ALE Formulation for Large Boundary Motion

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INTRODUCTION

Considerable research activities in vibration and seismic analysis for various fluid-structure systems have been carried out in the the past two decades. Most of the approaches are formulated within the framework of finite elements, and the majority of the work deals with inviscid fluids. However, the application of finite element analysis to viscous flows with large free surface motion is still rudimentary. This paper describes the arbitrary Lagrangian Eulerian (ALE) mixed formulation and applies it to the sloshing response of a two-dimensional idealization of a spent fuel storage tank of nuclear facilities.

Two classical descriptions are usually used in continuum mechanics. The first is Lagrangian, in which the mesh points coincide with the material particles. In this description, no convective effects appear and this simplifies considerably the numerical calculations; moreover, a precise definition of the moving boundaries and interfaces is obtained, recall that each element contains always the same amount of fluid. However, the Lagrangian description does not handle satisfactorily the material distortions that lead to element entanglement. On the other hand, the second description is the Eulerian viewpoint, which allows strong distortions without problems because the mesh is fixed with respect to the laboratory frame and the fluid moves through it. However, this latter approach presents two important drawbacks: *i)* convective effects, which introduce numerical difficulties, arise due to the relative movement between the grid and the particles; and *ii)* sophisticated mathematical mappings between the grid and the particles are required to follow the interface movement and they often lead to inaccuracies.

Because of the shortcoming of purely Lagrangian and Eulerian descriptions, arbitrary Lagrangian Eulerian techniques were developed, first in finite differences, Noh (1964) and Pracht (1975), among others; and then in finite elements, Belytschko et al. (1980), Donea (1983), Huerta and Liu (1988), and Liu et al. (1988), among others. This approach is based on the arbitrary movement of the reference frame, which is continuously rezoned in order to allow a precise description of the moving interface and to maintain the element shape.

The time integration scheme is based on the predictor-multicorrector algorithm of Liu and Ma (1982). On one hand, this algorithm reduces considerably the cost of more classical implicit methods; and on the other, it avoids diagonal elements of pressure equations equals to zero and thus, the need for penalty formulations. If the equations involved are continuity and momentum (isothermal flow), this algorithm solves at each step three systems of equations. Two of them are trivial (diagonal matrix) and have as number of unknowns the total number of velocity equations, while the third one is not trivial (symmetric banded positive definite matrix) but has a considerably smaller number of equations (total number of pressure unknowns) because a mixed pressure-velocity formulation is employed.

In the next section, the governing equations and fundamentals in ALE formualtion are presented. Then, the numerical results are discussed and compared with experimental test results. Finally, the concluding remarks are presented.

FUNDAMENTAL THEORY

Notation and Preliminaries

Two classic viewpoints are considered to describe the motion of a continuous medium. The first is Lagrangian, in which the region and coordinates of any point are denoted by R_X and $\boldsymbol{X}$, respectively. In the second, known as Eulerian, the spatial region is symbolized by R_x and the spatial coordinates by $\boldsymbol{x}$. In the ALE description, the computational frame is a reference independent of the particle movement and which may be moving with an arbitrary velocity in the laboratory system; the continuum view from this reference is denoted as R_χ, and the coordinates of any point are denoted as $\boldsymbol{\chi}$.

Both the material and the spatial domains are generally in motion. However, since R_χ is fixed throughout this formulation, it is convenient to express the material time derivatives in a referential form. Consider a physical property, $f(\boldsymbol{x}, t)$, expressed in a spatial representation; the material derivative of f, that is, the derivative of f with respect to t holding $\boldsymbol{X}$ fixed, may be written as

$$\left.\frac{\partial f}{\partial t}\right|_{\boldsymbol{X}} = \left.\frac{\partial f}{\partial t}\right|_{\boldsymbol{\chi}} + w_i \frac{\partial f}{\partial \chi_i} \tag{1}$$

where

$$w_i = \left.\frac{\partial \chi_i}{\partial t}\right|_{\boldsymbol{X}} \tag{2}$$

is defined as the particle velocity with respect to the referential coordinates. Standard indicial notation is adopted: lower-case subscripts denote the components of a tensor, and repeated indices imply sumation over the number of spatial dimensions.

