ABSTRACT

TEJEDA, HERNAN ALFONSO. Issues Regarding Price Risk in Agricultural Commodity Markets. (Under the direction of Barry K. Goodwin).

This study analyzes issues regarding the risk that crop and livestock production face with respect to price variation. Producers generally face an inelastic demand for their products, while their supply may encounter different unexpected production shocks – generating an increase in price variability. This price risk is compounded when it involves varying prices of production inputs, as in the case of livestock feed. The first study addresses crop production price risk mitigation via crop insurance – specifically crop revenue insurance contracts, by investigating the efficiency benefits of incorporating a model which captures the natural inverse relationship between crop prices and yields. The study begins by considering a parametric distribution which takes into account asymmetry and fat tails to model crop prices, in lieu of empirical evidence of prices having positively-skewed distributions and fatter tails than the conventional characterization obtained from a Log-normal distribution. Subsequently, a copula method is proposed to account for the natural inverse relation between crop prices and yields. Crops considered are corn, soybean, and wheat which form part of livestock feed. Copula methods have been used for managing risk in the financial sector, and results from this study show operational efficiency gains over present crop revenue insurance contracts. The next chapter begins by extending a multivariate time-series dynamic correlations model containing different correlation regimes with regime switching being governed by a Markov chain, via constant transition probabilities. Extension of the Regime Switching Dynamic Correlations (RSDC) model (Pelletier, 2006), permits identification of underlying economic fundamentals affecting the dynamic

interrelationships between markets, by incorporating state dependent transition probabilities in the regime switching process. The state dependent transition probabilities contain fundamental economic variables related to the process. The study then applies the developed model to empirically analyze the impact of increased corn demand from ethanol production - on the dynamic interrelationship between corn, soybean and cattle prices, including spillover effects from market frictions. Results identify specific periods in which the correlation levels among the markets are impacted by the surge in corn consumption conditions. Risk spillovers from one market to another are determined, and their impacts on market interrelationships are discussed. The final chapter makes use of a co-integrated vector autoregressive model to further gauge the dynamic relationships between grain and cattle markets, in lieu of the boost in corn demand from ethanol mandated production. In addition to corn and soybean prices, grain sorghum (milo) and wheat – which also serve as feed - are taken into account along with feeder and live-cattle markets. Results corroborate some of the previous findings regarding the dynamic interaction between grain and cattle markets, especially considering the impact on these market relationships during the period of increased corn demand from ethanol consumption.

Issues Regarding Price Risk in Agricultural Commodity Markets

by
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DEDICATION

To my wife – Luz Maria, my parents – Hernan & Monica, my sisters – Monica & Soledad and all family, friends and loved ones.

BIOGRAPHY

Hernan Alfonso Tejeda Jr. was born in Chile and studied at the Pontificia Catholic University of Chile in Santiago, were he earned a Bachelor Degree in Industrial Engineering. After working for a large forest product company in Chile he came to the U.S. and earned a M.B.A. at the University of Notre Dame. At this point he became aware of his true vocation and passion as an academic and thus switched his career path to doing research and also teaching undergraduate Algebra and Statistics classes at a junior college in South Bend, Indiana. He then entered his doctorate studies in Economics at North Carolina State University and earned his Ph.D. in Economics with a minor in Statistics.

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1.1 Introduction

This study analyzes issues regarding the risk that crop and livestock production face with respect to price variation. Commodity prices for agricultural products – such as crops and/or livestock – constitute a very important source of risk for farmers, livestock producers, consumers, and investors. A study by Goodwin and Kastens (1993) surveying crop producers in Kansas revealed that prices are by far the main source of risk for production decision-making. Fluctuation in these prices that include sharp spikes and plunges or just slight increases and drops, generates risk called volatility. This volatility changes steadily as new information becomes available.

Producers of crops and livestock generally face an inelastic demand for their products. That is, consumers tend to purchase nearly regular amounts of these crop and livestock products despite changes that may occur in their selling price. On the other hand, supply from these crop and livestock producers may encounter different unexpected production shocks. Particular examples of production shocks may be adverse weather conditions or new environmental policies being implemented, thus affecting their production or supply. These shocks affecting production generate a rise in price variability, bringing about price risk or volatility. This price risk faced is further increased when producers are subjected to varying prices of production inputs, as in the case of feed for livestock which consists of rations of different crops.

After the extensive overview of chapter one, the second chapter addresses matters regarding the reduction of price risk handled by crop producers through the use of crop insurance. More specifically, the chapter addresses shortfalls concerning crop revenue insurance contracts. These shortfalls refer to historical average yearly losses[1] - in hundreds of millions of dollars - resulting from crop revenue insurance contracts. The study first discusses distributional assumptions of crop prices being used by the most typical crop revenue insurance contracts: Crop Revenue Coverage (CRC) and Revenue Assurance with Base Price (RA-BP). Then with actual price data a goodness of fit test compares the assumed distributions of these contracts with respect to a different, more flexible distribution – the Burr XII distribution, able to capture in different parameters the distinct shapes of the distributions.

The study then introduces a Copula method to determine the inverse relationship between crop prices and yields by considering yields from corn and wheat from both a specific county and a specific state. The Copula method used is able to determine the relationship between many variables without requiring these variables to assume particular marginal distributions. Subsequently, simulations of the expected payout for different coverage levels of crop revenue insurance are made by using this Copula method, and these different coverage scenarios are contrasted to the case of using the CRC and RA-BP crop revenue insurance contracts. Results determine lower expected payouts when considering a Copula model, indicating potential advantages in efficiency with the Copula method for state crop yields.

The particular distribution proposed – the Burr XII – considers estimation of up to four parameters, including two parameters that are able to capture irregular shapes or higher moments such as skewness[2] and kurtosis. [3] The other two parameters are for location and for scale. Data

[1] In recent decades, many of these losses have been a result of the amount paid by Governmental subsidies, currently at over 50 % of the premium. However, these subsidies are not formally considered part of the losses in crop insurance programs.

[2] A measure of the asymmetry of the probability distribution of the random variable.

from futures prices are obtained from the Chicago Board of Trade (CBOT) through the Commodity Research Bureau (CRB). Results indicate a better characterization of the prices with the Burr XII distribution over the Normal distribution, yet not significant improvement over the Log-Normal distribution of prices.

The study then proceeds to model the joint relationship between crop prices and their yields by using two different Copula models. Copula models permit one to capture the relationship between many variables without requiring these individual variables to follow any pre-determined marginal distributions. The crop prices were modeled not only with the Burr XII distribution but also with a log-normal, and non-parametrically and the crop yields were modeled following a Beta distribution since they tend to be left-skewed. [4] Crop yields were considered for corn and soybeans from a large county in Iowa, as well as for the entire state of Iowa. Also considered were wheat yields for a large county in Kansas, as well as for the entire state of Kansas. These crop yields were obtained from the National Agricultural Statistics Service (NASS), USDA.

With simulated data from the previously estimated copulas, an expected payout function for different percentages of crop revenue coverage was computed and compared to expected payouts from using the CRC and RA-BP cases. A smaller expected payout was determined by introducing the Copula method in the crop revenue insurance program, i.e., it provides less-expected indemnity, when considering crops by state, than in the cases of the CRC and RA-BP insurance methods. This lower-expected indemnity may lead to lower premium rates than the

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[3] A measure of the "thickness" of the tails of the random variable's probability distribution. For higher kurtosis – more of the distribution's variance is due to infrequent extreme deviations.

