhance the likelihood of survival in the hustling/criminal sub-culture. During this period, narcotics use increases slowly. There are some mistakes and notions

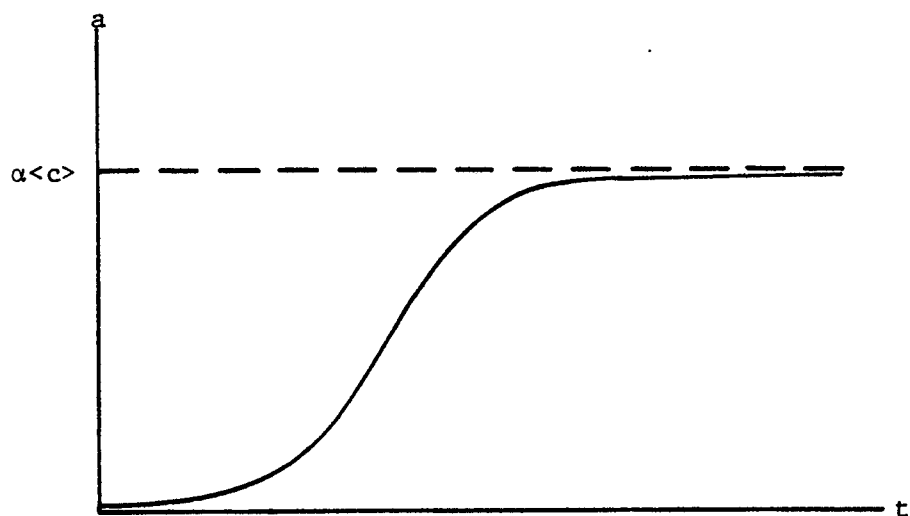


Figure 4. Drug usage approaching addiction goal

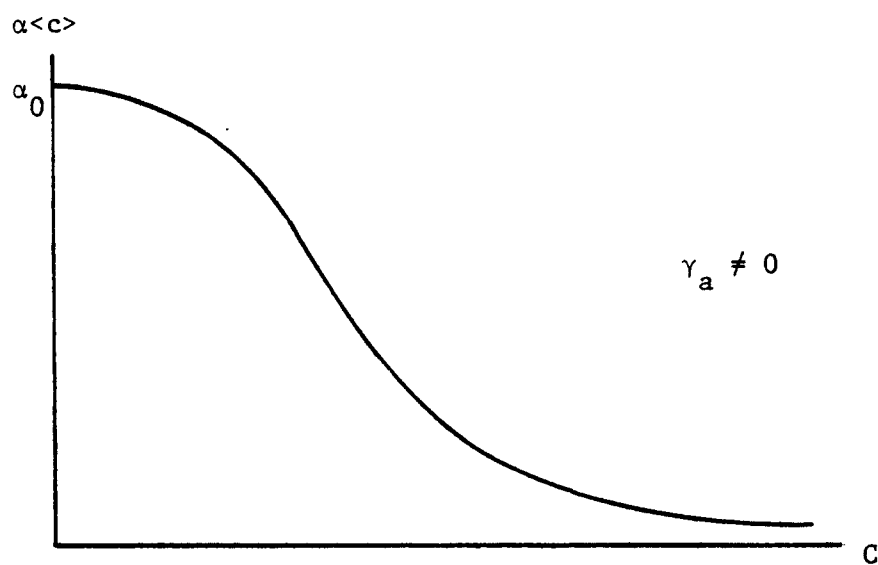


Figure 5. Addiction goal as a function of criminal earnings

of quitting the drug scene; i.e. the subject "builds a tolerance for addiction." After this "learning" period, narcotics use accelerates. These characteristics coincide with the early behavior of the logistic form.

Many addicts have been known to live nearly normal life spans with heavy and nearly constant dosage rate over many years [26]. Thus, it appears that use does not accelerate indefinitely, but after some accommodation period begins to decelerate and tends toward an upper bound, as is represented by α_{c} in the logistic formulation. Clearly, this bound is expected to be less than some lethal level or "physiological tolerance limit," here designated as α^* . Evidence [21] is, however, that many addicts tend to levels of maximum use considerably below the physiological tolerance limit.

In 1938 Lindesmith [16] reported that the euphoria accompanying initial experience with narcotics is not attained during subsequent use and that the addict increases the injection dosage at a rate required to avoid withdrawal symptoms as physiological changes occur. Dole et al. [8] assert that prolonged use of narcotics results in a permanent physiological change in which the characteristic craving, or "physiological demand," here designated as α^- , of addiction cannot be extinguished. McAuliffe and Gordon [21] have recently provided evidence, however, that qualifies Lindesmith's notion. They found that addicts fell into two fairly well-defined groups: "hard-core" and "week-enders." Interviews with "street" addicts showed that they were injecting more than the amounts necessary to avoid withdrawal

and, indeed, were able to produce the euphoric high several times a week. The "hard-core" addicts sought euphoric experiences on most drug-using occasions. But interviews with other addicts showed that, during the week (when participation in conventional activities was deemed necessary), they used only the amount of narcotic required to avoid withdrawal symptoms and then injected as much as 2.4 times the usual dose on weekends to achieve the desired euphoria. Apparently there was little crossover between the two groups once the decision about addict lifestyle had been made.

The subject-motivated upper bound on addiction level is represented by α_0 in (8). It is higher for "hard-core" addicts than for "week-enders," but in both cases it must lie below the "physiological tolerance limit," α^* , and, unless strong forces such as treatment intervene, above or equal to the "physiological demand," α^- . The value of α_0 no doubt reflects individual differences in reactivity to drugs, drug availability, and a compromise between involvement in conventional society and involvement in the drug culture; but these factors will be left implicit.

As shown in equation (8), the addiction level "goal," $\alpha < c >$, is made to depend on the amount of illicit earnings, c , necessitated by the subject's desire for drugs, as well as on α_0 . Analogous to the weekender's concern about jeopardizing his standing in conventional society and the lack of concern by the hard-core addict, subjects vary in the way they adjust the drug-taking level in consideration of the risk of detection from criminal activities to obtain income. The

formulation in (8) reflects a deterrent effect on drug use engendered by the increasing risk of arrest associated with augmentation of illegitimate income. As Winick [34, p. 97] has observed:

Chronic heroin users who are not arrested tend to have mixed dependencies; be careful about their 'works'; interact with fewer people; sustain fewer peer relationships; learn from 'close calls'; avoid 'dumb busts'; be less heavily 'strung out'; avoid overconfidence; be very sensitive to and wary of police activities; have ties to the straight world; and engage in illegal activities where the likelihood of arrest is slight, e.g., numbers runners.

Some of these adaptations are to reduce the risk of exposure as an addict and, thus affect α_0 . Others, however, by reducing the risk of arrest for illicit income-producing activities, are devices which tend to reduce drug use. To reflect variation in concern about arrest, we introduce the factor γ_a ($0 \leq \gamma_a \leq 1$) in (8); it represents the range from no adjustment for risk of arrest for criminal earnings to a full adjustment. The factor ξ_a in (8) thus defines the deterrent effect on $\alpha_{<c>}$ when the subject is most cautious (that is, when $\gamma_a = 1$).

The rate at which the general configuration of drug usage shown in Figure 4 unfolds will depend on a rate parameter, κ_a , as well as on $\alpha_{<c>}$. Although there is a complex of factors which determines the value of κ_a at any time, such as anxiety level of the subject, progress (or lack of progress) in other more conventional activities, proportion of drug-using associates, and so forth, the functional form of κ_a will not be specified since to do so would involve speculation beyond justification.

Legitimate Income Equation

Unlike the situation for addiction, there are hard data to support the form of the time-course of legitimate income. Mincer [24] found in cross-sectional data that, for each of various educational-attainment groups, the relation of earnings to years employed was well fitted by a Gompertz curve having the inflection point coincident with initiation of regular employment. The time trajectory of the Gompertz is sigmoid as is that of the logistic, but the ordinate value of the inflection point in the Gompertz is $1/e$ times the asymptote rather than one-half the asymptote. Mincer found in longitudinal data, however, that, even after adjustment for inflation, earnings increased with years employed at a faster rate than in the cross-sectional data. He states (p. 77):

Cohort profiles are similar in shape to the cross sections. They are displaced upward by some 20-30 percent per decade in most schooling groups and ages; that is, actual growth of income was that much greater in each cohort than in the cross section--a common effect of economy-wide secular growth.

In view of Mincer's findings, a basic Gompertz form, with a term, $\pi \lambda$ (π = productivity coefficient, not radian measure), added for productivity increases, is adopted for the "growth" of legitimate income, as is shown in equation (2). The result is still a Gompertz curve, but with asymptote λe^{π} instead of λ and inflection point at $\lambda e^{\pi-1}$ instead of λ/e .

Drug use often starts before the usual time of regular employment. But, even before regular employment, all individuals have real

income in the form of monetary and non-monetary support from parents or guardians and from part-time jobs. Lacking concrete information on this point, it is assumed that (2) describes legitimate earnings before, as well as after, regular employment.

In equation (9), the basic Gompertz "asymptote," λ , is expressed as a function of the addiction level, $\underline{a} < \alpha^*$; where, as earlier noted, α^* is the "physiological tolerance limit." The form of (9) is chosen so that, for $0 < \xi_\ell < \alpha^*$ legitimate earnings can be increased by light drug usage, but will then decrease as larger dosages interfere with job performance. The expression reflects the findings of Winick [34, p. 115], who reports:

. . . for some (heroin users), the drug may actually improve their level of functioning, or at least do so until their need for higher dosages asserts itself. Heroin may maintain a high level of motivation for people who are ambivalent about their work or dislike it.

Perhaps beginning users also recognize the impending financial burden of drug costs. In (9) the addiction level at which $\lambda < a >$ becomes maximum is $\underline{a} = \frac{1}{2} \xi_\ell$, and if $\xi_\ell = 0$, of course, the maximum is at $\underline{a} = 0$, no addiction.

The parameter, λ_0 , in (9) is the earning capacity toward which the subject effectively would strive in the absence of drug use. It is some function of innate abilities, education, regard of superiors and peers, opportunities in the community, and personal goals--to which point Woelfel and Haller's [35] "Significant Others, the Self-Reflexive Act and the Attitude Formation Process" is relevant. No attempt will be made to define a relation of λ_0 to such factors;

i.e., λ_0 , in the absence of counseling, will be assumed to be constant for an individual. It will be assumed, however, that most people do not realize their true potential, λ^* ; hence, $\lambda_0 \leq \lambda^*$.

It is conceivable, of course, that the subject maintains expectations of legitimate income above the maximum determined from consideration of background characteristics. That is, we may have $\lambda_0 > \lambda^*$. This circumstance would have important treatment implications, as discussed elsewhere (p. 38). In terms of modeling consequences, the choice of goal above the postulated achievable maximum can be accommodated by simply allowing the level of legitimate income, $\underline{\ell}$, to rise at the rate stipulated by the unrealistic objective, λ_0 , until the maximum, λ^* , is reached. At that time the derivative, $\dot{\underline{\ell}}$, would be set to zero for as long as $\lambda\langle a \rangle \equiv \lambda\langle a; \lambda_0 \rangle > \lambda^*$. There is further difficulty from the fact that, according to the model formulation, λ^* is an instantaneous maximum--set from cross-sectional data. Productivity increases can result in a maximum value which is greater than λ^* . To avoid this type of complication, particularly in the absence of treatment considerations by the model, λ_0 is taken to be $\leq \lambda^*$ for all applicable subjects.

The provision in (9) for the legitimate earnings goal, $\lambda\langle a \rangle$, to be stimulated by light drug usage requires the associated restrictions on ξ_ℓ shown in Table 2. This is because the maximum value for $\lambda\langle a \rangle$, occurring at $\frac{1}{2} \xi_\ell$, must be $\leq \lambda^*$. The restrictions can be shown as follows. Setting $\underline{a} = \frac{1}{2} \xi_\ell$ in (9) yields

$$0 \leq \max \lambda \langle a \rangle = \lambda_0 \left[1 + \frac{\xi_\ell^2}{4\alpha^*(\alpha^* - \xi_\ell)} \right] \leq \lambda^* . \quad (14)$$

Since $0 \leq \xi_\ell < \alpha^*$, the left-hand inequality is automatically satisfied. These restrictions on α^* and ξ_ℓ also permit expansion and rearrangement to write

$$\frac{\lambda_0}{\lambda^*} \xi_\ell^2 + 4\alpha^* \left(1 - \frac{\lambda_0}{\lambda^*} \right) \xi_\ell - 4\alpha^{*2} \left(1 - \frac{\lambda_0}{\lambda^*} \right) \leq 0 \quad (15)$$

in place of the right-hand inequality of (14). By virtue of the assumption that $0 \leq \lambda_0 \leq \lambda^*$, implying $0 \leq \frac{\lambda_0}{\lambda^*} \leq 1$, (15) is a parabola concave unless $\lambda_0 = 0$, in which case it is a line with positive slope. Thus, for $\lambda_0 > 0$, the inequality of (15) is satisfied by

$$2\alpha^* \left(\frac{\lambda^*}{\lambda_0} - 1 \right) \left[-1 - \sqrt{1 + \frac{\lambda_0}{\lambda^* - \lambda_0}} \right] \leq \xi_\ell \leq 2\alpha^* \left(\frac{\lambda^*}{\lambda_0} - 1 \right) \left[-1 + \sqrt{1 + \frac{\lambda_0}{\lambda^* - \lambda_0}} \right] , \quad (16)$$

and, for $\lambda_0 = 0$ by

$$\xi_\ell < \alpha^* \quad (17)$$

where the bounds in (16) and (17) are the roots of (15). Because $0 \leq \lambda_0 \leq \lambda^*$, the lower bound of (16) is clearly always less than or equal to 0, and the lower bound of (17) is $-\infty$. Thus, the defined zero lower bound for ξ_ℓ satisfies (14). In view of the restrictions on α^* , λ_0 and λ^* , the upper bounds of (16) and (17) are positive,

but there remains to check whether the right-hand inequality of (16) implies $\xi_\ell < \alpha^*$ as desired, not $\xi_\ell \leq \alpha^*$: i.e., whether

$$2\left(\frac{\lambda^*}{\lambda_0} - 1\right) \left[-1 + \sqrt{1 + \frac{\lambda_0}{\lambda^* - \lambda_0}}\right] < 1 \quad (18)$$

when $\lambda_0 > 0$. Rearrangement of (18) yields

$$\sqrt{1 - \frac{\lambda_0}{\lambda^*}} < 1 - \frac{1}{2} \frac{\lambda_0}{\lambda^*}, \quad (19)$$

a standard inequality which holds for the earlier noted inequality, $0 < \frac{\lambda_0}{\lambda^*} \leq 1$. Thus, the upper bound of (16) is the appropriate upper bound for ξ_ℓ when $0 < \lambda_0 \leq \lambda^*$, and the case $\lambda_0 = 0$ is handled by the initial requirement that $\xi_\ell < \alpha^*$ always.

A graph for $\lambda\langle a \rangle$ as related to λ_0 , λ^* , and addiction level, a , is shown in Figure 6. In addition to the possibility of an increase in legitimate earning goal with light addiction, the graph reflects the reasonable assumption that as addiction level approaches physiological tolerance, α^* , the asymptote, $\lambda\langle a \rangle$, for legitimate earnings approaches zero. Note that, if $\lambda_0 = \lambda^*$, then any level of heroin use lowers $\lambda\langle a \rangle$.

The rate parameter κ_ℓ in (2) is determined by what Lipset and Bendix [17, p. 36] call "the means of mobility," that is "access to education, acquisition of skills, . . . and motivation to seek higher positions." This should also include the general economic and job market growth potentials of the particular community. For the

present purposes, however, κ_l , like κ_a in (1) will be treated as a constant.

Standard-of-Living Equation

The non-heroin money outlets are pooled into what shall be called the "standard-of-living" variable, s . This includes paying the rent, buying food and clothing for self and family, providing transportation, education, adequate health care, recreation, etc., and the normal associated savings programs. This measures the addict's accommodation with the "straight world." Brotman and Freedman's [4] finding that the degree of involvement with conventional activities is one of the best prognostic indicators of success in breaking out of the addiction cycle indicates the relevance of this variable.

In the absence of drug/hustling complications, most people will allocate most of the money available from conventional employment to the purposes encompassed under standard of living, with additional savings in many cases. Ghez and Becker [13] develop a life-cycle model of consumption, including investments in human capital (i.e., education and job training), which predicts that "the shape of the consumption profile depends largely on the shape of the wage profile" (p. 23) and find an elasticity of .5 between the two. Hence, we adopt the Gompertz form for standard of living as shown in (3), as was done for legitimate income in (2). A productivity term is added in (3) as in (2); it is the productivity term of (2), namely, π_l , multiplied by an elasticity coefficient, ϵ .

Sadik and Johnson [29] have suggested that the market discount rate adjudged by a wage earner (a higher discount rate means a lower present value of future income) and his time preference for consumption (a measure of his impatience to spend income) are important determinants for the lifetime profile of individual consumption behavior. If patience is high enough relative to the discount rate, consumption will, assuming a balancing of accounts at death, increase more slowly than income in earlier years and faster in later years. If patience is low and the discount rate on earnings is high--a condition likely to be fulfilled by many heroin addicts--consumption will rise faster than income early and slower than income later. Equation (10) is designed to reflect the effects of discount rate on standard-of-living patterns.

The variable, \underline{x} , defined for (10) is a means of expressing the economic strain of the addict's drug habit. Since \underline{m} equals the monetary expenditures and \underline{c} will be used to make up the difference between expenses and legitimate resources, $x = \xi_s a/m$ when $c = 0$ and $x = \xi_s a/l$ when $c > 0$.

The parameter, σ^- , in equation (10) represents a subsistence level below which the physical endurance of the addict and his family cannot long be sustained. The γ_s is another personal choice parameter with a value between 0 and 1. If $\gamma_s = 1$, then the addict is quite willing to sacrifice on his living standard to obtain the money necessary for drugs. This may imply a willingness to reduce his involvement with a conventional life style. If the subject is

clinging to his expected living standard (by keeping γ_s close to 0), he values conventional involvement and limits his heroin use, rather than chance detection from higher levels of illicit earnings, c . Figure 7 gives the graph of $\sigma\langle a, m, c \rangle$ as a function of \underline{x} .

Again, the K_s of (3) is treated as a constant, although it should be recognized as some function of personal wishes, family pressures, the availability of goods and services, and so forth.

Other Equations in the Model

The "money requirement" variable, m , in equations (4), (5), (7), (10), (11), and (12) is simply the sum of drug expenses, a , and standard-of-living expenses, s . The variable R^* , defined, except for a constant of integration, by (5)--i.e., by $\dot{R}^* = l - m - \dot{R}$ controls use of reserves, \dot{R} , and change in criminal earnings, \dot{c} , as shown by (6) and (7).

