PROBABILISTIC FREQUENCY VARIATIONS OF CONCRETE STRUCTURES

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SUMMARY

The response of nuclear power plant structures to earthquake type excitations is not very sensitive to slight variations of the frequency characteristics of structures. This is so mainly because these frequencies lie in a range where current design criteria can simply be considered as a broad band input of uniform amplitudes. However, since these structures, during earthquakes, act as filters they greatly amplify the response of equipment whose frequencies are at or near their natural frequencies. Thus the estimation of these structure frequencies assumes significant importance both from a safety and cost points of view.

The actual in-situ frequencices of structures may differ from the calculated frequencies due to two major sources of uncertainty: modelling techniques and variation of material properties. To account for these frequency variations, the floor response spectra are usually broadened at their peak response. Presently this broadening is achieved by judgment decisions or by using extreme high and low parameter values to obtain upper and lower bounds on the frequency variation.

To place this decision process on a more rational basis, this paper studies the probabilistic distribution of the structure frequencies using data on the variation of the material properties only. Published empirical distributions of the modulus of elasticity and beam deflections are used in this process. The latter is used to obtain probabilistic distributions of the *EI* product (Modulus of elasticity × Moment of inertia). The fundamental distribution for *E* is determined through its relation to the compressive strength of concrete by $E = 33\ W^{3/2}\sqrt{f'_c}$.

Thus, for a prismatic beam, the frequencies are related to the fourth root of f'_c and the implied distribution of the frequencies may be derived. Also since beam deflections are lognormally distributed, the frequencies are to be proportional to the reciprocal of log-normal random variables. Tables for both the *E* only and the *EI* analyses are developed, showing the nominal frequencies, their expected values, variances and confidence intervals at 80%, 90% and 99%.

The solution is extended to lumped parameter systems using matrices where it is assumed that the stiffness matrix is affected probabilistically by a scalar multiple of *EI*. Furthermore, frequency variations of systems including soil-structure interaction effects are studied. An appropriate choice of coordinates uncouples the structure stiffness coefficients from the lumped soil-structure interaction compliances. This allows the independent application of variabilities to the super-structure and the soil-structure interaction lumped parameters. Additional examples are provided.

1. Introduction

The dynamic response of structures and the equipment located on them is a function of both the dynamic characteristics of the structure and equipment and the dynamic forcing environment. Although the results of the present discussion can be useful in many situations, this study is mainly concerned with the response of such equipment during earthquakes. Because of the great variety of equipment and systems with different frequencies, mode shapes, and damping characteristics in nuclear power plants, it has been found convenient to characterize the response of equipment by floor response spectra for several damping values (see, for example, Hadjian [1]).

The floor response spectra are functions of the input motion and the frequencies, mode shapes, and damping characteristics of the structure. The input motion is generally prescribed as the smoothed design spectrum curves for the site and will not be considered here. Although the amplitude of the floor response spectra is determined by the dynamic characteristics of the structure, the location of the peakedness of the floor response spectra is a function of the structure frequencies [1]. Variations from calculated natural frequencies are expected in the as-built structures, and therefore the floor response spectra are broadened at their peak response to account for this variation. Thus the determination of the in situ structure frequencies becomes important from safety and cost points of view.

Broadening can be achieved in many different ways. The designer can estimate a range of frequency variation by applying his experience and judgement. Alternately an upper or lower bound variation of the frequencies can be obtained by using extremely high and low values of those parameters that affect the structural frequencies. The present paper discusses a more rational approach based on probability distributions of the structural and soil parameters.

There are several sources of uncertainty that contribute to the variations of frequencies. Among these are nonlinear effects, variabilities of the compressive strength of concrete, the degree of quality control of the concrete mixes, aging of concrete, cross-sectional dimensions, cracking pattern of concrete sections, foundation soil properties, and mathematical modeling techniques.

The modeling techniques are a class of problems by themselves and will not be considered here. Although nonlinear effects are being researched, they are not as yet commonly used in the design of nuclear power plants mainly because under the conservative design loading criteria specified, stresses in structures, systems, and components are kept below yield levels. Thus, the present paper is restricted to only the study of the probabilistic effects of the properties of the stiffness of the structure and the foundation soil on the natural frequencies of the structure.

2. Data on the Variability of Concrete Properties

The variability of the modulus of elasticity, E, and the stiffness, EI, where I is the area moment of inertia, will be treated separately. It must be emphasized that the data on the variabilities used herein are obtained from static tests and the important assumption is made that, although the dynamic mean values would be different, the frequency distribution of the dynamic properties will have the same characteristics as the static properties. As more data on the dynamic properties become available, this assumption can be modified.

