A Proposal for Safety Factor Estimation on Seismic PSA of Structures by Stochastic FEM Technique

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1 INTRODUCTION

The seismic probabilistic safety assessment (hereinafter called "seismic PSA") to estimate the seismic integrated safety and reliability of structures during an earthquake has been actively performed mainly for important structures such as nuclear power plants and high rise buildings. This seismic PSA has mainly two methods: SSMRP method based on the Monte Carlo method, and the safety factor method based on conventional deterministic seismic design.

Since various uncertainties included in the input and system are expressed by an amount called "safety factor" in the safety factor method, the entire safety assessment is affected by the presumed accuracy of the safety factor. It is one of important problems in the enforcement of the safety factor method how accurately the safety factor as statistics should be determined.

The safety factor has been conventionally set mainly by using the Monte Carlo technique or empirically, and it has hardly been obtained by an analytical approach, in which it is easy to have a physical outlook.

This paper proposes a technique whereby a part of the above safety factor can be analytically estimated as an example of application of dynamic reliability analysis taking into consideration structural material uncertainty.

2 PROBABILISTIC SEISMIC RESPONSE ANALYSIS OF STRUCTURES WITH STOCHASTIC PARAMETERS

Assuming the power spectral density of seismic acceleration for input as $S_E(\omega)$, the power spectral density $S_F(\omega)$, of response acceleration at any point F in a general linear multi-degree-of-freedom structure can be expressed as below when the objective system is regarded as a stationary random process.

$$S_F(\omega) = G(\omega)S_E(\omega) \tag{1}$$

where $G(\omega)$ is a squared amplification of frequency response function of response absolute acceleration against the input acceleration. Then as an expression including uncertainty of the material characteristics of structure (Young's modulus, Poisson's ratio, etc.) X_k, the following form is assumed:

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$$X_k = X_k^0(1 + \alpha_k), \ (k=1,2,\ldots, n) \tag{2}$$

where X^0_k is an expectation of X_k, and α_k is a small random variable with 0 mean value.

If the second order perturbation method is applied to $G(\omega)$ when the material characteristics of structure vary as shown in equation (2), $S_F(\omega)$ in the structures with stochastic parameters takes the following form:

$$S_F(\omega) = S_F^0(\omega) + \sum_{k=1}^{n} S_{F_k}^{I}(\omega)\alpha_k + \frac{1}{2}\sum_{k=1}^{n}\sum_{\ell=1}^{n} S_{F_{k\ell}}^{II}(\omega)\alpha_k\alpha_\ell \tag{3}$$

where

$$S_F^0(\omega) = G^0(\omega)S_E(\omega), \quad S_{F_k}^{I}(\omega) = G_k^{I}(\omega)S_E(\omega), \quad S_{F_{k\ell}}^{II}(\omega) = G_{k\ell}^{II}(\omega)S_E(\omega)$$

An expectation μ_{x_T} of statistics of maximum values and variance $\sigma^2_{x_T}$, which are required to calculate the safety factor, can be readily determined by the statistical theory of extremes if the material characteristics of structure are definite values. If they have uncertainties, it is normally very troublesome to handle. To avoid this, an approximate expression is introduced by utilizing the following total representation theorem in the probability structure (Mochio et al. 1990).

$$\mu_{x_T} = \tilde{E}[E[x_T|A=a]] \tag{4}$$

$$\sigma^2_{x_T} = \tilde{E}[V_{ar}[x_T|A=a]] + \tilde{V}_{ar}[E[x_T|A=a]] \tag{5}$$

In equations (4) and (5), $E[x_T|A=a]$ and $V_{ar}[x_T|A=a]$ are the conditional expectation and conditional variance when the stochastic parameters are fixed at certain values. To actually calculate equations (4) and (5), the FOSM method to approximate around the expectation of stochastic parameters is used as follows:

$$\mu_{x_T} = E[x_T|A=0] \equiv \mu^0_{x_T} \tag{6}$$

$$\sigma^2_{x_T} = V_{ar}[x_T|A=0] + \sum_{k=1}^{n}\sum_{\ell=1}^{n} \left(\frac{\partial g}{\partial \alpha_k}\right)_{A=0} \left(\frac{\partial g}{\partial \alpha_\ell}\right)_{A=0} C_{\alpha_k\alpha_\ell} \tag{7}$$

The above equations show that $C_{\alpha_k\alpha_\ell}$ is the covariance for stochastic parameters α_k and α_ℓ, and g shows the expectation for statistics of maximum values when the stochastic parameters are fixed at certain values as can be seen from equations (4) and (5). As regards the form of g, the estimated equation (Der Kiureghian 1980) proposed by Der Kiureghian is adopted here.