The variable R denotes the amount of reserve resources available to cover expenses when legitimate earnings are not adequate. It represents extra savings not encompassed under standard of living, s ; i.e., readily accessible to cover financial crises plus all other legitimate sources of cash from which utilization involves no risk of detection for deviant behavior. Funds from rich uncles and solicitous wives could fall into this category. Illicit, or "criminal" earnings, c , cover a wide variety of income-yielding activities for which the subject is liable to arrest.

The side condition of equation (11) holds by definition. Those of (12) assume that the subject will not engage in criminal earning

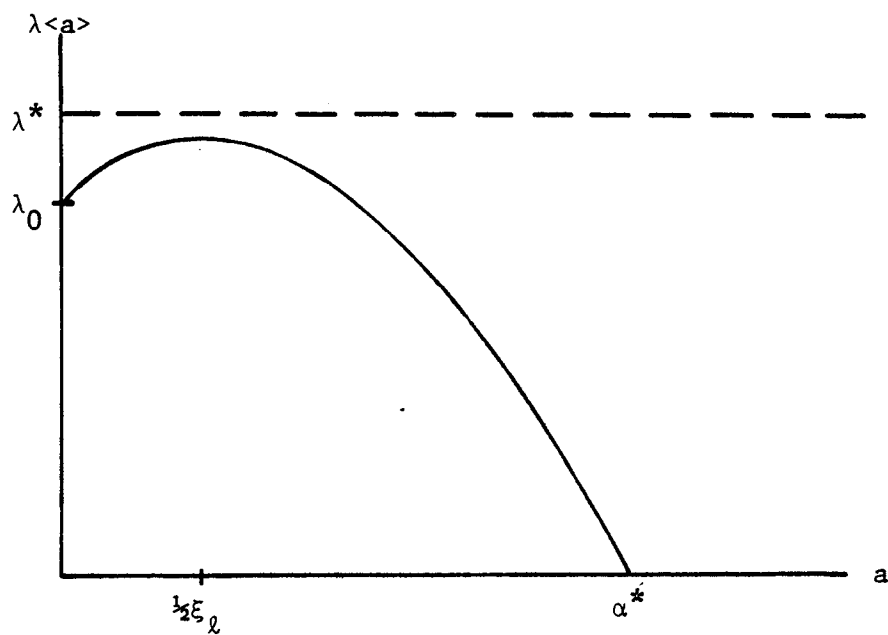


Figure 6. Legitimate income goal as a function of drug usage

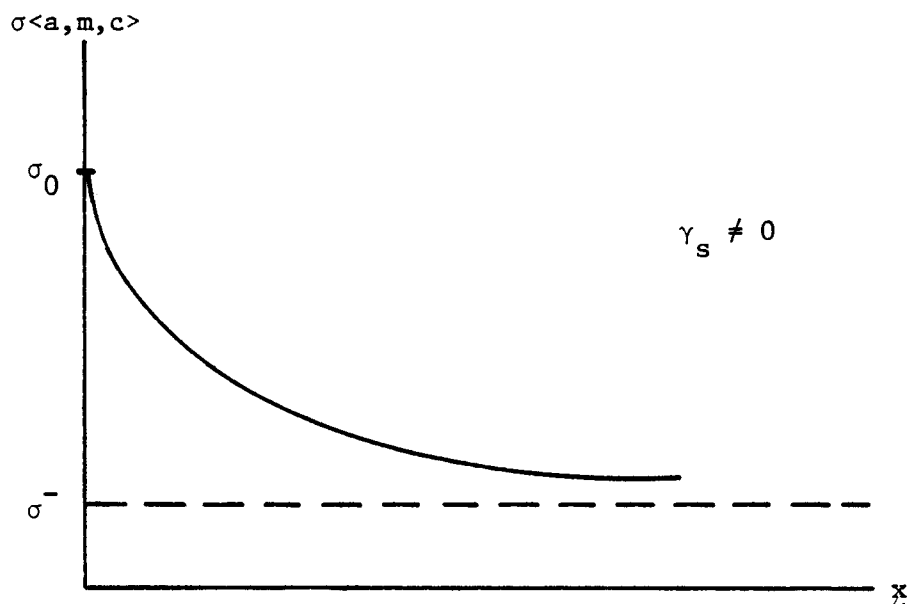


Figure 7. Standard of living goal as a function of financial strain

activities as long as legitimate earnings plus reserves will cover his total money needs. The condition of (13) reflects that when \dot{R}^* is positive there is money available to add to reserve resources, \underline{R} , for future spending. This is extra savings and is cumulative. However, when \dot{R}^* is negative and reserves exist ($R > 0$), reserves will be drawn upon until they are depleted ($R = 0$). If, after \underline{R} became zero, \dot{R}^* remains negative, then illicit income, \underline{c} , becomes necessary to meet expenses. Therefore, when the deficit ends (i.e., \dot{R}^* becomes ≥ 0), R^* is reset to 0 so that it can again be identical to \underline{R} .

Further Comments

It has perhaps been implied that the state variables \underline{a} , $\underline{\ell}$, and \underline{s} are always below their respective "asymptotes" $\alpha\langle c \rangle$, $\lambda\langle a \rangle$, and $\sigma\langle a, m, c \rangle$ and increasing toward them. This is not necessarily true, even if these limits are initially greater than the states of the system, because the limits are functions of the state variables. Keeping in mind that one ultimate aim of this model is as a tool in prescribing treatment for addicts, the interaction between treatment and subject may work to reduce drug-taking objectives. For instance, treatment could reset $\alpha\langle c \rangle$, possibly through α_0 , below the level of \underline{a} just preceding treatment. Or it might be determined that job aspirations are set at such a high value that drugs are being used to assuage the fears of failure. In this case, λ_0 could be lowered and $\lambda\langle a \rangle$ could fall below $\underline{\ell}$. A more complete representation of the graph of \underline{a} with time than that shown in Figure 4 is illustrated by the alternative curves drawn in Figure 8A. However, a more realistic

reflection of system dynamics would have $\alpha\langle c \rangle$, as well as \underline{a} (in the absence of adequate resources), changing with time, as depicted in Figure 8B. The foregoing applies, as well, to the variables \underline{l} and \underline{s} and their respective goals, $\lambda\langle a \rangle$ and $\sigma\langle a, m, c \rangle$.

Since treatment for addiction is not a matter to be considered in the present model, we shall choose the initial values of these variables to be below their respective limiting values. It should be pointed out that if either \underline{a} or \underline{l} has an initial value of zero, the corresponding time derivative will be zero for all \underline{t} , thereby generating the uninteresting constant (zero) state for that variable. We shall circumvent that irregularity by requiring \underline{a} , \underline{l} , and \underline{s} to be greater than zero. This is consistent with previous statements that the subject is already using narcotic drugs (page 13) and that a child can be thought of as receiving income in the form of financial support from his parents--all of which is applied to his standard of living (page 30). To accommodate initiation to positive values of the variables, we can simply define those variables as being identical to the zero function until the time of initiation, at which time jumps to the appropriate values occur.

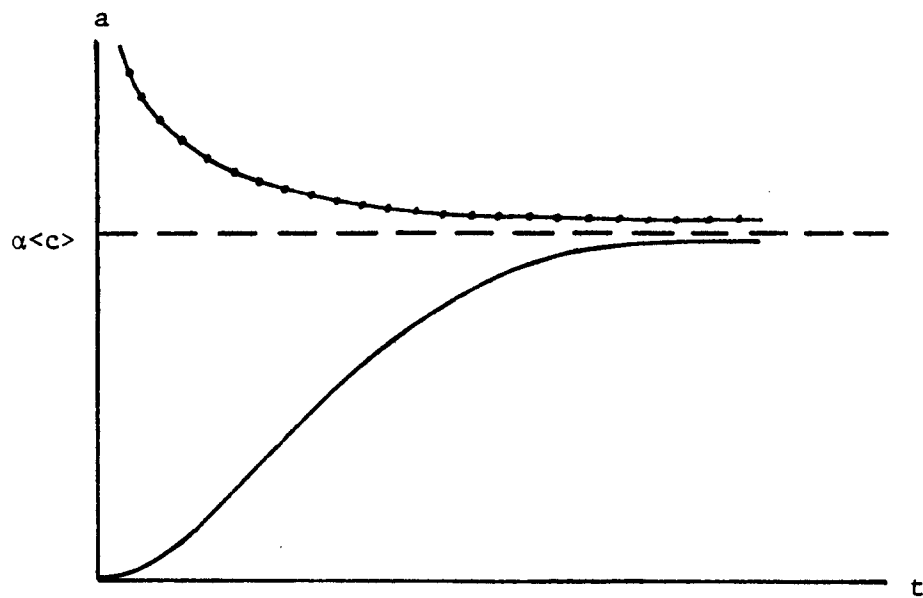


Figure 8A. Drug usage approaching constant addiction goal

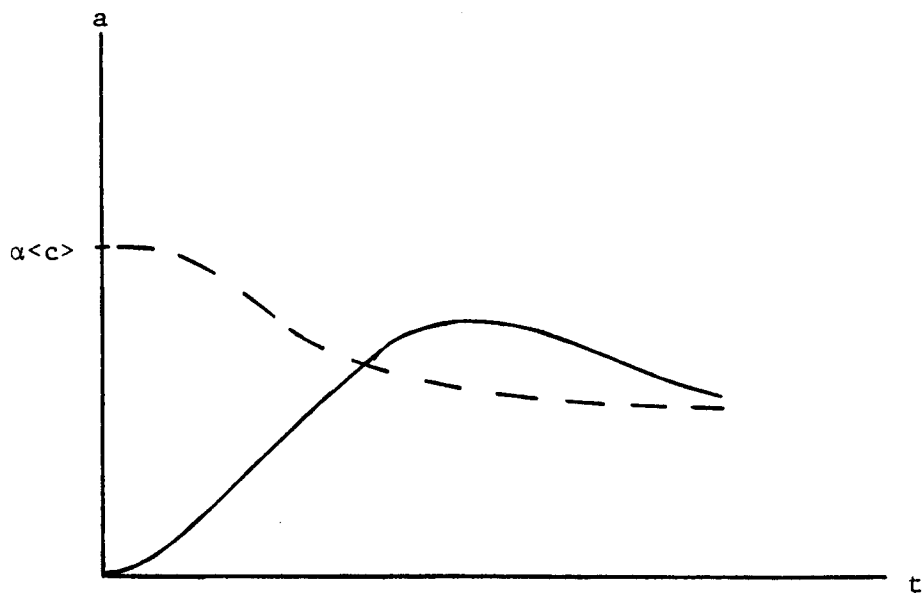


Figure 8B. Drug usage approaching variable addiction goal

STUDY OF MODEL PERFORMANCE

To make the mathematical system described above operational, values must be assigned to each of the parameters in equations (1) through (10) and initial conditions for the state variables must be chosen. System behavior under a variety of initial conditions and parameter settings are herewith observed by means of a computer-assisted procedure [28] in which a fourth-order Runge-Kutta numerical integration routine has been programmed to generate approximate solution trajectories in convenient steps over a desired interval--in our case, for five years from the onset of heroin use. The method is a variation developed by Gill to minimize computer storage requirements during program operation. A subroutine for plotting any of the state variables and/or their derivatives against another or against time is also used.

Parameter Values for Addiction Equation

Parameters for which values must be assigned in the addiction equation (1) are K_a and $\alpha_{<c>}$; but, to evaluate $\alpha_{<c>}$, values of α_0 , ξ_a , and γ_a of (8) must first be assigned.

We first arrive at some values for α_0 , the addiction "proclivity" in the absence of risk of detection for illicit earnings. Moore [25], from analyses of the New York City drug scene, designates a "large habit hustler" as one using nine 100-mg bags/day of heroin of 10 percent purity, or 90 mg/day of pure heroin. For New York City, he cites a price of \$5/bag (50¢/mg of heroin). Assuming such users are

insensitive to risk of detection for criminal earnings ($\gamma_a = 0$), this yields a value for α_0 of \$315/wk.

In Chicago, at a price of \$2.34/mg, the same habit would cost \$1475/wk. The National Institute on Drug Abuse [15] reports that the national average retail price of heroin during the first quarter of 1975 was \$2.45 per milligram and that the average national purity of heroin during the same quarter was 10.7 percent. Actually, most heroin is not purchased at these retail prices [25], but this represents the best information available. Fluctuations in price, which can be considerable during disruptions of the distribution network, will be ignored.

It may be, however, that the Large Habit Hustler ($\alpha_0 = 315$) is representative of neither the hard-core addict nor the weekender. To remedy this possible incongruity, values for α_0 will also be chosen to correspond to the "Large Habit Dealers" and the "Medium Habit Hustlers" of Moore's classification. Henceforth, we shall confine our attention to the conditions of New York City. This means that the Large Habit Dealer will require 18 bags (\$90) per day and the Medium Habit Hustler is expected to use 5 bags (\$25) per day. Then α_0 will be assigned the respective values \$630/wk and \$175/wk.

To arrive at a value for K_a , the parameter for rate of increase in use, we invoke Chein et al.'s [7] findings that most addicts try heroin by age 16, spending \$2-12/day for the drug, and that 90 percent become regular users within a year of first trial. We then assume that beginners are the "Joy Poppers" of Moore's [25] classification,

that is, "intermittent" users, with an average \$2/day or \$14/wk habit, and that after 50 weeks they become the "Medium Habit Hustlers" of Moore ("more skilled and getting larger tolerances", p. 61) using 5 bags a day. The integrated form of (1) with $\alpha_{<c>} = \alpha_0$ (i.e., with $\gamma_a = 0$) is

$$a = \frac{\alpha_0}{1 + \frac{\alpha_0 - a_0}{a_0} e^{-\alpha_0 K_a t}}$$

Substituting $\alpha_0 = 315$, $a_0 = 14$, $a = 175$, $t = 50$, and solving yields $K_a = 0.00021$ for the Large Habit Hustler. Lacking justification for alteration of K_a in the case of the Large Habit Dealer or the Medium Habit Hustler, we shall retain the value of 0.00021 for that addiction-rate parameter. Because of the larger α_0 value, drug usage can be expected to advance more rapidly for the Large Habit Dealer than for the Large Habit Hustler. Conversely, the Medium Habit Hustler will advance more slowly than either of the other two.

The value for ξ_a will be decided in a rather arbitrary manner. It will be assumed that $\alpha_{<c>} = \frac{1}{2} \alpha_0$ when illegitimate income, c , is one-half the maximum potential legitimate income, λ^* , and when $\gamma_a = 1$ (i.e., when sensitivity to detection of illegitimate earnings is maximal). With values of λ^* given later, this results in $\xi_a = 0.0004$ for grade-school dropouts and $\xi_a = 0.0001$ for college graduates regardless of drug-usage-level proclivity.

Parameter Values for Legitimate Income Equation

Here, values must be assigned for K_ℓ and π in (2) and for λ_0 , α^* and ξ_ℓ of (9), which in turn determine $\lambda_{<a>}$ of (2). Values for K_ℓ and λ_0 can be found from the data of the Bureau of Labor Statistics presented by Mincer [24], who shows weekly earnings in 1959 of white, nonfarm men at several educational levels as related to years of job experience. To match the profile for workers with 5 to 7 years of schooling (grade-school dropouts), λ_0 and K_ℓ are, respectively, 100 and 0.00282. For a college graduate, λ_0 and K_ℓ are, respectively, 200 and 0.00305. To correspond with a productivity growth rate of 25 percent per decade, $\pi = 0.000479$.

To arrive at the values of ξ_a previously chosen for the addiction equation, it was assumed that $\lambda^* = \lambda_0$; i.e., maximum legitimate earning capacity and legitimate earning goal in the absence of drug use are the same.

It is not possible to determine accurately the value of α^* , the minimum lethal dosage for experienced heroin users. Experimentation is precluded, and analyses of overdose deaths would not greatly clarify the picture. Tolerance levels are not determined in autopsies, lethal dosages are not likely to be minimum, and the dilution agent (usually quinine) plays a significant role in drug use complications. From Moore's [25] analysis of heroin habit costs, an appropriate numerical choice for α^* may be made. It is reported there that 11 percent of New York City's addict population has what is known as "Dealers' Habit" of 18 bags (180 mg of pure heroin) per

day. It seems not unreasonable then to assume that 200-250 mg/day is the minimum lethal dosage. We choose 200 mg (20 bags) per day which, at the earlier-used cost of \$5/bag assigns α^* the value \$700/wk.

Since λ_0 was chosen equal to the maximum expected earnings for a particular educational category, ξ_ℓ is necessarily zero.

Parameter Values for Standard-of-Living Equation

Here values are needed for K_s and ϵ of (3) and for σ_0 , σ^- , γ_s and ξ_s of (10), which in turn determine $\sigma_{\langle \underline{a}, \underline{m}, \underline{c} \rangle}$ of (3).

To have standard-of-living expenses follow legitimate earnings closely when drug use is not involved, K_s is chosen equal to K_ℓ and $\epsilon = 1$. Likewise, the goal for standard of living in the absence of drug usage, σ_0 , is set equal to λ_0 .

The subsistence level, σ^- , is set at the weekly equivalent of a yearly poverty-level income of \$2000. That is, $\sigma^- = 40$. Somewhat arbitrarily, a value of $\xi_s = 2$ is chosen. This drops the standard-of-living goal, $\sigma_{\langle \underline{a}, \underline{m}, \underline{c} \rangle}$, to $\frac{\lambda_0 + \sigma^-}{2}$ when the addict is fully willing to sacrifice on standard of living ($\gamma_s = 1$), and when drugs account for one-half of the subject's expenditures in the absence of criminal earnings or one-half of his legitimate earnings when \underline{c} is greater than zero; i.e., when $\underline{a} / (\underline{m} - \underline{c}) = \frac{1}{2}$ if $\underline{c} = 0$ or $\underline{a} / \underline{l} = \frac{1}{2}$ if $\underline{c} > 0$.

Cases Studied

To illustrate the salient features of the behavior of the model, realizations for a number of combinations of parameter values and

initial conditions are shown in the next section. Certain of the parameters were not varied; their values are shown in Table 5A. Other parameters were varied to correspond to addict type; their values are shown in Table 5B. In addition, the values of the personal choice parameters, γ_a , γ_s , were alternated between 0 and 1 in various combinations for each of the several addict types. Initial conditions were fixed for the drug usage variable; initial conditions for other variables were varied to correspond to combinations of education level and initiation to heroin use at an early versus a later stage in his legitimate employment career; the values adopted are shown in Table 5C. In addition, in some instances, reserve resources, R , were alternated between none and ample to eliminate the need for illegitimate earnings. Consequences to model performance of parameter values other than those shown in Tables 5A and 5B are considered under "Discussion."

Table 6 provides an index of all the cases for which realizations were computed. Since evidence cited earlier [7, 25] indicates that the level of heroin consumption tends to approach the maximum dictated by addiction proclivity about two years after initiation to heroin use, results will be shown only for the five-year period following initiation.