2.1 Data on the Variations of E

The design of a structure according to the ACI 318-71 Building Code [2] starts with a specified value of the unit compressive strength of concrete, f'_c. The modulus of elasticity E, is then calculated according to

$$E = 33w^{1.5}\sqrt{f'_c} \qquad (1)$$

Since there are variations in actual f'_c and actual unit weight of concrete, w, there will be variations in actual E. Further, eq. (1) is not an exact relation, being a least squares fit of sampled data as developed by Pauw [3].

The effect of the variability of w on E is relatively minor compared to the effect of the variability of f'_c and will therefore be neglected. However, two other sources of variability should be considered, and each must be considered separately to determine the total effect on E. Throughout the analysis, actual values (i.e., random variables) will be denoted by a tilde (∿). The notations Exp (·) and Var (·) denote expectation and variance operators, respectively.

As reported by ACI Committee 214 [4], the actual compressive strengths tend to be normally distributed about a mean which is somewhat higher than the specified value, f'_c. To insure both quality control and adequate compressive strength, concrete specifications usually stipulate the number of concrete test specimens that could be allowed to fail below the specified strength (usually one in ten specimens) and a coefficient of variation of the compressive strength itself (usually between 0.1 and 0.15). Thus the average compressive strength of concrete, f_{cr}, can be obtained from

$$f_{cr} = \mathrm{Exp}\left[\tilde{f}'_c\right] = \frac{f'_c}{\left(1 + z_p V\right)} \qquad (2)$$

where z_p = the fractile value of a standard normal distribution with cumulative value p, and V = coefficient of variation of f'_c.

The related standard deviation, σ, is found directly from $\sigma = \sqrt{\mathrm{Var}\ \tilde{f}'_c} = V\cdot f_{cr}$. For example, a concrete mix with a specified strength of 3000 psi, a coefficient of variation of 0.12, and having 10% of the tests failing below the specified strength would have an actual strength the average value of which would be

$$f_{cr} = \frac{3000}{1 + (-1.28)\ (0.12)} = 3545 \text{ psi and } \sigma = (0.12)\ (3545) = 425 \text{ psi}$$

Having determined the variability of $\tilde{f}'_c$, based on specification requirements of f'_c, V and the number of test specimens allowed to fail below f'_c, the variability of E can be calculated if eq. (1) were exact.

Studies by Pauw [3], found the least squares equation to be

$$E_c = 33.6w^{1.5}\sqrt{f'_c} \qquad (3)$$

The data used in this study are reproduced in Fig 1. Two points should be noted: the revised scale constant from 33.6 in eq. (3) to 33 in eq. (1) and the scatter of data around the least squares line in Fig. 1. To account for both deviations an error term $\tilde{u}$ must be considered:

$$\tilde{E} = 33w^{1.5}\sqrt{\tilde{f}'_c} + \tilde{u} \qquad (4)$$

To find the mean and variance of $\tilde{u}$, only the data corresponding to the region of normal weight concrete in Fig. 1 are used. This was decided mainly because nuclear power plant construction uses normal weight concrete.

To determine the distribution of $\tilde{E}$, the Exp $(\tilde{u})$ and Var $(\tilde{u})$ are needed. Direct numerical calculations of unbiased estimates using the 31 points in Fig. 1 for normal weight concrete show that Exp $(\tilde{u})$ = 99,000 psi and Var $(\tilde{u}) = 108 \times 10^9$ $(\text{psi})^2$.

2.1.1 Aging of Concrete

Concrete unit compressive strengths are usually specified at the age of 28 days. The progressive hardening and increase in strength of concrete has been studied for many projects. Fig. 2 from Dunham [5] shows a typical relation of compressive strength to age. Since statistical data from these studies are lacking, the reasonable assumption is that the variance is constant for all times after 28 days. This assumption leads to the use of a modified specified unit compressive strength, f'_{ct}, related to f'_c by

$$f'_{ct} = k_t f'_c \qquad (5)$$

where k_t is the ratio of the mean or average compressive strength at time t after concrete placement to the mean at 28 days. k_t values from Figure 2 are as follows, where the subscript denotes time elapsed since placement in years:

$k_{0.5} = 4600/3800 = 1.21$, $k_1 = 4720/3800 = 1.24$, $k_3 = 5000/3800 = 1.32$, $k_5 = 5160/3800 = 1.36$

Since fuel loading of a nuclear power plant occurs about three years after concrete placement of the containment dome, it seems that any additional strength gained after the fuel loading is insignificant.

