Application of Laplace Transform to Industrial Problems: Determination of Crack Initiation and Usage Factor

D. J. M. Dubois, J. Vagner
FRAMATOME Heavy Industry Division, Saint Marcel, France

1 - INTRODUCTION

This paper presents 2 industrial applications of a new methodology based on Laplace Transform properties which has been implemented in an industrial finite element program (SYSTUS).

In structures endowed with thermal and mechanical properties constant with the temperature, the stresses are computed for unit thermal shocks applied on the areas which are actually affected by the temperature variations. The analytical formulation and the general feature of this implementation are presented in [1].

If the response of a structure to a unit thermal shock is noted Ru(tk) at time tk and if the structure is subjected to an external temperature T(tj) at time tj, both evolutions are approximated by piecewise linear functions and the response is a piecewise parabolic function build on the derivatives of Ru and T :

$$R(t)=\sum_{k}\sum_{j}(\dot{R}u(tk+1)-\dot{R}u(tk)).(t-tk-tj).[DTj+1/2.(\dot{T}(tj+1)-\dot{T}(tj)).(t-tk-tj)]\delta(t-tk-tj)$$

The unit responses are post-processed into numerical values corresponding to :

case 1 - circumferential stresses at a given distance from a crack tip, for various angular positions. (Primary Pump Fig. 1 and 2).

case 2 - linearized (membrane and bending) and peak stresses in the cross-sections retained for a fatigue analysis (Reactor Vessel Outlet Nozzle Fig. 3).

The response to any thermal transient is further obtained by the "integration" of equation (1) in order to determine :

case 1 - the risk of crack initiation in singular geometric areas

case 2 - the usage factor in current and discontinuous parts

This paper shows how this methodology has been applied in order to :

- firstly, compute and check the validity and accuracy of the unitary response,
- secondly, solve all the transient thermal loadings retained for the analysis,
- thirdly, check the conformity of the component analysed.

2 - DETERMINING THE UNIT RESPONSE

The determination of the unit response is performed using the standard finite element procedures :

- compute the temperatures for any independent external load,
- compute the resulting displacements,
- if necessary, average the elemental stresses to obtain nodal stresses
- eventually post-process results into specific output files.

The calculation is performed for a varying time step which is based on mesh sizes, diffusion coefficient and heat transfer conditions. The time step selection is based on 2 factors : finding an accurate solution for temperature evolutions, minimizing the number of steps for economic reasons since the calculation of any mechanical variable is naturally determined with the same time step.

For that purpose, abaci have been introduced in an interactive pre-processing procedure (Fig. 4) in order to determine a varying time step that :
- ensures an accurate description of the temperature and stresses variations,
- avoids oscillating temperatures,
- seizes the time at which maximum stresses occur in a thermal shock,
- predicts when the steady state can be imposed.

This procedure determines automatically the time steps in each cross-section selected ; for all the sections retained the final time steps are sorted in order to reduce the integration time and still ensure accurate results.

The time step selection of fig. 3 is indicated in table 1 with a restart in order to impose the steady state.

Once the unit response is computed, the results are post-processed and stored into specific files.

These post-process files, whatever they contain, have the same structure : they contain a table of addresses for each chain of results ; a chain contains a string of points with their local coordinates, the load titles and the corresponding numerical results.

For Fracture Mechanics evaluation the file contains 2 chains of results. The first chain contains a fictitious point (with the actual crack-tip coordinates) at which are stored the values of KI, KII and the crack-tip temperature.

The second chain contains the values of the circumferential stress around the crack tip at a distance d (characteristic of the material) for different angular positions around the crack tip and the coordinate of the corresponding fictitious points are the angles (Fig. 2).

For ASME applications (design by analysis as described by ASME code, section III §NB 3200) the file contains stresses in cross-sections of the structure (Fig. 5) ; each cross section contains 5 points (Fig. 6) :
- the point 1 is located at the inner radius of the structure ; its stresses correspond to the inner wall TOTAL STRESSES,
- the point 2 is a fictitious point, located close to point 1, at which are stored the linearized stresses corresponding to the bending plus membrane stresses (noted PL + PB for mechanical loads),
- the point 3 is a fictitious point, located in the middle of the cross-section, at which are stored the linearized stresses corresponding to the membrane stresses (noted PL for mechanical loads),
- the point 4 is a fictitious point, located close to point 5, at which are stored the linearized stresses corresponding to the bending minus membrane stresses (noted PL - PB for mechanical loads),
- the point 5 is located at the outer radius of the structure ; its stresses correspond to the outer wall TOTAL STRESSES.

