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A reliability approach for optimal design of structural components

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ABSTRACT: The objective of this paper is to present a reliability approach for optimal design of structural components. The approach eliminates the uncertainty in the design when employing the factor of safety concept. We consider the failure distribution, cost of replacements, and cost of manufacturing of the the component to determine the optimal principal dimension and the optimal replacement cycle (if exists) which minimize the total cost per cycle.

1 INTRODUCTION

Conventional engineering design uses a deterministic approach. It disregards the fact that material properties, the dimensions of the components, and the externally applied loads are statistical in nature (Dieter 1983). In conventional design, these uncertainties can be reduced by employing the concept of factor of safety, however, the design procedures based on this factor do not consider these uncertainties in a rational manner. In fact, the selection of the factor of safety is quite often arbitrary (Rao and Reddy 1977). The probability of failure exists regardless of the value of the factor of safety, which has led to the extensive study of probabilistic concepts and reliability-based design procedures.

Since reliability is the best available quantitative measure of the performance integrity of the designed components, researchers have investigated the use of the reliability theory in the design of structures. In designs based on reliability, a structure is designed for a specified probability of failure depending on the type of structure and consequences of failure. Mischke (1970) presents a relationship between the factor of safety and reliability of a mechanical element. Rao (1979) presents a reliability analysis and design of epicyclic gear trains which he considers as a series-parallel network. His design criterion is that the reliability of the gear train either in bending and surface wear modes are considered. Elmaraghy and Siddall (1976) develop a model for fatigue failure, which considers the variability of the material properties as well as the variability of the applied load. Rao and Reddy (1977) analyze the reliability of machine tool structures in various failure modes. They (Rao and Reddy 1977) assume that the random design parameters follow normal distribution, and the reliability of the machine tool structure is obtained by considering the structure as a weakest-link system. Siddall (1983) estimates the

reliability of a single member of a structure for given loads and strengths. Latcha and der Hovanesian (1993) show the links between the factor of safety and the reliability methods of design. Recently, Pham-Gia and Turkkan (1994) developed classical and Bayesian approaches to estimate the median safety factor for the general stress-strength model.

Although components are designed based on either the concept of factor of safety or reliability, they still experience failures. The choice of the factor of safety or the acceptable reliability level affects the size of the principal dimension of the component to be designed. A large factor of safety or reliability level will result in a larger principal dimension which will require higher cost to manufacture. This will also increase the mean time to failure (MTTF) of the component resulting in less frequent replacements of the component. On the other hand, a small factor of safety will result in a smaller principal dimension requiring less cost to manufacture and more frequent replacements over the life of the component. There exists an optimal principal dimension which minimizes the cost of replacements and the cost of component manufacturing. In other words, the optimal principal dimension is a function of the initial cost of the component, cost of failure, cost of repair, and losses due to failure. The cost of parts and components may be expressed as a function of the principal dimension of the component. It is found (Kogan 1976) that the cost of some parts or components can be expressed as

$$(1) \quad C = aF^2 + bF + c_0$$

where

| | |
|-------------|---|
| C | is the cost of the component |
| a, b, c_0 | are numerical coefficients |
| F | is the principal dimension of the component |

Few researchers have investigated the use of the minimum cost criterion to achieve the optimum reliability of the components. Fenton (1974) derives an analytical expression for the total cost of equipment, including initial capital cost and the present value of the escalated cost of the expected failures throughout the service life of the equipment. The total cost expression is a function of the reliability of the equipment. Nelson and Hayashi (1974) present an approach whereby the design and replacements of finite life equipment are considered. Jardine (1973) develops an optimal preventive policy for equipment subject to breakdown. Sherif (1982) presents a comprehensive literature survey of the optimal inspection and maintenance schedules of failing systems.

In this paper, rather than using a factor of safety to design parts, components, and structures, we introduce a new approach for the optimal design of a structure by considering its failure distribution, cost of manufacturing, and cost of failure replacements. In this approach we determine the optimal preventive replacement schedule and the principal dimension of the structure corresponding to the optimal mean time to failure. In other words, we determine the principal dimension of the structure (using experimental data) without arbitrarily choosing a factor of safety. Finally, we present a numerical example to demonstrate the use of this approach.

2 THE OPTIMUM RELIABILITY MODEL

In this model, the relationship between the principal dimension and the MTTF of the structure or part to be designed is determined. For example, experiments can be conducted by subjecting samples similar to the structure to be designed but having different principal dimensions to the same loading conditions which will be applied in real life. A relationship between the principal dimension and MTTF can then be determined as shown in Figure 1. Next, we develop a reliability model which considers such a relationship.