In order to work with spatial derivatives instead of derivatives of f with respect to $\boldsymbol{\chi}$, Eq. (1) is further simplified by means of the following definitions of material velocity, $\boldsymbol{v}$, and mesh velocity, $\hat{\boldsymbol{v}}$:

$$v_i = \left.\frac{\partial x_i}{\partial t}\right|_{\boldsymbol{X}} \tag{3a}$$

and

$$\hat{v}_i = \left.\frac{\partial x_i}{\partial t}\right|_{\boldsymbol{\chi}} \tag{3b}$$

respectively. If the physical property is the spatial coordinate $\boldsymbol{x}$, (1) and (3) yield

$$v_i = \hat{v}_i + w_j \frac{\partial x_i}{\partial \chi_j} \tag{4}$$

or

$$c_i = v_i - \hat{v}_i \tag{5}$$

where

$$c_i = w_j \frac{\partial x_i}{\partial \chi_j} \tag{6}$$

is the convective velocity. Finally, substituting (6) into (1) and using the chain rule, yields the classical relationship between material time derivative and referential time derivative:

$$\left.\frac{\partial f}{\partial t}\right|_{\boldsymbol{X}} = \left.\frac{\partial f}{\partial t}\right|_{\boldsymbol{\chi}} + c_i \frac{\partial f}{\partial x_i} \tag{7}$$

For simplification purposes, the field equations are integrated over the spatial domain (all the derivatives are with respect to $\boldsymbol{x}$, not $\boldsymbol{\chi}$). Thus, Eq. (7) is needed to determine the material derivatives because the time derivatives are kept in referential form to simplify the mapping procedures. However, the mapping itself (between $\boldsymbol{\chi}$ and $\boldsymbol{x}$) is based on Eq. (5), or Eq. (4). That is, apart from the field equations, Eq. (5) must be also verified over the domain. The definition of $\hat{\boldsymbol{v}}$, needed in (5), is the basis of the remeshing techniques and they will be discussed later.

Governing Equations

In this section super-star (*) stands for dimensioned variables. The continuity and equilibrium equations which must be satisfied, under an ALE formulation are written as

$$\frac{\partial v_i^*}{\partial x_i^*} = 0 \tag{8a}$$

$$\rho \left.\frac{\partial v_i^*}{\partial t^*}\right|_{\boldsymbol{\chi}} + \rho c_j^* \frac{\partial v_i^*}{\partial x_j^*} = -\frac{\partial P^*}{\partial x_i^*} + \mu \frac{\partial}{\partial x_j^*}\left(\frac{\partial v_i^*}{\partial x_j^*} + \frac{\partial v_j^*}{\partial x_i^*}\right) + \rho g_i^* \tag{8b}$$

for isothermal incompressible flows, where P^* is the pressure, ρ is the density, μ is the viscosity, and $\mathbf{g}^*$ is the acceleration of gravity.

In order to reduce the number of parameters and determine those governing the problem, the dimensionless problem should be studied. The characteristic dimensions chosen in a free surface flow poblem such as the one presented here are: length D, velocity $\sqrt{gD}$, acceleration g, pressure $\rho g D$, and time $\sqrt{\frac{D}{g}}$. Where D is a characteristic length of the problem, usually the height of the still water level. All the other characteristic dimensions are automatically deduced from these; for instance, the characteristic viscosity and density are, respectively, $\rho\sqrt{gD}D$ and ρ. The problem is rewritten in terms of dimensionless variables:

$$v_i = \frac{v_i^*}{\sqrt{gD}} \qquad x_i = \frac{x_i^*}{D} \qquad i = 1,\ldots,\mathtt{NSD} \qquad t = \frac{t^*}{\sqrt{\frac{D}{g}}}$$
$$g_i = \frac{g_i^*}{g} \qquad P_i = \frac{P_i^*}{\rho g D} \tag{9}$$

After substitution of Eqs. (9) into Eqs. (8) the following dimensionless equations are obtained,

$$\frac{\partial v_i}{\partial x_i} = 0 \tag{10a}$$

$$\frac{\partial v_i}{\partial t} + c_j \frac{\partial v_i}{\partial x_j} = -\frac{\partial P}{\partial x_i} + \frac{1}{R_e}\frac{\partial}{\partial x_j}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) + g_i \tag{10b}$$

where R_e is the Reynolds number,

$$R_e = \frac{\sqrt{gD}D}{\frac{\mu}{\rho}} \tag{11}$$

Apart from external conditions, the fully dimensionless problem is governed by: all the geometrical data scaled by D and the Reynolds number. Notice that the dimesionless fluid constants are the density 1 and the viscosity $1/R_e$, and the field equations, Eqs. (10), only depend on the Reynolds number.