[4] Nelsen and Preckel (1989).

CRC and RA-BP programs incur and thus improve efficiency in the crop revenue insurance programs.

The next chapter extends a model which measures price risk or price volatility in a multivariate time-series setting. This Regime Switching Dynamic Correlations (RSDC) model[5] estimates the risk or volatility of multiple time series of prices; yet more importantly, the model is able to estimate the changing or dynamic correlations between these price series. This RSDC model considers the price time-series switching from one correlation regime to a different one as being governed by a Markov chain with constant transition probabilities determining the change from one regime to another.

The chapter makes use of this newly extended RSDC model – the state-dependent regime-switching model of dynamic correlations – to determine dynamic price transmissions or linkages between corn, soybean, and feeder- and fed- cattle markets. It aims to identify the specific impact produced by the recent increase in corn consumption used for ethanol production on the dynamic price relationships among these markets. This spike in ethanol production using corn as an input, responds to the Energy Acts of 2005 and 2007 which mandated substantial increases in ethanol consumption.

In our estimation results, the model captures asymmetric correlations between grains and livestock prices, including volatility spillovers. These volatility spillovers are characterized by the resulting persistence of markets remaining at certain correlation levels instead of switching to a different correlation level. In addition, a potential inhibition is determined in the transmission

[5] Pelletier (2006).

of prices between markets, which may occur as a result of adjustment costs between these markets.

Additional results determined are consistent with past literature such as the case of positive dynamic correlations between corn and soybean prices, since these two crops share planting acreage. Likewise, positive dynamic correlations between prices for feeder-cattle and live-cattle are verified, as these are the main components of cattle production. These results are determined for both periods estimated, pre- and post- mandated ethanol corn consumption. For the period of post- mandated ethanol consumption (i.e., when corn and soybeans experience sharply rising prices), inverse or negative dynamic correlations are determined between both corn and feeder cattle and soybean and feeder cattle. The inverse corn and feeder cattle relationship is likewise consistent with previous literature, where increases in prices of corn, a main regular feed component for livestock, leads to a decrease in the price of feeder cattle in order to maintain cattle production profitability. Similar results are obtained for soybeans, which is another relevant feed component.

The final chapter makes use of a Co-Integrated Vector Auto- Regressive model (Co-Integrated VAR), also known as a Vector Error Correction model (VEC), to further gauge the dynamic relations between grains and cattle markets. The co-integration factor refers to an error correction term among the variables that estimates the long-run relationship between the markets considered. This model also permits the determination of Granger Causality among the variables, but more importantly, it allows for the analysis of response functions on each market from the effects of individual shocks done to a market; in other words, what type of response a market has when shocking a related variable.

Some results here are consistent with past literature, as in the case of corn and grain sorghum having bi-directional Granger Causality among them during the first or pre-mandated ethanol period (i.e. changes in prices of corn lead the changes in prices of sorghum, but also changes in prices of sorghum lead the changes in prices of corn). Yet for the post-mandated ethanol period, it is only price changes in corn which produce Granger Causality in price changes in sorghum. A plausible result from substituting corn at higher prices for sorghum in livestock feed rations.

Another result anticipated from the literature was that feeder cattle and live cattle price changes have bi-directional Granger Causality during both the first period and the second period estimated. This result was anticipated as previous studies have shown that these prices are the main risk components in cattle production profitability, and hence affect each other. This result is also corroborated by the estimation of the prior model, where there is a significant dynamic correlation between feeder cattle and live cattle markets.

A surprising result was that for the first period estimated, there was no Granger Causality in any direction between price changes of corn and soybean. During the second period a long-run positive relationship was determined between corn and soybean, given by the error correction term for the VEC model. This long-run relationship between price changes of corn and soybeans was expected since these crops shared planting acreage for depending on the crop's price and profitability, farmers will regularly choose to plant either corn or soybean.

1.2 Overview

Crop insurance contracts began formally in 1938 with the passage of the Crop Insurance Act which created the Federal Multi-Peril Crop Insurance (MPCI) program aimed at reducing crop yield variability. These insurance contracts were initially designed and limited to major crops within major producing areas; e.g., wheat or corn in mid-western states. However, through the years, crops and area coverage have been increasingly extended. Until 1980, participation by eligible crop producers was generally low. More importantly, throughout that period the crop insurance program incurred substantial yearly average losses - both as a result of average annual loss ratios[6] being higher than one and from the farmer's insurance premiums being subsidized to promote participation.

With passage of the Federal Crop Insurance Act in 1980, crop insurance extended coverage to most farmers in all crop-producing areas. Participation was still promoted with premium cost subsidies of up to 30 %,[7] and the programs covered average yields of 50, 65, and 75 % of a farmer's crop. Despite participation rates increasing during this period from 10 % to about 30 %, substantial losses increased as loss ratios remained above one. Yearly average losses during this period were in the order of hundreds of millions of dollars.[8] In 1994, passage of the Federal Crop Insurance Reform Act led to mandatory participation by farmers in insurance programs in order to be eligible for deficiency payments under price or income support programs. In 1996 with passage of the Federal Agriculture Improvement and Reform Act, mandatory participation was repealed, and new crop insurance programs became available that included protection from both yield and price changes under market conditions, i.e. new crop revenue insurance programs were established. Participation rates climbed to over 50 % with federal support maintained by subsidizing a farmer's premium payment.

[6] Loss Ratio is defined as total indemnities paid out (received by crop producer) over the premium paid.
[7] For this case, the farmer may incur up to a maximum of 65 % coverage of their average crop yield.
[8] Goodwin and Smith (1995)

Among these new crop revenue insurance programs, two of the most widely used since 1996 have been Crop Revenue Coverage (CRC) and RA-BP (Revenue Assurance with Base Price). These programs assure a farmer's crop revenue and consider a shortfall from both crop prices and yields. Both CRC and RA programs consider separate distributions for crop yields and their prices in order to compute the estimated premium rate.[9] In the case of the CRC program, the changes in crop futures prices are considered to follow a Normal distribution. On the other hand, in the case of the RA-BP program, futures prices are considered to follow a Log-normal distribution, which is equivalent to the log of prices following a Normal distribution. Despite these previous assumptions, empirical evidence holds that price returns tend to have a positively skewed distribution and fatter tails (i.e. leptokurtosis) than Log-normal distributions. These two programs have also experienced substantial losses, either from a few loss ratios being greater than one or from the subsidies at over 50% of the premium being granted to promote participation, or likewise from the loss ratios being less than one.

The next study extended and developed a dynamic friction model based on the RSDC model, by modifying the transition probabilities that govern the switching process between regimes from constant probabilities to state dependent or time-varying probabilities[10]. The model's extension introduces underlying fundamental economic, weakly exogenous variables in determining the probabilities of switching from one regime to another. These new regime switching probabilities become state dependent or time-varying. The underlying economically-related variables in the regime switching probabilities identify specific friction levels which have particular effects on

[9] For (an Actuarial) Fair Rate – the Premium rate should be equal to the Loss (or Indemnity) rate.
[10] Following Diebold et al. (1994).

the dynamic process. For example, the economically related variables have a direct impact on the series remaining at one correlation regime or another.

