THERMAL EFFECTS IN CONCRETE MEMBERS

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SUMMARY

When subjected to temperature changes and restrained from free movement, a member develops stresses. Restrained members are sometimes assumed to act independently of other members. A method of analysis and design for thermal stresses in such members is provided. This method, based on the principles of ultimate strength design, is also applicable to frames and other structures.

Unlike gravity and some other loadings for which the use of relative stiffness is adequate, dynamic and thermal loadings are directly related to the actual member stiffness. A characteristic of thermal forces in concrete members is that these forces tend to be self-relieving. High tensile stresses cause cracking of concrete and lower the loads induced by temperature changes. Thus an iterative approach to the problem is generally called for.

The proposed method of analysis for concrete members subjected to temperature changes is consistent with the requirements of ultimate strength design. This also facilitates the provision of the same safety margin as for other loads.

Due to cracks and creep in concrete, thermal stresses are nonlinear; they are dependent on the effective member stiffness, which in turn vary with the magnitude of loading. Therefore it is inconsistent to have an ultimate strength design in conjunction with an analysis based on the linear elastic theory.

It is proposed that when the requirements of serviceability are met, the neutral axis corresponding to the ultimate load capacity conditions be considered for temperature-induced loadings. This conforms with the fact that the thermal load, because of creep and formation of cracks in the member, can be self-relieving as the failure load condition or ultimate capacity is approached. The maximum thermal load that can develop in dependent on the effective cross section of the member.

Recommendations are made for determining the average effective member stiffness, which lies between the stiffness corresponding to the cracked (at ultimate condition) and the uncracked sections. These recommendations reduce iterative efforts associated with conventional analyses for thermal effects in concrete members. The method is not necessarily unconservative as it is statically consistent and desired safety margins are provided. In the proposed method, thermal stresses are not considered completely self-relieving. The stresses are considered simultaneously with stresses resulting from other causes. A step-by-step approach is presented for analysis and design of concrete members subjected to temperature changes.

The method of analysis, based on the ultimate strength concept, greatly reduces the computational efforts for determining thermal effects in concrete members. Available charts and tables and the recommendations given herein simplify the design.

1. Introduction

Any member, subjected to temperature changes and restrained against free movement, develops stresses. In some cases, members in a structure develop temperature stresses due to mutual restraint. In certain other cases, the affected members, though restrained, can be assumed to act independent of other members.

The frame interaction in the case of steel structures has been studied by Kar [1], and West and Kar [2, 3]. In the case of concrete structures, contributions have been made, among others, by Khan and Fintel [4]. All the interaction studies are limited to elastic analysis. Several methods [5, 6], based on elastic analysis, are available for the design of members, subjected to temperature variations. The method of Ref. [5] is questionable for use in safety-related nuclear power plant structures.

Unlike gravity and some other loadings, for which the use of relative stiffness is adequate, dynamic and thermal loadings are directly related to actual member stiffness.

A characteristic of thermal forces in concrete members is that these forces can tend to be self-limiting. High tensile stresses cause cracking of the concrete, and thus lower the temperature loads. Thus an iterative approach to the problem is generally called for.

A new method of analysis for individual concrete members having uniform capacities along their lengths, and subjected to differential (different at two faces) temperature, is presented in this paper. This method shortens iterative efforts. The recommendations are useful also in the case of members with non-uniform sections, and subjected to uniform temperature change.

2. Cracking and Member Stiffness

The Building Code Requirements for Reinforced Concrete, ACI 318-71 [7] recommends that concrete members be designed for ultimate load conditions whereas the design load determinations are based on elastic behavior, and the effects of different loads be combined on a factored basis. This is a questionable practice when we remember that dynamic and thermal forces are nonlinear, being dependent upon effective member stiffnesses.

The basic difference in design requirements and analysis creates an ambiguity in the case of thermal loadings due to temperature changes. The temperature loading depends upon the stiffness of the member, which in the case of bending depends upon the location of the neutral axis of the member, and this location is different at working conditions and at ultimate load resistance conditions. This raises a question. Should the neutral axis be considered at working conditions or at ultimate load capacity conditions?