Boundary Conditions

A special section is defined because of the importance of such conditions on large free surface motion problems. In fact, the boundary conditions are related to the problem, not to the description employed. Thus, the same boundary conditions used in Eulerian or Lagrangian descriptions are employed in an ALE formulation. That is, along the boundary of the domain, kinematical and dynamical conditions must be defined. Usually, this is formalized as:

$$v_i = b_i \qquad \text{in } \partial R_x^b \tag{12a}$$

$$\sigma_{ij}\ n_{xj} = h_i \qquad \text{in } \partial R_x^h \tag{12b}$$

where $\boldsymbol{b}$ and $\boldsymbol{h}$ are the prescribed boundary velocities and tractions, respectively; $\boldsymbol{n}_x$ is the outward normal to ∂R_x, and ∂R_x is the piecewise smooth boundary of the *spatial* domain, R_x. ∂R_x is in fact composed of two distinct subsets ∂R_x^b and ∂R_x^h. As usual, stress conditions on the boundaries are "natural boundary conditions", and thus, they are automatically included in the weak, or variatonal form of Eq. (10).

However, if part of the boundary is composed by a material surface, then a mixture of both conditions is sometimes required. Firstly, the conditions required on a material surface are: *a)* no particles can cross it, and *b)* stresses must be continuous across the surface (if a net force is applied to a surface of zero mass the acceleration is infinite). And two types of material surfaces are usually present: free surfaces, and solid wall boundaries which may be frictionless or not.

Along the solid wall boundaries the particle velocity is defined (or coupled to a structure system in fluid-structure interaction problems). The requirement that no particles can cross it, can be simply verified if $\boldsymbol{w}$ is prescribed equal to zero along that boundary, i.e. $\boldsymbol{v} = \hat{\boldsymbol{v}}$ from Eq. (4). However, this condition may be relaxed imposing only the necessary condition: $\boldsymbol{w}$ equal to zero along the normal to the boundary. The latter allows remeshing tangent to the boundary and it is used for the container walls in the example shown later. The dynamic condition, is automatically verified along rigid boundaries, but it presents the classical difficulties in fluid-structure interaction problems when compatibility at nodal level in velocities and stresses is required.

Along the other type of material surface, i.e. free surface, problems arise because its position is unknown. Thus, the kinematic and dynamic conditions must be imposed and solved. The kinematic condition can be formalized as:

$$\boldsymbol{v} \cdot \boldsymbol{n}_x = 0 \tag{13}$$

in an Eulerian description. However, since the boundary is moving, its equivalent in referential form is prefered because the referential domain is fixed, namely

$$\boldsymbol{w} \cdot \boldsymbol{n}_\chi = 0 \tag{14}$$

where $\boldsymbol{n}_\chi$ is the exterior normal to the *referential* domain. While the dynamic condition, expresses the stress free situation

$$\sigma_{ij}\ n_{xj} = 0 \tag{15}$$

and, as mentioned earlier, it is directly taken into account by the weak formulation.

In conclusion, the free surface problems are the only ones that introduce a new equation, Eq. (14), that must be verified along that boundary. It is obvious that this new equation has a strong influence on the remeshing techniques employed.

Remeshing

The equations that must be solved are Eqs. (10) with Eqs. (5), but this is only possible if the mesh velocities are given in the domain. The remeshing techniques are concerned with the definition of $\hat{\boldsymbol{v}}$. For instance, if $\hat{\boldsymbol{v}} = \boldsymbol{0}$, the Eulerian description is imposed, but if $\hat{\boldsymbol{v}} = \boldsymbol{v}$ is prescribed, the Lagrangian description is used. It is obvious that finding the "best" choice for these velocities and a low cost algorithm for updating the mesh constitutes one of the major problems of the ALE description (cost meaning here, computer time and computer storage).

As a consequence of the previous section discussion, two cases are considered: free surface flow or confined flow. The latter implies that the boundary motion of the material surfaces is known (or may be calculated) at every instant. The former is, in fact, reduced to the confined flow case, once the free surface, at any given instant, is known. This means solving Eq. (14). However, instead of solving for $\boldsymbol{w}$, if Eq. (4) is used with (14), one can solve directly for $\hat{\boldsymbol{v}}$ along the free surface, see Huerta and Liu (1988). Three important remarks should be done:

- Eq. (14) together with (4) is not solved in the complete domain but only along the free surface;
- it is written in referential form, thus integrated over the **fixed** referential domain and consequently the finite element matrices are generated once and for all at the beginning of the computations; these two remarks are obviously associated with an important reduction of computer cost;
- and, the general formulation presented here does not describe the free surface with an elevation (over a datum level) parameter, thus vertical surfaces (such as the ones present in coating flows), for example, may be studied (see also the dam break problem in Huerta and Liu, 1988).