During periods of increasing changes in price levels and rising volatilities, it is especially relevant to determine both the dynamic market linkages among related assets or markets and the evolution of the transmission of price changes or variations between these related markets. This model permits a better portrayal of the price relationship and transmissions between markets for operational improvement, as well as providing better information for risk-management purposes. This extended model determines the dynamic correlation values between multiple-related markets, including the particular effect that underlying fundamental economic variables have in the evolution of these markets. Capturing the impact of these underlying variables also enables to determine the response effect that specific shocks on these variables may produce on the dynamic process of these series. The model is also applicable for managing risk through multiproduct hedging and may provide better information for potential increases of efficiency in the operation of related markets.

The market prices of grain and oilseeds rose sharply from 2006 until mid 2008; they increased steeply for soybeans and approximately two-fold in the case of corn. This may have impacted livestock markets by increasing prices and leading to significant volatility shocks since more than half of the corn production is used as animal feed and soybean is also an important feed source of protein. During the estimated period considered between 1998 and 2008, parameter estimation was partitioned into two separate time periods – with the latest period beginning late 2003 until 2008 when corn faced a consumption boost from ethanol production. Data for grains and cattle come from the Chicago Board of Trade (CBOT) and the Chicago Mercantile Exchange (CMEX), respectively.

Increasing grain commodity prices coupled with changes in their volatility have implications for many decision makers. Agents who have a direct relation with grain markets – specifically agents dealing with corn and also with soybeans (oilseeds) are particularly affected by these price variations. In general, corn and soybean farmers share production acreage; therefore, crop producers must decide between growing either corn and/or soybeans on their farm, seeking to obtain higher profitability. At the same time, livestock producers require these crops as inputs; hence, their costs and profitability are directly affected by the change and volatility in these crop prices. These agents benefit from an appropriate determination of the dynamic interrelationships among these markets, as it may lead to efficiency gains in their operation. In addition, policy makers must determine the impact that recent energy policies – in this case, those directly affecting corn consumption – have on the prices and markets related to this grain.

No significant correlation was obtained between corn prices (used as feed) and fed – or live – cattle markets for both periods considered. Thus, an existing permanent friction was identified, responding to a transaction or adjustment cost which inhibits the transmission of price variations between corn prices and live cattle prices. This transaction cost may respond to information and/or negotiation costs[11] from cattle producers selling slaughter cattle and is materialized in the form of modifications of the feed rations given to cattle for weight gain in periods of increasing corn price, thus preventing increases in the price of these crops to be transmitted to the live cattle prices.

[11] Hobbs (1997)

Information costs for cattle producers may involve price uncertainty for the case of fed cattle being sold at cash or spot markets[12] leading producers to switch feed rations by increasing allocation to feed components with lower costs in order to maintain costs down during production. Negotiation costs may respond to the limited number of cash or spot markets faced by producers when fed cattle are ready to be slaughtered, thus raising the transaction cost by using that channel. For producers selling directly with alternative marketing arrangements[13] to packers, negotiation costs may rise because of having only a few different packers to bargain with, thus resulting in the packer exercising marketing power and establishing price and conditions of delivery. This again leads the cattle producer to seek feed rations that do not experience cost increases, i.e., modifying rations when there is a price rise in some of the components.

The impact or effect of significant underlying economic factor(s) is identified by taking into account (1) the effect of the 'use to stocks' ratio of corn which comprises the market's dynamic demand and supply conditions for corn and (2) the ratio of soybeans to corn futures' harvest prices, a measure contemplated by crop producers when deciding the acreage portion to plant either corn or soybeans. Both of these factors have a role in spillover effects among the markets. Results previously mentioned are determined separately by two different models: a restricted or parsimonious version and the full unrestricted version. The model with a mild better fit considers the 'use to stock' ratio from corn as the relevant underlying economic variable.

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[12] Cash or spot markets here refer to transactions 'on the spot.' These include auction barn sales; video or electronic auction sales; sales through order buyers, dealers and brokers, and direct trades as per RTI (2007).

[13] These arrangements accounted for less than 40 % of all Fed Cattle volume from October of 2002 to March of 2005 and includes forward contracts, marketing agreements, procurement or marketing contracts, production agreements, packer ownership, custom feeding, and slaughter: per RTI (2007)

For the mildly preferred model, a relevant point determined during the post-ethanol corn-consumption scenario is the identification of the significant effect of corn's "use to stock" ratio in the state-dependent transition probabilities. This results in the determination of volatility spillover effects, or markets (e.g., corn and soybeans, or feeder- and fed- cattle) remaining in certain correlation regimes. These spillover effects are identified by explicitly taking into account the variable that includes the dynamic (up-to-date) market demand and supply conditions of corn. This variable – the use to stocks ratio of corn – has specifically incorporated the impact of the increase in demand of corn resulting from the surge in ethanol production. Thus, the effect of ethanol production on corn's rapidly growing consumption is contained in the dynamic use to stock ratio of corn, during the post-ethanol corn-consumption period. Likewise, the explicit impact of this increased corn consumption, from ethanol production, is captured and/or identified by the resulting spillover effects produced among the markets.

The final chapter is the application of a Co-Integrated Vector Auto-Regressive model (Co-Integrated VAR), or Vector Error Correction model (VEC), to further explore the dynamic relations between grains and cattle markets during the pre- and post-mandated ethanol periods. In this study, daily data was considered instead of the prior study with weekly data, and included not only the variables considered before (i.e., corn, soybeans, feeder-cattle and live- cattle), but also grain sorghum (known as milo) and wheat. It is important to note that these latter grains are main elements of carbohydrate substitution for corn used in feed rations for livestock. The time periods considered were once again for intervals of pre- and post- mandated ethanol production, which has relied mostly on corn consumption. Estimations were made from January 1998 to December 2004 and from January 2004 to April 2009, with grains data obtained from the CBOT and cattle data obtained from the CMEX. Two different unit roots tests conducted confirm the presence of a unit root for each of the series during both periods considered. In other words, the series are non-stationary for each period.

Some results here are consistent with past literature, as in the case of corn and grain sorghum having bi-directional Granger Causality among them during the first or pre-mandated ethanol period (i.e. changes in prices of corn lead the changes in prices of sorghum, but also changes in prices of sorghum lead the changes in prices of corn). Yet for the post-mandated ethanol period, it is only price changes in corn which produce Granger Causality in price changes in sorghum. Thus, in the second period corn price changes are leading those of sorghum; this being a plausible result from substituting corn at higher prices for sorghum in livestock feed rations. These results are corroborated by the response function on sorghum from shocks to corn, and from the response function on corn from shocks to sorghum, for each period estimated. There is a higher response on sorghum obtained from a corn shock in the second period when compared to the response during the first period.

Another result anticipated from the literature was that feeder cattle and live cattle price changes have bi-directional Granger Causality during both the first period and the second period estimated. This result was anticipated as previous studies have shown that these prices are the main risk components in cattle production profitability, and hence affect each other. This result is also corroborated by the estimation of the prior model, where there is a significant dynamic correlation between feeder cattle and live cattle markets.