Several other questions should be answered before an answer can be found for the above question. Is the true loading at working conditions, which is different from design conditions, known? The answer is obviously no. This is so because the real load may not be known at design stages. Also, a part or whole of the design live load may be present when the temperature changes occur.

Different levels of loading lead to different amounts of cracking of the section. Again, are the present practices adequate in determining the amount of this cracking? ACI 349-72 [8] suggests that temperature moments can be calculated separately by assuming that dead and live load moments are absent. Since the presence of non-thermal forces influence the cracking or stiffness of the concrete sections, such practices are questionable. Depending upon the case, the recommendations of ACI 349-72 [8] can be conservative or nonconservative. This depends upon whether the moments due to dead and live load are opposite or similar to those produced by the change in temperature. The same can be said of the age-long conventional working stress method, according to which the location of the neutral axis, whether of the gross section or of the cracked section, depends upon the original section properties only, and thus no consideration is given to the level, type or location of loadings. ACI 318-71 [7], however, relates the effective moment of inertia, I_e , to the presence of moments, causing cracks in a section.

ACI 349-76 [9] is based on the strength design method, and it recommends that thermal stresses be evaluated based on the cracked section properties, provided certain conditions are met. The recommendations of Ref.[9] include the determination of thermal stresses on the basis of the conventional working conditions and cracked section properties, and a design for factored load conditions. In general, temperature loadings induced at different levels of loading, due to other causes, will be different. Thus, if working conditions are to be considered, many different analyses may be necessary even if the effects of existing loads are accounted for in the determination of the sectional stiffness.

Is it adequate to design a member by ensuring that the ultimate capacity of the member is at least equal to elevated factored amounts of anticipated forces? If serviceability is not a problem, i. e., if serviceability is assured by limited deflection, etc., the answer is yes, and the present ACI Code [7] is based on this structural philosophy. For the containment shell during a temporary loss-of-coolant accident and for various other cases the serviceability requirements are easily met.

On the basis of the design principles, described in the preceding, it is recommended that when the requirements of serviceability are met, the neutral axis, corresponding to the ultimate load capacity conditions, be considered in the design for temperature-induced loadings. This is in conformity with the fact that in certain cases the temperature load may tend to be self-limiting as a result of the formation of cracks in the member, and as the failure load condition or ultimate capacity is approached, the maximum temperature load that can occur is equal to that which is possible in the effective portion of the member-section at this load condition. The analysis, presented later, is based on this hypothesis.

The new approach eliminates or greatly reduces the necessity of iterations in the design for thermal loads. The hypothesis is not necessarily nonconservative because it is statically consistent and an appropriate load factor is used.

There still remains another question to be answered. The moment in an individual member, subjected to temperature changes depends upon the average moment of inertia or effective area of the cross-sections along the length of the member. But a member may not develop cracks at sections for the full depth of the tensile zone. Also, the cracks do not develop for the same depth for the entire length of the member. Thus, the average stiffness of a member along its length is more than the stiffness at a section with the least remaining uncracked depth or concrete area. This is recognized in eq. (9-4) of ACI 318-71 [7].

Since the restraint moment due to differential temperature is a function of the section stiffness along the entire length of a member, it is assumed that the average stiffness is γ times the stiffness corresponding to the section at the ultimate stress state. This average stiffness, however, need not be greater than that corresponding to the gross moment of inertia, I_g. The recommendation has the effect of reducing or eliminating unconservatism, if any at all, in determining the thermal stresses at ultimate load conditions. In this paper, it is recommended that for individual members γ be equal to 1.5. A value of 1.1 to 1.25 for γ would normally be adequate, if the stiffness of the cracked section would have been determined on the basis of a cracked section whose neutral axis is located by the principle of elastic theory and without any consideration given to the presence of loads. In the case of analysis for a frame or a large structure, it is recommended that the frame stiffness be based on effective-to-gross-section stiffness ratio of one-third for beams and a ratio of one-half for columns.

The following analysis of individual concrete members, subjected to temperature variations, is based on the hypothesis and assumption of sectional stiffness, mentioned earlier. It should, however, be noted that in the method, proposed here, thermal stresses are not considered as completely self-limiting. These stresses are considered together with stresses due to other causes. Also, attention is given to serviceability. This is done by satisfying the requirements of Chapter 9 of ACI 318-71 [7] or any other relevant specifications.