It must be remembered that Fig. 2 is based on tests of laboratory cured specimens. Such ideal conditions of moist air at 70°F cannot be duplicated in the field. Thus, the amount of moisture under field conditions may accelerate or retard the rate of gain of strength. However, it is felt that this effect would be negligible after a period of about three years.

2.2 Data on the Joint Variations of E and I

Although it would be desirable to evaluate the variability of I alone, empirical data are available on the inverse of the EI product only. Two sources of variability in I come from the cracking of concrete cross sections and dimensional tolerances. The latter is the less significant, and the former is very difficult to assess analytically because of the uneven pattern of cracking. However, since E and I appear as a product in the stiffness coefficient calculations, only a univariate distribution of the product is actually needed. This can be derived from a recent report by ACI Committee 435 [6] on beam deflections. The data from that report are reproduced here in Fig. 3. The distribution of the ratio of actual to specified deflection is shown to be lognormal [6].

The data in Fig. 3 are obtained from laboratory studies on beam deflections. The beams were simply supported with a concentrated load at midspan. The justification for extending these results to cantilever beams is based on the fact that the shear and moment profile along a cantilever beam with a concentrated force at its tip is identical to the shear and moment profiles along the half span of the test beams. Furthermore, the cracking extent and pattern along a cantilever beam with a force at the tip and a cantilever beam with inertial loadings are not dissimilar. As more appropriate data become available, these assumptions can be eliminated.

Deflection of a simply supported beam with concentrated load P at midspan is related to the EI product according to

$$\Delta = \frac{PL^3}{EI} \qquad (6)$$

where L is the span of the beam. Under laboratory controlled conditions, the predominant effect of the variability of the deflection is only from EI, and the lognormal distributions of deflection ratios can be used to determine the distributions of actual-to-calculated ratios of EI.

The cumulative distribution $F_{\tilde{x}}(x)$, of the ratio

$$\tilde{x} = \frac{\tilde{\Delta}}{\Delta} = \frac{\text{actual beam deflection}}{\text{calculated beam deflection}} \qquad (7)$$

and the cumulative distribution, $F_{\tilde{y}}(y)$, of the ratio

$$\tilde{y} = \frac{\widetilde{EI}}{EI} = \frac{\text{actual value of EI}}{\text{calculated value of EI}} \qquad (8)$$

are related to each other. Since $\tilde{x}$ and $\tilde{y}$ are reciprocals of each other by eq. (6), then

$$F_{\tilde{y}}(y) = P\,[\tilde{y} \le y] = P\left[\frac{1}{\tilde{x}} \le y\right] = P\left[\tilde{x} \ge \frac{1}{y}\right] = 1 - F_{\tilde{x}}\left(\frac{1}{y}\right) \qquad (9)$$

The complete derived distribution of $F_{\tilde{y}}(y)$ is shown in Fig. 4.

It should be emphasized that both E and I in Ref. [6] were calculated according to the ACI 318-71 Code [2]. The calculations of E were based on the actual value of f'_c rather than the specified value of f'_c [7]. Therefore, to make this data useful for the design environment where only specified strengths are available, the distribution in Fig. 4 must be modified to include the variability of actual compressive strength relative to its specified value. That is, the conditional cumulative of Fig. 4, $F(y|f'_c)$ must be modified to

$$F(y) = \int_0^\infty F(y|f'_c)\, P(f'_c)\, df'_c \qquad (10)$$

where $P(f'_c)$ is the normal probability density function of the compressive strength described in Section 2.1. Such an unconditional distribution has been derived for V = 0.12 and p = 0.10 numerically integrating eq. (10) and is shown by the dotted line in Fig. 4.

Although the data in Fig. 3 and 4 are related to calculating the moment of inertia, I, according to the ACI 318-71 Code, it seems to be more practical to use the data in Ref. [6] that relates to the ACI 318-63 Code, mainly because the moment of inertia calculation for under-reinforced beams is based on the concrete cross section. This is highly desirable from a design point of view, and becomes evident when it is realized that for a complicated structure it is almost impossible to evaluate the moment of inertias according to ACI 318-71, since these calculations are meant to be used for individual beams only.

3. Data on the Variability of Foundation Soil Properties

The representation of the soil in the natural frequency calculations of the soil-structure system will be based on discrete parameters as given by Richart et al [8]. The spring constants are functions of the foundation dimensions, Poisson's ratio and shear modulus of elasticity. Little is known concerning the frequency distribution of the latter two soil properties. Furthermore, the problem of applying independent variabilities to each element of a matrix for the eigenvalue problem is a formidable task.