Two unexpected results were obtained, on the one hand not determining any Granger Causality relation between changes in prices of corn and feeder cattle for the second period estimated. This unusual result occurred during the substantial increase in the price of corn during this second period. Another surprising result was that for the second period estimated, there was no Granger Causality in any direction between price changes of corn and soybean. The lack of relationship between these two markets is corroborated by the null- response on a market from price shocks to the other market. However, during this second period a long-run negative relationship was determined between corn and feeder cattle, and a positive relationship was determined between corn and soybean. Both of these results were given by the error correction term for the VEC model. This long-run relationship between price changes of corn and cattle feeder was expected, especially in lieu of the spike in corn prices, affecting cattle production profitability and thus driving feeder cattle process down. Likewise, a long-run positive relationship between corn and soybeans was expected, especially during rapidly changing prices, since these crops share planting acreage and depending on the crop's price and profitability, farmers will regularly choose to plant either corn or soybean.

The two previous unexpected results also occurred during the first period, but in this case there was no long-run relationship identified between the markets. Perhaps during this first time period, it may be due to the lack of a spike in demand of corn with respect to soybeans, which did occur during the second period estimated. In this sense, the lack of large shift in acreage from soybean to corn and then back again which occurred during the second period reaffirmed the long-run relationship between these two crops; yet the relationship was not identified during the first period. In the same way, during the second period and specifically in a scenario of increasing corn prices, the long run inverse relationship between corn and feeder cattle prices was corroborated. However, lacking this expansion in corn prices during the first period may have prevented the relationship between these markets to be identified.

CHAPTER 2

Crop Price modeling with a Burr Distribution and Correlation Analysis between Prices and Yields with Copula Models for Crop Insurance Rating

2.1 Introduction

Crop insurance is of critical importance in the farming business to properly address production and/or revenue shortcomings for farmers. A substantial drop in crop production may be the result of random shocks caused by weather, pests or other forms of natural disasters including fire. These unexpected events lead farmers to bear substantial losses in their yields, subsequently producing a significant drop in their revenues for that period. The outcome of these losses may be compounded when farmers are facing large fixed costs – due to capital-intensive production techniques – thus putting them in an even more perilous situation.

Historically, the federal government has sought to address these unexpected circumstances – which result in substantial lower yields – before they occur, through various crop insurance programs. Incentives for farmers to participate in crop insurance programs have generally been supplied by subsidizing their premium payment. In addition, several federal disaster relief programs have operated in response to significant yield shortcomings by providing disaster payments to farmers who were initially not participating in a crop insurance program. Payments from these disaster relief programs – at first with no requirement of being previously enrolled in an insurance program – were generally accompanied by a slim participation in the formal insurance programs, which did require upfront premium payment (Goodwin and Smith, 1995).

A vast amount of literature on different venues regarding crop insurance exists up to date. Studies of farmer's characteristics and their risk position in the willingness to purchase particular types of crop insurance have been made by Goodwin (1993), Smith and Baquet (1996), Coble et al. (1996), Coble and Knight (2002), Mishra and Goodwin (2003), Serra et al. (2003), Sherrick et al. (2003) and Chambers (2007). Changes in marketing programs as a result of crop insurance choice is addressed by Coble et al. (2000). Another notable venue studied concerns the moral hazard and adverse selection effects on the adoption of crop insurance as researched by Miranda (1991), Babcock and Hennessy (1996), Smith and Goodwin (1996), Just et al. (1999), and Roberts et al. (2006). In addition, Knight and Coble (1997) conducted an extensive survey of papers that have examined issues relating to the Multiple-Peril Crop Insurance (MPCI) program, following the 1980 Federal Crop Insurance Act. More recently, a general overview of crop insurance studies from 1980 onwards was presented by Glauber (2004).

A major problem with the use of crop insurance since its formal inception in 1938 has been the excessive payouts compared to the premiums paid by farmers; i.e., loss ratios[1] being higher than one, as addressed extensively in Goodwin and Smith (1995) and Goodwin (2001), therefore generating substantial losses to the federal government. Since the previous decade many of these ratios have been far less than one hence losses may result from excessive premium rates including subsidies paid by the federal government, currently at over 50% of the premium rate.[2] Thus another venue of research addresses matters relating to proper calculation of the premium

[1] The Loss ratio is defined as the amount of indemnity or payout from the insurance program over the premium received by the insurance program. An actuarial fair Loss Ratio is equal to one (i.e., the price of the policy or premium is equal to the expected payout or loss).

[2] Premium Rates are defined as the quotient between the expected indemnity/payout (loss) over the minimum amount insured. Both these latter terms are defined and computed under specific conditions, in the Discussion section ahead.

rates, as these are essential for an efficient implementation of any crop insurance program (Coble and Knight, 2002). Many of these studies concern area yield insurance such as Miranda (1991), Skees et al. (1997), Goodwin and Ker (1998), Ker and Goodwin (2000), Ker and Coble (2003), Babcock et al. (2004), Barnett et al. (2005), and Ozaki et al. (2008). Area yield insurance provides effective risk management for areas where yield risks are largely systemic. This is applicable for crops in the Midwest, Plains and other geographical areas where the topographical and climate characteristics of the terrain are similar over a vast amount of the planted acreage.

Currently, two of the primary crop revenue insurance programs in place are the Crop Revenue Coverage (CRC) and the Revenue Assurance (RA) programs, which calculate premium rates considering the estimated distributions of both crop yields and their prices. These rating methods must give consideration to the correlation between annual price returns and yields, and the natural hedge that is implied by them. This study aims to provide a new perspective in the method of analyzing crop revenue by determining the inverse relationship between crop yields and their prices with the application of copula methods. In contrast to prior studies considering lower tail dependence (Embrechts et al., 2003)[3] between crop yields and prices, this chapter applies copula models to specifically capture the negative relationship present between crop yields and their prices. Subsequently, the identified value of this inverse relationship is incorporated in a simulated crop revenue insurance context, obtaining improved efficiency results over current crop revenue insurance methods.

[3] Concept relates to the amount of dependence in the lower-left-quadrant tail of a bi-variate distribution. Upper tail dependence is the amount of dependence in the upper-right-quadrant tail. Definition details ahead in section of Empirical Methods.

This chapter first adopts a Burr type XII distribution to characterize price returns and compares the goodness of fit of these prices relative to the Normal and Log-Normal distributions, which are currently being applied in the CRC and RA programs, respectively. Empirical evidence and conventional wisdom holds that price returns tend to have a positively-skewed distribution, with fatter (leptokurtosis) tails than Log-normal distributions. Goodwin and Ker (2002) offer an extensive review in this matter. The relevance of properly portraying the series of price returns for this study is that it may assist in subsequently obtaining a better depiction of the relation between prices and yields – the second analysis of this chapter. The benefit conveyed by the Burr XII distribution is that it specifically considers parameters that capture higher moments observed in the data, enabling skewness[4] and kurtosis[5] properties to be properly portrayed.