Table 5A. Values assigned to unvaried parameters

<u>Parameter</u>	<u>Value</u>
α^*	700.0
K_a	0.00021
ξ_l	0.0
π	0.000429
σ^-	40.0
ξ_s	2.0
ϵ	1.0

Table 5B. Values assigned to varied parameters

<u>Addict Type</u>	<u>Large Habit</u>	<u>Large Habit</u>	<u>Medium Habit</u>	<u>Medium Habit</u>	<u>Dealer's Habit</u>	<u>Dealer's Habit</u>
	<u>Grade-School</u>	<u>College Graduate</u>	<u>Grade-School</u>	<u>College Graduate</u>	<u>Grade-School</u>	<u>College Graduate</u>
<u>Parameter</u>						
α_0	315.0	315.0	175.0	175.0	630.0	630.0
λ_0	100.0	200.0	100.0	200.0	100.0	200.0
σ_0	100.0	200.0	100.0	200.0	100.0	200.0
K_l	0.00282	0.00305	0.00282	0.00305	0.00282	0.00305
K_s	0.00282	0.00305	0.00282	0.00305	0.00282	0.00305
ξ_a	0.0004	0.0001	0.0004	0.0001	0.0004	0.0001

Table 5C. Values assigned to varied initial conditions

$$C(0) = a(0) = 14, \text{ no resources case}$$

$$= 0, \text{ ample resources case}$$

$$l(0) = 40 \text{ or } 85, \text{ if } \lambda^* = 100$$

$$= 40 \text{ or } 170, \text{ if } \lambda^* = 200$$

$$s(0) = l(0)$$

Table 6. Index to graphs

Resource Situation	Education Level	Initiation Time	Aversion to		Addiction Proclivity		
			Criminal Earnings	Lowered Living Standard	Medium Habit	Large Habit	Dealers' Habit
None	Eighth grade	Early	Yes	Yes	15D	9D	12D
			Yes	No	15C	9C	12C
			No	Yes	15A	9A	12A
			No	No	15B	9B	12B
		Late	Yes	Yes			
			Yes	No			
			No	Yes			
			No	No			
	College	Early	Yes	Yes	16D	10D	13D
			Yes	No	16C	10C	13C
			No	Yes	16A	10A	13A
			No	No	16B	10B	13B
		Late	Yes	Yes	17D	11D	14D
			Yes	No	17C	11C	14C
			No	Yes	17A	11A	14A
			No	No	17B	11B	14B
Ample	Eighth grade	Early	Yes	Yes			
			Yes	No			
			No	Yes			
			No	No			
		Late	Yes	Yes			
			Yes	No			
			No	Yes			
			No	No			
	College	Early	Yes	Yes			
			Yes	No		18A	18C
			No	Yes			
			No	No			
		Late	Yes	Yes			
			Yes	No		18B	18D
			No	Yes			
			No	No			

Graphs for Large Habit Hustler

We examine now the projected time-course of the variables \underline{a} , \underline{s} , $\underline{\ell}$, and \underline{c} over the five-year period immediately following the initiation to heroin for the Large Habit Hustler. Most of the features of the model will be illustrated with this type of addict. Graphs for other categories of addict are presented in the next two sections and will highlight differences in model performance resulting from changes in addiction proclivity level. In Figures 9A-9D we first consider the Large Habit Hustler with an eighth-grade education. Initiation to heroin is postulated at the addiction level $\underline{a}_0 = \$14/\text{week}$ for all cases studied. Assuming the subject is at an early stage of his legitimate employment career, we choose the initial value of $\underline{\ell}$ to equal $\$40/\text{week}$ and set the level for standard of living, \underline{s} , also at 40. This is consistent with the expectation that, in the absence of drug usage, standard-of-living expenses will absorb all legitimate income. If it is further stipulated that there are no reserve resources, an amount of illicit earnings, \underline{c} is required to match the initial cost of drugs. The case of reserve resources which are ample to cover all drug costs during those five years is considered later. Projections for the situation in which reserve resources can help cover money requirements during only part of the five-year period, as illustrated in Figure 3, are considered mixtures of the "no resources" and "ample resources" examples and are, therefore, not presented.

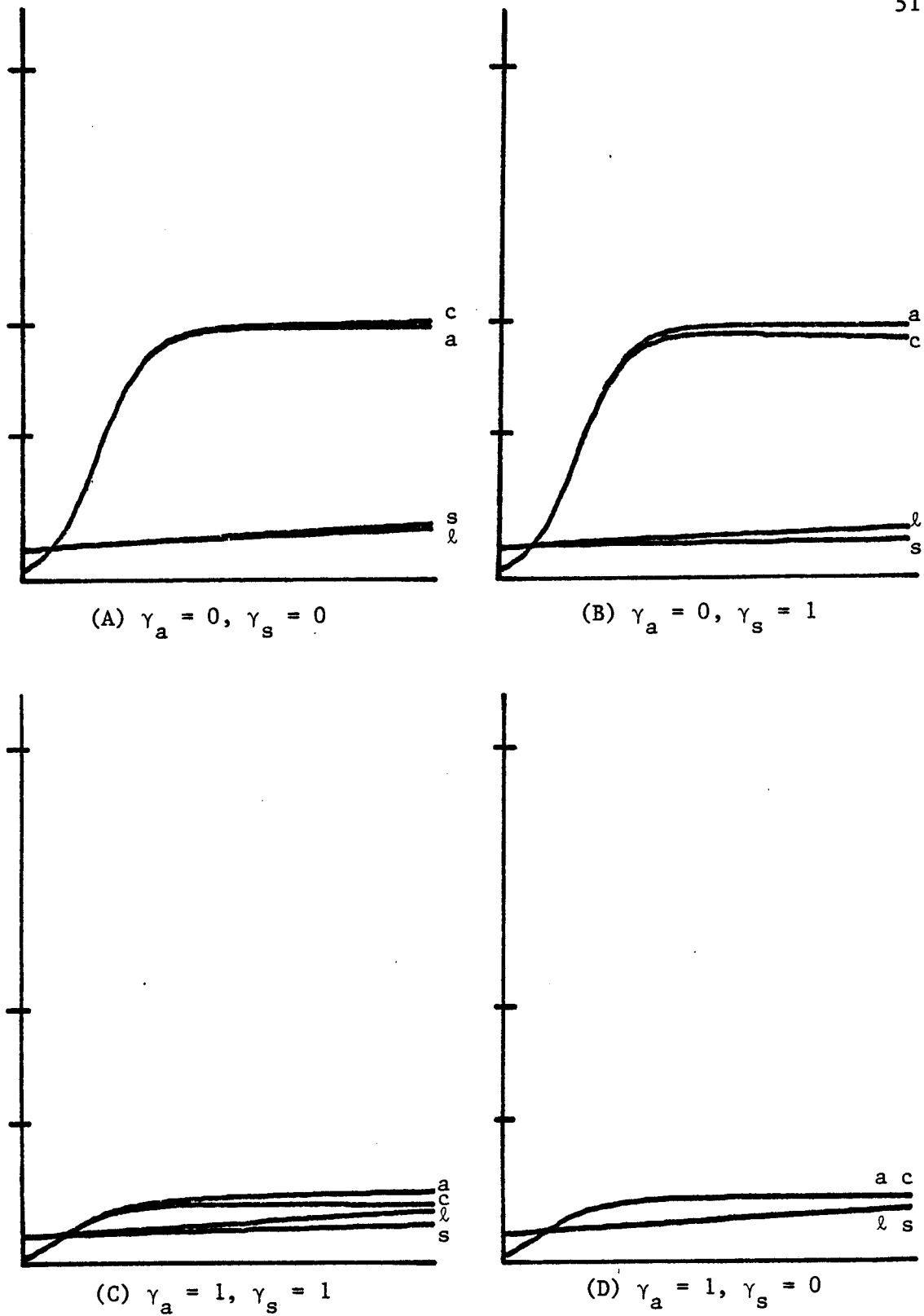


Figure 9. Large habit hustler, grade-school drop-out, no resources, early initiation

Figures 9A-9D show how the values of the "personal choice" parameters γ_a and γ_s affect the course of the four primary state variables over the five-year period of inspection. Refusal to adjust drug usage when criminal earnings are necessitated or to adjust standard of living when drug usage is absorbing an increasing fraction of earnings (i.e., $\gamma_a = \gamma_s = 0$), results (Figure 9A) in addiction level virtually reaching its maximum. Criminal earnings, \underline{c} , rise above the drug expenditure level, \underline{a} , because standard of living is increasing unabated while legitimate income slackens in response to drug usage effects. When $\gamma_s = 1$ (while $\gamma_a = 0$, Figure 9B), adjustments for drug expenditures are made and the standard-of-living value trails legitimate income throughout the period. Criminal earnings are thereby decreased by about 7 percent. If full adjustments are allowed ($\gamma_a = \gamma_s = 1$, Figure 9C), drug usage and criminal earnings are curtailed drastically; because of the reduced inhibitory effect of drugs on job performance, legitimate income is approximately 10 percent higher at the end of 5 years than for the cases where γ_a equaled 0. With $\gamma_a = 1$ and $\gamma_s = 0$ (Figure 9D) standard of living, \underline{s} , follows legitimate income, \underline{l} , closely. Likewise, criminal earnings, \underline{c} , cover only drug costs, \underline{a} . Since this subject is sensitive to criminal involvement, and more illicit income is required when no sacrifices on standard of living are made, the addiction level here is lower than for the subject represented in Figure 9C.

In Figures 10A through 10D parameter λ_0 is set at 200, and the college graduate narcotics addict is represented. All other parameter

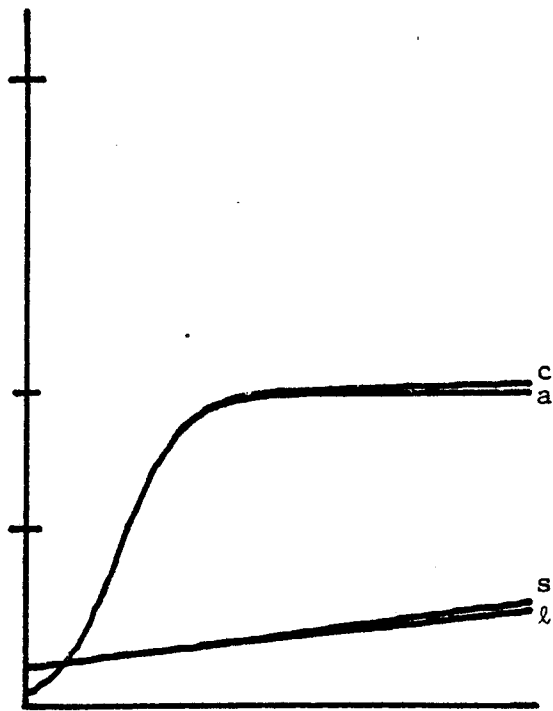
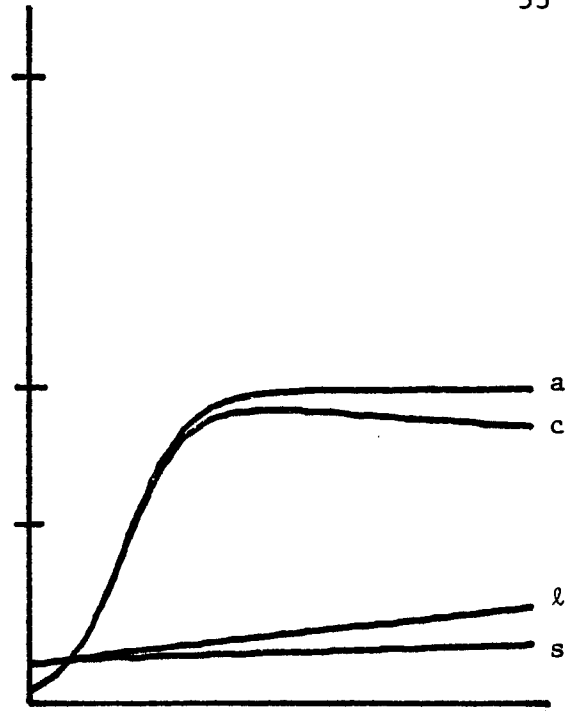
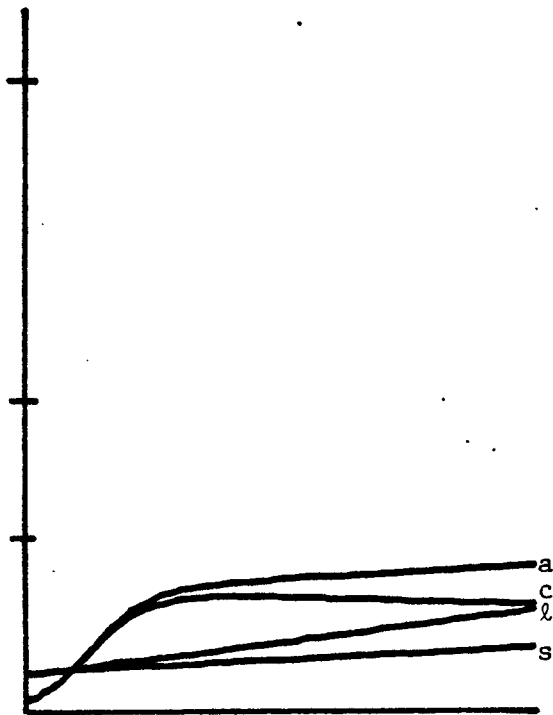
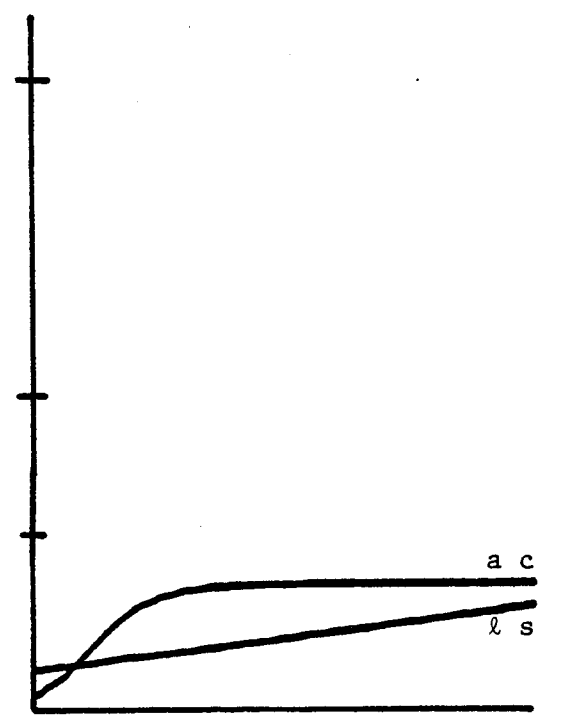
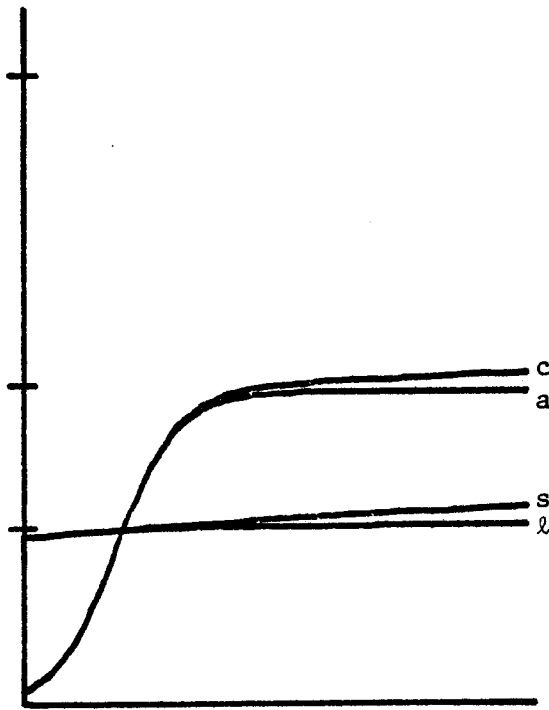
(A) $\gamma_a = 0, \gamma_s = 0$ (B) $\gamma_a = 0, \gamma_s = 1$ (C) $\gamma_a = 1, \gamma_s = 1$ (D) $\gamma_a = 1, \gamma_s = 0$

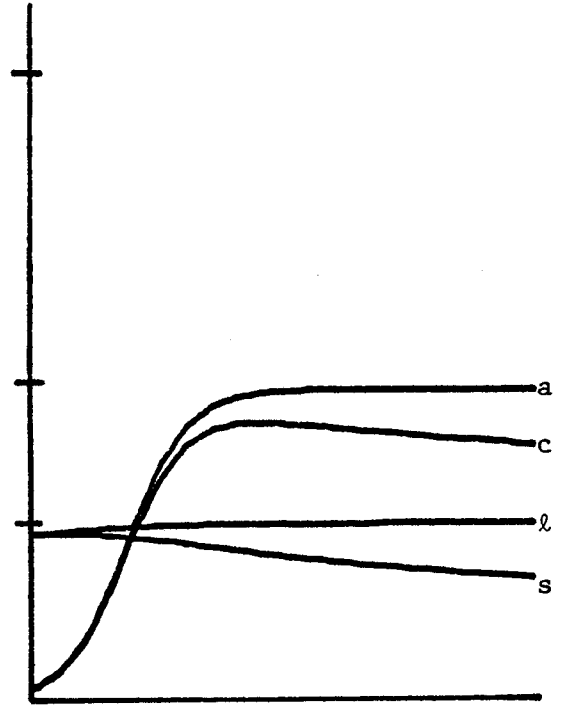
Figure 10. Large habit hustler, college graduate,
no resources, early initiation

values are the same as in Figures 9, and the arrangement of graphs allows a case-by-case comparison with the grade-school drop-out by combination of γ_a and γ_s . There is very little qualitative difference in the two sets of graphs. Legitimate income and standard of living rise more quickly for the college graduate and at the end of five years the difference between $\underline{\ell}$ and \underline{s} is greater, requiring slightly more criminal income when $\gamma_a = \gamma_s = 0$ (Figure 10A) and somewhat less criminal income when $\gamma_a = 0$ but $\gamma_s = 1$ (Figure 10B). When $\gamma_a = \gamma_s = 1$ (Figure 10C) all variable levels are greater than on the corresponding graphs of Figure 9C. The larger addiction level values are attributable to the larger value of $(\ell - s)$, which makes more legitimate resources available for drug purchases. The aversion to criminal activity does not fully offset the increased drug usage resulting from the greater legitimate income. Figure 10D (with $\gamma_a = 1$ and $\gamma_s = 0$) shows the addiction level, \underline{a} , reaching a higher level than on the corresponding graph of the less educated addict (Figure 9D), even though criminal earnings, \underline{c} , are greater on 10D than on 9D. This is explained by the selection criterion for ξ_a in the formula defining $\alpha < c >$ (equation (8)). The choice of ξ_a is made to depend on λ^* , and λ_0 is set equal to λ^* .