Considering the above difficulties, the problem of the variability of the soil properties has been simplified to a selection of a percent variation of the soil-structure stiffness coefficients. For the examples solved herein, it is assumed that the soil-structure stiffness coefficients can reasonably be expected to vary by ±25% of the values selected for design. As more data on soil property variations become available, a probabilistic approach to this problem can be implemented.

4. Frequency Distributions of Continuous Beams

The natural frequencies of prismatic cantilever beams are given by

$$\omega_n = (\beta\ell_n)^2 \sqrt{\frac{EI}{m\ell^4}} \tag{11}$$

where m = mass per unit length of beam, ℓ = length of beam and $\beta\ell_n$ = the eigenvalues of the characteristic equation (n = 1,2,3...). The first three eigenvalues are given by: $\beta\ell_1 = 1.875$, $\beta\ell_2 = 4.694$ and $\beta\ell_3 = 7.855$ and the succeeding eigenvalues can be approximated by $\beta\ell_n = (n-1/2)\,\pi$ for n = 4,5,...

Although all of the structural parameters in eq. (11) have random variable characteristics, the modulus of elasticity, E, and the moment of inertia, I, (mainly due to the cracking of concrete) contribute most of the variability of ω_n. The effects of the E variation only and the more general case of the joint EI variations will be considered separately.

4.1 Variability of $\tilde{\omega}_n$ due to $\tilde{E}$ Variations

Eq. (11) gives the nominal natural frequencies of the cantilever beam. The actual frequencies will be given by

$$\tilde{\omega}_n = (\beta\ell_n)^2 \sqrt{\frac{\tilde{E}I}{m\ell^4}} = \gamma_n \sqrt{\tilde{E}} \tag{12}$$

where γ_n is a constant.

Knowing the Exp $(\tilde{\omega}_n)$ and Var $(\tilde{\omega}_n)$, the distribution of the natural frequencies of the beam can then be determined. Thus from eq. (12)

$$\text{Exp}\ (\tilde{\omega}_n) = \gamma_n\ \text{Exp}\left(\sqrt{\tilde{E}}\right) \tag{13}$$

and

$$\text{Var}\ (\tilde{\omega}_n) = \gamma_n^2\ \text{Var}\left(\sqrt{\tilde{E}}\right) \tag{14}$$

The Exp $\left(\sqrt{\tilde{E}}\right)$ and Var $\left(\sqrt{\tilde{E}}\right)$ are calculated numerically from the Exp$(\tilde{E})$ and Var$(\tilde{E})$. From eq. (4)

$$\text{Exp}\ (\tilde{E}) = 33w^{1.5}\ \text{Exp}\left(\sqrt{\tilde{f}'_c}\right) + \text{Exp}\ (\tilde{u}) \tag{15}$$

and, assuming that $\tilde{f}'_c$ and $\tilde{u}$ are independently distributed,

$$\text{Var}(\tilde{E}) = (33w^{1.5})^2\text{Var}\left(\sqrt{\tilde{f}'_c}\right) + \text{Var}\ (\tilde{u}) \tag{16}$$

The Exp $\left(\sqrt{\tilde{f}'_c}\right)$ can be obtained directly from its distribution as discussed in Section 2.1 and the Var $\left(\sqrt{\tilde{f}'_c}\right)$ is related to the first and second probabilistic moments by

$$\text{Var}\left(\sqrt{\tilde{f}'_c}\right) = \text{Exp}\ (\tilde{f}'_c) - \text{Exp}\left(\sqrt{\tilde{f}'_c}\right)^2 \tag{17}$$

Note that the ratio of actual to nominal frequency is given by

$$\frac{\tilde{\omega}_n}{\omega_n} = \frac{\gamma_n\sqrt{\tilde{E}}}{\gamma_n\sqrt{E}} = \sqrt{\tilde{E}/E} \tag{18}$$

Thus from eq. (18), the distribution of actual-to-nominal frequencies is seen to be independent of the mode number n. That is, for a given confidence level the frequency variation from the nominal calculated value is constant for all natural frequencies of the beam.

For a concrete cantilever beam of length ℓ = 180 ft, moment of inertia = 2,400,000 ft^4, weight per unit length = 350,000 lb/ft, specified unit compressive strength = 3000 psi (not modified as discussed in section 2.1.1) and a unit weight of concrete = 150 lb/ft^3, the nominal fundamental frequency is calculated by eq. (11) as being 5.61 Hz. The variability in the natural frequencies will be discussed from the point of view of the broadening of the floor response spectra as described in Section 1.0. Thus, the variations of the actual frequency are stated as a percentage of the latter. Moreover, these percentage variations are calculated at specified probabilities of either exceeding or being less than the nominal frequency. The results for the sample beam problem are given in Table I.