The study then makes use of copula models to assess the inverse relationship between crop price returns and their yields. Copulas are a convenient statistical method of measuring the correlation between variables by only taking into account the parametric marginal distributions of these variables or their non-parametric marginal distributions (in the empirical case of having a large number of observations). Two different copula models are estimated by using the previous mentioned distribution for price returns, i.e.; Burr XII, and by applying the Beta distribution for crop yields. The use of this latter distribution for crop yields is in accordance with Nelson and Preckel (1989) and Gallagher (1987). Additional estimations of these copula models are made considering Log-Normal distribution for prices and likewise considering a non-parametric cumulative fit for both prices and yields. Crops considered are corn and soybean in the state of

[4] A measure of the asymmetry of the probability distribution of the random variable.
[5] A measure of the "thickness" of the tails of the random variable's probability distribution.

Iowa and wheat in the state of Kansas. Results obtained show that a significant inverse relationship between crop prices and yields is duly captured with a Copula method.

Simulations of expected indemnities for a crop producer – considering average area yields and expected price returns – are made under current revenue insurance contracts and compared to the case of incorporating a copula model that accounts for the negative relationship between crop prices and yields. Results provide a lower expected payout for the application of the copula model in comparison to the two current crop revenue insurance contracts mentioned previously. These lower expected indemnities may result in efficiency improvements of revenue insurance programs, since it leads to the computation of lower premium rates. The chapter proceeds by providing a general background on crop insurance followed by a section on empirical methods where the characteristics of the Burr XII distribution are discussed, as well as the copula methods and specific copula models are presented. Subsequently, the data is presented along with the results and discussion about the potential advantages of applying a Copula method in the rating of crop insurance. Future veins of research are also mentioned.

2.2 Background

Since the inception of the Federal Multiple-Peril Crop Insurance (MPCI) in the 1930s, there has been continuous updating of federal programs to improve the application and efficiency of the crop insurance programs. Another means of supporting crop producers during unanticipated devastating events has been to provide aid through federal disaster relief programs. The theory of relation between this federal disaster aid and crop insurance is addressed in Goodwin and Smith (1995), and an overall historical review of federal disaster relief programs is studied by Goodwin and Vados (2007). Regarding the Federal MPCI, the latest program change occurred in the year 2000, with the enactment of the Agricultural Risk Protection Act (ARPA) from Public Law 106-224. This change includes comprehensive sections in crop insurance coverage including incentives to introduce new insurance products, as well as more agricultural assistance by increasing government subsidies.

Formally crop insurance begun in 1938 with the Federal Crop Insurance Act that established the Federal Crop Insurance Corporation (FCIC). This entity created the Federal Multiple-Peril Crop Insurance program (MPCI), which assisted farmers with multiple-peril or all-risk crop insurance. This insurance coverage program was initially designed and limited to major crops within major producing areas; e.g., wheat or corn in Midwestern states. However, for many years since its inception, the crop insurance program has incurred substantial losses coming both from the farmer's insurance premium being subsidized to promote participation, as well as from the result of annual average loss ratios being higher than one.[6]

From 1939 to 1943 the average loss ratio was 1.65 as noted by Goodwin and Smith (1995). For the next year, 1944, and until 1955, the average loss ratio fell to 1.16, and the average annual premium subsidy was approximately \$3.4 million which was smaller than the average annual premium subsidy of the previous period of \$11.7 million, yet the insurance program still incurred in losses. The following years saw the loss ratio drop below one due to innovations in crop coverage by extending to more crops, yet areas covered by these new crops were limited.

Subsequently, a major drive to extend coverage for a much larger number of crops and a much broader extent of farming land was established with the 1980 Federal Crop Insurance Act.

[6] Government subsidies that promote the purchase of crop insurance are not formally considered as a loss in the loss ratios.

This act involved major revisions to the crop insurance program, seeking mainly to address multiple cases of major federal disaster payouts that had previously been made since 1973. These federal disaster payments operated in response to the Agricultural and Consumer Protection Act from the farm bill of 1974 and were maintained in the 1977 farm bill with the Food and Agriculture Act. These acts considered mandatory disaster coverage program for crop producers, resulting in disaster payments of $3.39 billion (Goodwin and Smith 1995). The 1980 Act sought to deal with these enormous free disaster payments by sharply promoting the purchase of crop insurance and encouraged the participation of farmers by subsidizing up to 30% of the premium rates for 65% or 50% coverage of their average yields. The brief federal disaster relief program was to be replaced by a more broadly covered federal crop insurance program, which was offered extensively by private companies.[7]

Participation rates in crop insurance by farmers grew from about 10% in 1980 to about 40% in 1990 and then dropped back to 32% in 1992. Nonetheless, average annual loss ratios continued to be above one and many times close to two, including many years for which the average yearly payment of premium subsidies and indemnities exceeded half a billion dollars in total (Goodwin and Smith, 1995). During the late 1980's and early 1990's various disaster relief payments were authorized by Congress due to major droughts experienced by farmers during these periods. These ad-hoc disaster bills allowed farmers without crop insurance to claim relief, though in some cases they were obliged to purchase MPCI the following year to subsequently claim benefits. Another disaster relief program was enacted by Congress in 1993 due to a prolonged wet and cool season, leading the federal government to realize substantial additional

[7] Previously, private companies offered crop insurance backed by federal insurance for only a limited number of crops and counties.

payments and hence, losses. Some of these relief programs were in direct competition with the federal crop insurance program, driving Congress to enact the Federal Crop Insurance Reform Act of 1994.

The Crop Insurance Reform Act of 1994 not only addressed lower yields obtained during production, but also dealt with lower prices being faced by farmers at harvest time. Participation was made mandatory in order to obtain price-support programs, special loans or other benefits, including catastrophic coverage.[8] Later in 1996, crop insurance mandatory participation was repealed, yet farmers where able to receive other benefits including disaster relief only if insurance was previously purchased, since ad-hoc disaster relief payments were eliminated. The 1994 Act had three coverage levels of 50, 65, or 75 % of crop producer's average yields made available with a price election between 50 to 100 % of the expected market price.[9] Average yields were determined according to an actual historic production (APH)[10] for the farmer's crops of up to ten years. In cases of insufficient yield-data available for a farmer seeking insurance, the National Agricultural Statistics Service (NASS) provided data on program yields.

Participation rates climbed to over 50 %, with federal support being maintained by subsidizing a farmer's premium payment. Among these new crop revenue insurance programs, two of the most widely used since 1996 onwards have been Crop Revenue Coverage (CRC) and RA-BP (Revenue Assurance with Base Price). These programs insure a farmer's crop revenue and consider a shortfall from both crop prices and yields. Both CRC and RA programs consider a separate distribution for crop yields and their prices to compute the estimated premium rate. Yet

[8] Losses exceeding 50% of crop yields where compensated at 60% of that year's established price.
[9] Previously with the 1980 Act, only three price election levels were available for most crops.
[10] APH - Actual Historic Production is used to determine the guarantees of insurance. This APH yield is determined by averaging at least four subsequent yields from an insuring unit. Transition yields are used in case the four subsequent yields are not available. These transition yields encompass up to ten years of historical yields.

the two programs have also experienced losses, with loss ratios far below one (i.e., premium payments much higher than the payout) during some periods or higher than one for other instances.

