Crack-Depth Effects in the Cylindrically Guided Wave Technique for Bolt and Pump-Shaft Inspections

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1 INTRODUCTION

Nuclear power plants have experienced failures of bolting and pump shafts. The industry is concerned about nondestructive evaluation (NDE) techniques that can be applied to these components. The cylindrically guided wave technique (CGWT) has been developed to detect simulated circumferential defects in long bolts and studs (Light et al. 1988). The ultrasonic CGWT employed zero-degree longitudinal waves constrained to travel within the boundary of the cylindrically shaped components during inspection. Since the ultrasonic wave is guided down the length of the component, mode conversion occurs during the propagation of the wave. These mode-converted signals are dependent upon the diameter of the component under inspection and the longitudinal and shear-wave velocities of the component material.

The decay and growth of an ultrasonic pulse and the associated trailing pulses as they passed down a solid cylindrical wave guide were calculated in terms of the cylinder dimensions and the elastic constants of the material (Varey, 1976). The dispersion of elastic waves in solid circular cylinders was investigated for a wide range of wave lengths (Hudson, 1943).

When a longitudinal wave is guided to travel along a cylinder and impinges onto a circumferential defect, the wave is scattered at the crack in the cylinder surface. In the present work, the wave scattering at the circumferential crack in a long cylinder is investigated. An exact formulation is employed to satisfy the stress-free conditions at the crack surfaces and the cylinder surface. The guided waves are shown to be dominant in the place where the ultrasonic receivers are usually located. The transfer factor of the scattered wave is calculated for a wide range of the frequency spectrum. The scattered waveforms at a distance away from the crack are calculated. The effects that the crack depth has on the waveforms in CGWT are presented.

2 CGWT SIGNALS

The CGWT signals used to detect the circumferential defects in a cylinder are expected to be dependent upon the depth of the crack. In the theoretical development, a long cylinder is assumed to contain a circumferential crack. The crack tip has an arbitrary radius d, which is less than the cylinder radius a. A time-harmonic incident wave with frequency ω is assumed to impinge onto the crack. The scattered field is governed by the following equations of motion (Tsai and Kolsky, 1967)

$$(\lambda + 2\mu)\ \nabla\nabla\cdot\underline{u} - \mu\nabla \times \nabla \times \underline{u} = \rho\frac{\partial^2\underline{u}}{\partial\tau^2} \tag{1}$$

where τ is the time variable, $\underline{u}$ is the displacement vector, ρ is the density of the medium and λ and μ are the Lamè constants. The problem considered is axisymmetrical and cylindrical coordinates (r,θ,z) are used, with the z-axis directed along the axis of symmetry. There are only two non-vanishing components of the displacement: U_r and U_z. The equations of motion are reduced to a set of two wave equations for the dilatation χ and the rotation Ω.

The surfaces of the circumferential crack are located at z=0 and free from stresses. The boundary conditions for the scattered wave field at z=0 can be written as (Tsai, 1989)

$$\sigma_{zr} = 0 \tag{2}$$

$$\sigma_{zz} = -p_o e^{i\omega\tau} \qquad d<r\leq a \tag{3}$$

$$U_z = \begin{cases} o & r<d \quad (4) \\ w(r)e^{i\omega\tau} & d\leq r<a \quad (5) \end{cases}$$

Furthermore, the surface of the cylinder is also free from stresses. The surface conditions can be described as

$$\sigma_{rr} = 0\ , \quad \sigma_{rz} = 0 \qquad\qquad r = a \tag{6}$$

The solutions for χ and Ω are obtained from the wave equations. The stresses and displacements are calculated in terms of χ and Ω and used to satisfy the boundary conditions (4) - (6) and (2). The results are written in terms of a parameter function $\phi(n)$. Satisfying the stress boundary condition (3) yields an integral equation for $\phi(n)$, which can be determined in terms of the input parameters.