4.2 Variability of $\tilde{\omega}_n$ Due to Joint EI Variations

Fig. 4 gives the cumulative distribution $F_{\tilde{y}}(y)$ where $\tilde{y}$ was defined in eq. (8).

Thus, $\widetilde{EI} = EI\ \tilde{y}$. (19)

Substituting eq. (19) into eq. (11) gives

$$\tilde{\omega}_n = (\beta \ell_n)^2 \sqrt{\frac{EI}{m\ell^4}} \sqrt{\tilde{y}} \qquad (20)$$

and therefore from eqs. (11) and (20), $\tilde{\omega}_n/\tilde{\omega}_n = \sqrt{\tilde{y}}$.

From Fig. 4, fractile values, y_p, can be obtained such that $P\left[\tilde{y} \le y_p\right] = p$.

By successive manipulations, these can be converted to confidence intervals for $\tilde{\omega}_n/\tilde{\omega}_n$ ratios:

$$P = P\left[\tilde{y} \le y_r\right] = P\left[\sqrt{\tilde{y}} \le \sqrt{y_r}\right] = P\left[\frac{\tilde{\omega}_n}{\omega_n} \le \sqrt{y_p}\right] \qquad (21)$$

The results from eq. (21) for the sample beam problem are shown in Table 1.

When interpreting the results in Table I it must be remembered that the data on the variability of E in the E-only and EI variations come from different sources. For the E-only variation, the results show that the mean value of the frequency is larger than the nominal. This is as expected since the mean value of the compressive strength of-concrete is larger than the nominal value by design. However, when the EI variation is considered, the mean value of the frequency is less than the nominal, indicating that the impact of the variation of the moment of inertia is greater than that of the variations in E. It must be noticed though that the total variation for any confidence interval is always larger for the EI variation. This is as one would expect.

5. Frequency Distribution of Lumped Parameter Systems

In the previous section the variation of beam frequencies was studied. Although the study of continuous beams helps in understanding the response of more complicated systems, the main objectives in the previous sections were the development of the raw data and its application to a simple system. In this section the data developed will be applied to more complicated systems that are usually represented by mass and stiffness matrices.

The application of the probabilistic model to a fixed base structure should not create any problems if it is assumed that the variability is the same for all the elements of the dynamical matrix. Furthermore, making similar assumptions as in Section 4, it can be assumed that the variability of the mass matrix is negligible relative to the variability of the stiffness matrix. Thus the problem reduces to finding the distribution of the random eigenvalues of a fixed matrix which is multiplied by a random variable.

Letting A denote the inverse of the dynamical matrix D, λ_j the jth eigenvalue of A, and $\tilde{\mu}_j$ the jth random eigenvalue of $\tilde{A} = \tilde{y}A$, it is readily shown that $\tilde{u}_j = \tilde{y}\lambda_j$. The roots of $\tilde{A}$ must satisfy the following eq.

$$0 = |\tilde{A} - \tilde{\mu}I| = |\tilde{y}A - \tilde{\mu}I| = \tilde{y}\left|A - \frac{\tilde{\mu}}{y} I\right| \qquad (22)$$

for all positively distributed $\tilde{y}$. Hence from eq. (22), $\tilde{\mu}_j/\tilde{y} = \lambda_j$, and the distribution of each $\tilde{\mu}_j$ is a scalar multiple of $\tilde{y}$. Further, if fractile values μ_j are desired such that $P\left[\tilde{\mu}_j \le \mu_{j,p}\right] = p$ they can be found directly from $u_{j,p} = \lambda_j y_p$.

A more interesting problem arises when soil-structure interaction effects are included in the dynamic model. As mentioned in Section 3, lumped parameters are used to represent the soil-structure interaction compliances. The choice of the usual physical coordinate system for a lumped mass model would statically couple and dynamically uncouple the system. That is, the mass matrix would be diagonal. However, since different variabilities are associated with the structure stiffness and soil-structure interaction parameters, it is desirable to have a coordinate system that statically uncouples the structure from the interaction parameters. Such a coordinate system is shown in Fig. 5, where x_j = horizontal displacement of the superstructure at the jth floor relative to the foundation, excluding roations; ν_g = free-field surface displacement; ν_o = translation of the foundation relative to the free field motion; ϕ = rotation of the foundation; h_j = height of the jth story above the foundation; ν_j^t = total horizontal displacement of the jth mass with respect to a fixed vertical axis, i.e., $\nu_j^t = \nu_g + \nu_o + h_j\phi + \nu_j$; m_j = mass of the jth floor; m_o = mass of the foundation, I_t = sum of the centroidal moments of inertia of the m+1 masses. The solution coordinates are given by the vector $(x_1, x_2, \ldots x_n, \nu_o, \phi)^T$.