The following notations will be utilized in the development of the reliability model:

| | |
|-----------------|---|
| a, b, c_o | constants |
| \bar{C} | expected cost per cycle |
| C_f | cost of a failure replacement |
| C_p | cost of a preventive replacement ($C_p < C_f$) |
| C_f^t | cost of a failure replacement at time t |
| C_p^t | cost of a preventive replacement at time t |
| $C(t_p, MTTF)$ | expected cost per unit time for given MTTF and t_p |
| e | effective interest rate per unit time of t_p |
| F | the principal dimension of the structure |
| $F(t)$ | failure distribution function of the structure |
| $f(t)$ | failure density function of the structure |
| $g_f(t)$ | repair density function of the unplanned replacement (failure replacement) |
| $g_p(t)$ | repair density function of the preventive replacement |
| \bar{t} | expected cycle length |
| t_p | length of the planned (preventive) replacement cycle |
| T_f | mean time to perform an unplanned replacement $\int_0^{\infty} t g_f(t) dt$ |
| T_p | mean time to perform a planned replacement $\int_0^{\infty} t g_p(t) dt$ |
| T_o | operating time of the structure |
| α, β | constants |
| θ | ratio between C_f and C_p , ($\theta > 1$) |

In order to determine the principal dimension and the optimal length of the preventive replacement cycle, we propose the following replacement policy: perform a preventive replacement once the component has reached a specified age t_p . In

addition, perform failure replacements when failures occur. We now develop a total cost criterion which we seek to minimize.

The expected cost per unit time, $C(t_p, \text{MTTF})$ is

$$(2) \quad C(t_p, \text{MTTF}) = \bar{C} / t$$

where \bar{C} , the expected cost per cycle, is composed of:

1. Cost of a preventive replacement as a function of the principal dimension. This cost is obtained as follows:

$$(3) \quad C_p^{t_p} = C_p(1 + e)^{t_p} P(t_p)$$

where C_p is assumed to be equal to the initial cost of the structure, i.e., using Eq. (1), one can express C_p as

$$(4) \quad C_p = a(\text{MTTF})^2 + b(\text{MTTF}) + c_0$$

where $P(t_p)$ is the probability of a preventive replacement $\int_{t_p}^{\infty} f(t) dt$.

$$\text{MTTF} = \int_0^{\infty} t f(t) dt$$

2. Cost of a failure (unplanned) replacement which is obtained as follows:

$$(5) \quad C_f^t = C_f(1 + e)^t [1 - P(t)]$$

For some components, it is found (Kogan 1976) that the time (which implies cost) required to perform an unplanned replacement is related to the time required to perform a preventive (planned) replacement by:

$$(6) \quad \frac{T_f}{T_o} = \alpha + \beta e^{T_p / T_o}$$

Utilizing Eq. (6), one can express C_f in terms of C_p (for given T_f , T_p , and T_o) as

$$(7) \quad C_f = \theta C_p$$

The expected cost per cycle, \bar{C} , is then obtained as

$$(8) \quad \bar{C} = (1 + e)^{t_p} \left((1 - \theta) \int_{t_p}^{\infty} f(t) dt + \theta \left(a \left[\int_0^{\infty} t f(t) dt \right]^2 + b \left[\int_0^{\infty} t f(t) dt \right] + c_o \right) \right)$$

The expected cycle length, \bar{t} , is composed of

1. Length of a preventive cycle including the time required to perform the preventive replacement, i.e.,

$$(9) \quad (t_p + T_p) P(t_p)$$

$$\text{where } T_p = \int_0^{\infty} t g_p(t) dt$$

2. Length of a failure cycle including the time required to perform the unplanned replacement, i.e.,

$$(10) \quad \left(\int_{-\infty}^{t_p} t f(t) dt / [1 - P(t_p)] + \int t g_f(t) dt \right) \left(\int_{\infty}^{t_p} f(t) dt \right)$$

The expected cycle length is the sum of Eqs. (9) and (10). Substituting Eqs. (8), (9), and (10) into Eq. (2), we obtain:

$$(11) \quad C(t_p, \text{MTTF}) = [(1 + e)^{t_p} \left((1 - \theta) \int_{t_p}^{\infty} f(t) dt + \theta \left(a \left[\int_0^{\infty} t f(t) dt \right]^2 + b \left[\int_0^{\infty} t f(t) dt \right] + c_o \right) \right)] / \left(t_p + \int_0^{\infty} t g_p(t) dt \right) \left(\int_{t_p}^{\infty} f(t) dt \right) + \left[\int_0^{t_p} f(t) dt \right] \left[\int_0^{\infty} t g_f(t) dt \right] + \left[\int_0^{t_p} t f(t) dt \right]$$

3 NUMERICAL EXAMPLE

In this example, we apply the proposed approach to structures with Weibull distributions. Again, the objective is to determine the optimal replacement schedule and the optimal MTTF of the component. We choose the Weibull distribution because it is widely used for many engineering problems. Elsayed (1982) shows that the failures of many of the mechanical components of the low profile cranes follow

Weibull distributions. Rabon (1981) indicates that Weibull distribution is the second most common distribution (following the lognormal) associated with mechanical and electro-mechanical reliability. We now apply the new approach to an axle of 1000 mm length made of grade 45 steel.