Now that both cases (free surface and confined flows) are reduced to one, the remeshing technique is completely defined once $\hat{\boldsymbol{v}}$ is given in the domain. This can be done by simple ad hoc formulae, solving potential equations that maintain element regularity, see Benson (1987) and Liu et al. (1988), or any other mesh smoothing algorithms. Most of these remeshings are based in defining new locations for the nodes, and then to compute the mesh velocities by finite difference aproximation (i.e. increment of displacement over increment of time). If structured meshes are used, the authors recommend to use simple ad hoc formulae where the mesh velocity is linearly interpolated between the velocities at both ends of the inter-element lines. This is an extremely simple an efficient algorithm which maintains element regularity and it is obviously more cost effective than solving potential equations previous to the evaluation of the mesh velocities.

Usually, in Lagrangian codes, remeshing is done when the elements are overly distorted. This occurs after several time steps, and when the remapping is done all the variables must be interpolated at the new nodal points. This over-damps the solution. The low cost strategy presented here allows remeshing to take place at every time step; thus, there is no need for interpolation of variables at the new nodes because the transport of these variables is already "built in" the field equations by means of the convective velocity.

NUMERICAL TEST

Model Definition

The goal here is to demonstrate the applicability of the ALE formulation. To illustrate the finite element code, the sloshing response of a two-dimensional idealization of a spent fuel storage tank of nuclear facilities is evaluated. That is, the problem studied experimentally by Muto et al. (1985) is reproduced numerically.

A particular feature of these tanks is the presence of submerged storage racks that are usually modelled by under water rigid blocks. Solving the field equations, i.e. momentum and continuity, for incompressible fluids at medium to large Reynolds numbers in a fluid domain which has a moving boundary (free surface) and fixed boundaries (rigid blocks) requires the ALE description.

It should be noticed that the presence of submerged rigid blocks increases numerical difficulties. The submerged blocks impose an Eulerian description in their vicinity in contrast with the vertical Lagrangian description of the free surface. These requirements can be fulfilled by the ALE formulation: *i)* at the free surface the mesh follows the vertical movement of the surface and remains fixed horizontally, *ii)* around the blocks the mesh is fixed, and *iii)* in between, a transition zone is defined with an arbitrary mesh motion that maintains the regularity of the element shape. Moreover, the blocks do disturb the flow path, generating strong recirculations, principally at medium to large Reynolds numbers; this difficulty can be overcome, for example, using a streamline upwind Petrov–Galerkin formulation, see Brooks and Hughes (1982).

The geometry is completely given in Figure 1. The two-dimensional rigid container has a depth D and a width W, the submerged blocks dimensions are also defined in Figure 1.

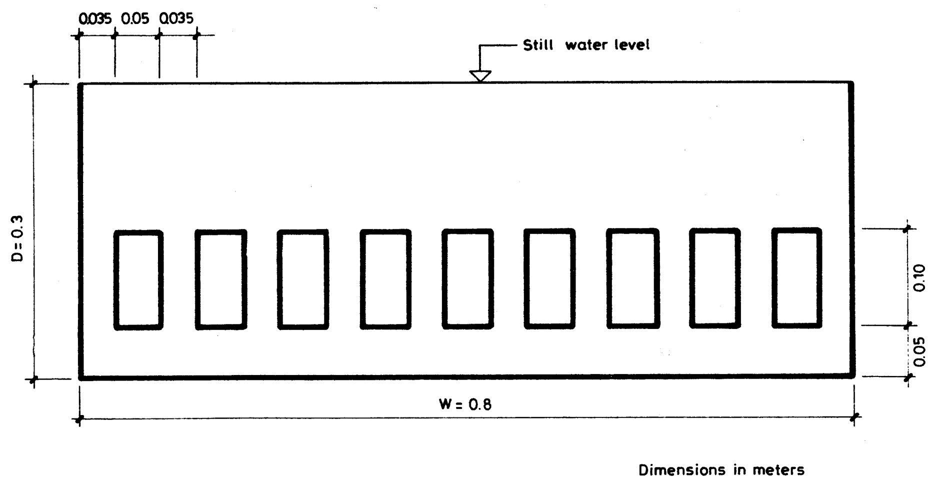

Figure 1.– Two-dimensional model configuration.