The choice of the above coordinate system results in the following mass and stiffness matrices

$$M^* = \left[\begin{array}{c|c} M_{11} & M_{12} \\ \hline M_{21} & M_{22} \end{array}\right] \text{ and } K^* = \left[\begin{array}{c|c} K_c & 0 \\ \hline 0 & K_s \end{array}\right]$$

Initially, the submatrix $[K_s]$ associated with the interaction elements will be held constant. If the variations in stiffness, $\tilde{y}$, are applied to the K_c submatrix, the resulting random submatrix is:

$$\left[\tilde{K}_c\right] = \tilde{y}\left[K_c\right] \qquad (23)$$

Since the natural frequencies of the original structure depend on the characteristic roots of $[M^*]^{-1}$ $[K^*]$, the distributions of probabilistic frequencies will depend on the distribution of the random characteristic roots of $[M^*]^{-1}$ $[\tilde{K}^*]$. This is a more complicated problem than the fixed base system since only a submatrix of $[K^*]$ is being made random and, correspondingly, only the left most columns of $[M^*]^{-1}$ $[K^*]$. From eq. (23), the elements of $[\tilde{K}_c]$ are all positive multiples of $[K_c]$, hence the roots of the characteristic equation of $[M^*]^{-1}$ $[K^*]$ will be continuous monotone functions of the stiffness multiplier y. Thus while analytic distribution of the $\tilde{\omega}_n$'s are extremely difficult to obtain, their fractile values and confidence intervals may be found numerically. For example, to find ω_n such that $P[\tilde{\omega}_n \le \omega_n] = p$, multiply the original $[K_c]$ by y_p (the pth fractile of $\tilde{y}$) and numerically find the nth natural frequency associated with

$$[M^*]^{-1}\left[\begin{array}{c|c} y_p \cdot K_c & 0 \\ \hline 0 & K_s \end{array}\right]$$

This has been done for a lumped mass system shown in Figure 5, and the results are shown in Table II.

Considering each column separately, the most notable feature is that there is no longer the constant variation across all frequencies which was noted in the continuous beam and fixed base systems. Further, for each frequency, the fractile limits are tightest for the lowest frequencies, and grow successively larger at the higher frequencies. However at no time do they exceed the fractile limits established for the beam problem. In fact, at the lower frequencies, the limits are significantly less than for the beam problem. This is due to the fact that $[K_s]$ is being held constant.

As for the variations in soil parameters in $[K_s]$, there is insufficient data on their distribution. Until such data is available, a "worst-case" analysis can be made using ±25% variations along the lines described in Section 3. That is, to obtain design lower and upper limits, the $[K_s]$ elements are multiplied by 0.75 and 1.25, respectively. Their percentage relations to the nominal frequencies (at y = 1, K_s unchanged) are given in Table III for 0.1 and 0.9 fractiles.

The outermost columns of Table III are of primary interest for design purposes. The innermost columns demonstrate that a study of simultaneous effects of soil and structure parameters should be done once a distribution for the latter becomes available.

6. Discussion and Conclusions:

This present paper considered only one aspect of the effect of frequency variation on floor response spectra. The problem of the amplitude of the peak response was not considered. Many assumptions were made either because pertinent satistical data were not available or simply because the influence of the parameter in question was considered to be negligible. The method presented can, of course, be refined and extended as additional data become available or are considered to be important.

The frequency distribution tables include left and right tail fractile values corresponding to $y_{0.001}$ and $y_{0.999}$. It should be recognized that these calculations are derived from the hypothesis of a lognormal distribution for deflection ratios. This hypothesis should be verified and confidence intervals derived for the distribution parameters to sharpen the precision of such extreme fractile values.

The method described to account for the effects of the soil parameter variations is admittedly not rigorous due to the lack of data on the variability of soil properties. With the choice of the proper coordinates, the structure and soil have been decoupled and it is a simple matter to apply separate variabilities to each submatrix.

The numerical results were included to illustrate the use of the method in a specific instance. Other cases with different values of f'_c, V and failure specifications should be considered separately before generalizing the frequency fractiles to them.