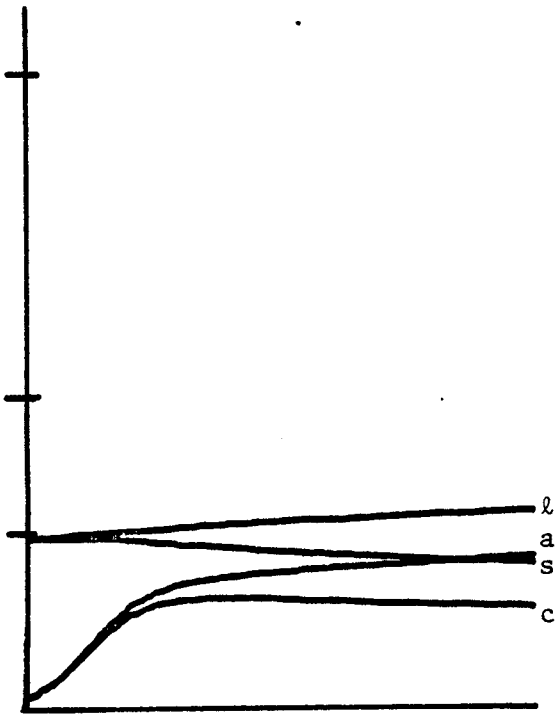
The college graduate who becomes an addict is more likely to be initiated to heroin when his legitimate income is above that of the grade-school drop-out. Therefore, Figures 11A through 11D showing initiation to narcotics late in the college graduate's career, are, perhaps, more pertinent. There is an upward transposition of the



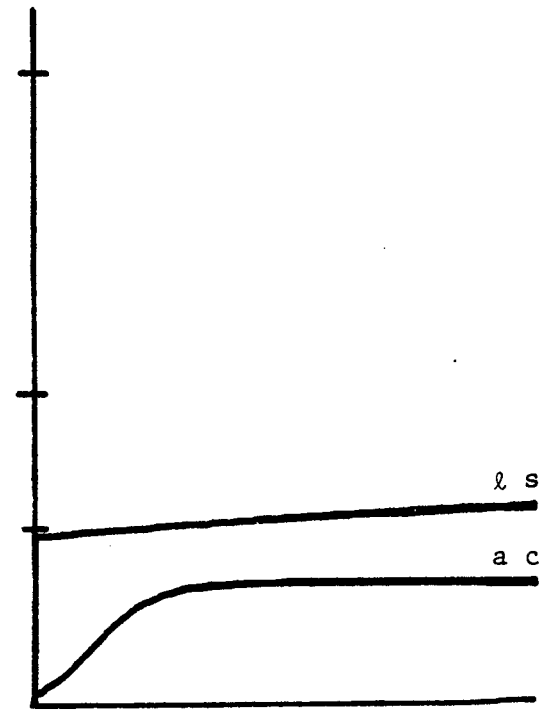
(A) $\gamma_a = 0, \gamma_s = 0$



(B) $\gamma_a = 0, \gamma_s = 1$



(C) $\gamma_a = 1, \gamma_s = 1$



(D) $\gamma_a = 1, \gamma_s = 0$

Figure 11. Large habit hustler, college graduate,
no resources, late initiation

standard-of-living and legitimate income curves and a resultant change in relative position of the variables in Figures 11C and 11D. In comparison with the corresponding graphs of Figure 10, the differences once again reflect primarily the magnitude of $(s - \ell)$. Note that persistence of standard of living ($\gamma_s = 0$) in Figure 11D results in nearly a 20 percent reduction in drug usage in contrast to that in Figure 11C. Figures 18A and 18B present ample reserve resources cases for the early- and late-starting college-graduate Large Habit Hustler whose personal choice parameters, γ_a and γ_s , are both equal to 1. In the absence of a criminal income requirement, the drug usage variable, \underline{a} , reaches the proclivity level, α_0 . There are consequent reductions in the levels of the variables $\underline{\ell}$ and \underline{s} .

Graphs for Large Habit Dealer

Since "Large Habit Dealer" is taken here to mean hard-core addict, the trajectories for the cases in which $\gamma_a = 0$ (no adjustment to risk-of-detection for criminal earnings) are most appropriate. However, for comparison with the graphs of the other addict types, all combinations of 0 and 1 for γ_a and γ_s will be examined. Figures 12 show the range of response for the early-starting grade-school drop-out. Legitimate earnings, $\underline{\ell}$, trails standard of living, \underline{s} , in Figure 12A when $\gamma_a = \gamma_s = 0$, as it did for the Large Habit Hustler (Figure 9A); but the greater drug usage of the Large Habit Dealer has reduced the achievement in legitimate employment by nearly 50 percent after five years, and in fact, has resulted in legal income below its

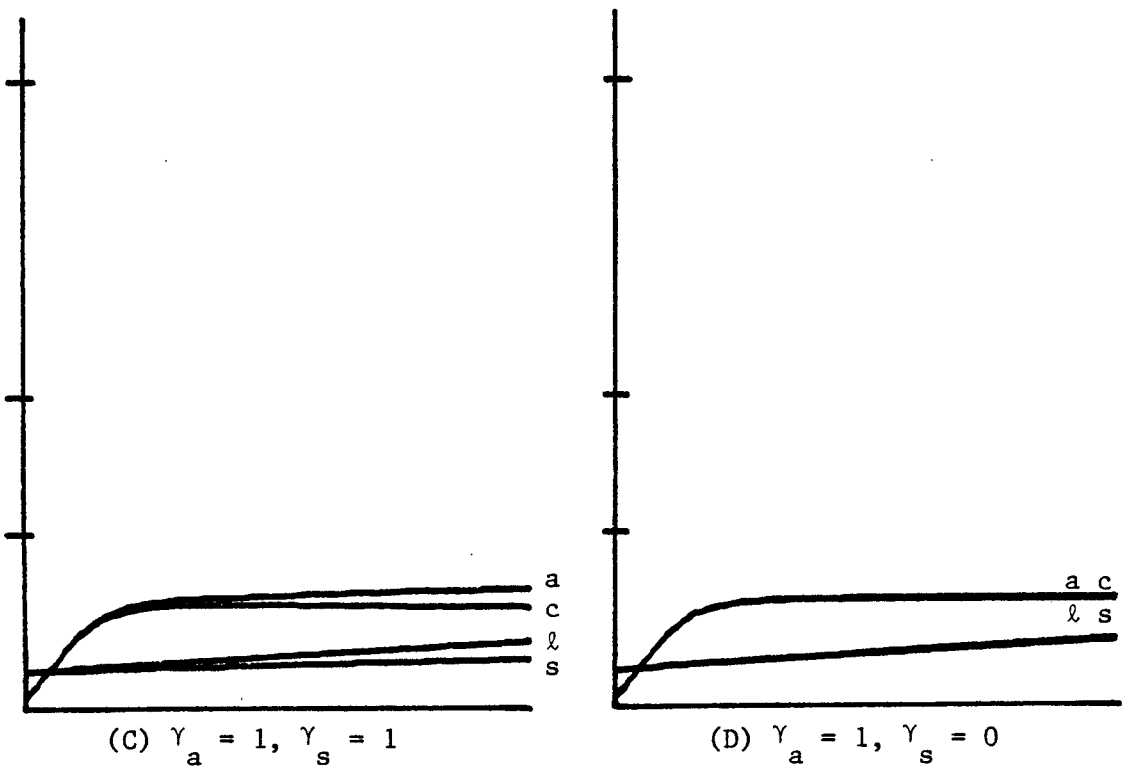
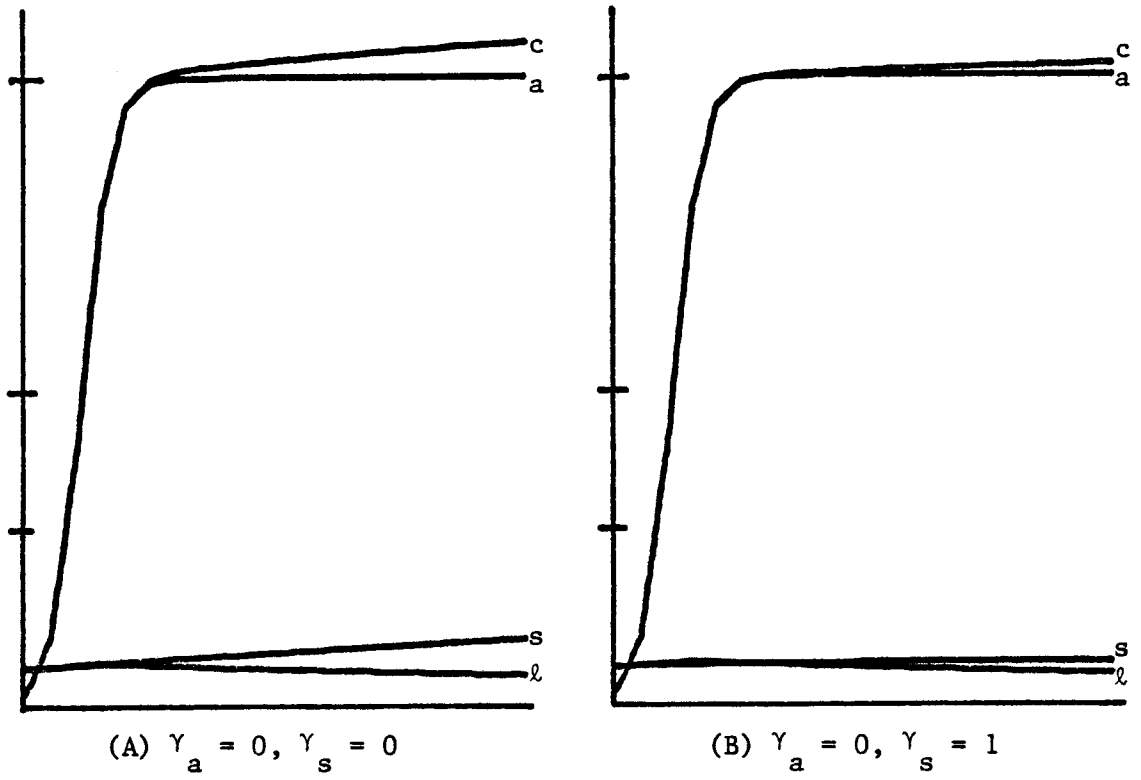


Figure 12. Large habit dealer, grade-school drop-out, no resources, early initiation

value at initiation to heroin. Similar reductions of legitimate earnings are noted in Figures 12C and D.

Graphs for the college-graduate Large Habit Dealer comparable to those of the Large Habit Hustler are presented in Figures 13 and 14 (early-starting and late-starting, respectively). With $\gamma_a = 0$ (Figures 13A and 13B, 14A and 14B), the magnitude of drug habit level, a , again suppresses advancement of legitimate income, ℓ , and, in contrast with the Large Habit Hustler, quickly causes a diminution of legal earnings. Figures 13C and 13D and Figures 14D and 14D show the expected responses when the criminal aversion mechanism is permitted to operate. The reduction in drug habit is dramatic; the legitimate earnings profiles are markedly improved over those produced when $\alpha_a = 0$.

Figures 18C and 18D give ample resources cases comparable to those displayed for the Large Habit Hustler. Drug usage reaches α_0 and legitimate income is consequently reduced.

Graphs for Medium Habit Hustler

If the Medium Habit Hustler is intended to represent the week-ender who is concerned about detection for criminal-earning activities and who maintains a desire for a high degree of conventional involvement, then the most appropriate values of γ_a and γ_s are, respectively, 1 and 0. Again, parameter settings and initial conditions are made comparable to those for the Large Habit Hustler and Large Habit Dealer. The various outputs of the model under the same sets of

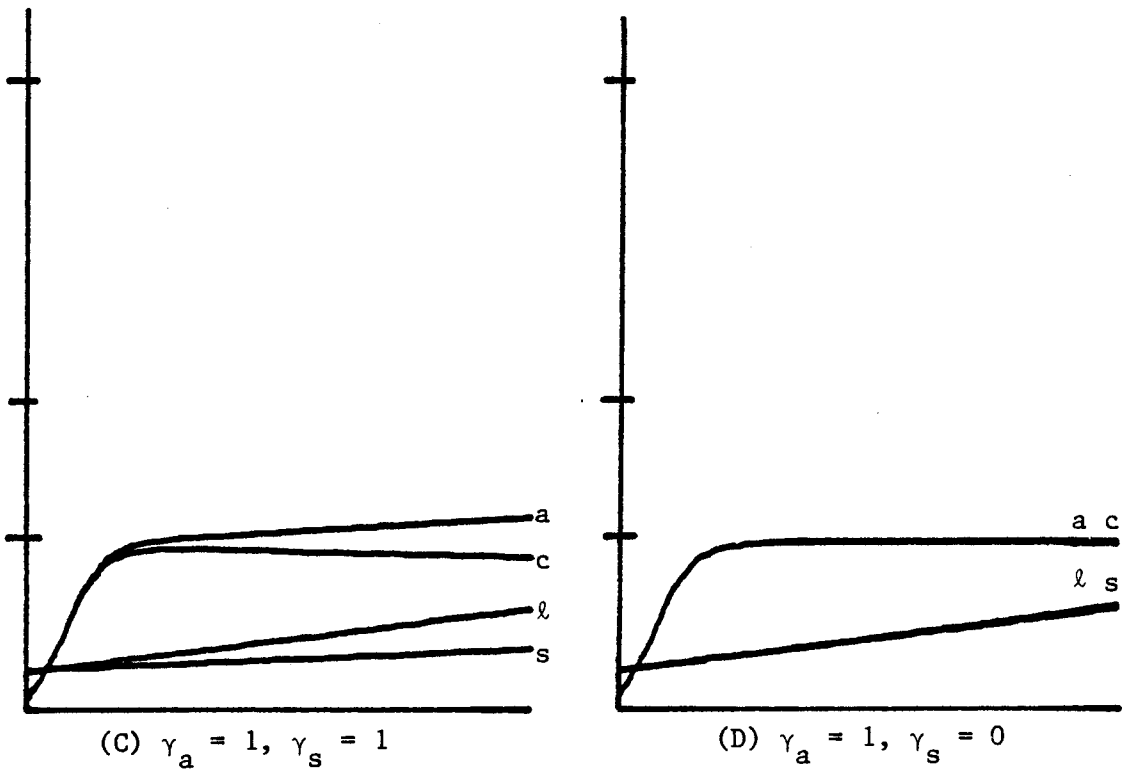
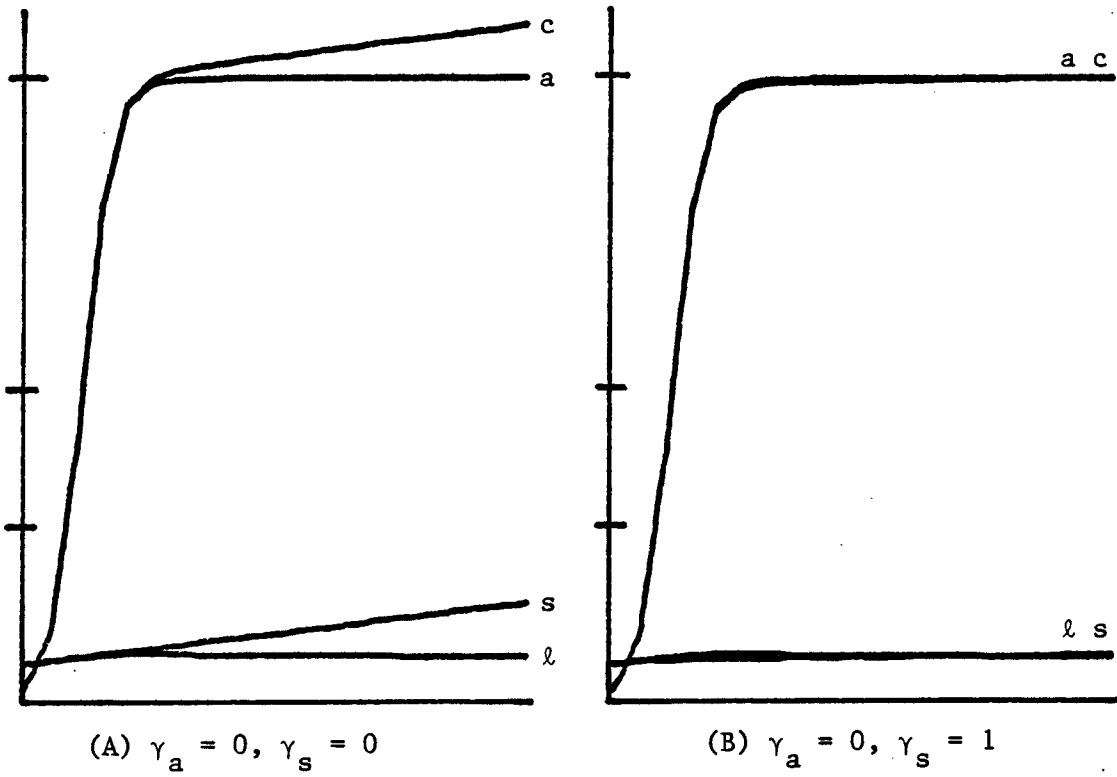
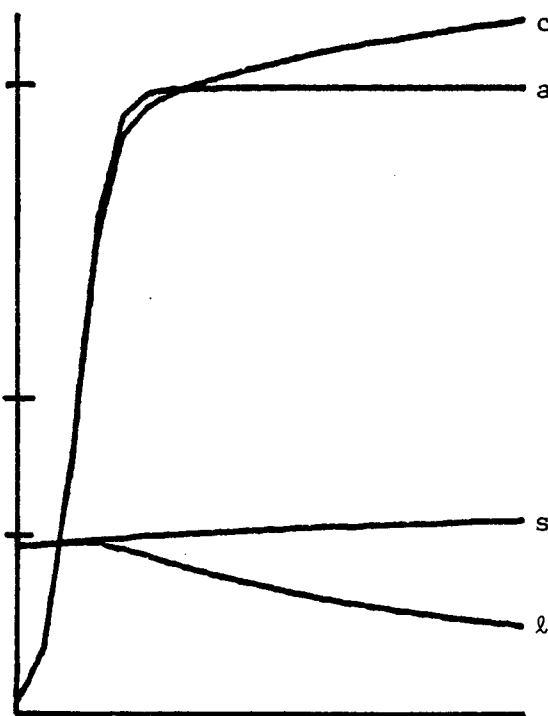
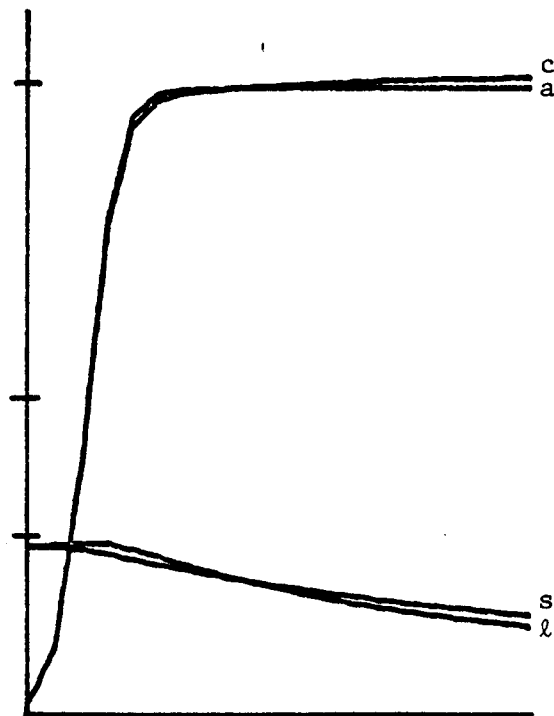