2.3 Empirical Methods

Annual crop price returns considered are the product of 100 times the difference of logs, i.e. $100 * (ln(P_t^{dph\_f}) - ln(P_t^{ea\_f}))$, where $P_t^{dph\_f}$ refers to the futures market price at the post-harvest delivery date and $P_t^{ea\_f}$ is the future price 'ex-ante' (i.e., this latter future price is at the planting stage). In the present case of CRC, premium rate calculations assume that annual price differences between $P_t^{dph\_f}$ and $P_t^{ea\_f}$ follow a normal distribution according to a procedure developed by Botts and Boles (1957). For the case of RA with the base price option, i.e., RA-BP, premium rates are computed under the assumption that 'ex-ante' crop prices follow a log-normal distribution. This is equivalent to log prices following a normal distribution. For extensive details of these methods and discussion of their shortcomings see report from the U.S. General Accounting Office (1998, GAO/RCED 98-111).

The Burr distribution family – includes from Burr I to Burr XII – has densities that account for a broad range of shapes making them useful in capturing higher moments from the data. The Burr I is the uniform distribution with no parameters, and both the Burr III and the Burr XII include up to four parameters – one for location, one for scale and two for shape. The density and distribution functions are of closed form (Kotz and Johnson, 1981). The rest of Burr distributions consider only up to one shape parameter; hence, the Burr III or XII have a more flexible specification to capture skewness and kurtosis.

In addition to estimating crop prices with a Burr XII distribution, estimation was made with the Burr type III distribution. This latter distribution considers the data variable being equal to the inverse of that used for the Burr type XII distribution. Estimation of parameters was done via method of moments (Lindsay et al., 1996). A comparison of results from mapping moment estimates obtained from simulations in both models revealed that the Burr type XII distribution characterized better the data.

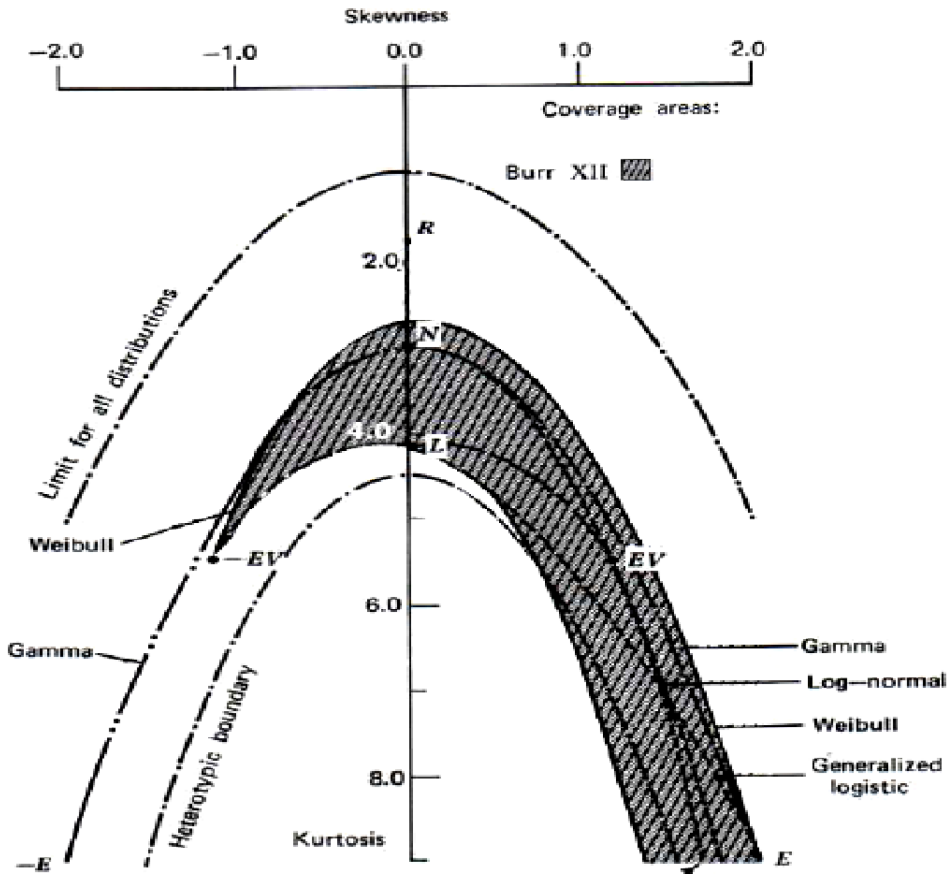
The *Burr type XII distribution* for the crop price returns has the following characteristics: (4 parameters – 2 for shape, 1 scale, 1 for location[11])

(1) c.d.f. $F_{B12}(y;\alpha,\tau,\phi) = 1 - \left\{1 + \left(\frac{y}{\phi}\right)^{\tau}\right\}^{-\alpha}$ for $y \geq 0$, $\tau: shape1$; $\alpha: shape2$; $\phi: scale$

(2) p.d.f. $f(y) = \left(\tau * \alpha * \left(\frac{y}{\phi}\right)^{\tau}\right)\{y^{-1} * [1 + (\frac{y}{\phi})^{\tau}]^{-(\alpha+1)}\}$

This Burr XII distribution comprises a broad range of values for the estimates of both skewness and kurtosis (Rodriguez, 1977). The skewness-kurtosis points or regions covered by the Logistic, Normal, Exponential, Log-normal, and Weibull distributions are all included, or mostly included – as in the case for the Gamma distribution – by the Burr XII distribution as seen in figure 2.1. The parameters of this distribution are estimated via a Maximum likelihood method following Watkins (1999) and Johnson (2003).

[11] For simplicity, in this case the location parameter has been set equal to zero.



N: Normal, L: Logistic, E: Exponential, EV: Extreme value 1. The symbol – denotes reflection.

Source: Kotz and Johnson, 1981 – Volume 1

Figure 2.1: A Skewness - Kurtosis Chart with Different Distributions.

Both Normal and Log-Normal distributions are likewise estimated via maximum likelihood, and contrasted to that obtained with the Burr type XII distribution. The non-nested Vuong test, from Vuong (1989), is used to assess the improvement of the Burr XII distribution over the previous two distributions. The Normal distribution has the following known characteristics –

c.d.f. $F_N(y;\ \mu,\sigma) = \Phi(\frac{y-\mu}{\sigma})$ for $y \epsilon \mathbb{R}$, μ: mean $\in \mathbb{R}$, σ: standard deviation > 0, and the p.d.f. $f(y) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{\frac{-(y-\mu)^2}{2\sigma^2}\right\}$; and the Log-normal distribution: c.d.f. $F_{LN}(y;\ \mu,\sigma) = \Phi(\frac{\ln(y)-\mu}{\sigma})$ for $y > 0, \sigma > 0$ μ (location),σ (scale): in log space, and p.d.f. $f(y) = \frac{1}{y\sqrt{2\pi\sigma^2}} \exp\{\frac{-(\ln(y)-\mu)^2}{2\sigma^2}\}$.