3. Analysis

The individual members, subjected to temperature changes, may be of two principal types:

a) Members subjected to bending moment only, and
b) Members subjected to combined bending and axial forces. Members, subjected to axial forces only, are treated in this type.

The two cases are treated separately.

3.1 Members Subjected to Bending Moment Only

In a member with flexurally restrained ends, the moment M_{te}, due to differential temperature T with constant slope between two faces is given by

$$M_{te} = \frac{\alpha E I_e T}{t} \tag{1}$$

where α is the coefficient of thermal expansion of concrete, t is the total depth of member, I_e is the effective moment of inertia, and E is the effective modulus of elasticity, as described earlier. However, as mentioned earlier, the effective average stiffness can be 1.5 times as much as the stiffness of the cracked section. Thus,

$$M_{te} = \frac{\alpha E \gamma I_{cr} T}{t} = \frac{1.5 \alpha E I_{cr} T}{t} \tag{2}$$

where I_{cr} is the moment of inertia of the cracked section, determined with considerations given to loads. The moment of inertia I_{cr} of the cracked section is determined for the sections shown in Figs. 1 or 2. In determining I_{cr} it is recognized that the full depth of the section below the neutral axis would not be cracked. If concrete cracks when the tensile stress is approximately equal to one-tenth of the ultimate compressive stress, then

$$x \approx \frac{11}{10} c \tag{3}$$

where c is the depth to the neutral axis of the cracked section, and x is the depth of the actual uncracked portion of the member (Fig. 1). The value of c or x is dependent on the presence of axial loads, including those due to temperature changes. It should be mentioned here that eq. (3) may not be fully consistent with the method of determining the value of c. But it is practical.

Since I_e cannot be greater than the gross moment of inertia I_g, M_{te} is the smaller of

$$M_{te} = \frac{1.5 \alpha E I_{cr} T}{t} \tag{4}$$

and

$$M_{te} = \frac{\alpha E t^2 b T}{12} \tag{5}$$

For the design of the section, the moment M_{te} is used simultaneously in combination with moments due to other causes. Finally, it should be verified that the ultimate moment capacity M_u should be at least equal to the total ultimate design moment, U, due to all causes. Thus

$$M_u \geqq U \tag{6}$$

3.2 Members Subjected to Combined Bending and Axial Forces

When the section is subjected to axial forces in addition to bending, the analysis becomes more involved. Still, iteration can be virtually avoided or greatly reduced. Two failure conditions, as compression or tension may govern, are possible. These are enumerated below.

a) If the ultimate eccentric load P_u is greater than the balanced value P_b or if the eccentricity e is less than the balanced value e_b, the capacity of the section is controlled by compression in the concrete. In this case, the tensile force P in Fig. 1 will be based on a stress less than yield and may even be a compressive force.

b) If the ultimate eccentric load P_u is less than the balanced value P_b, or the eccentricity e is greater than the balanced value e_b, the capacity of the section is controlled by tension in the steel. In such a case, the ultimate strain in the steel, most distant from the neutral axis, is greater than the yield strain ϵ_y $(= f_y/E_s)$. A case where tension governs, is shown in Fig. 2.

3.3 Step-by-Step Analysis

The complete analysis of a section of the member, subjected to temperature variations, can be made by the use of the following seven steps:

1. For the ultimate load condition, calculate

$\left(\frac{e}{t}\right)_{\substack{\text{for all causes except}\\ \text{temperature differential}}}$. In determining this value, include axial loads, if any, due to temperature variation. Increase (or decrease, if necessary) this value by a quantity $\left(\frac{e}{t}\right)_{\text{temperature}}$. Thus the resulting eccentricity is given by

$$\frac{e}{t} = \left(\frac{e}{t}\right)_{\substack{\text{for all causes except}\\ \text{temperature differential}}} \pm \left(\frac{e}{t}\right)_{\text{temperature}} \tag{7}$$

2. When possible, use interaction curves, e. g., ACI-SP 17A [10]. Use $\frac{P_u}{f'_c bt}$ and $\left(\frac{e}{t}\right)$ from Step 1 to compute $p_t m$, where p_t is the steel ratio $\frac{A_s}{bd}$, and $m = \frac{f_y}{0.85\ f'_c}$. In using the interaction curves, effects of member slenderness should be considered.

or 2a. Determine $\left(\frac{\delta e}{h}\right)$, where δ is the moment magnification factor for slenderness, and h is the thickness t. Compute $\frac{P_u}{A_g}$. A_g is the gross concrete area of the section. Determine ρ_g from column tables in Ref. [11]. ρ_g is the ratio of longitudinal reinforcement in a column to the gross concrete area.