The failures of such an axle follow a Weibull distribution with one mode of failure. The optimal parameters (MTTF and the optimal length of the preventive replacement interval) are determined using Eq. (11). The optimal diameter corresponding to these parameters can be determined from a figure similar to Figure 1. The probability density function of the two-parameter Weibull distribution is

$$(14) \quad f(t) = kt^m e^{-\frac{kt^{m+1}}{m+1}}$$

and

$$(15) \quad \text{MTTF} = \frac{\Gamma\left[\frac{1}{m+1}\right]}{(m+1)\left[\frac{1}{m+1}\right]^{\frac{1}{m+1}}}$$

where

k is the scale parameter

m is the shape parameter

The coefficients of Eq. (9) for the cost of manufacturing the axle are: $a = 0.00177$ \$/mm², $b = -0.70$ \$/mm, $c_o = \$125.0$, and $e = 0.05$ per unit time.

Let $C = C_p$, $g_p(t) = \lambda_1 e^{-\lambda_1 t}$, $g_f(t) = \lambda_2 e^{-\lambda_2 t}$, $\lambda_1 = 100$, and $\lambda_2 = 33.33$.

Substituting the above values and Eqs. (14) and (15) into Eq. (3), we obtain

$$(16) \quad C_p^{t_p} = (1 + 0.05)^{t_p} \left[0.00177 \int_0^{\infty} t f(t) dt - 0.70 \int_0^{\infty} t f(t) dt + 125.0 \right] P(t_p)$$

where

$$(17) \quad P(t_p) = \frac{-kt_p^{m+1}}{e^{m+1}}$$

Also, the term $\int_0^{t_p} t f(t) dt$ needs to be evaluated:

$$(18) \int_0^{t_p} t f(t) dt = \left[\frac{m+1}{k} \right]^{\frac{1}{m+1}} \gamma \left[\frac{m+2}{m+1}, \frac{kt_p^{m+1}}{m+1} \right]$$

where $\gamma(x)$ is the incomplete gamma function of x .

Substitutions of Eqs. (14) through (18) into Eq. (11) result in a complex expression for $C(t_p, \text{MTTF})$. The optimal values of t_p and MTTF are then determined numerically. The procedure for determining these values involves finding the failure distribution for each point in Figure 1. The parameter of these distributions are then utilized in Eq. (11) and the total cost per cycle is estimated for each point in Figure 1. The optimal MTTF and t_p are those corresponding to the minimum cost equation.

Figure 2 shows the relationship between t_p and $C(t_p, \text{MTTF})$ for a given range of MTTF (see Figure 1). Extensive numerical results show that the optimal principal dimension corresponds to components whose MTTF and t_p are equivalent (see Figure 2 for $m = 4, k = 1, \text{MTTF} = 1.267, t_p = 1.24$).

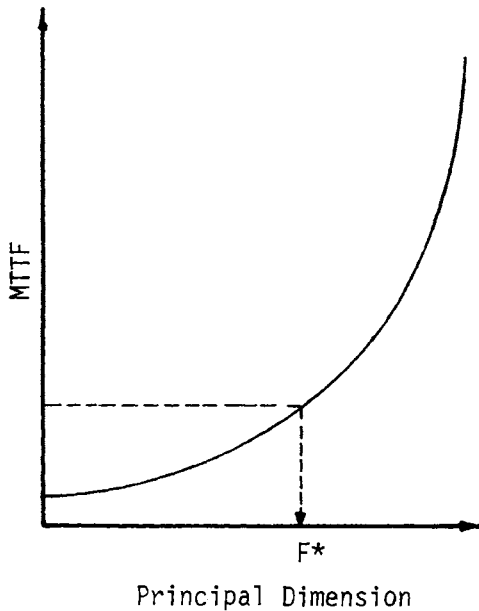


Figure 1. Experimental relationship between the principal dimension and MTTF

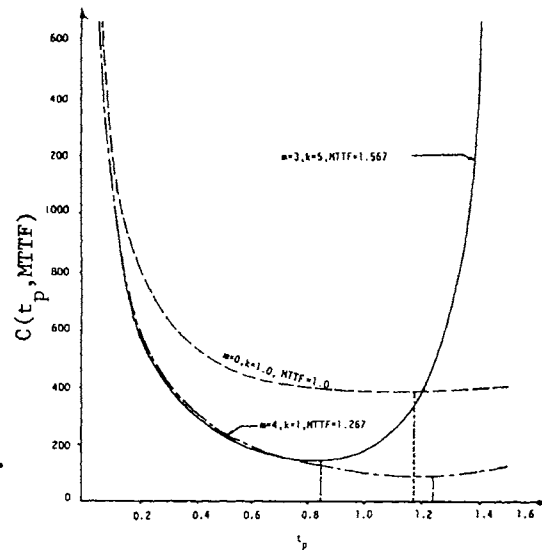


Figure 2. t_p versus $C(t_p, \text{MTTF})$ for different failure distributions

4 CONCLUSIONS

In this paper, we have presented a new approach for determining the optimal principal dimension of components with one or multistates of failures. This approach eliminates the use of factor of safety concept. The optimal principal dimension is obtained based on a cost criterion which includes the cost of replacements and cost of manufacturing of the component.

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