The 2 examples presented here cover all the possibilities of the method : an external load (temperature) may be variable (the most general case) or constant (in areas which are cooled by a fluid at a constant temperature).

In the case 1, the primary pump is subjected to 3 independent loads :
- load 1 is a varying temperature (cold leg primary temperature described with 33 time steps),
- loads 2 and 3 correspond to 2 constant (but different) temperatures for which only the steady state is computed.

In the case 2, the nozzle can be subjected to both hot and cold leg temperatures ; both loads are computed with the same time steps.

3 - DERIVING THE UNIT RESPONSE

These post-process files are then derived using specific procedures intended to avoid errors : as a matter of fact the determination of displacements and stresses in thermoelastic cases is obtained by solving a linear and static system in which the applied load (in addition to boundary conditions) is reduced to the internal thermal strains (there is no notion of time dependence). To derive these results it is therefore necessary to indicate at which time each result has been computed ; 3 possibilities are available :
- input time steps in the derivation,
- input time steps in load titles when computing the displacements,

- indicate which thermal transient file corresponds to the analysis and read the time steps which are naturally stored in transient files.

In the derivation process, results are not derived if they correspond to constant temperatures but merely copied to restore directly their participation in further integrations.

4 - INTEGRATING THE UNIT RESPONSE

The integration is straightforward, it is no longer necessary to solve temperature evolution, compute displacements and average stresses ; the post-process files are integrated directly and stored on files with identical marks (for quality assurance reasons).

The only input necessary is the title of the transient analyzed (to know what is what) and the piecewise linear description of the external temperature (Table 2). When computing interactively (it is possible to type data or read them in a file), the graphic evolution is plotted :

For case 1, the data of table 2 are used :

- it takes 6 seconds cpu (cpu time = elapsed time on a VAX VS 2000 station!) to load data and plot Fig. 7,
- it takes 9 seconds to compute and plot KI, KII (KJ is also computed as the square root of $KI^2 + KII^2$) and the temperature at the crack-tip (Fig. 8),
- it takes 10 seconds to compute and plot the evolution of $\sigma_\theta(d)$ for 16 angular positions around the crack tip (fig. 9).

This calculation is performed at 62 different times ; the same calculation would take more than 3600 seconds (VAX 785) with the standard Finite Element method !

In order to check the accuracy yielded by the unit response, it is recommended to perform a sample finite element analysis with the lowest heat rate transient applied to the structure (it has been previously tested that the larger discrepancy between the 2 methods was obtained for slow ramps) ; the same transient is directly computed with the standard finite element method for a selected number of times ; the iso ux displacement (intervening in the calculation of the stress intensity factors) are plotted at the same time for the 2 methods (Fig. 10) ; the relative difference is less than 0.1 % (at this level it is hard to state which one is the most accurate !). If the accuracy is not reckoned sufficient, additional results can be computed and inserted in the unit response.

For case 2, the response contains 2 independent unit loads ; the integration of a response is based on the sum of the responses (transient part plus initial steady state) of each unit load. In many cases, the same temperature is applied to both unit loads : it is possible to combine linearly the results of each unit load corresponding to the same times, post-process and derive these results so that the integration will be performed twice as fast !

For the input presented in table 3, the stress evolution is computed in the sections of Fig. 5 :

- it takes 6 seconds cpu to load data and plot the temperature evolution,
- it takes 4 seconds to compute and 6 seconds to plot the evolution of the 4 stresses of each fictitious point (Fig. 11) (respectively 2 and 6 seconds if the unit loads were merged !).

The total cpu time for computing the stresses in the 5 sections is 54 seconds for 31 different times ; the standard finite element method would take about 5000 seconds ! The cpu time necessary for computing the unit response corresponds to about twice the time spent in computing the response to a standard thermal transient so that this methodology reduces the total cpu time if at least 3 thermal transients are analyzed.

5 - CHECKING THE CONFORMITY

Once the results have been computed for all the thermal transients applied to the structure, the corresponding post-process files are merged (eventually with mechanical loads corresponding to external forces and internal pressure) and analyzed by the corresponding sub-program.

For case 1, we check that the geometric singularity at the junction with the diffuser does not turn into an initiated crack (the criterion is based on the variation of circumferential stress $\sigma_\theta(d)$ where d is a characteristic value of the material [2,3].
For case 2, we check that the normal and upset conditions which have been analyzed meet the requirements of ASME III division 1 design conditions (NB-3222.2, NB-3222.4 and NB-3222.8).
These methodologies were already implemented in SYSTUS ; the only modification was the creation of new entries for files containing "ready for use" information.