Separately, a Beta distribution with two parameters is used to model the crop yield data as it is usually negatively skewed. For the alternative of having large amounts of observations, non-parametric methods may be more suitable (Goodwin and Ker, 1998). The Beta distribution applied has c.d.f.: $F_B(y;\ \alpha,\beta,\emptyset) = I_{\frac{y}{\emptyset}}(\alpha,\beta) = \frac{B_{\frac{y}{\emptyset}}(\alpha,\beta)}{B(\alpha,\beta)}$, with $B_{\frac{y}{\emptyset}}(\alpha,\beta) = \int_0^{\frac{y}{\emptyset}} t^{\alpha-1}(1-t)^{\beta-1}dt$, $B(\alpha,\beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$ for $\alpha,\beta > 0$, and $\Gamma(n) = (n-1)! = \int_0^{\infty} t^{n-1}e^{-t}dt$ and the p.d.f. $f(y) = \frac{1}{B(\alpha,\beta)}(\frac{y}{\emptyset})^{\alpha-1}(1-\frac{y}{\emptyset})^{\beta-1}$ for $0 < y < \emptyset$

Regarding the copula method, it is a function that 'couples' together a multivariate joint probability function to their one-dimensional marginal distributions. In other words, a copula is a multivariate distribution function that has one-dimensional marginal functions that are uniform on the interval [0, 1] (Nelsen, 1999). This chapter considers the application of two different copulas to the data - the Normal Copula and the Frank Copula, both which will be defined and contrasted below.

Copula Definition (derived from Sklar, 1959)

Let *H* be an n-dimensional distribution function with marginals $F_1, \dots \dots \dots, F_n$.Then there exists an *n*-copula C such that for all $\boldsymbol{x}$ in $\mathbb{R}^n$:

(3) $H(x_1, \dots \dots \dots, x_n) = C(F_1(x_1), \dots \dots \dots, F_n(x_n))$.

If $F_1, \ldots \ldots \ldots, F_n$ are all absolutely continuous, then C is unique. Conversely, if C is an *n*-copula and $F_1, \ldots \ldots \ldots, F_n$ are (cumulative) distribution functions, then the function *H* defined above is an *n*-dimensional (cumulative) distribution function with marginals $F_1, \ldots \ldots \ldots, F_n$. For a univariate distribution function F, the generalized inverse of F is $F^{-1}(t) = \inf\{x \, \epsilon \, \mathbb{R} \mid F(x) \geq t\}$ for all *t* in [0,1]. Then for any $\boldsymbol{u}$ in $[0,1]^n$:

(4) $$C(u_1, \ldots \ldots \ldots \ldots \ldots \ldots, u_n) = H\left(F_1^{-1}(x_1), \ldots \ldots \ldots \ldots \ldots .. F_n^{-1}(x_n)\right)$$

The multivariate density can be obtained directly by differentiating (3), such that:

(5) $$h(x_1, \ldots \ldots \ldots, x_n) = c[(F_1(x_1), \ldots \ldots \ldots, F_n(x_n)] \prod_{i=1}^{n} f_i(x_i),$$

where $c(u_1, \ldots \ldots \ldots \ldots \ldots .., u_n) = \frac{\partial^n C(u_1, \ldots \ldots \ldots, u_n)}{\partial u_1, \ldots \ldots \ldots, \partial u_n}$ and $f_i(x_i) = F_i^{'}(x_i)$ for $i = 1, \ldots \ldots n$

Elliptical copulas (Normal or t-student) are restricted to radial symmetry. The Normal Copula distribution (Freez and Valdez, 1998) is as follows:

(6) $$C_\theta(u, v) = H_\theta\left[\Phi^{-1}(u), \Phi^{-1}(v)\right] \quad \text{for } \theta \, \epsilon \, [-1,1] \quad \text{and}$$

(7) $$c_\theta(u, v) = \frac{\varphi_{X,Y,\theta}(\Phi^{-1}(u), \Phi^{-1}(v))}{\varphi(\Phi^{-1}(u))\varphi(\Phi^{-1}(v))} \quad \text{with } \varphi_{X,Y,\theta}$$ [12]

Other types of copula families exist, in particular, the Archimedean class of copulas. These copulas are not derived directly by applying Sklar's theorem to multivariate distributions (Nelsen, 1999), and some of them are able to capture asymmetric correlation between the tails of the marginal distributions (Embrechts et al., 2003). However, given that our purpose in this study is to capture the proper inverse relationship between crop yields and prices instead of the tail dependence

[12] $\varphi_{X,Y,\theta} = \frac{1}{2\pi\sqrt{(1-\theta^2}} \exp\{\frac{-(x^2+y^2-2\theta xy)}{2(1-\theta^2)}\}$ is the Standard Bivariate Gaussian density function and θ the Pearson's correlation factor.

between them, the estimation of these types of copulas will serve as a goodness-of-fit comparison to the estimated Elliptical copula.

Archimedean Copulas:

Let X and Y be continuous random variables, with joint bivariate distribution H and marginal distribution functions F and G, respectively. Consider a strictly increasing continuous function $\lambda : [0,1] \rightarrow [0,1]$ such that $\lambda(0) = 0$ and $\lambda(1) = 1$, and suppose that $\lambda(\Pr\{X \leq x, Y \leq y\}) =$ $= \lambda(\Pr\{X \leq x\})\lambda(\Pr\{Y \leq y\})$, i.e.; $\lambda(H(x,y)) = \lambda(F(x))\lambda(G(y))$. If we let $\phi(t) = -\log\lambda(t)$ for $0 < t \leq 1$ (i.e. $\phi(t)$ is a convex decreasing function such that $\phi(1) = 0$), then the previous equation becomes: $\phi\big(H(x,y)\big) = \phi(F(x)) + \phi(G(y))$, and arranging in copula terms, the distribution becomes: $\emptyset\big(C(u,v)\big) = \emptyset(u) + \emptyset(v)$, thus:

(8) $C_{\emptyset}(u,v) = \emptyset^{-1}[\phi(u) + \phi(v)]$, or $H_{\phi}(x,y) = \phi^{-1}[\phi(F(x)) + \phi(G(y))]$

is generally referred to as an "Archimedean" Copula (ϕ being a convex decreasing function).

Three typical bivariate, one-parameter Archimedean copulas (Nelsen, 1999) are:

Clayton family:

$\phi(t) = \big(t^{-\theta} - 1\big)/\theta$; for $\theta \in [-1,\infty]\backslash\{0\}$, and $H_{\phi}(x,y) = Max[(x^{-\theta} + y^{-\theta} - 1)^{-\frac{1}{\theta}},\ 0]$; or

(9) $H_{\phi}(x,y) = (x^{-\theta} + y^{-\theta} - 1)^{-\frac{1}{\theta}}$ for $\theta > 0$,

with following Frechet-Hoeffding bounds (Nelsen, 1999):[13]

(10) $\lim_{\theta \to -1} H_{\phi} = \max(x + y - 1, 0)$; $\lim_{\theta \to 0} H_{\phi} = x * y$; $\lim_{\theta \to \infty} H_{\phi} = \min(x,y)$

Frank family:

[13] Universal Bounds for Copulas, i.e., for *any* copula C and for all (u,v) in (0,1), then: $W(u,v) = \max(u+v-1, 0) \leq C(u,v) \leq \min(u,v) = M(u,v)$. And if X, Y are random variables with joint distribution H and marginal distributions F and G , then by Sklar's theorem: $\max(F(x) + G(y) - 1, 0) \leq H(x,y) \leq \min(F(x), G(y))$. Since *W and M* are copulas, the previous bounds are joint distributions referred to as Frechett-Hoeffding bounds for a joint distribution H with margins F and G.