As, therefore, can be seen from equation (3), equations (6) and (7) will be able to be calculated if $G^0(\omega)$, $G^{I}_k(\omega)$ and $G^{II}_{k\ell}(\omega)$ can be determined. $G^0(\omega)$ can be easily determined within normal frame of deterministic analysis. On the other hand, $G^{I}_k(\omega)$ and $G^{II}_{k\ell}(\omega)$ generally require rather complicated calculations if the structure is of the multi-degree-of-freedom system, but they can be simply and effectively calculated by using the stochastic FEM (Hisada and Nakagiri 1981).

3 CHECKING THE ANALYTICAL METHOD FOR ADEQUACY

To check the analytical method specified in the previous paragraph for adequacy, this paper performed theoretical calculation for such a lumped mass system model as shown in Fig. 1 and numerical simulations by means of the Monte Carlo technique. The power spectral density $S_E(\omega)$ for the input acceleration used has the following Kanai-Tajimi's equation.

$$S_E(\omega) = S_0 \frac{1 + 4\zeta_E^2(\omega/\omega_E)^2}{\{1-(\omega/\omega_E)^2\}^2 + 4\zeta_E^2(\omega/\omega_E)^2} \qquad (8)$$

where $S_0 = 43.4$, $\omega_E = 9\pi$, $\zeta_E = 0.6$

The amount of structural material uncertainty to be analyzed is in the following two cases:

(1) Case 1 (In the case of stiffness uncertainty)
The Young's modulus of the building and spring constant of the ground are uncertain as shown in equation (2). However, the statistics for α_k are assumed to be 0.1 in standard deviation and 1 in correlation coefficient (full correlation).

(2) Case 2 (In the case of damping uncertainty)
The damping of the building and ground is uncertain as shown in equation (2). The statistics for α_k are the same as in Case 1.

For the above two cases, Table 1 shows the comparison between theoretical calculation for T=20 seconds and simulation results due to 5000 artificial seismic waves, for the maximum value of response absolute acceleration at mass point 2 in Fig. 1. Both show good agreement, and thereby it is considered that the adequacy of the analytical method could be verified.

4 CALCULATION OF SAFETY FACTOR IN SEISMIC PSA ANALYSIS

This paper describes a method to analytically determine a part of the structural response factors F_R among the safety factors, by utilizing the analytical results for the structure with stochastic parameters. It is assumed that the form of structural response factor F_R can be described by the following expression:

$$F_R = F_{SA} \cdot F_D \cdot F_{MS} \cdot F_{MF} \cdot F_{MC} \cdot F_{SS} \cdot F_{EC} \qquad (9)$$

where F_{SA} = spectral shape factor, F_D = damping factor
F_{MS} = modal shape factor, F_{MF} = modal frequency factor
F_{MC} = mode combination factor
F_{SS} = soil/structure interaction factor
F_{EC} = earthquake component combination factor

Further if F_D, F_{MS} and F_{MF} in equation (9) are assumed as factors to express only the uncertainty occurring from the structural material uncertainty, the results obtained in paragraph 2 express the statistics for values of the denominator (x_T) in the following definition as a new structural response factor F_{DM} by the products of F_D, F_{MS} and F_{MF}.

$$F_D \cdot F_{MS} \cdot F_{MF} \equiv F_{DM} = \frac{\text{Deterministic response value for design}}{\text{Response value obtained by probabilistic analysis}} = \frac{x^D}{x_T} \qquad (10)$$

The method in paragraph 2 is mainly based on the analysis in the frequency domain which uses power spectral density for seismic input acceleration as the analytical input, and further uses equation (8) as power spectral density for input in the numerical calculation. For this reason, to apply the method mentioned in paragraph 2 to problems for presuming the response factor, it is necessary to presume the Kanai-Tajimi's equation parameters which meet a given design response spectra. For the method, this paper adopts the technique in reference (Shinozuka et al. 1988).