6 - CONCLUSION

The advantages of the methodology are evident :
- reducing computation time,
- reducing man time,
- reducing the number of files and saving space on disks,
- plotting the evolution for a better selection of the times retained in the conformity analysis,
- reducing the risk of error by reducing the necessary input,
- allowing the analysis of more transients to reduce a conservatism that was sometimes too stringent,
- improving consequently the quality assurance of the analysis.

Most of the time spent in the analysis of a structure previously concerned the verification of input and the file management ; now most of the time is spent in controlling where and when the conformity of the structure must be analyzed.

REFERENCES

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A criterion for Analysing Fatigue Crack Initiation in geometrical Singularities.
5th SMIRT, paper G8.1.

[3] BOSSER M., MERCIER F., AMZALLAG G., PONSOT J.
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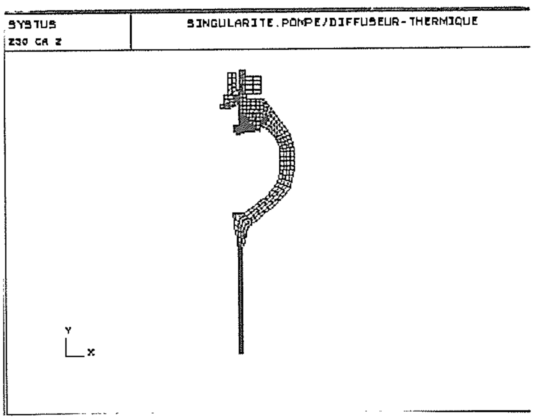
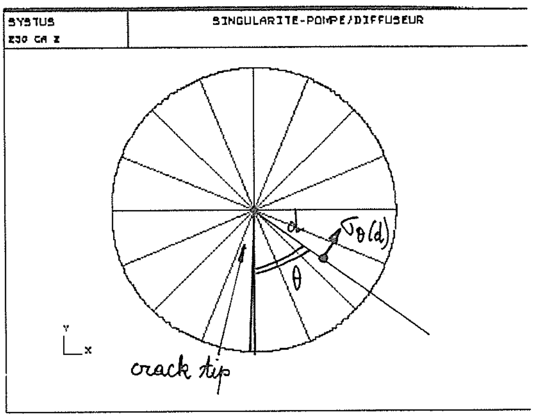
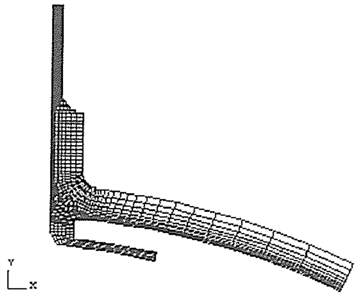
5th SMIRT, paper G8.2.



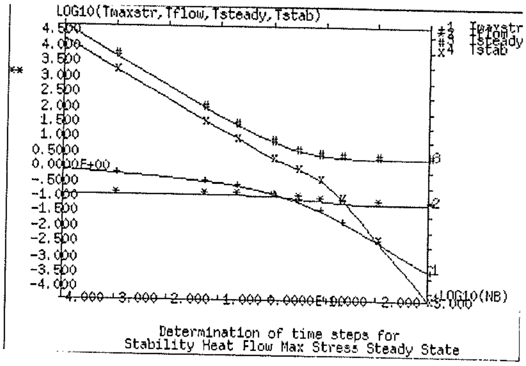
PRIMARY PUMP Fig. 1



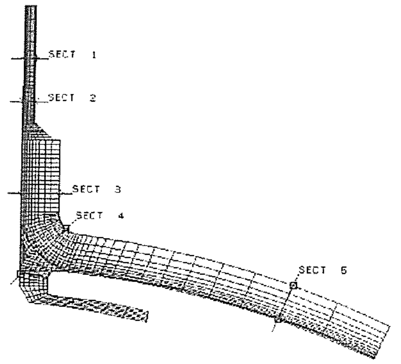
PROCESSING RESULTS AROUND CRACK TIP Fig. 2