The fluid is assumed Newtonian, hence, only density, ρ, and viscosity, μ, are needed. The body forces are the standard acceleration of gravity (only the vertical component is non zero) and the acceleration due to the imposed vibration (only the horizontal component is non zero). The latter is a prescribed sinusoidal motion, thus, the body forces are:

$$\begin{cases} g_1^* = Ag\sin(\omega^* t^*) \\ g_2^* = -g \end{cases} \tag{16}$$

where g, ω^*, and t^* are the acceleration of gravity, circular frequency, and time, respectively, and A is an arbitrary dimensionless constant governing the amplitude of the excitation.

The dimensionless external excitation is also obtained when the scaling definitions, Eqs. (9), are substituted into Eq. (16), namely,

$$\begin{cases} g_1 = \dfrac{g_1^*}{g} = A\sin(\omega t) \\ g_2 = \dfrac{g_2^*}{g} = -1 \end{cases} \tag{17}$$

where t is the dimensionless time defined in Eqs. (9) and the dimensionless circular frequency ω is

$$\omega = \omega^* \sqrt{\frac{D}{g}} = 2\pi f \sqrt{\frac{D}{g}} \tag{18}$$

f being the frequency of the excitation.

The density of the fluid is equal to that of water, $\rho = 10^3 \mathrm{Kg/m^3}$, while the values of viscosity range from $\mu = 10^{-3}$Pa·s for water to 1 Pa·s because the fluid constants are not especified on the cited paper and this study wants to assess the influence of the material viscosity on the pool response. The amplitude of the external excitation is 10 Gal. Obviously, the resonance frequency depends on the prescribed damping, i.e. Reynolds number. Therefore, it seems that for every Reynolds number under study the resonance frequency must be evaluated. However, it is well known that for low values of the damping (large Reynolds numbers) the resonance frequency does not vary significantly, and consequently, the forcing frequency is always prescribed equal to 0.79 Hz which corresponds to the resonance frequency determined experimentally by Muto et al. (1985).

Apart from the geometry, it has been seen that the sloshing response of the pool is only governed by the Reynolds number and the excitation constants. The Reynolds number varies between 500000 and 500, the dimensionless amplitude is $A = 0.01$, and the dimensionless circular frequency is obtained after substitution of $f = 0.79$ Hz in Eq. (18).

Figure 2 shows the discretization of the fluid domain used for the computations. It consists of 2758 constant pressure-bilinear velocity elements (25 layers) which imply 3052 nodes. Notice the concentration of elements around the blocks needed to capture recirculation. Two areas are defined in the domain, see Figure 2, an Eulerian description is implemented in the bottom one (i.e. the mesh is fixed) while the ALE formulation is used in the upper zone.

The time step choosen is such that 200 time steps are needed in every cycle; that is Δt is obtained dividing 2π by $200 \times \omega$. However, for $R_e = 500000$, 300 time steps must be used in the tenth cycle due to large convection.

With respect to the boundary conditions, zero velocities are prescribed around the blocks, while perfect sliding boundaries are assumed for the bottom and sides of the tank. The material surface conditions described in the previous section, are imposed on the free surface.

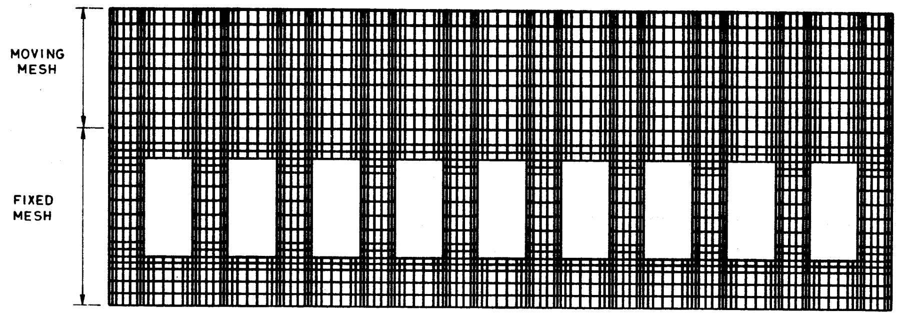

Figure 2.– Mesh discretization.

Numerical Results

The case corresponding to pure water, i.e. $R_e = 500000$, is studied first. Instantaneous configurations of the domain with velocity vectors and streamlines are plotted in Figure 3 for the tenth cycle. At $\omega t = n\pi$ (n integer), i.e. at maximum amplitude and minimum velocity, the recirculation around the blocks is obvious. At minimum amplitude and maximum velocity, most of the flow is concentrated in the upper half of the tank, nevertheless, the flow perturbation due to the presence of blocks is clear. The nonlinear evaluation of the free surface is reflected by its vertical motion at the center of the tank in spite of the vertical material velocity at that point being equal to zero. The presence of blocks induces a reduction in the wave height and a magnification of the horizontal particle oscillation at the center portion of the tank.