3. Provide reinforcement according to p_t, calculated in Step 2. From interaction curves, recompute $\left(\frac{e}{t}\right)$ according to reinforcement provided, if necessary. Recompute $\left(\frac{e}{t}\right)_{temperature}$ according to eq. (7), if necessary.

or 3a. If Step 2a has been followed instead of Step 2, provide reinforcement according to ρ_g. From the column tables in Ref. [11], recompute $\left(\frac{e}{h}\right)$ according to reinforcement provided, if necessary. Recompute $\left(\frac{e}{t}\right)_{temperature}$ according to eq. (7), if necessary. In this equation, use $\left(\frac{e}{h}\right) = \left(\frac{e}{t}\right)$.

4. Calculate the temperature moment capacity as

$$M_{tp} = P_u\ t\left(\frac{e}{t}\right)_{temperature} \tag{8}$$

Calculate $M'_{tp} = M_{tp}/$ (appropriate load factor) (9)

The load factor is for ultimate strength design, and it is taken from the relevant code or specifications.

5. Determine c (Fig. 1) from statics. Compute $x \approx \frac{11}{10}\,c$.

6. Calculate ultimate temperature moment M_{te} according to eq. (4) or (5). If $M_{te} \leq M'_{tp}$, the section is safe. Otherwise, increase reinforcement and repeat Steps 3 to 6.

7. Check serviceability. This is done by satisfying the requirements of Chapter 9 of ACI 318-71 [7] in the case of structures for which this code is applicable. For other cases, use any applicable code or specification.

Steps 1 to 6 are for symmetrically reinforced sections. For sections with unsymmetrical reinforcements, analyze the sections as a column for any arbitrary steel distribution or a beam, as the case may be, and determine M_u . Use of Ref. [11] will lessen the computational efforts in such cases. The temperature moment capacity M_{tp} is determined by subtracting factored moments for all other causes from M_u . The unfactored temperature moment capacity M'_{tp} is determined from eq. (9). If temperature moment M_{te} , determined from eq. (4) or (5), is less than or equal to M'_{tp} , the section is safe. Otherwise, increase reinforcement and reanalyze the section.

In Step 1, the axial load due to temperature variation is that due to average change in temperature T_{ave} in the section. This load depends upon the effective cross-sectional area A_{eff} of the member. The following approach for the determination of A_{eff} will be practical and generally conservative.

(a) For rise in average temperature, use the full section as uncracked section if the axial load for other causes is greater than P_b ; otherwise consider half of the section to be effective.

(b) For fall in average temperature, use the uncracked part of the section at zero axial load as shown in Step 5, if axial load for other causes is greater than P_b ; otherwise consider the smaller of x determined in Step 5 or half of the section to be effective.

The axial load P_{te} at ultimate condition due to an average change of temperature T_{ave} is

$$P_{te} = \alpha \, E A_{eff} T_{ave} \text{ (load factor)} \qquad (10)$$

In eq. (10), the load factor is for ultimate strength design. It is obtained from the appropriate code or specification.

4. Conclusions

Constrained members, subjected to temperature changes, develop stresses. In the case of concrete members, these stresses tend to be self-limiting as a result of creep and crack. For the design or analysis of such members, an iterative approach is generally called for.

A new method of analysis for individual concrete members, subjected to temperature changes, has been presented. The stiffness of a member at ultimate load capacity condition is used in the analysis. The effective moment of inertia is that for a cracked section. The depth of the uncracked portion is larger than the depth to the neutral axis. Also, the effective moment of inertia for the full member is larger than the moment of inertia of the cracked section. Recommendations are made for the effective moment of inertia of concrete members. The recommendations reduce the necessity of iterations. In addition, available handbooks simplify the design. A step-by-approach has been presented for the analysis and design.