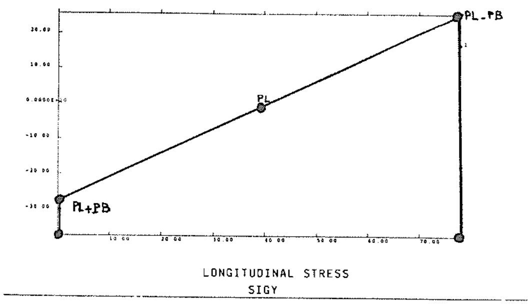
EXIT NOZZLE Fig. 3



ABACI FOR TIME STEP SELECTION Fig. 4



CROSS-SECTIONS ANALYZED Fig. 5



FICTITIOUS POINTS Fig. 6

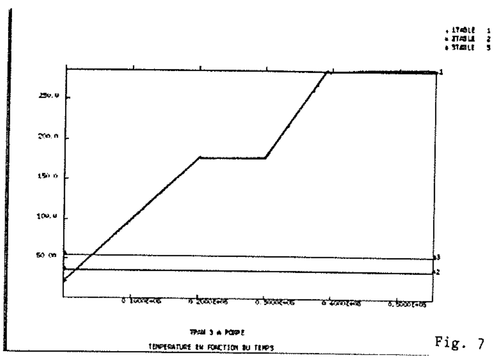


Fig. 7

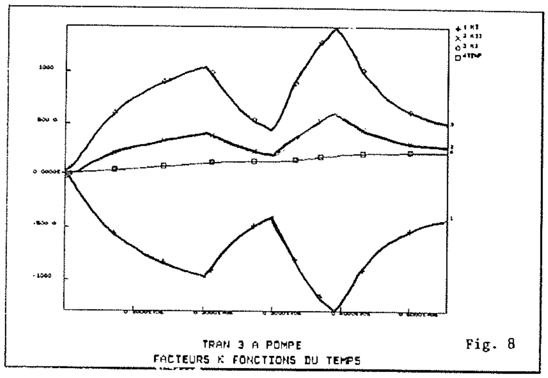
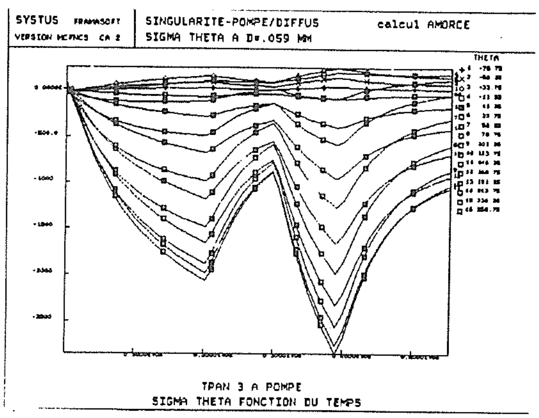
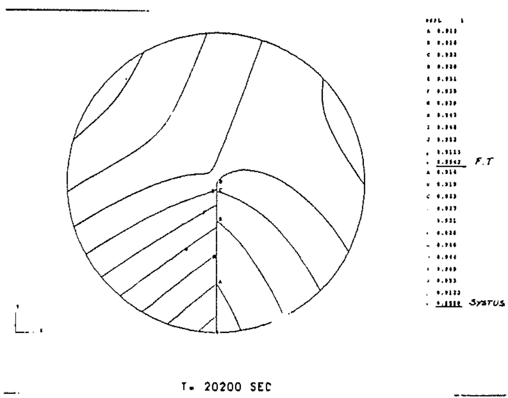