Then the structural response factor F_{DM} to be determined in this paragraph is regarded as lognormal distribution, and its median value $\hat{F}_{DM}$ and logarithmic standard deviation β_{DM} should be presumed. Since $\hat{F}_{DM}$ and β_{DM} are approximately equal to expectation of F_{DM}, F^0_{DM} and coefficient of variation V_{DM} respectively, the following can be derived from equation (10) by using expectation μ_{x_T} and standard deviation σ_{x_T} for x_T.

$$\hat{F}_{DM} \simeq F^0_{DM} = \frac{x^D}{\mu_{x_T}} \tag{11}$$

$$\beta_{DM} \simeq V_{DM} = \frac{\sigma_{x_T}}{\mu_{x_T}} \tag{12}$$

From the foregoing, the median value of structural response factor and logarithmic standard deviation, which are required for failure estimation, could be analytically obtained. A simple example for numerical calculation is shown as below. It is assumed that the response absolute acceleration at mass point 2 in Fig. 1 is the object for failure estimation and there exists only stiffness uncertainty in the structure. If it is further assumed that the Kanai-Tajimi's equation parameters which meet a design response spectra for input are given by equation (8), μ_{x_T} and σ_{x_T} in equations (11) and (12) are given from Table 1 as follows:

$$\mu_{x_T} = 637, \ \sigma_{x_T} = 70 \tag{13}$$

On the other hand, x^D has been determined by ordinary (deterministic) seismic design using the design response spectra, and assuming this value to be, for example, x^D=720, the median value $\hat{F}_{DM}$ and logarithmic standard deviation β_{DM} of response factor F_{DM} are given from equations (11) and (12) as follows:

$$\hat{F}_{DM} = 1.13, \quad \beta_{DM} = 0.110 \tag{14}$$

Since there are many reports in which the logarithmic standard deviations for F_{MS} and F_{MF}, which constitute F_{DM}, are normally classified as ones showing uncertainty variability, it seems reasonable to consider that β_{DM} is similarly a logarithmic standard deviation showing uncertainty variability. Strictly speaking, however, it is necessary to quantitatively estimate based on the actual definition and distinction between uncertainty variability and random variability, and much is expected among the researches in this field in the future.

5 CONCLUSIONS

As a method to estimate the response uncertainty of a structure with stochastic parameters during an earthquake, an analytical method using the random vibration theory and stochastic FEM is developed. The adequacy of the analytical method is verified by the Monte Carlo simulation.

Further as an example of the application of the developed analytical method, a new technique capable of analytically presuming a part of safety factor in the seismic PSA is proposed.

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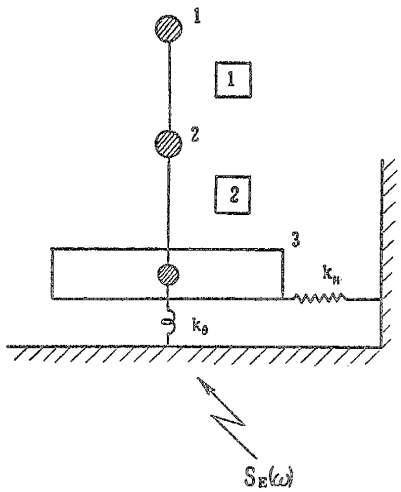

Fig. 1 Benchmark Model

Table 1. Comparison of analytical results with Monte Carlo simulation

		Analytical results (a_A)	Simulation (a_S)	Ratio for comparison (a_A/a_S)
Case 1 Stiffness uncertainty	Expectation of maximum response μ_{x_T}	637	609	1.046
	Standard deviation of maximum response σ_{x_T}	70	63	1.111
	Coefficient of variation of maximum response	0.109	0.104	1.048
Case 2 Damping uncertainty	Expectation of maximum response μ_{x_T}	641	612	1.047
	Standard deviation of maximum response σ_{x_T}	76	70	1.086
	Coefficient of variation of maximum response	0.119	0.114	1.044