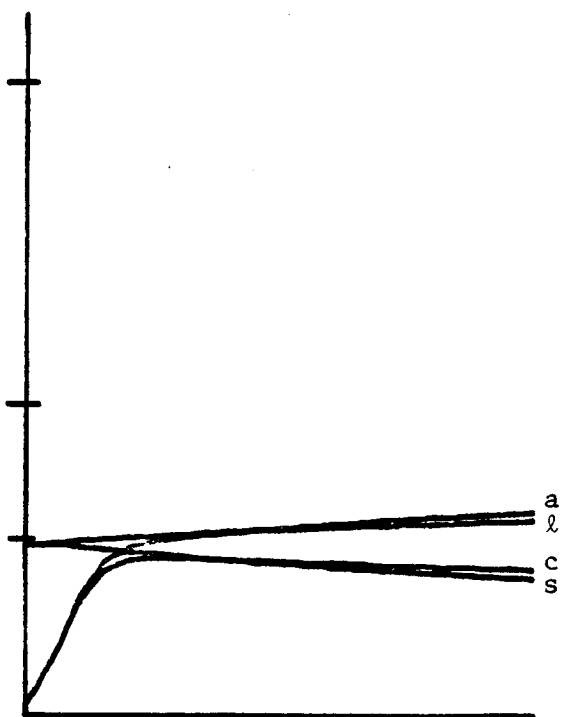
Figure 13. Large habit dealer, college graduate,
no resources, early initiation



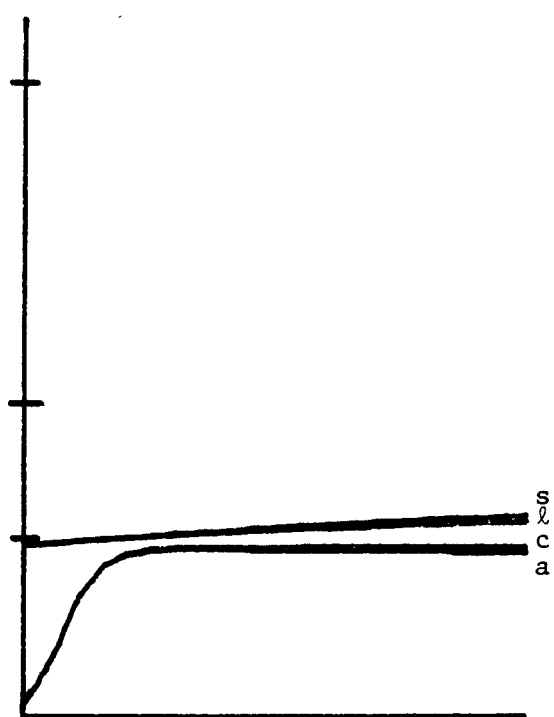
(A) $\gamma_a = 0, \gamma_s = 0$



(B) $\gamma_a = 0, \gamma_s = 1$



(C) $\gamma_a = 1, \gamma_s = 1$



(D) $\gamma_a = 1, \gamma_s = 0$

Figure 14. Large habit dealer, college graduate, no resources, late initiation

considerations are shown in Figures 15, 16, and 17. The graphs do not differ qualitatively from those for the Large Habit Hustler, but, as should be expected, drug usage levels are lower for the Medium Habit Hustler, and that, in turn, allows legitimate income to progress more rapidly.

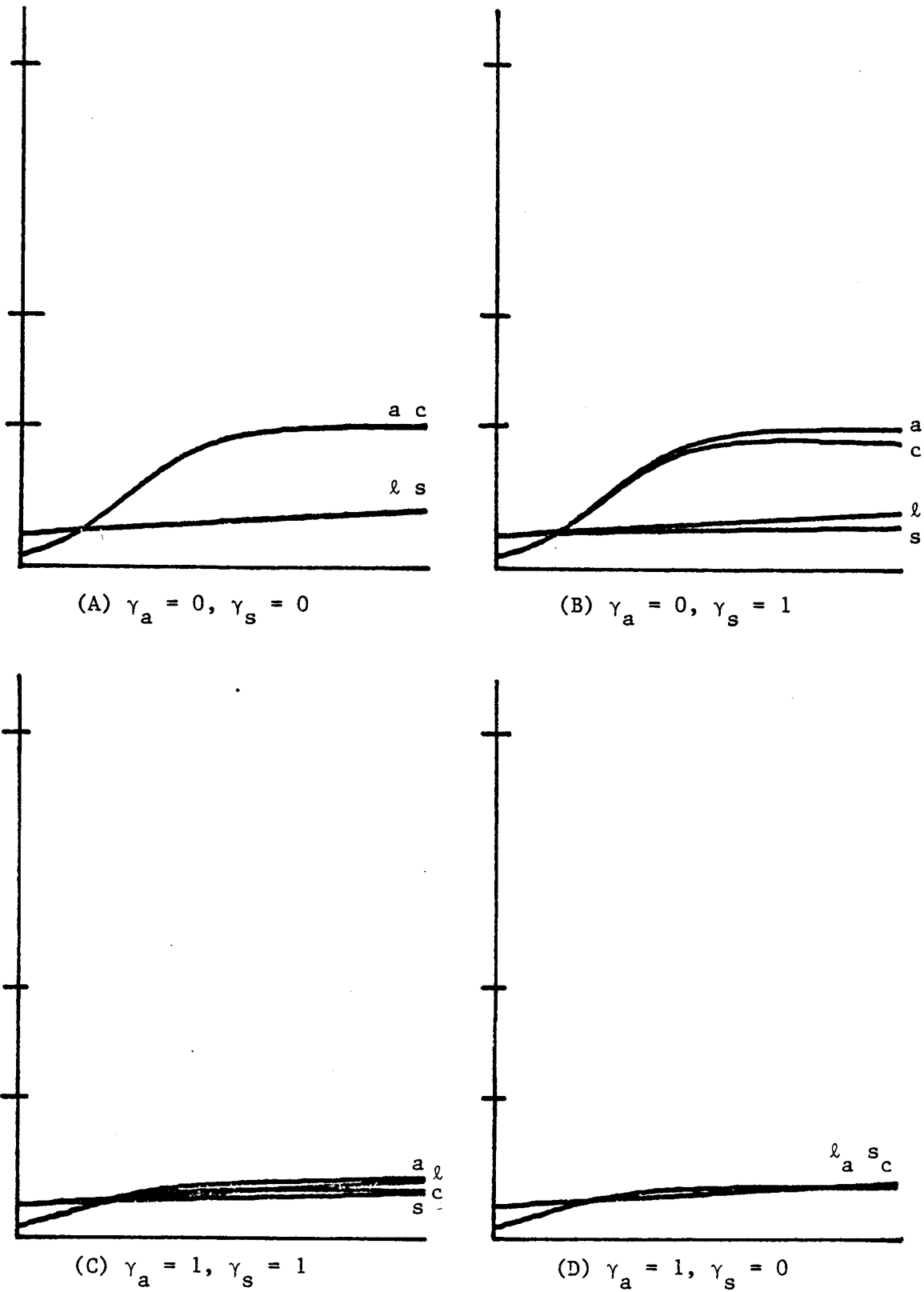


Figure 15. Medium habit hustler, grade-school drop-out, no resources, early initiation

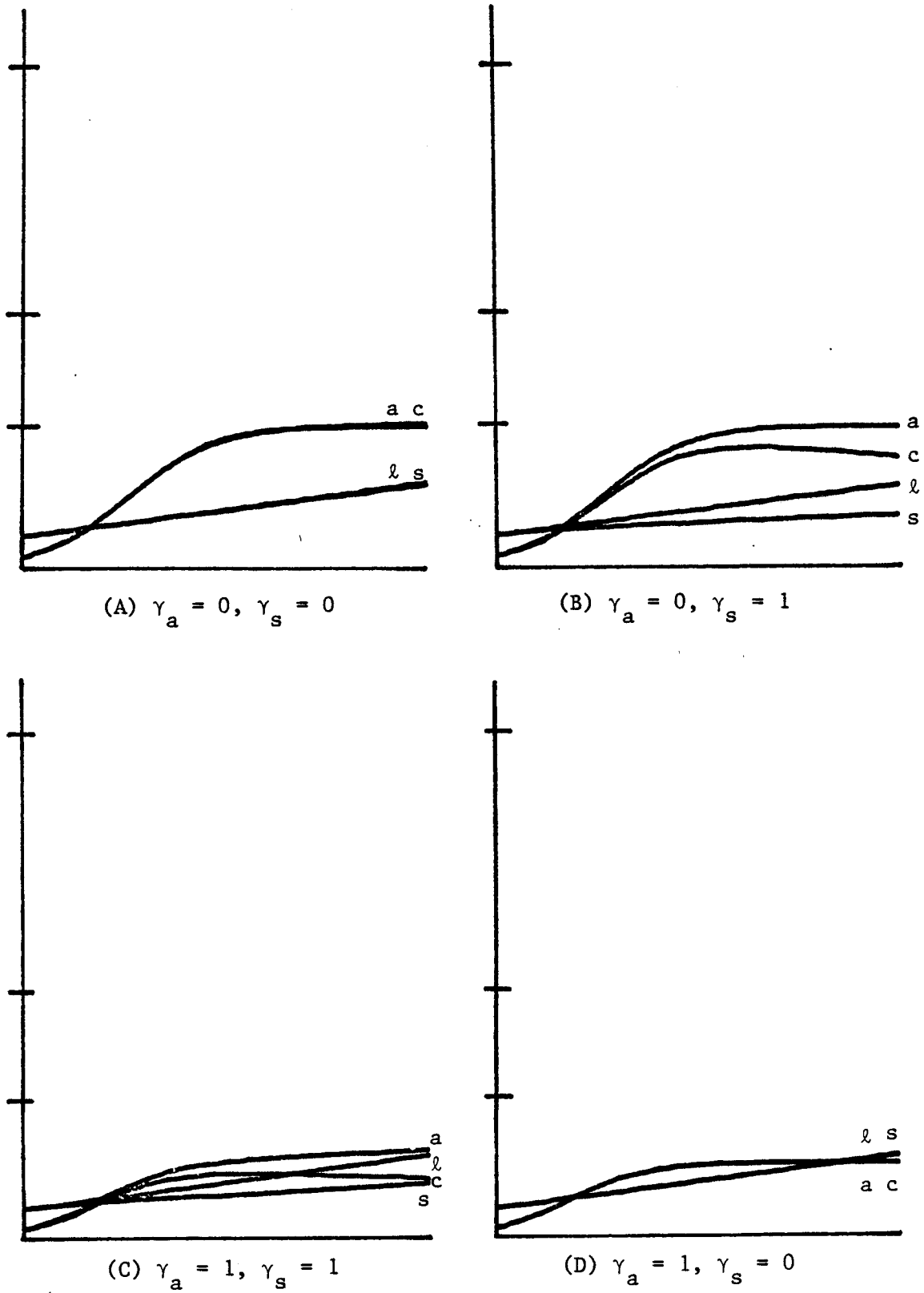
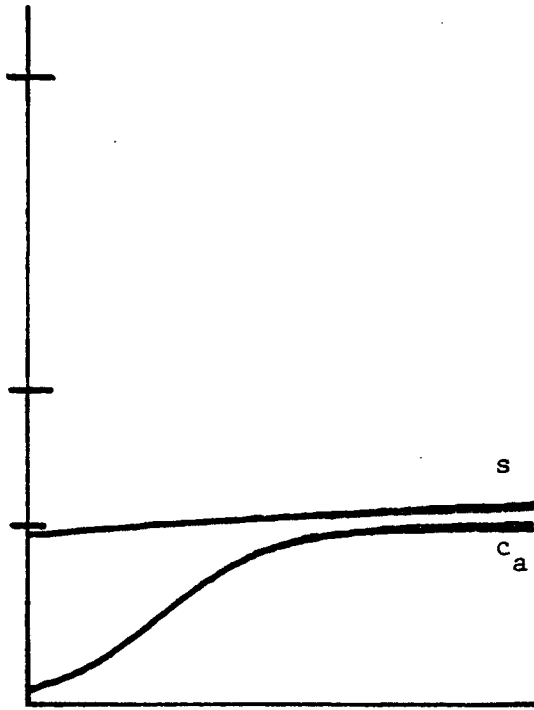
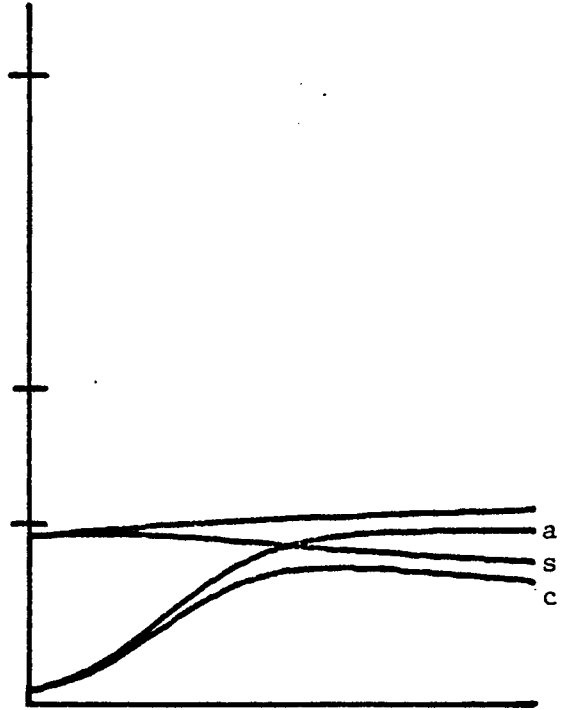


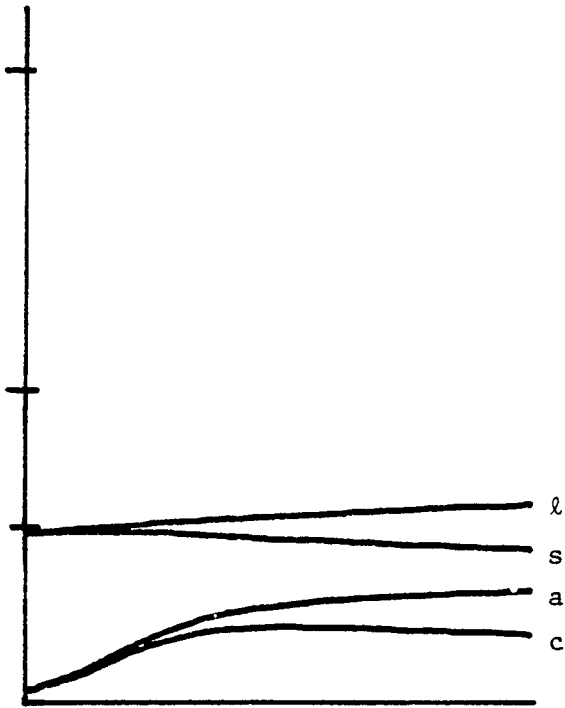
Figure 16. Medium habit hustler, college graduate,
no resources, early initiation



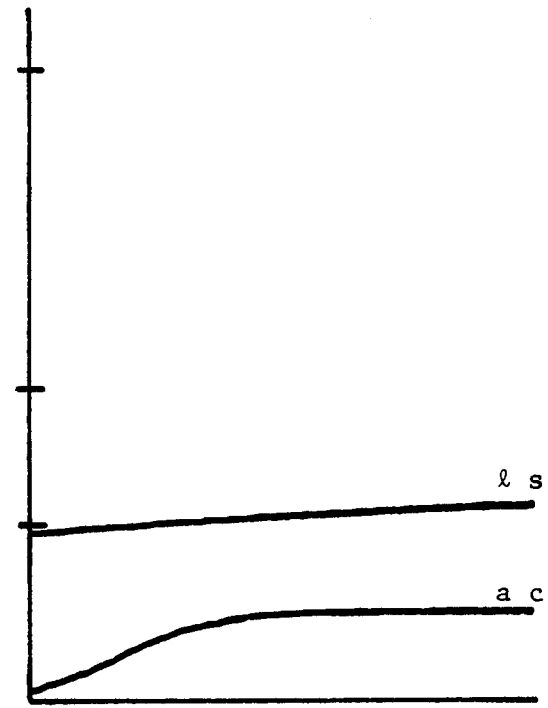
(A) $\gamma_a = 0, \gamma_s = 0$



(B) $\gamma_a = 0, \gamma_s = 1$

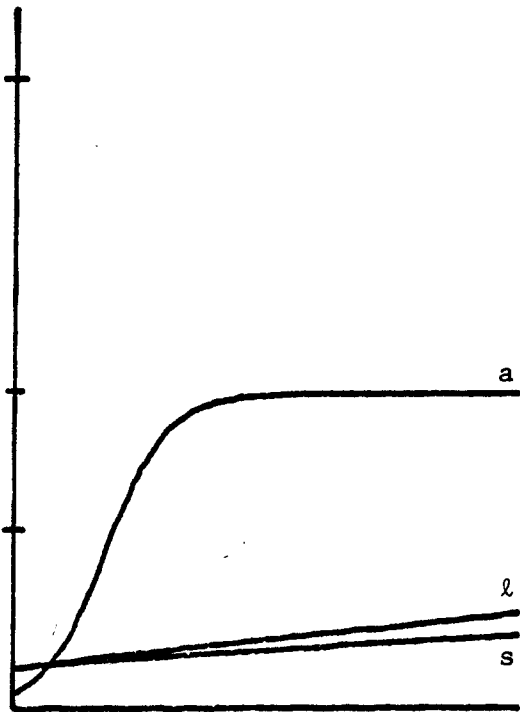


(C) $\gamma_a = 1, \gamma_s = 1$

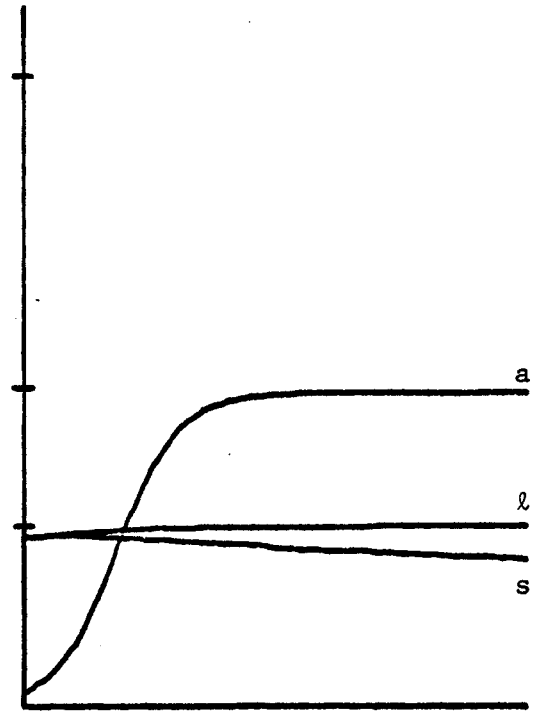


(D) $\gamma_a = 1, \gamma_s = 0$

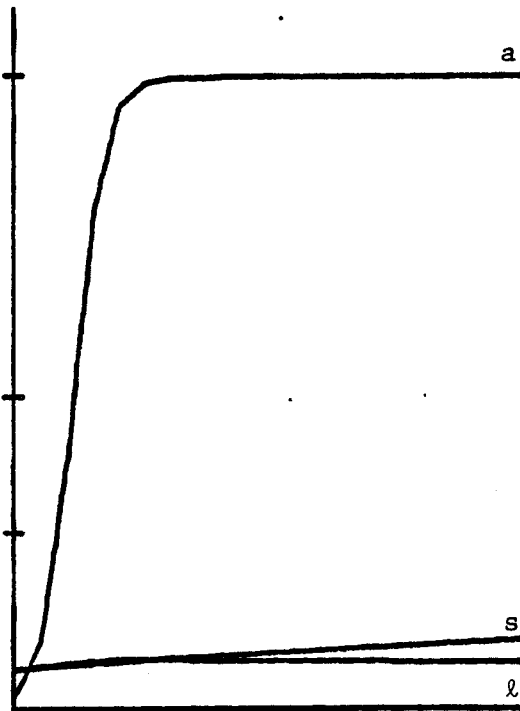
Figure 17. Medium habit hustler, college graduate, no resources, late initiation