The time history of the free surface displacements at both ends of the pool is shown in Figure 4 for the first ten forced cycles. Notice that the amplitude at the left end is 0.41 (i.e. 12.4 cm in the model); this implies that the reduction of wave height due to the presence of blocks is very small, 15% after the tenth cycle. These results are in accordance with the previous ones obtained by the authors, Huerta and Liu (1988), where in a pool without blocks and filled with water, no apreciable damping was observed after the excitation ceased (see Figure 5).

In fact, these conclusions ought to be expected since the only energy losses, see Eqs. (10), are due to the viscous effect. The presence of blocks creates recirculation which increases the viscous losses; however, with such a large Reynolds number the viscous damping is small. This does not agree with the experimental results obtained by Muto et al. (1985). Several possible reasons may be advanced. Apart from the gap between blocks and glass which contradicts the hypothesis of pure two-dimensional flow, and the springs subjecting the blocks which create a flow disturbance and allow some movement of the submerged blocks; it is believed, that the the loci of conffetti like brass-foil introduced in the fluid to visualize the particle movement by Muto et al. (1985) increased the viscosity of the fluid. Therefore, the Reynolds number assumed, i.e. $R_e = 500000$ (pure water), is not the actual one.

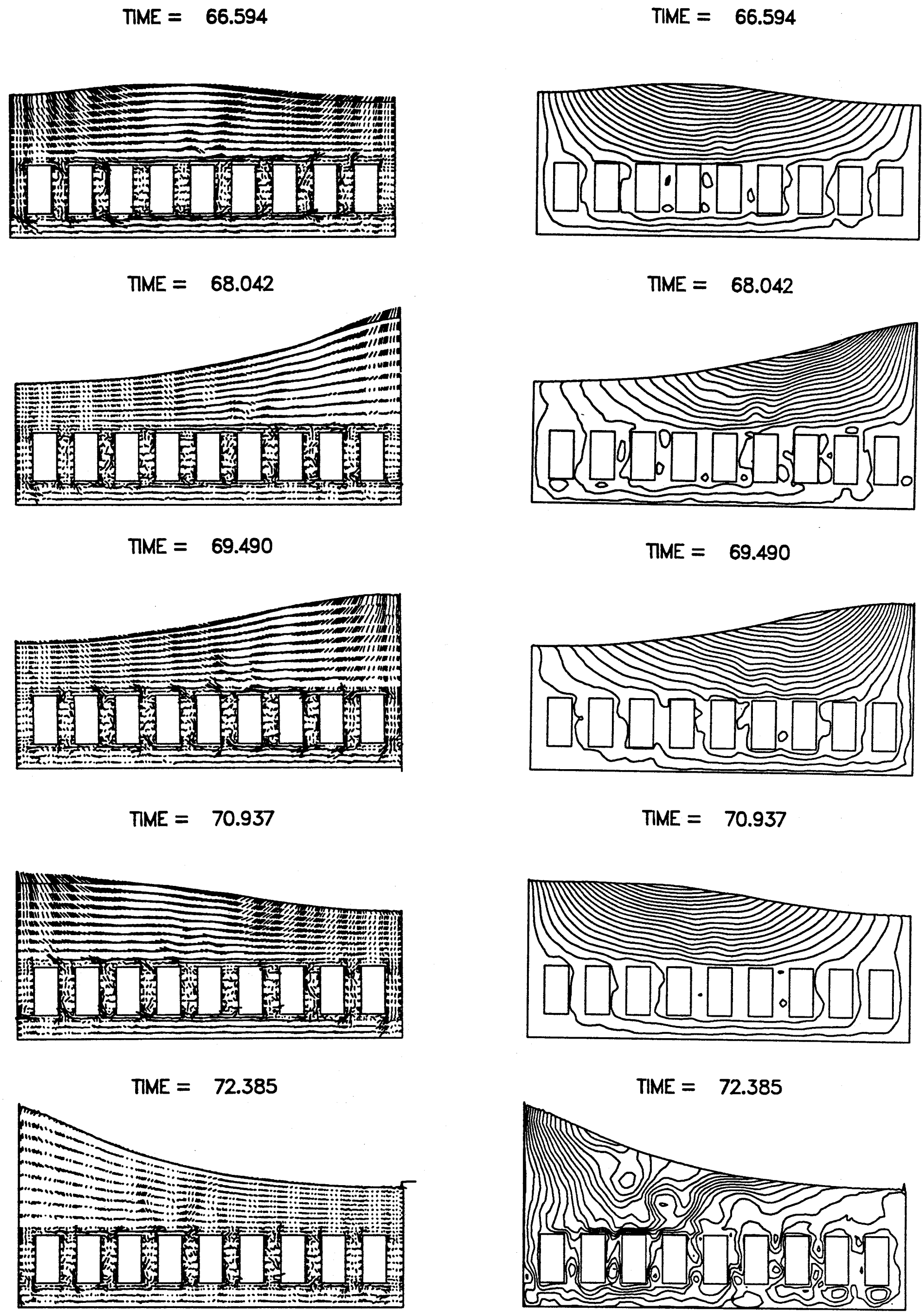

Figure 3.- Instantaneous configurations of the domain velocities, and streamlines for the tenth cycle.

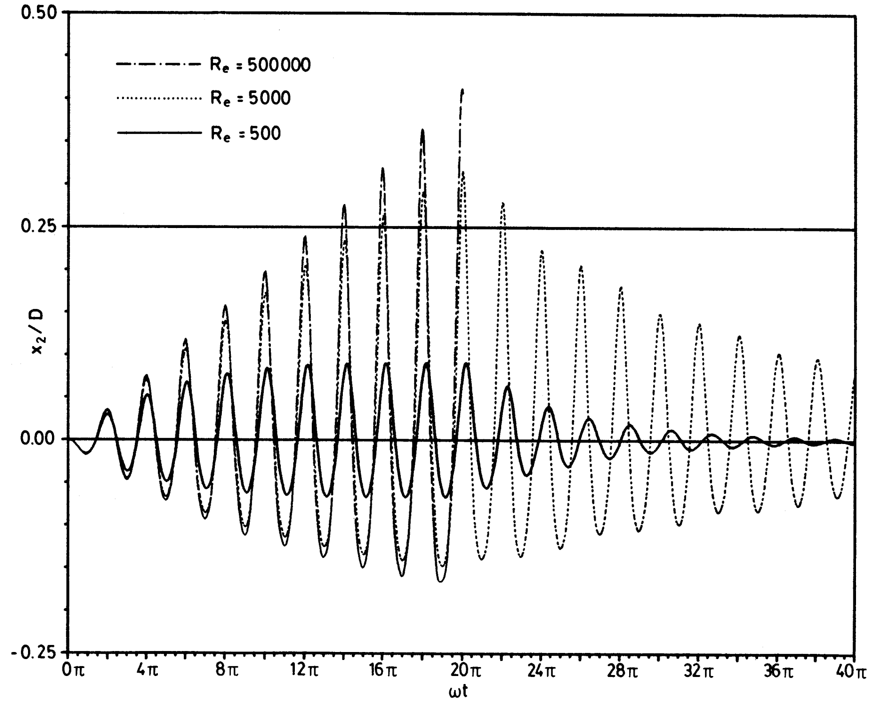

Figure 4.– Time history of the wave height at the left wall with submerged blocks.