It can be seen from the results that the terms of the displacement at a large distance from the crack are vanishing except for the guided wave terms. The guided waves become predominant in the place where the ultrasonic receivers are usually located. The displacement U_z at the distance z from the crack in the direction of the cylinder axis is written in terms of the guided waves in the following form of progressive waves

$$U_z = \frac{1-\nu}{\mu}\ \frac{2}{\pi}\sum_{j=1}^{\infty} A_j \cos[\omega(t - \gamma_j z/c_2)] \tag{7}$$

$$A_j = R_{1j}(\bar{k}f_{j1} - 2f_{j2}) + 2\bar{k}\ R_{2j}\ f_{j3} \tag{8}$$

$$f_{j1} = \int_D^{\infty} \phi(n) \int_0^1 (1-m^2)^{-1/2} I_{j1} dmdn \tag{9}$$

$$f_{j2} = \int_D^{\infty} \phi(n) \int_0^1 (1-m^2)^{-1/2} I_{j2} dmdn \tag{10}$$

$$f_{j3} = \int_D^{\infty} \phi(n) \int_0^1 (1-m^2)^{-1/2} I_{j} dmdn \tag{11}$$

where ν is Poisson's ratio and D is equal to d/a. The shear wave speed is c_2 and the frequency factor is $\bar{k} = a\omega/c_2$. The quantity γ_j is the root of the following cylinder frequency equation:

$$\Delta(\xi) = (2\xi^2-1)^2 I_o(\bar{k}\alpha) I_1(\bar{k}\beta) - 4\alpha\beta\xi^2 I_o(\bar{k}\beta) I_1(\bar{k}\alpha) + 2\alpha I_1(\bar{k}\alpha) I_1(\bar{k}\beta)/\bar{k} \tag{12}$$

$$\alpha^2 = \xi^2 - 1/k^2, \quad \beta^2 = \xi^2 - 1, \quad k^2 = 2(1-\nu)/(1-2\nu) \tag{13}$$

The functions $I_o(x)$ and $I_1(x)$, respectively, are the zeroth and the first order modified Bessel functions of the first kind. The values of γ_j are equal to the inverse of the ratio of the cylinder phase velocity to the shear wave speed. For the fundamental mode of wave propagation, the characteristic root is γ_1. For $\gamma_1 > 1$, the quantity I_{12} in (10) for j=1 has the following form

$$I_{12} = \begin{cases} \bar{D}e^{-\bar{k}nd_1} \sinh(m\bar{k}d_1) - 2\gamma_1^2 e^{-\bar{k}nd_2} \sinh(m\bar{k}d_2), & n > m \\ 2\gamma_1^2 e^{-\bar{k}md_2} \cosh(n\bar{k}d_2) - \bar{D}e^{-\bar{k}md_1} \cosh(n\bar{k}d_1), & m > n \end{cases} \tag{14}$$

$$d_1^2 = \gamma_1^2 - k^{-2}, \quad d_2^2 = \gamma_1^2 - 1 \tag{15}$$

$$\bar{D} = 2d_1^2 + 1 \tag{16}$$

The quantities f_{j1}, f_{j3}, R_{1j} and R_{2j} are functions of the characteristic root γ_j.

3 WAVEFORM CALCULATION

The expression for the displacement U_z in (7) is used to calculate the received waveforms at distance z, which is measured from the crack location. The scattered wave emitted from the crack is dispersive. To understand the dispersive nature of the scattered wave, a simple waveform as shown in Fig. 1 is assumed to be scattered at the crack. The waveform