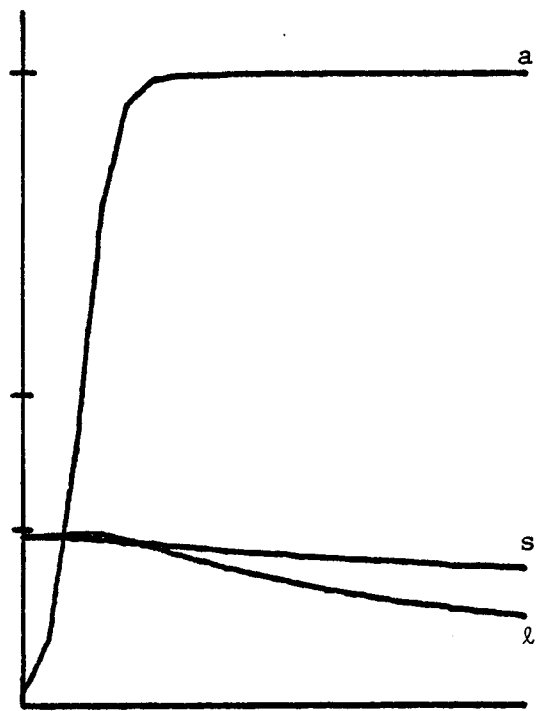
(A) Large Habit Hustler,
Early Initiation



(B) Large Habit Hustler,
Late Initiation



(C) Large Habit Dealer,
Early Initiation



(D) Large Habit Dealer,
Late Initiation

Figure 18. Ample resources, college graduate ($\gamma_a = \gamma_s = 1$)

DISCUSSION

Model Performance

Figures 9A through 18D reflect, of course, the substantive assumptions underlying the model's mathematical form and the particular choices of parameter values and initial conditions. While for some combinations of parameter values and initial conditions the graphs are not simply monotonic functions of time, all curves are quite "regular" in pattern and appear to be directly approaching equilibrium values; i.e., with proper allowance for the degree of resolution of and the mechanical "noise" in the plotting apparatus, the graphs show no pronounced changes in slope or evidence of oscillatory behavior. These features are associated with the deterministic character of the model and the relatively simplistic representation of the multiplicity of factors affecting an individual's behavior. More faithful representations would in some way take into account the turbulence in the life style of an addict related to risks and dangers encountered on the drug scene and the attendant psychological vacillations. If, nevertheless, one regards the model as an "on-the-average" or "median" representation for each of the several classes of addicts, the graphs suggest that the dominant forces determining the time-course of addiction and life style have been encompassed in a reasonably sound way and that the model is a solid stepping stone on which to base further efforts. Attention will be given some of the complicating factors as discussion proceeds. In

particular, some results about "stability" of the model and for the introduction of stochastic features are given in Appendices A and B.

Although the consequences of varying the parameters and initial conditions might not be completely intuitive from the model's structure, there were no particular surprises among the graphic outputs. Yet, certain qualities of the model become evident that are worthy of further comment. Most striking of these is the extent by which, in the absence of ample resources, aversion to criminal income-producing activities limits the level of drug usage attained, regardless of usage proclivity level (i.e., medium-, large-, or dealers' habit). Aversion to reduction of standard of living is a much weaker deterrent. In the absence of ample resources, the model yields somewhat higher drug usages for late starters than for early starters and for those with a college education than for those with only a primary education. In the context of the model, this property simply reflects the higher legitimate income levels. In any event, the magnitudes of effect are small compared to those produced by the criminal-earnings aversion. With ample resources, drug usage consistently reaches the proclivity level, regardless of other factors. The range of values to be observed for the drug usage variable is, of course, determined by the choice of α_0 .

Alternative Parameter Assignments

The parameters listed in Table 5A were maintained at the indicated (constant) levels throughout the preceding realizations. The

parameters in Table 5B take on more than one value, and the graphs can be thought to encompass the range of model response between the maximum and minimum values of those parameters. This section addresses the possibility that values for the parameters in Table 5A other than those shown may be more appropriate and discusses the consequences to model performance.

Since α^* , the physiological tolerance limit of addiction, is incorporated into the model as the drug usage level at which no productive work can be performed, a decrease in its value would cause the legitimate income goal, $\lambda < a >$, to peak at a lower level of drug usage and have a positive value for a shorter period of time, thereby necessitating faster depletion of reserve resources, R , or greater amounts of criminal activity--unless there is a strong aversion (i.e., $\gamma_a = 1$). A value of α^* greater than 700 would prolong the period of productivity and reduce levels of the variable c .

Large values of κ_a would cause the addiction level, a , to advance more quickly. Without a comparable improvement in the legitimate earnings profile, resources will be depleted more quickly or higher levels of illicit income and sacrifices on standard of living will be required. Again, the magnitude of effect is dependent upon the personal choice parameters γ_a and γ_s . With lower values of κ_a , the advancement of the variable a will be slower, and progress of legitimate income will more easily accommodate the drug costs.

The value of ξ_λ must be 0 when $\lambda_0 = \lambda^*$. When $0 < \lambda_0 < \lambda^*$, ξ_λ must satisfy the inequality immediately under equation (9) on page 19.

If ξ_ℓ equals the upper limit, then $\lambda < a >$ will attain the maximum, λ^* , for some value of a ; otherwise, the maximum will be less than λ^* . In the latter event, lower legitimate earnings will again strain financial resources.

The productivity parameter, π , enters the asymptotic value of legitimate income as λe^π , in the absence of drug effects. Therefore, a lower value of π will result in a lower equilibrium value of ℓ and lower lifetime earnings. Conversely, for larger values of π .

If σ^- , the subsistence level, were set at a lower value, greater sacrifice in standard of living would be possible when drug costs strain income. This would tend to reduce criminal income requirements in the absence of reserve resources when $\gamma_s > 0$. With higher values of σ^- , the options of the subject to adjust expenditures for standard of living are narrowed.

A zero value for ξ_s results in a constant value of $\sigma < a, m, c >$ at σ_0 . As ξ_s increases, $\sigma < a, m, c >$ falls to the subsistence level, σ^- , at faster rates for each non-zero value of γ_s , thus tending toward earlier sacrifices in standard of living and a reduction in pressure for criminal income requirements.

With ϵ less than 1, standard of living would not advance as rapidly as when $\epsilon = 1$, enabling the addict to better afford the cost of his drug habit.

Treatment implications of changing parameter values are considered further on pages 72-77.

Model Significance

The basic framework established by the foregoing mathematical model, rather than the specific nature of its outputs, constitutes its primary contribution to the analysis of addiction phenomena. Combined within a coherent perspective of community-wide scope are important attributes of the addict and his life style. Although many factors relating to his activities are expressed as parameters of constant value, the points at which other relevant characteristics (of the addict, his family and associates, treatment agencies with which he is involved, and of community policy managers) may be entered are rather clear. As already indicated, the manner in which further elaborations may be made is discussed under suggestions for further work.

Because its various features help focus attention on the components of the addiction system at several functional levels--individual background and actions, family considerations, treatment interactions, community policies and market features--the model represents an advance toward the objective of developing a significant tool for research and administration. The judicious selection of state variables to represent what are judged to be principal components of addict behavior and the accompanying units by which they are quantified has given some clarity to the complex mix of personalities, missions, and activities on the drug scene and suggests the possible utility of a more elaborate version. Expressing variables in terms of money flow rates is, as discussed earlier ("Perspective," pages 12, 13).

a convenient means of unifying behavioral attributes which are motivated by diverse mechanisms.

At the level of the individual, activities can essentially be divided into those which are regarded as constructive--legitimate employment and provision for standard-of-living needs--and those which are destructive of successful involvement with conventional society--drug usage and crime. It is therefore feasible, if the community's interest is not narrowly defined as the eradication of drug users, to chart an individual's progress in terms of increases in constructive activities and decreases in destructive activities.

If treatment intervention is the primary concern, a good model can highlight certain individual attributes, and due attention could be given the implications of specific variable and parameter values at the time of a treatment admission interview. Receiving attention might be maximum capacities expected in the various activities (λ^* , etc.), the subject's inclinations, in the absence of complicating factors, with regard to maximum activity levels (α_0 , etc.), the drive to achieve the proclivity levels as expressed by the rate parameters κ_a , etc., sensitivity to detection risk and financial strain (expressed by γ_a and γ_s), and the history of acquisition and depletion of monetary and technical resources since initiation to narcotic drugs. It may be found that some parameters already take values consistent with treatment objectives, in which case no systematic attempt to alter the addict characteristics to which they are related ought to be made. Instead, focus should be on modifying parameters

reflecting attributes antagonistic to treatment objectives. The next section contains a more detailed discussion of treatment implications.

For community administrators a good model offers a way of screening policy alternatives before implementation of control programs and a framework to aid in assessing policies and programs already implemented. More specifically, law enforcement strategies and the referral of addicts to particular treatment modalities could be designed with more assurance of near optimum effect. Behavioral expectations of the community's drug culture participants may be surmised from repeated applications of the model utilizing knowledge of parameter distributions among the identified addict population. From such an analysis, the necessity of certain community-sponsored provisions--for example, treatment facilities, educational and employment opportunities, recreational arenas, and the attraction of sundry goods and services--may be manifested.

While the model has been developed in the context of narcotics usage, the framework, including the mechanisms identified and the actual mathematical formulation, seem suitable in principle for addictive and deviant behavior generally. Thus, the model, especially with some modifications to be suggested, seems to have relevance to many social problem areas that are foci of psychological research or of societal concern.

Treatment Implications

While the mathematical model presented in the preceding pages is not--and was not intended to be--a precise decision-making tool

without further elaborations and refinements (to be discussed in the next section), the performance dynamics of the system defined by equations (1) through (13) suggest a number of addict characteristics toward which treatment methodologies might be directed and their expected consequences.

The addiction proclivity, α_0 , is an obvious focal point for intervention. If α_0 is reduced, then, even under the least favorable combination of other conditions, the drug usage level, \underline{a} , would equilibrate at a value close to α_0 ; and, with prudent manipulation of other parameters, the value of \underline{a} could be considerably less than α_0 . Changes of addiction proclivity would probably be best managed by some form of individual or group psychotherapy. Feelings of frustration, failure, hopelessness, or excessive expectations in the other activities, which may condition the addict's choice of life style, can be put in a different perspective and a new balance of activity priorities struck by this form of treatment. As comparable graphs for the three proclivity levels (for example, Figures 9A, 12A, and 15A) indicate, the drop in drug usage improves the subject's legitimate work performance and requires less criminal earnings for drugs.

The maximum expected legitimate earnings capacity, λ^* , is also subject to treatment intervention, with formal education and training in specific job skills as the primary tools. Although λ^* is not an element of the model, it does establish the range of values for λ_0 , the individual goal for legitimate income. An increase in λ^* may

alleviate feelings of hopelessness with regard to substantial achievement in a legitimate career--thus encourage larger values of λ_0 . It may be desirable in an extended model version to have success in employment result in a lowering of the drug addiction proclivity value. An increase in λ^* may also encourage an increase in aversion to criminal activity (i.e., have γ_a take a value closer to 1) because the subject has more to lose if arrested. But increases of legitimate income capacity and goal do not assure a reduction in criminal income-producing activity. If the subject's goal for standard of living, σ_0 , is close to or greater than λ_0 at the same time he seeks to maintain maximum progress for the s variable (i.e., γ_s at or close to 0), there can be a monetary shortfall, as Figures 9A and 10A illustrate.

Since improvements in legitimate earnings capacity and involvement in conventional living (represented by λ_0 and γ_s , respectively) may cause an increase in crime with no reduction in drug usage, a key element of the treatment strategy may aim at reducing the standard of living goal, σ_0 . This may utilize counseling and/or financial management advice to make the addict less susceptible to peer and family pressure for higher rates of consumption. While this version of the model removes the inhibitory mechanism on drug usage when there are ample resources, it may be appropriate in a more extensive model to reduce the addiction proclivity when reserves are being enhanced.

The graphs show most clearly the consequences of altering the personal choice parameters, γ_a and γ_s . Both are intervention-sensitive

and obvious targets in a treatment setting. Values of γ_a near 0 represent the addict's unwillingness to reduce drug usage significantly when criminal income is required to cover expenditures. When γ_a takes the value 1, drug usage is cut by 50 to 70 percent. Comparison of Figures 10C and 10D illustrates the interaction of γ_s with γ_a . In Figure 10C $\gamma_a = \gamma_s = 1$ and spending for standard of living is curtailed as drug costs increase. The consequent reduction in criminal income requirements causes drug usage, \underline{a} , to reach somewhat higher values than in 10D, where $\gamma_s = 0$ and $\gamma_a = 1$. In the latter case, lack of responsiveness to the financial burden causes standard of living to increase toward the maximum desired level. Legitimate income trails standard of living because of adverse drug effects on job performance. Higher levels of criminal activity result, which, because of the addict's sensitivity to risk of detection for criminal activity, brings a reduction in drug usages values. There is, then, a trade-off over the range of γ_s between lower addiction level and greater participation in crime. The most direct solution may involve reduction in standard of living goal, σ_0 . There are forms of aversion therapy which depict crime as repugnant while enhancing the subject's perception of the desirability of conventional living modes.

The legitimate income profile is affected, to a degree, by the value of the productivity parameter, π , which can be modified by some form of motivational counseling and indications that greater efforts at the place of employment will, with high probability, be suitably rewarded. Greater legal earnings will reduce the need for illegal

earnings, but may, thereby increase drug usage (if $\alpha_a > 0$). Once again a decision regarding the relative importance of areas of improvement would have to be specified before therapy is undertaken. The elasticity parameter, ϵ , may also respond to treatment efforts but, as with π , larger positive values of $(\ell - s)$ tend to reduce crime while increasing drug usage.

The rate parameters κ_a , κ_ℓ , and κ_s offer treatment personnel an especially effective way to control the primary earning and spending activities if intervention is applied early enough in the addict's career. For example, a very small positive value of κ_a could prevent the drug usage level from changing significantly over a number of years. Slowing the rate of increase of addiction in the face of a developing tolerance and psychological dependence, could represent a major breakthrough in getting the subject to take control of his life. Furthermore, a fairly stable addiction level would allow more attention to be given to the proper balancing of the other activities. For example, increasing the rate of progress in a legitimate career and slowing standard of living consumption could eliminate the necessity for criminal earnings and (particularly in a more elaborate version of the model) may enhance the contribution of other parameters to the reduction of drug use proclivity.

To review briefly the flexibility of the present model in achieving objectives of treatment, let us consider the steps which model performance indicates would help reduce drug usage over some time period of length t^* . If the variable \underline{a} is small, then establishing

κ_a closer to zero would result in less drug usage (integrated) over t^* . Reducing the addiction proclivity α_0 would likewise slow the progression of drug usage and would further cause \underline{a} to equilibrate at a lower value. Greater sensitivity to the risk of detection for criminal activity (i.e., increasing the value of γ_a) would cause $\alpha_{<c>}$ to shift downward more rapidly as criminal activity increases. Further reductions in drug usage are achieved if α_s is brought closer to 0 (i.e., an increased desire to maintain a maximal living standard) when γ_a is close to 1.

Suggestions for Further Work

There are a number of ways in which elaborations or refinements of the model presented in equations (1) through (13) can be made. If the foregoing framework is to have justifiable application as an objective decision-making or research tool, then attributes of the mathematical system must be based on appropriate data from carefully designed experiments rather than on an assembly of observations and impressions, as is necessary at this stage of its development. In particular, this means setting, by proper selection of test subjects or conditions for their observation, those variables which are deemed relevant to the study of addiction behavior at several levels over a plausible range of values and controlling those factors which are recognized as impinging upon system performance but are not given explicit consideration in the model formulation. All other factors are assumed to be randomly distributed in the experimental work.

Characteristics of the model framework which can profitably be altered are herewith discussed.

The most direct data requirements are with respect to confirmation of specific features of the model in its present form. For example, is the logistic curve most appropriate for progression of drug usage and the Gompertz for legitimate income and standard of living? Over a broad range of addict types, what is the time-frame for true addiction as opposed to experimental or occasional use? What precisely is the nature of the elasticities between standard of living and legitimate income or between drug habit level and criminal earnings? There is also the problem of operationalizing the several limiting values of the model (α^* , α_0 , λ^* , λ_0 , etc.) as well as choosing more precise values for the graph-shaping parameters ξ_a , ξ_l , and ξ_s .

From a mathematical point of view, it is desirable to go beyond the general considerations of system stability which are presented in Appendix A and obtain the particular range of values for the various parameters and initial conditions of the state variables which assure model equilibration. Because the state equations themselves are rough approximations for an underlying reality, application of more sophisticated mathematics to the analysis of model stability would not be especially informative. As the model is developed further, such mathematical elaborations become more appropriate.