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Table I. Frequency Variation - Sample Beam

		Probability of being either less than the lower limit or greater than the upper limit	Percent variation of actual frequency to calculated frequency	
			Lower Limit	Upper Limit
E-only Variation	$\frac{\text{Mean Freq}}{\text{Calculated}} = 1.056$	0.10	1.2%	12.1%
		0.05	3.1%	14.0%
		0.01	6.6%	17.6%
		0.001	10.6%	21.6%
EI Joint Variation	$\frac{\text{Mean Freq}}{\text{Calculated}} = 0.961$	0.10	16.0%	8.8%
		0.05	19.0%	12.7%
		0.01	24.4%	21.0%
		0.001	30.4%	26.9%

Table II. Ratios of p-Fractile Frequencies to Specified Frequencies of a Lumped-Mass System

P: / ω_n	.001	.01	.05	.10	.90	.95	.99	.999
ω_1	.866	.901	.928	.941	1.025	1.035	1.053	1.065
ω_2	.837	.874	.905	.921	1.037	1.053	1.083	1.103
ω_3	.792	.838	.878	.899	1.050	1.072	1.117	1.148
ω_4	.819	.848	.878	.895	1.064	1.094	1.158	1.206
ω_5	.701	.759	.813	.842	1.087	1.126	1.207	1.265
ω_6	.714	.769	.820	.848	1.084	1.121	1.201	1.257

Table III. Percentage Differences in Fractile Frequencies at Extreme Limits of Soil Parameters

Fractile / ω_n	Soil = .75 of Design		Soil = 1.25 of Design	
	.1	.9	.1	.9
ω_1	-14.2%	-7.9%	+0.4%	+10.8%
ω_2	-14.6%	-4.9%	-2.6%	+10.4%
ω_3	-14.9%	-1.1%	-6.6%	+9.8%
ω_4	-15.2%	+3.6%	-5.5%	+9.6%
ω_5	-15.9%	+8.5%	-15.3%	+8.8%
ω_6	-15.8%	+7.8%	-14.1%	+8.9%

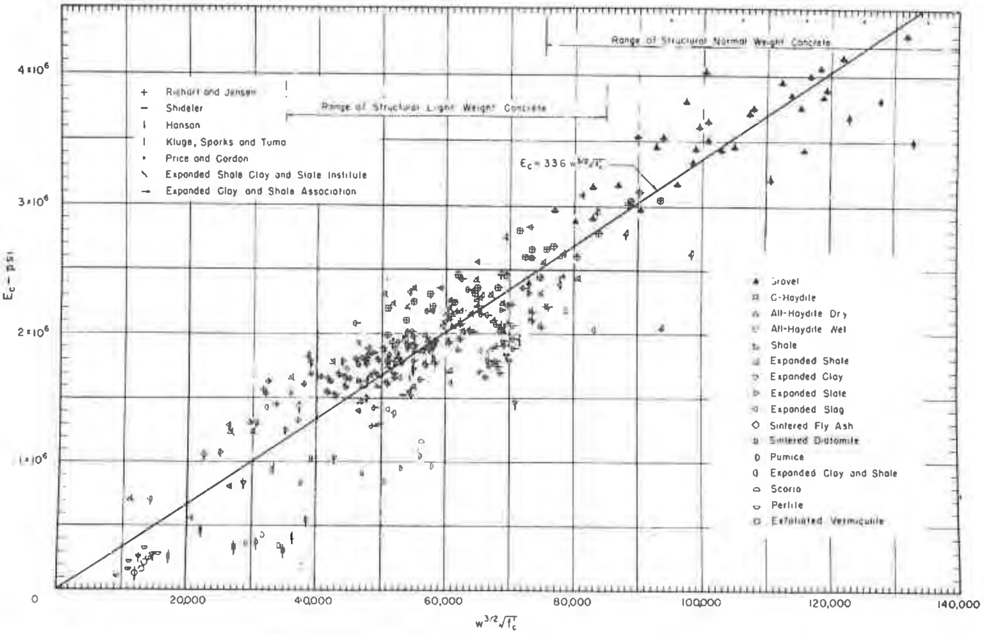


Fig. 1 - Correlation of Data Based on Static Tests. (After Pauw [3])

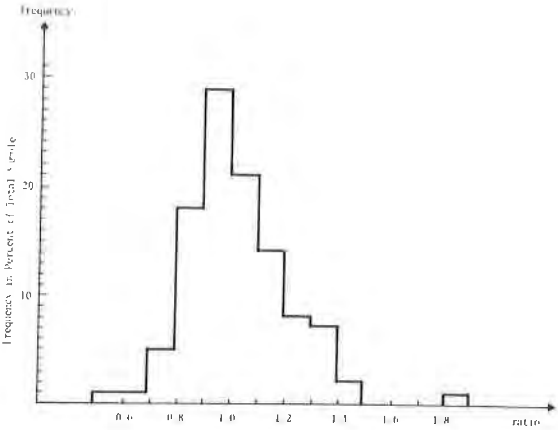


Fig. 2 - Histogram of Ratio of Actual Short Term Deflection to Deflection Calculated by ACI 318-71 Method. (After ACI Committee 435 [6])