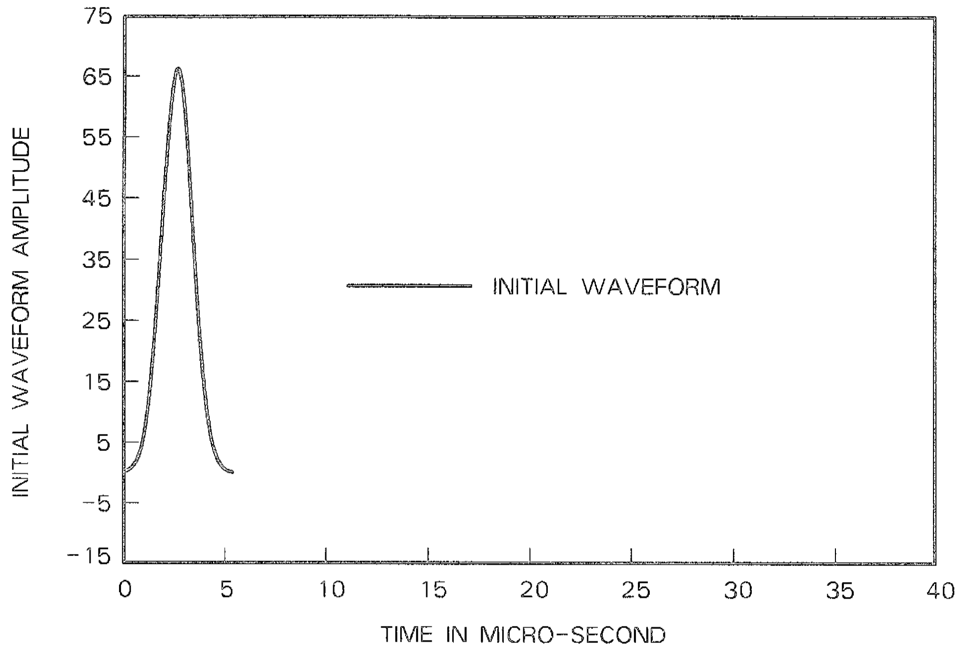

Fig. 1. Initial waveform used in the calculations for different values of D which is the ratio of the crack-tip radius d to the cylinder radius a.

involves a representative range of Fourier spectrum. For this range of Fourier spectrum, the transfer factor A_1 is calculated for three different values of the ratio D = d/a for the fundamental mode of the guided wave. The values of the transfer factor are shown in Fig. 2 as a function of the frequency factor $\bar{k} = a\omega/c_2$. To have a better understanding of the range of the frequency factor, the inverse of the phase velocity normalized by the shear wave speed is also shown in the same figure. The first order approximation of the parameter function $\phi(n)$ is used in the calculations. For the transfer factors shown in Fig. 2, the waveforms at the same distance from the crack are calculated and shown in Fig. 3. The waveforms at two different distances away from the crack are also calculated and the results show the dispersive nature of the scattered waveforms as a function of the propagation distance.

Figure 3 clearly shows the effects that the crack depth has on the amplitudes of the scattered waves. The amplitude and width of the main scattered pulses in Fig. 3 are seen to be different from those of the initial waveform in Fig. 1.

4 CONCLUSION

The cylindrically guided wave technique (CGWT) has been developed and applied to the inspections of bolt and pump shaft in a nuclear power plant. The CGWT has been shown to be able to detect defects after an ultrasonic wave has traveled through a long metal path in a metallic bolt or shaft.