On the other hand, the restrictive assumptions imposed upon this conceptualization, either initially or by the specifics of model

development, could be relaxed. For example, drugs other than narcotics--not as addictive or laden with risk of arrest--might be incorporated into the model framework. In that case vector quantities would necessarily be formulated for which the respective costs and relative substitutability of the different drugs would have to be specified. Fluctuation of drug prices could be allowed.

Likewise, the parameters γ_a , ξ_a , γ_s , and ξ_s , which regulate the aversion reactions to criminal earnings and to reduction in standard of living, as well as the proclivity parameter, α_0 , are regarded in the model to be fixed characteristics of an individual. A more realistic formulation should possibly allow a weakening of the aversions to criminal earnings and to reduced standard of living as level of drug usage increases--particularly as cumulative drug consumption increases; i.e., γ_a and ξ_a should be decreasing functions of drug consumption and γ_s and ξ_s should be increasing functions of drug consumption. Furthermore, the proclivity parameter should possibly be an increasing function of drug consumption.

In a similar fashion, an addict who, as a result of treatment, has ceased to use drugs, may not have the same tendencies as if he had never been an addict. If, through some circumstance, he again starts to use drugs, his usage may well increase at a rate faster than before. Also, as the time since usage ceased increases, the aversions to criminal earnings and to reduced standard of living may become stronger and his usage proclivity may decrease. To take effects of matters such as cumulative drug consumption, treatment,

readdiction, etc. properly into account would require augmentation of the model in the direction, for example, of regarding the aversion and proclivity parameters to be additional state variables and expressing their rates of change with appropriate differential equations.

The requirement that criminal activities be engaged in only as a response to the shortfall of legitimate earnings and other resources could be abandoned in favor of a new "deviant activity" variable which would be independent of financial considerations. Likewise, a mechanism for preserving resources even when legitimate income is inadequate could be included.

Such considerations, not to mention other factors, are important in the context of agency treatment of individual addicts and of societal programs for addiction management.

As indicated under Model Performance, there are many unsettling influences on the drug scene which make precise prediction of an addict's behavior pattern virtually impossible. There are fluctuations in drug prices, the risk of arrest for obtaining income illegally to purchase drugs, deterioration of health, changes of mood and motivation, differences in drug purity, vengeance of a heroin distributor or buyer, among others. One way to incorporate these variations in addict life style conditions is by stochasticization of the state equations. Examples of this kind of model alteration are given in Appendix B.

At still another level of complexity, the numerous forces-- discussed earlier, but not specifically accounted for by the model--

which influence the rate parameters and the goal parameters could be given explicit expression. With the introduction of several new model features, it would be advisable to perform "sensitivity analyses" to determine the extent to which each parameter of the system influences the respective variables' time trajectories. If there is little change in output over the entire potential range of a parameter's values, then it may be appropriate to simplify the model by eliminating that parameter or by consolidating and restructuring a number of such minor factors.

Receiving consideration as new variables of the mathematical system would undoubtedly be those relating to family pressures, peer pressures, drug availability, relaxational activities, effects of continuing education, and rates of achievement in the activities already incorporated into the model. Data needs are obviously great for this kind of model extension, and careful introduction of these new elements is requisite. It may also be desirable to formulate mechanisms for activities which are predecessors to or successors of those encompassed by the model. Particular reference is made to the process of initiation to use of narcotic drugs and, at the other end of the addiction cycle, the process of "maturing out" of the drug scene--a phenomenon described by Winick [33].

Finally, accommodation of postulated treatment effects will be necessary to give the model utility as a decision-making tool for intervention analysis.

Treatment often results in the disruption of an addict's life style. Imprisonment and hospitalization are two dramatic examples, in which cases the subject is removed from his usual environment and placed under agency supervision. Other forms of treatment--with their rap sessions, urinalyses, dispensaries, interviews, etc.--impose new demands on the addict's time and attention. There is a qualitative difference in daily regimen which comparative studies of pre- and post-treatment behavior sometimes ignore. In our model such qualitative changes are equivalent to jump discontinuities of the state variables. If the subject is withdrawn from drugs, the drug usage variable, \underline{a} , is reset to zero; if close monitoring makes criminal activity impossible, then \underline{c} becomes zero; and so forth.

Some of the components of the goal parameters $\alpha\langle c \rangle$, $\lambda\langle a \rangle$, and $\sigma\langle a, m, c \rangle$ could be altered in the following ways: Chemical agents, such as cyclazocine and naloxone, can block or make repellent the effects of narcotics ingestion. This type of intervention could result in a lower value for α^* , the heroin tolerance limit. On the other hand, people with low achievement potential (reflected by λ^*) can have the λ^* value enhanced by agency encouragement or offerings of further education and job training. Individual counseling or group therapy sessions may determine some of the psychological driving forces which contribute to the personal goals/proclivities α_0 , λ_0 , and σ_0 . Here the roles of anxiety, peer pressure, and satisfaction motivators in addictive behavior would have to be elucidated. Treatment can influence the personal choice/sensitivity parameters

(γ_a and γ_s) by having the subject become more concerned about the risk of detection in criminal activities or to help him become more determined to maintain a high level of conventional involvement, as indicated by a high standard of living. Chemotherapy (e.g. methadone in regulated dosage and at low cost), could allow progress in a legitimate career to proceed and could alleviate the need for high-priced, black market drug sources. The expanded model might also prescribe how the goal parameters would be re-established below the prevailing levels of the state variables for drug use, legitimate income, and standard of living in response to treatment supervision, as discussed on pages 38 and 39.

One may structure the equations of interaction between treatment and subject by close examination of the patterns of economic behavior and background characteristics among addicts who were treated successfully under certain regimes. This could be followed by an analysis of common features of treatment methodology. If intuition can be a guide, one should expect--as with the influence of peers on drug-taking and career goals, etc.--that the intimacy, form, and duration of the interactions between agency and addict require examination. The variables of treatment most likely to receive consideration are length of commitment to the program, time per week with treatment personnel (the depth and diversity of the involvement), time between contacts with the treatment agency, provisions for training clients in special skills, level of chemotherapeutic support, time for incarceration, and flexibility of the regimen to individual needs.

Study of the post-treatment contact between the agency and important elements of the subject's environment (such as his family) should indicate the degree to which constructive changes of the system's parameters can be reinforced by the agency and maintained by the subject.

If the model incorporates a sufficient number of relevant variables and the validity of assumptions imposed by the mathematical formulation can be assured, then decision-makers charged with the care and management of addicts will have a useful product to guide their course.

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APPENDICES

Appendix A. Stability Analysis

It would be desirable to find an explicit mathematical expression (as a function of time) for each of the state variables in the system of differential equations constituting the addiction model. When the system is linear, it is usually possible to write the closed-form solution and chart the variables over time in the corresponding state space. If system stability, rather than explicit solution, is of primary interest, the procedures are, likewise, straightforward.

Comparable, and generally applicable, procedures have not been established for the much larger class of non-linear differential equation systems, of which the model presented here is a member. Unless the system is of a special type, other methods of examination are required. A common alternative is the approximation, under suitable conditions, of the non-linear equations by linear ones in the neighborhood of isolated singular points of the original system. Although the paths of the variables over time become distorted by this process, the stability properties of both systems at the singular points are the same. This method determines the points in the state space at which the differential system may equilibrate.

Four of the seven variables in the model-- m , R^* , R , and c --serve essentially a bookkeeping role; the analysis of singular points is, therefore, restricted to the remaining variables-- a , l , and s .

The system of equations is specified by

$$\begin{aligned}\dot{a} &= Q_1(a, \ell, s) = \kappa_a a \left[\frac{\alpha_0}{1 + \xi_a \gamma_a (a+s-\ell)^2} - a \right] \\ \dot{\ell} &= Q_2(a, \ell, s) = \kappa_\ell \ell \left[\log_e \left(\lambda_0 \left[1 - \frac{a(a-\xi_\ell)}{\alpha^*(\alpha^*-\xi_\ell)} \right] \right) - \log_e(\ell) \right] + \pi \ell \\ \dot{s} &= Q_3(a, \ell, s) = \kappa_s s \left[\log_e \left(\frac{\sigma_0^{(m-c)} + \sigma_s^- \gamma_s \xi_s a}{m-c + \gamma_s \xi_s a} \right) - \log_e(s) \right] + \epsilon \pi \ell.\end{aligned}\tag{A1}$$

The approximating linear differential system would then be given as the truncated Taylor's expansion

$$\begin{aligned}\dot{a} &= B_1(a - a_1) + C_1(\ell - \ell_1) + D_1(s - s_1) \\ \dot{\ell} &= B_2(a - a_1) + C_2(\ell - \ell_1) + D_2(s - s_1) \\ \dot{s} &= B_3(a - a_1) + C_3(\ell - \ell_1) + D_3(s - s_1)\end{aligned}\tag{A2}$$

where (a_1, ℓ_1, s_1) is a singular point of the system (A1), derived by simultaneously solving $Q_1 = Q_2 = Q_3 = 0$, and

$$B_1 = \frac{\partial Q_1}{\partial a} \Big|_{(a_1, l_1, s_1)} = -2\kappa_a \left\{ a_1 - \frac{\alpha_0}{2} \frac{1 + \xi_a \gamma_a (a_1 + s_1 - l_1) (s_1 - l_1 - a_1)}{[1 + \xi_a \gamma_a (a_1 + s_1 - l_1)]^2} \right\}$$

$$C_1 = \frac{\partial Q_1}{\partial l} \Big|_{(a_1, l_1, s_1)} = \frac{2\kappa_a \alpha_0 \xi_a \gamma_a a_1 (a_1 + s_1 - l_1)}{[1 + \xi_a \gamma_a (a_1 + s_1 - l_1)]^2}$$

$$D_1 = \frac{\partial Q_1}{\partial s} \Big|_{(a_1, l_1, s_1)} = -C_1$$

$$B_2 = \frac{\partial Q_2}{\partial a} \Big|_{(a_1, l_1, s_1)} = \frac{-\kappa_l l (2a_1 - \xi_l)}{\alpha^* (\alpha^* - \xi_l) + a_1 (a_1 - \xi_l)}$$

$$C_2 = \frac{\partial Q_2}{\partial l} \Big|_{(a_1, l_1, s_1)} = -\kappa_l \left\{ 1 + \log_e \left(\frac{l_1 \alpha^* (\alpha^* - \xi_l)}{\lambda_0 [\alpha^* (\alpha^* - \xi_l) - a_1 (a_1 - \xi_l)]} \right) \right\}$$

$$D_2 = \frac{\partial Q_2}{\partial s} \Big|_{(a_1, l_1, s_1)} = 0$$

$$B_3 = \frac{\partial Q_3}{\partial a} \Big|_{(a_1, l_1, s_1)} = - \frac{\kappa_s \gamma_s \xi_s (\sigma_0 - \sigma^-) l_1 s_1}{(\sigma_0 l_1 + \sigma^- \gamma_s \xi_s a_1) (l_1 + \gamma_s \xi_s a_1)}$$

$$C_3 = \frac{\partial Q_3}{\partial l} \Big|_{(a_1, l_1, s_1)} = \frac{\kappa_s \gamma_s \xi_s (\sigma_0 - \sigma^-) a_1 s_1}{(\sigma_0 l_1 + \sigma^- \gamma_s \xi_s a_1) (l_1 + \gamma_s \xi_s a_1)} + \epsilon \pi$$

$$D_3 = \frac{\partial Q_3}{\partial s} \Big|_{(a_1, l_1, s_1)} = -\kappa_s \left[1 + \log_e \left(\frac{s_1 l_1 + \gamma_s \xi_s a_1 s_1}{\sigma_0 l_1 + \sigma^- \gamma_s \xi_s a_1} \right) \right]$$

By letting $X = \begin{pmatrix} a - a_1 \\ \ell - \ell_1 \\ s - s_1 \end{pmatrix}$ and $A = \begin{pmatrix} B_1 & C_1 & D_1 \\ B_2 & C_2 & D_2 \\ B_3 & C_3 & D_3 \end{pmatrix}$, the equations

in (A2) can be written as $\frac{dX}{dt} = AX$. When all solutions for δ in the equation

$$\det(A - \delta I) = 0 \tag{A3}$$

are negative real numbers or are complex numbers with negative real parts, the singular point will be stable. It could then be concluded that if the variable coordinate values, (a, ℓ, s) , are close enough to those of the singular point, (a_1, ℓ_1, s_1) , the system will settle down (equilibrate) at that point.

Because of the way \underline{x} is defined in model equations (10) and (12), it is necessary to consider the cases $\underline{c} = 0$ and $\underline{c} > 0$ separately.

If $\underline{c} = 0$, the possible set of singular points are described in Appendix Table 1.

If no resources are available, the criminal earnings, \underline{c} , may be greater than zero. In that case Appendix Table 2 describes the possible singular points of the system.

From inspection of the elements B_1, C_1 , etc., of the coefficient matrix A and the items listed in Appendix Tables 1 and 2 as singular point alternatives, methods for explicit solution for the δ of the determinant equation are clearly intractable in all but the most trivial and unrealistic cases.

Appendix Table 1. Singular points for ample resources case ($c = 0$)

	$a_1 = 0$	or	$a_1 = \alpha_0$
$\lambda_1 = \lambda_0 e^{\pi/\kappa_\ell} \left[1 - \frac{a_1(a_1 - \xi_\ell)}{\alpha^*(\alpha^* - \xi_\ell)} \right]$	$s_1 = 0$ or $s_1 = \sigma_0$		$s_1 = 0$ or $s_1 = \frac{1}{2} \{ \sigma_0 - \alpha_0 - \gamma_s \xi_s \alpha_0 + \sqrt{(\alpha_0 + \gamma_s \xi_s \alpha_0 - \sigma_0)^2 + 4\alpha_0(\sigma_0 + \gamma_s \xi_s \alpha_0)} \}$
	s_1 such that $s_1 [\log_e(s_1) - \log_e(\sigma_0)]$ $= \frac{\epsilon \pi \lambda_0}{\kappa_s} e^{\pi/\kappa_\ell}$		

Appendix Table 2. Singular points for no resources case ($c > 0$)

	$a_1 = 0$	or	$a_1 = \frac{\alpha_0}{1 + \xi_a \gamma_a (a_1 + s_1 - \ell_1)^2}$
$\ell_1 = 0$	$s_1 = 0$		$s_1 = 0$ or $s_1 = \sigma^-$
or	s_1 such that $s_1 [\log_e(s_1) - \log_e(\sigma_0)]$ $= \frac{\lambda_0 \epsilon \pi}{\kappa_s} e^{\pi/\kappa_\ell}$		a_1, ℓ_1, s_1 such that $a_1 = \frac{\alpha_0}{1 + \xi_a \gamma_a (a_1 + s_1 - \ell_1)^2}$ $\ell_1 = \lambda_0 e^{\pi/\kappa_\ell} \left[1 - \frac{a_1 (a_1 - \xi_\ell)}{\alpha^* (\alpha^* - \xi_\ell)} \right]$ $= \frac{\kappa_s}{\epsilon \pi} s_1 [\log_e(s_1) - \log_e(\frac{\sigma_0 \ell_1 + \sigma^- \gamma_s \xi_s a_1}{\ell_1 + \gamma_s \xi_s a_1})]$
$\ell_1 = \lambda_0 e^{\pi/\kappa_\ell} \left[1 - \frac{a_1 (a_1 - \xi_\ell)}{\alpha^* (\alpha^* - \xi_\ell)} \right]$			

For a particular system of equations, in which the parameter values have been assigned, numerical methods can be employed, in many instances, to solve for the singular point coordinates given in Appendix Tables 1 and 2 in terms of unspecified parameter values. Further computations may be required to solve for δ in equation (A3) and thereby determine the system's stability properties.

A danger in using implicit functions with numerical analysis (particularly in conjunction with computer processing) is the magnification of errors due to decimal truncations, choices of initial values, and so forth. Negative values of δ may be due to such procedural errors rather than being truly indicative of system stability. Usually, however, the degree of error is specified along with the methodology.

On a far less sophisticated level, one may take the alternative course of gauging the stability of the system for a particular configuration of parameter values and initial conditions by observing the trajectories of the variables over a long time period. Examination of the graphs in Figures 9 through 18 suggests that the system does, indeed, stabilize at nontrivial values of the variables for the given parameter levels and initial conditions chosen. Appendix Figure 1 which simply presents extensions, through 60 years, of the indicated (five-year) graphs, appears to confirm the impression of stability for graphs which showed the greatest potential for instability in the original figures.

Appendix B. Stochasticized Model

Recalling that equation (1) of the deterministic system was written as $\dot{a} = \kappa_a a[\alpha\langle c \rangle - a]$, where $\alpha\langle c \rangle = \alpha_0 / (1 + \xi_a \gamma_a c^2)$, there are a number of ways to make that equation, and the corresponding equations pertaining to changes of \underline{l} and \underline{s} , stochastic. Clearly, any of the parameters or variables in equation (1) could have a random element associated with it by multiplication or addition. For example, rationale can be provided for assigning random variations to α_0 and γ_a in the expression for drug usage "goal," $\alpha\langle c \rangle$. While there may be some operational measures for drug usage proclivity and sensitivity to risk of arrest, such measures could only incorporate a finite number--and probably a very small number, at that--of factors upon which to base the value of either α_0 or γ_a . Then the subject's attributes not accounted for by the measures can be considered randomly distributed. The variable \underline{a} , through changes in \dot{a} , could fluctuate as drug prices and purity affect the actual quantity of heroin consumed.

For purposes of illustration, random elements will be assigned to the rate constants κ_a , κ_l , and κ_s and will be added to the right-hand sides of equations (1), (2), and (3). Thus, the stochastic version of equation (1) would appear as $\dot{a} = \kappa_a(\epsilon_1) a[\alpha\langle c \rangle - a] + \delta_1$, with comparable equations for \dot{l} and \dot{s} . To highlight the resulting changes in time-course behavior of the graphs, the effects of ϵ_1 and δ_1 will be examined separately.

The δ_1 is formulated as an approximately normal term by averaging 20 uniformly distributed random numbers generated by computer. Since the purpose of this section is to illustrate, rather than to analyze, a stochastic system, the range of the repeatedly-called random variable was arbitrarily set (to avoid negative arguments of the logarithms in equations (2) and (3)). If $\delta_1 = \frac{\sum_{i=1}^{20} \delta^{(1i)}}{20}$, then each $\delta^{(1i)}$ was designed to equal $.8(\text{"random"}-.5)\min(a, \alpha_0 - a)$ where "random" is the computer-generated random variable uniformly distributed over the interval $[0,1]$.