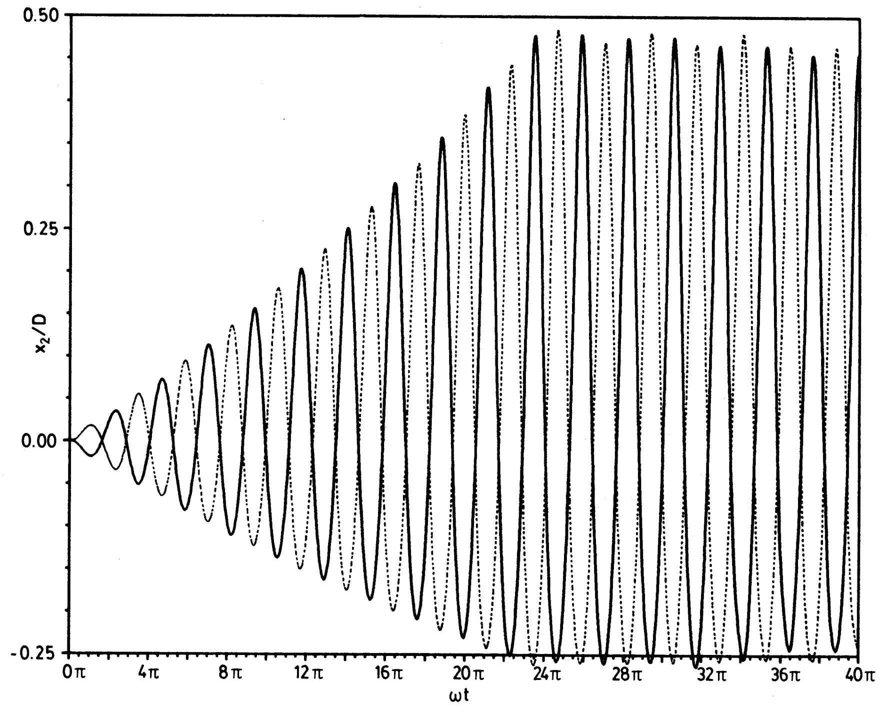

Figure 5.– Time history of the wave height at the wall without submerged blocks, $R_e = 500000$.

Because of the lack of information on the fluid constants (i.e. density and viscosity), a simple sensitivity analysis on the viscous effects is presented; that is, the Reynolds number is prescribed equal to 500 and 5000. Figure 4 shows the time history of the amplitude at the left end for all the Reynolds numbers. To be consistent with the previous analysis, during the first ten cycles forced vibrations are imposed while free vibrations are allowed after the tenth cycle for ten more cycles.

The influence of the viscous effects is clearly observed; for $R_e = 500$ a "2π steady state" is readily obtained after the first five cycles, and the maximum amplitude is 0.09 (i.e. 2.7 cm); this is a reduction of the 80% compared to the case of $R_e = 500000$. Moreover, the vibrations are clearly damped during the free vibration cycles, the measured damping factor is 6%.

For $R_e = 5000$ no stationary maximum wave amplitude is observed after ten forced cycles. The measured value after the tenth cycle is 0.32 (i.e. 9.6 cm) which is already larger than the stationary value observed by Muto et al. (1985), i.e. 6 cm; compared to the results for pure water the reduction is, however, of 29%. Logically the damping factor obtained for $R_e = 5000$, 2%, is lower than the experimental one, 3%.

Figure 4 also shows clearly the effect of the viscosity on the resonance frequency. Notice the out of phase motion of the surface for $R_e = 500$ while no phase lag relative to $R_e = 500000$ is apreciable for $R_e = 5000$ vibrating at the resonance frequency. This shows that for low damping the resonace frequency does not vary considerably and validates the choice of a constant excitation frequency.

CONCLUSIONS

Based on the results described in this study of large amplitude sloshing with submerged blocks, the following conclusions can be advanced:

1.– The pressure-velocity mixed formulation and the streamline upwind Petrov–Galerkin techniques are now usual methods to compute confined incompressible flows with large Reynolds numbers and strong convective effects. The implementation of the arbitrary Lagrangian–Eulerian formulation into such preexisting codes is simple. And moreover, this formulation allows an efficient and accurate description of large free surface motions.

2.– The generality of the ALE approach presented here allows to treat similarly confined flow with large boundary motion of free surface flows, it does not introduce an important computational burden and it is also valid for more complicated free surface problems (e.g. free surface with vertical and horizontal areas).

3.– While resonance frequencies are easily obtained by linear codes in sloshing problems, the surface displacements depend strongly on the non linearities if large motions occur. The ALE description takes into account the non linear effects on both the surface and interior of the domain. Moreover, numerical tests such as the one presented herein reduce considerably the experimental costs and allow, if needed, a complete sensitivity analysis on the material properties or geometry of the problem. That is, numerical simulations which were confined to "*qualitative*" results, can now be used to obtain "*quantitative*" results.

4.– While in the experimental analysis the reduction of wave height is only attributed to the presence of blocks. The present analysis shows clearly the combined effect of the submerged blocks and the viscosity of the fluid. The presence of blocks reduces in 15% the wave height after ten cycles when $R_e = 500000$. If $R_e = 5000$ the reduction in wave height is 34%, and if $R_e = 500$ it is over 80%. It seems obvious, after this numerical simulation, that the presence of submerged blocks amplifies the viscous losses because of the recirculation they induce. In the experimental analysis, the loci conffetti like brass-foil introduced in the fluid to observe the particle motion, may have increased the viscosity and consequently reduced the wave height. From a practical point of view, this may imply that, if the viscosity is small (large Reynolds number), the presence of blocks alone may not reduce enough the wave height to preclude a potential seismic hazard in spent fuel storage tanks.

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