5. Appendix I. — Determination of Neutral Axis

The location of the neutral axis is determined separately for the two cases when compression or tension governs.

Case A: Compression Governs (Fig. 1)

Using statics, it is seen from Fig. 1 that

$$C_c = 0.85\ f'_c\ (\beta_1 c)b \tag{A.1}$$

where f'_c is the specified compressive strength of concrete.

$$C_s = A'_s\ (f'_s - 0.85\ f'_c);\ \text{generally}\ f'_s = f_y \tag{A.2}$$

where f'_c is the stress in compressive reinforcements and f_y is the yield stress of steel.

$$P = A_s f_s = A_s \frac{E_s \epsilon_c\ (d-c)}{c} \tag{A.3}$$

where f_s is the stress in tensile reinforcement.

The value of c, the depth to the neutral axis, can be determined by equating the algebraic sum of moments of C_c , C_s and P about P_u to zero, and solving for c. This can also be determined from conditions of axial equilibrium: $P_u + P = C_c + C_s$.

Should the full section be under compression, obviously $c = t$, and $x = t$.

Case B: Tension Governs (Fig. 2)

Using techniques, as in Case A, taking moment about the tension steel A_s instead of about P_u , the value of c is determined with the aid of Fig. 2 as

$$c = \frac{d-e'}{\beta_1} + \left[\left(\frac{d-e'}{\beta_1}\right)^2 + \frac{2d\{p'\ (m-1)(d-d') + e'p' + e'm(p-p')\}}{\beta_1^2}\right]^{0.5} \tag{A.4}$$

where p = tension steel ratio, $\dfrac{A_s}{bd}$

p' = compression steel ratio, $\dfrac{A_s}{bd}$

$$m = \frac{f_y}{0.85\ f'_c}$$

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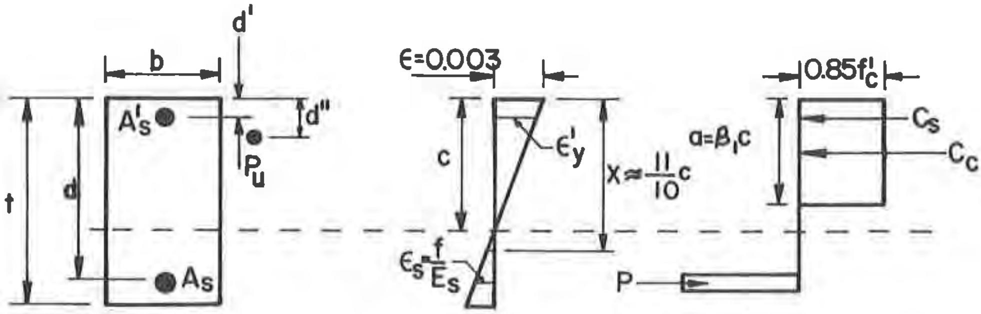
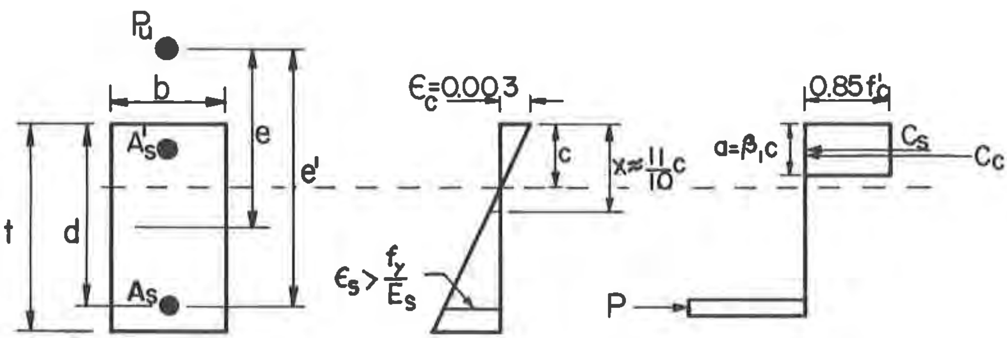


1. Strain and Stress Diagram when Compression Governs



2. Strain and Stress Diagram when Tension Governs