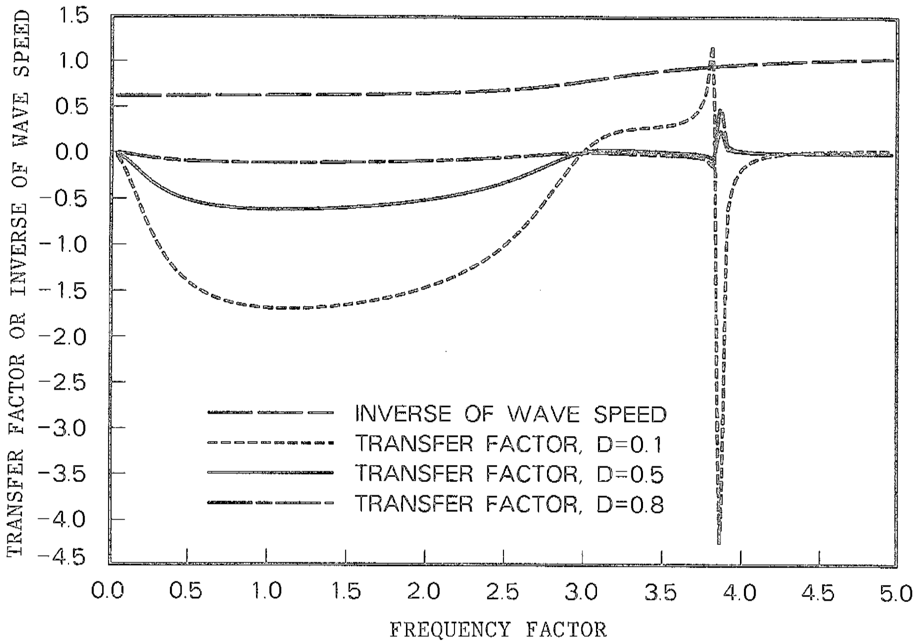

Fig. 2. Transfer factor of three different defect sizes: D=0.1, D=0.5 and D=0.8.

A theoretical study has been made on the wave scattering at the circumferential crack in a long cylinder. The crack tip has an arbitrary radius d, which is less than the cylinder radius a. A time-harmonic incident wave with frequency ω is assumed to impinge onto the crack. The scattered field involves several terms and has satisfied the stress-free conditions at the crack surfaces and the cylindrical surface.. In this exact three-dimensional formulation, it is shown that the terms of the displacement at a large distance from the crack are vanishing except for the guided wave terms. The guided waves become predominant in the place where the ultrasonic receivers are usually located. The displacement in the direction of the cylinder axis is written in the form of guided progressive waves.

In the process of solution, an integral equation is established. The highest order approximation of the solution is obtained and used in the numerical calculations of the scattered waveforms. The transfer factor of the scattered wave is calculated for a wide range of the frequency spectrum. The scattered waveforms at a distance away from the crack are calculated and presented for various values of the ratio of d to a. Also described is the variation of the scattered waveform with respect to the distance of travel. The result clearly reveals the effects that the crack depth has on the amplitude and width of the scattered waveforms. It also enhances the capacity of CGWT to determine the depth of a crack.

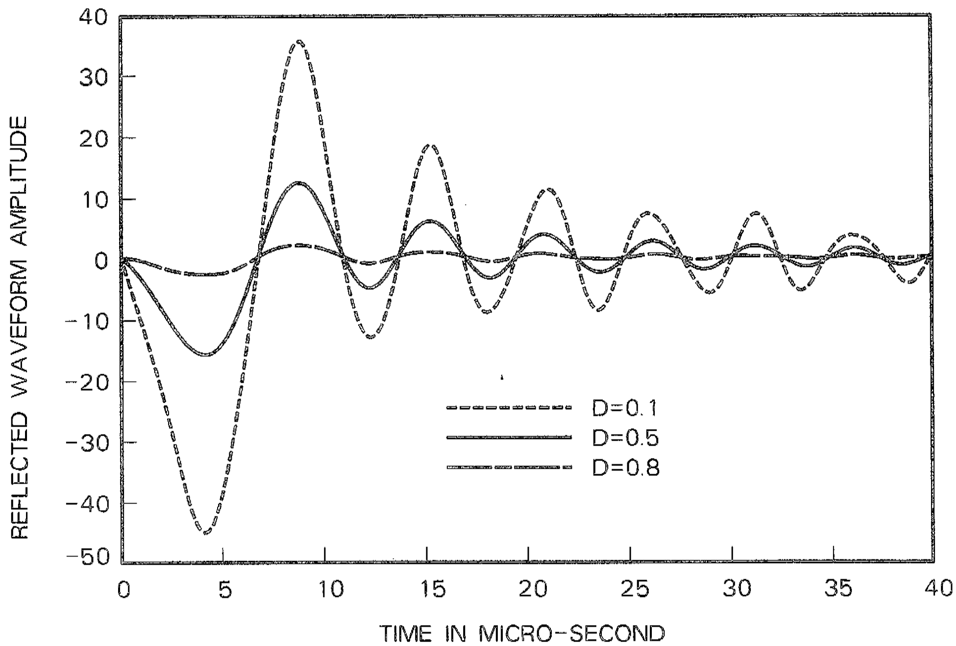

Fig. 3. Reflection waveform from a crack as a function of time for crack sizes D=0.1, D=0.5 and D=0.8.

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