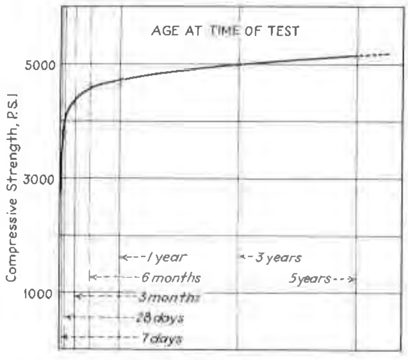


Fig. 3 - Relation of Compressive Strength to Age. Average of Twenty-five 6-by-12-inch Laboratory Cylinders Cured in Moist Air at 70° F. (After Dunham [5])

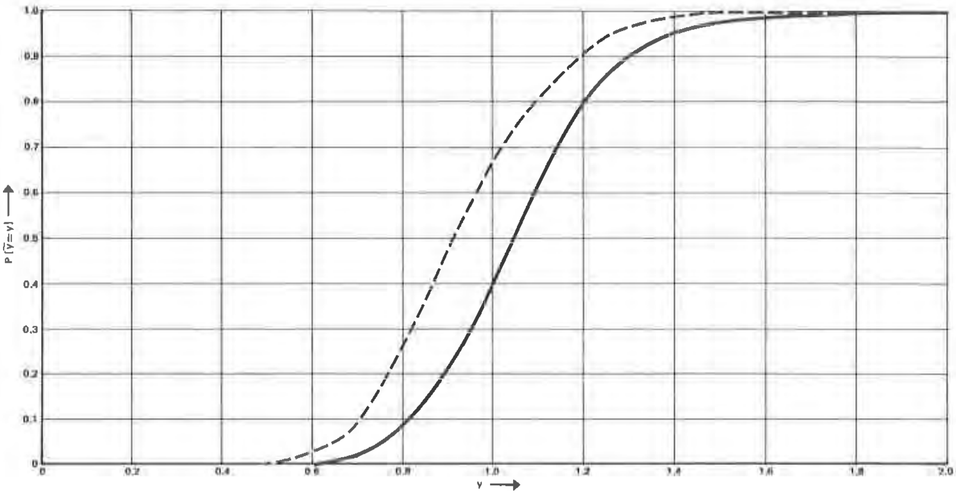
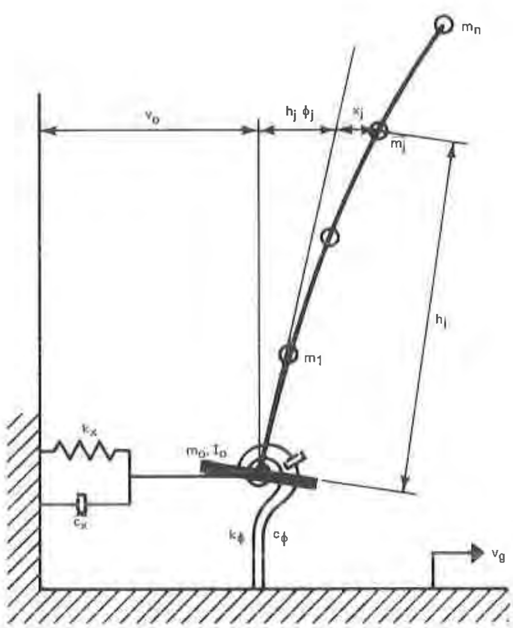


Fig. 4 - Cumulative Distribution of Ratios of Actual-to-Calculated EI-Products



DATA FOR SAMPLE PROBLEM

$w_1 = 9 \times 10^6$ lbs $w_2 = w_3 = w_4 = 18 \times 10^6$ lbs ($w_j = m_j g$)

I FOR ALL MEMBERS = 2.4×10^6 ft^4

$h_j = 45$ ft, $h_2 = 90$ ft, $h_3 = 135$ ft, $h_4 = 180$ ft

$k_x = 6 \times 10^9$ lbs/ft, $k_\phi = 14 \times 10^{12}$ lbs-ft/rad

m_o and I_t NEGLIGIBLE

Fig. 5 - Model of Soil-Structure System