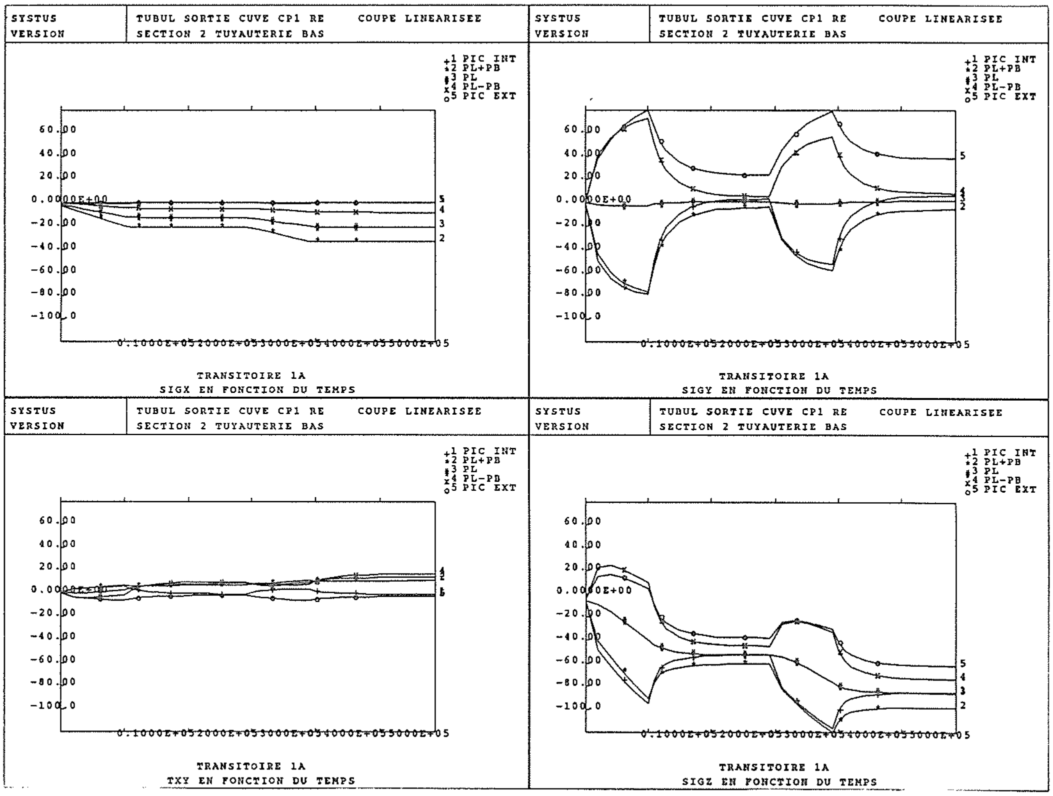
Fig. 8



EVOLUTION OF $\sigma(\theta)$ Fig. 9



COMPARISON BETWEEN THE 2 METHODS Fig. 10



MULTI-WINDOW GRAPHIC OUTPUT OF STRESS EVOLUTIONS Fig. 11

```
$SYSTUS
SEARCH DATA STIFFNESS MASS 100
TRANS THERMAL
FUNCTIONS 1 0
TABLE
 1/1 0 0 .0001 1 30000 1
TIME INITIAL 0
  0/STORE 1
       0.0661/ STORE 1
       0.331/ STORE 1
       0.661/ STORE 1
       0.844/ STORE 1
       1.233/ STORE 1
       2.056/ STORE 1
       2.672/ STORE 1
       4.728/ STORE 1
       7.400/ STORE 1
       12.13/ STORE 1
       19.53/ STORE 1
       31.66/ STORE 1
       51.19/ STORE 1
       82.84/ STORE 1
       134.0/ STORE 1
       216.9/ STORE 1
       350.9/ STORE 1
       567.8/ STORE 1
       918.7/ STORE 1
       1486./ STORE 1
       2405./ STORE 1
       3892./ STORE 1
       6297./ STORE 1
      10188./ STORE 1
      15999./ STORE 1
RETURN
SAVE DATA TRANSIENT 101
SEARCH DATA 100
ASSIGN 19 TRAN101.TIT BINARY
TRANS THERMAL
FUNCTION 1 0
TABLE
 1/1 0 0 .0001 1 30000 1
CONDITION INITIAL RESTART CARD  26
TIME INITIAL       20604.
      20604./ STORE 1
RETURN
```

TABLE 1

```
$SYSTUS
SEARCH INIT 102
LAPLACE
TITLE
  tran 3 A : PRIMARY PUMP  CPY
FUNCTIONS 1 2 3
TABLES
 1 / 1 0 20 180 21.4 20200 177 29800 177 33200 217 36920 261 37550 268.4
* 39000 286 52000 286 55500 286
 2/1 0 35.4
 3/1 0 54
TIME INITIAL 0
 0
 2000
 20200 STEP 1000
 29800 STEP 860
 33200 STEP 850
 36200 STEP 500
 36920
 37550
 39000 STEP725
 55000 STEP 1000
 55500
RETURN
SAVE INIT 103
END
```

TABLE 2

```
$SYSTUS
SEARCH CROSS_SECTION  502
LAPLA
TITLE
  TRANSIENT 1A
FUNCTION 1 1
TABLE
 1/1 0. 20.  10276. 177.  29400. 177.  39210. 286.  58763. 286.
TIME INITIAL 0.
 0
 10000 STEP 2000
 15000 STEP 1000
 39000 STEP 2000
 42000 STEP 1000
 50000 STEP 2000
 58763
RETu
SAVE CROSS_SECTION 503
END
```

TABLE 3