The particular example chosen to demonstrate the stochasticization process is that of the addict with a Dealer's Habit and no resources, a college education, and sensitivity to both risk of arrest for criminal activity and the financial burden of drug usage. This is represented by Figure 13C, which is reproduced here. This case was chosen because of its maximum potential range of drug usage levels, its more rapid rate of legitimate income and standard-of-living advancement, and its maximum suppression (by having $\gamma_a = \gamma_s = 1$) of the variables \underline{a} , $\underline{\ell}$, and \underline{s} . Appendix Figure 2 shows the consequence of adding δ_1 to equation (1) at each iteration of the numerical procedure described earlier (page 41). As was the case with the deterministic graphs, printout and plotting values were generated at thirteen-week intervals and there were three repetitions of the Runge-Kutta routine in each interval. This means that δ_1 (and each of the other random elements) was applied every four and one-half weeks, with the resulting

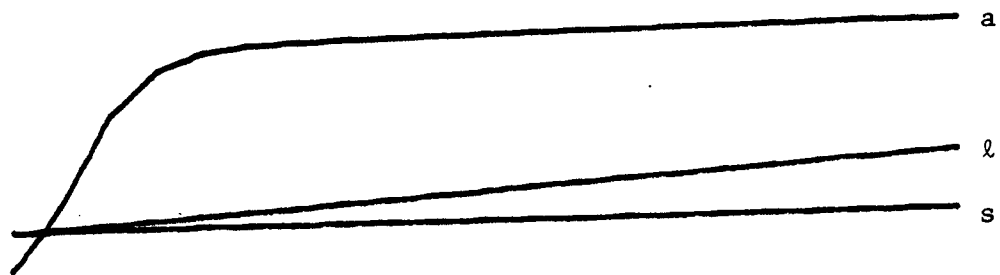
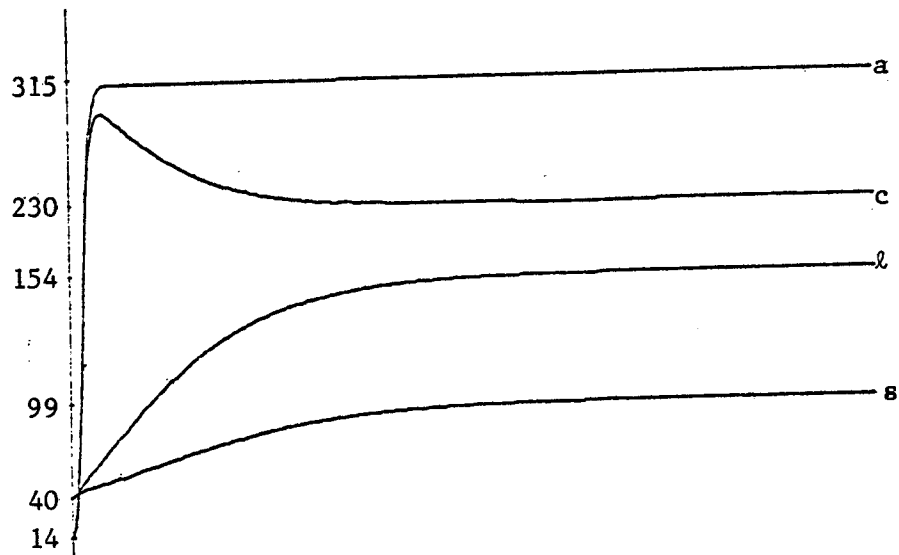
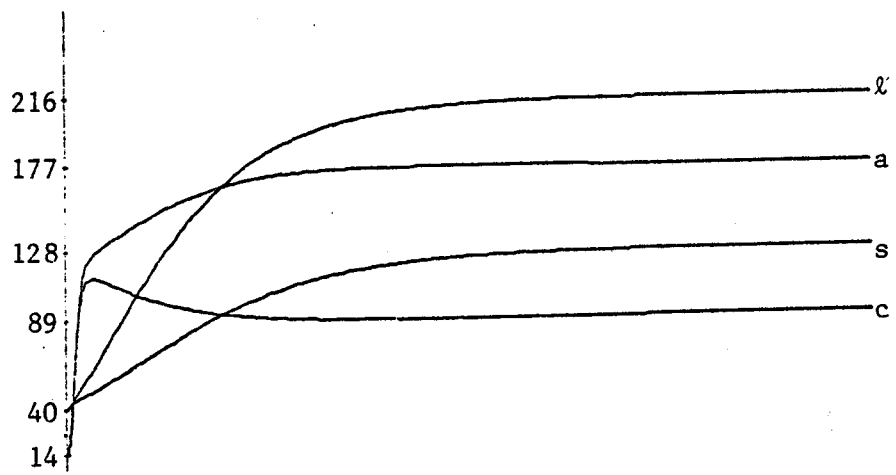


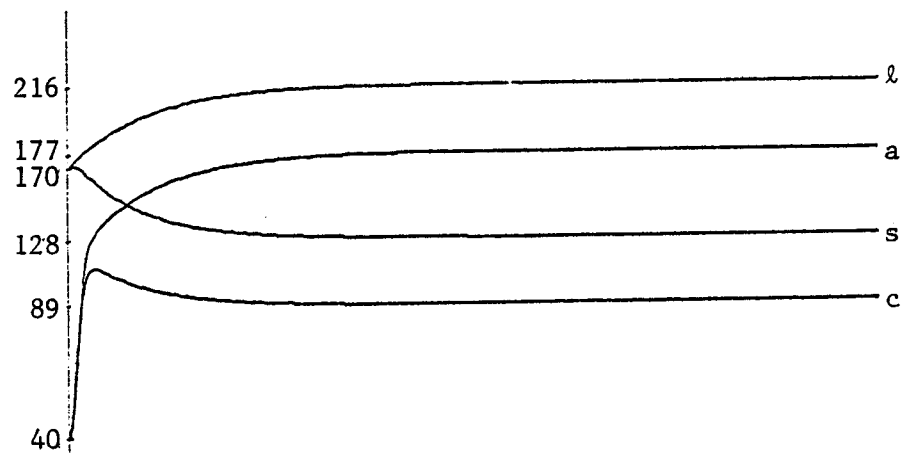
Figure 13C (repeated)



(A) Extension of Figure 10B

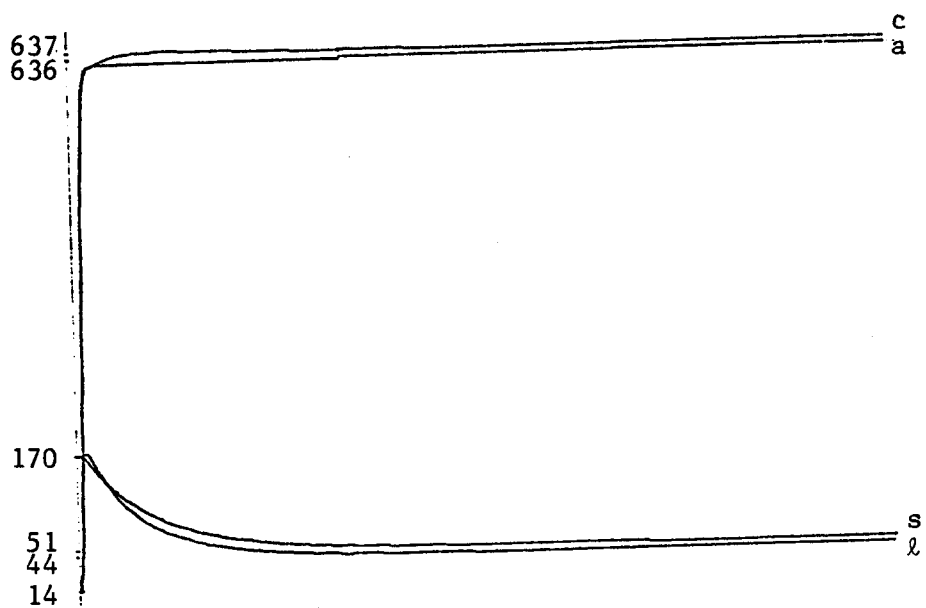


(B) Extension of Figure 10C

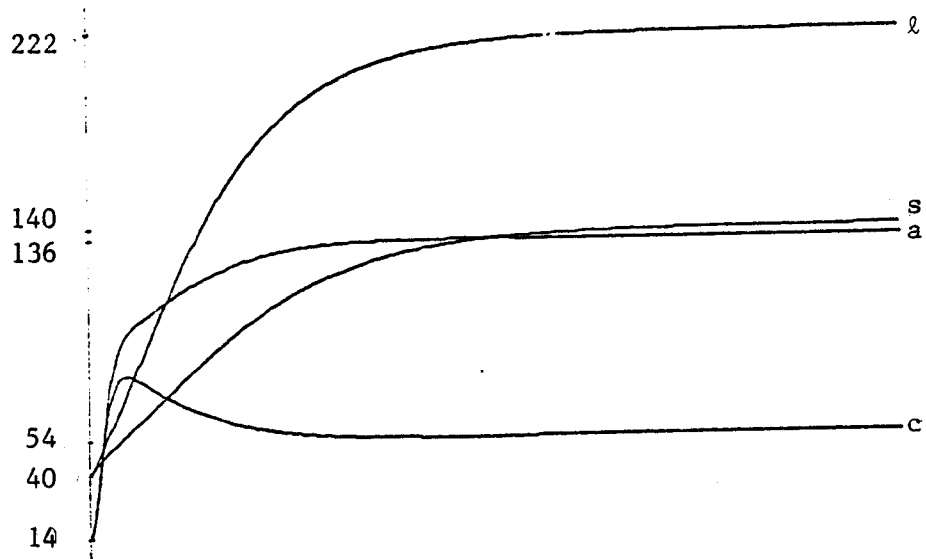


(C) Extension of Figure 11C

Appendix Figure 1. Extension of five-year graphs to sixty years



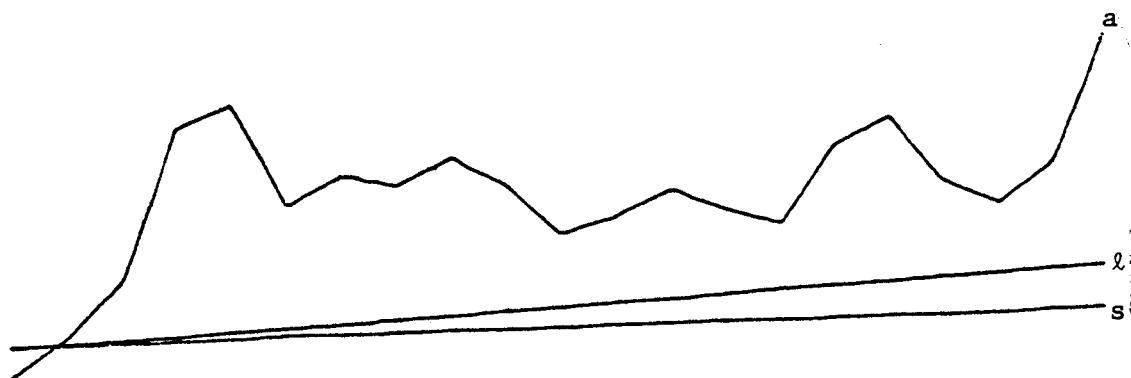
(D) Extension of Figure 14B



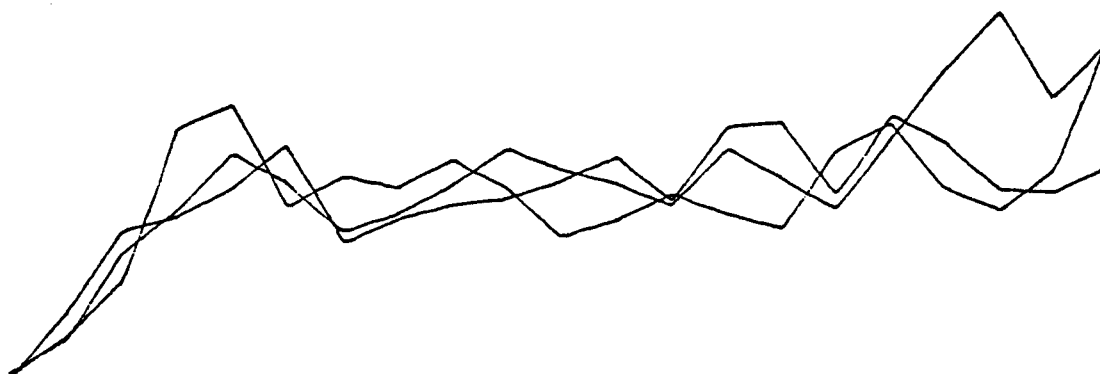
(E) Extension of Figure 16C

straight-line graph segment being an average change over thirteen weeks. The variables \underline{l} and \underline{s} are seen to be little affected by the fluctuation in the level of \underline{a} . This is, in large measure, due to the fact that δ_1 has a direct effect on changes in \dot{a} while changes in the value of \underline{a} have only an indirect effect on \dot{l} and \dot{s} , through changes in $\lambda\langle a \rangle$ and $\sigma\langle a, m, c \rangle$, respectively. The graph for criminal earnings, while operative and available as usual, is not shown in any of these graphs, since the variable \underline{c} has served a bookkeeping role throughout, and inclusion of the additional graphs would needlessly clutter the figures. Appendix Figure 3 has simply added to the graph of \underline{a} from Appendix Figure 2 two more realizations of \underline{a} only. Appendix Figure 4 resulted from the addition of stochastic elements, δ_2 and δ_3 , to equations (2) and (3).

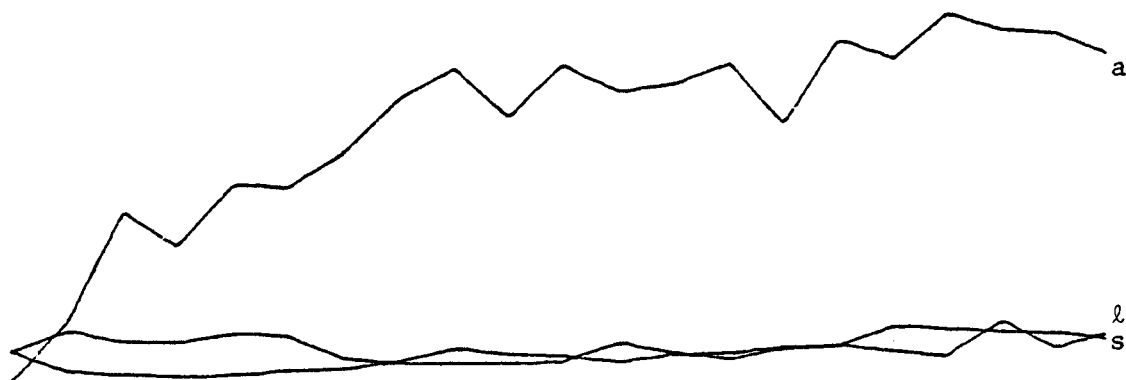
Appendix Figures 5 through 7 are examples of model performance with the κ_s replaced by $\kappa(\epsilon_1)$'s. For the graph in Appendix Figure 5, κ_a was multiplied by an approximately lognormal random variable. In this example the multiplier of $a[\alpha\langle c \rangle - a]$ is $\kappa(\epsilon_1) = .00021 \exp(\epsilon_1)$, where $\epsilon_1 = \frac{20}{\sum_{i=1}^{20} \epsilon^{(1i)}}/20$ and each $\epsilon^{(1i)} = 10$ ("random" - .5). The non-negative nature of κ_a in equation (1) makes the lognormal distribution preferable to the normal; and, to have $\kappa(\epsilon_1)$ distributed around the original κ_a , the stochastic coefficient was multiplied --at each iteration (every four and one-third weeks, as above)-- rather than added. Little difference is noted between Appendix Figure 5 and the deterministic graphs of Figure 13C.



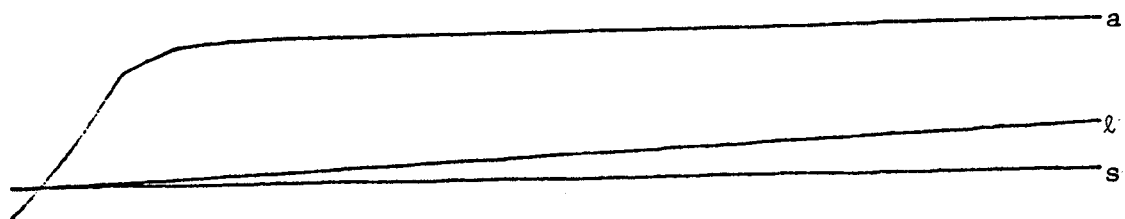
Appendix Figure 2. Realization with δ_1 added



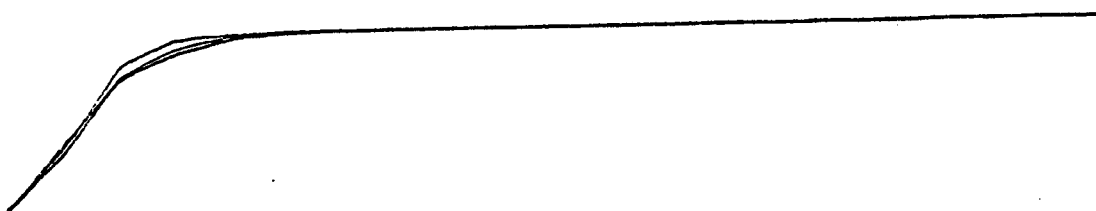
Appendix Figure 3. Three realizations with δ_1 added



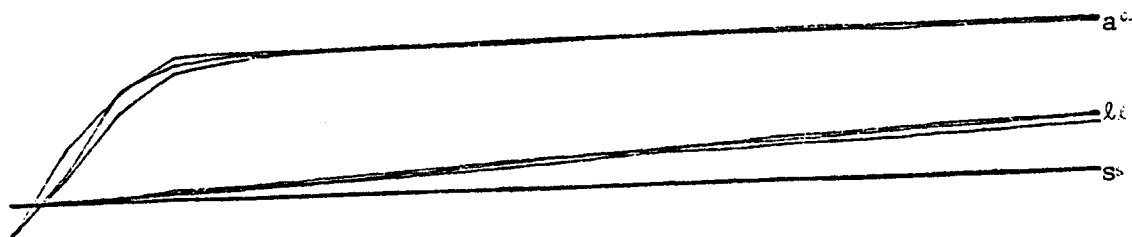
Appendix Figure 4. Realization with δ_1 , δ_2 , and δ_3 added



Appendix Figure 5. Realization with ϵ_1 as multiplier



Appendix Figure 6. Three realizations with ϵ_1 as multiplier



Appendix Figure 7. Realization with ϵ_1 , ϵ_2 , and ϵ_3 as multipliers

Appendix Figure 6 shows the graph of \underline{a} from Appendix Figure 5 and two other realizations. For Appendix Figure 7 three independent realizations of the model are performed with $\kappa(\epsilon_1)$, $\kappa(\epsilon_2)$, and $\kappa(\epsilon_3)$ replacing, respectively, κ_a , κ_ℓ , and κ_s . Again the differences in corresponding components of the figure are slight.

Little significance should be ascribed to the magnitude of the random fluctuations caused by the δ 's as contrasted with the ϵ 's since the variances of the latter are several hundred times greater than those of the former through most of the performance period. With further experimental work, more precise specification of the stochastic terms may be possible. In that event, it would be appropriate to examine variance characteristics more closely.