$\phi(t) = -\ln\left((e^{-\theta t} - 1)/(e^{-\theta} - 1)\right)$; for $\theta \in \mathbb{R} \setminus \{0\}$, and:

(11) $$H_{\phi}(x,y) = -\frac{1}{\theta}\ln\left(1 + \frac{(e^{-\theta x}-1)(e^{-\theta y}-1)}{e^{-\theta}-1}\right),$$

having the same Frechet-Hoeffding bounds as the Clayton family.

Gumbel family:

$\phi(t) = (-lnt)^{\theta}$; for $\theta \geq 1$, then

(12) $$H_{\emptyset}(x,y) = \exp\left(-[(-lnx)^{\theta} + (-lny)^{\theta}]^{\frac{1}{\theta}}\right),$$

having only the latter two Frechet-Hoeffding bounds, since parameter space is non-negative.

Lower tail dependence (Embrechts et al. 2003)[14] is captured by the Clayton family for $\theta > 0$, which due to its limited parameter space results in this copula only being able to capture positive correlation for such lower tail dependence (Freez & Valdez, 1998). Upper tail dependence (Embrechts et al. 2003)[15] is captured or determined in the Gumbel family for $\theta > 1$, which once again due to its parameter space, holds only for positive co-dependence. Nonetheless, negative dependence may be obtained in both previous copulas by initially pre-multiplying either series by -1, thus the pair (-*X, Y*) or (*X, -Y*) may be modeled as a joint distribution, however; this also modifies accordingly the copula function result. More importantly, all three models include the specific case for independent or unrelated marginal distributions between *x* and *y*, which is at $\theta = 0$. In other words, for $\theta = 0$ all three Copula families simply become the product of the two variables *x* and *y* (Genest & Rivest, 1993):

(13) $$H_{\phi} = x * y$$

[14] For bi-variate copula C: if coefficient $\lambda_L = \lim_{u\to 0}\frac{C(u,u)}{u}$ exists, then C has lower tail dependence if $\lambda_L \in (0,1]$

[15] For bi-variate copula C: if coefficient $\lambda_U = \lim_{u\to 1}\frac{1-2u+C(u,u)}{1-u}$ exists, then C has upper tail dependence if $\lambda_U \in (0,1]$

For the case of two variables, the Frank Copula family is able to model both positive and negative correlations. As mentioned previously, this is not the case for the Clayton or Gumbel families which have parameter limitations to support an inverse relationship. Hence the Frank family copula is used in modeling the inverse relation between crop yields and their prices, along with an Elliptical class copula, the Normal family, to serve as goodness-of-fit comparison.

Kendall's Tau coefficient is applied as a dependence, association, or correlation measure between the marginal distributions in a copula. This is a rank coefficient that does not depend on the specification of the marginal distributions, but only on the copula model estimated. The coefficient Tau is the probability of concordance (positive relationships) minus the probability of discordance (inverse relationship) (Nelsen, 1999).

(14) $\tau_{xy} = P[(X_1 - X_2)(Y_1 - Y_2) > 0] - P[(X_1 - X_2)(Y_1 - Y_2) < 0]$

For the Normal copula (Freez and Valdez, 1998), $\tau_{xy} = \frac{2}{\pi}\arcsin\theta$ with $\theta \in [-1, 1]$ and for the Frank copula (Genest, 1987), $\tau_\alpha = 1 + \frac{4}{\theta}\{D_1(\theta) - 1\}$; with $\alpha = e^{-\theta}\ or\ \theta = -log\alpha$; and $D_1(\theta) = \frac{1}{\theta}\int_0^\theta \frac{t}{e^t - 1} dt\ with\ \theta \in \mathbb{R} \setminus \{0\}$.

2.4 Data

Modeled prices (as mentioned in the empirical methods) consist of the differences or returns between the monthly averages of February futures prices with delivery in December for corn and November for soybeans, and the monthly averages of November futures prices for corn and October for soybeans, also with delivery in December for corn and November for soybeans. In other words, the model prices are the difference or return between an *ex-ante* price (during

February for end of year delivery) and an *ex-post* price (end of year price with end of year delivery). The data is from the Chicago Board of Trade (CBT) with observations running from 1960 to 2006 and obtained through the Commodity Resources Bureau (CRB).

Wheat returns considered the average yearly prices taken from the Kansas City Board of Trade (KCBOT) for the period between 1971 and 2006. In this case, the *ex-ante* prices taken into account are from mid-August to mid-September from the previous harvesting year for delivery in July of the present year. The ex-post prices were for either June or mid-July, depending on the revenue insurance contract. For the case of RA-BP insurance, ex-ante crop prices were considered in the same manner. Summary statistics for Futures prices are in table 2.1, and their returns or differences are in table 2.2.

Table 2.1: Futures Prices - Summary Statistics

	Future Prices[16]						
	Corn- Delivery Dec (Z)		*Soybean –* Delivery Nov (X)		*Wheat –* Delivery July (N)		
	Feb.	Dec.	Feb.	Nov.	Mid-Aug/Sept.[17]	June[18]	July[19]
Mean	221.68	215.36	510.70	514.84	338.67	336.28	336.64
Std Dev	71.30	74.64	178.03	180.87	78.42	82.66	84.49
Max	376.57	381.39	826.43	859.04	488.98	575.91	531.55
Min	110.19	104.46	207.62	214.88	136.80	142.13	146.66

[16] Price units are in Cents per bushel (bu.). Minimum contract is for 5,000 bushels.
[17] For Planting - Previous calendar year. (KCBOT)
[18] For CRC - Harvest Price (KCBOT)
[19] For RA, Harvest Price - 1st half of July (KCBOT)

Table 2.2: Futures Prices Returns - Summary Statistics

	Returns					
	Corn		*Soybean*		*Wheat*	
	Level	Log+0.5[20]	Level	Log+0.5	Level	Log+0.5
Mean	-6.31	0.465	4.14	0.520	-2.39	0.493
Std Dev	45.62	0.202	99.68	0.158	68.85	0.201
Max	113.97	1.142	229.15	0.876	185.21	0.855
Min	-99.58	0.111	-169.95	0.237	-122.71	0.091

Histograms for the differences or returns between the price levels, and also between the price logs, are presented below for corn, soybean, and wheat. These histograms include the current distribution being used in crop insurance contracts. That is, the differences of level prices include the Normal distribution, and the differences of log prices include the Log-Normal distribution. Figures 2.2.1 and 2.2.2 are for returns of price levels and price logs of corn, followed by figures 2.3.1 and 2.3.2 for soybean and figures 2.4.1 and 2.4.2 for wheat.

The data for crop yields were from observations belonging to the largest corn and soybean producing county in Iowa, Kossuth, obtained from the NASS dataset (USDA). Other corn yield data from a regular- sized corn and soybean producing county, Marshall County, were also taken into account in order to compare the estimated parameters from the distribution. Results obtained for the estimated parameters of the Beta distribution are quite similar between both sets of data. The wheat data corresponds to the largest producing county in Kansas, Sumner, and in order to compare these estimated parameters, the Beta parameters where likewise estimated for wheat from Sedgwick County in Kansas, obtaining very similar estimated parameter results.

[20] Here 0.5 was added to difference of log prices to obtain observations > 0

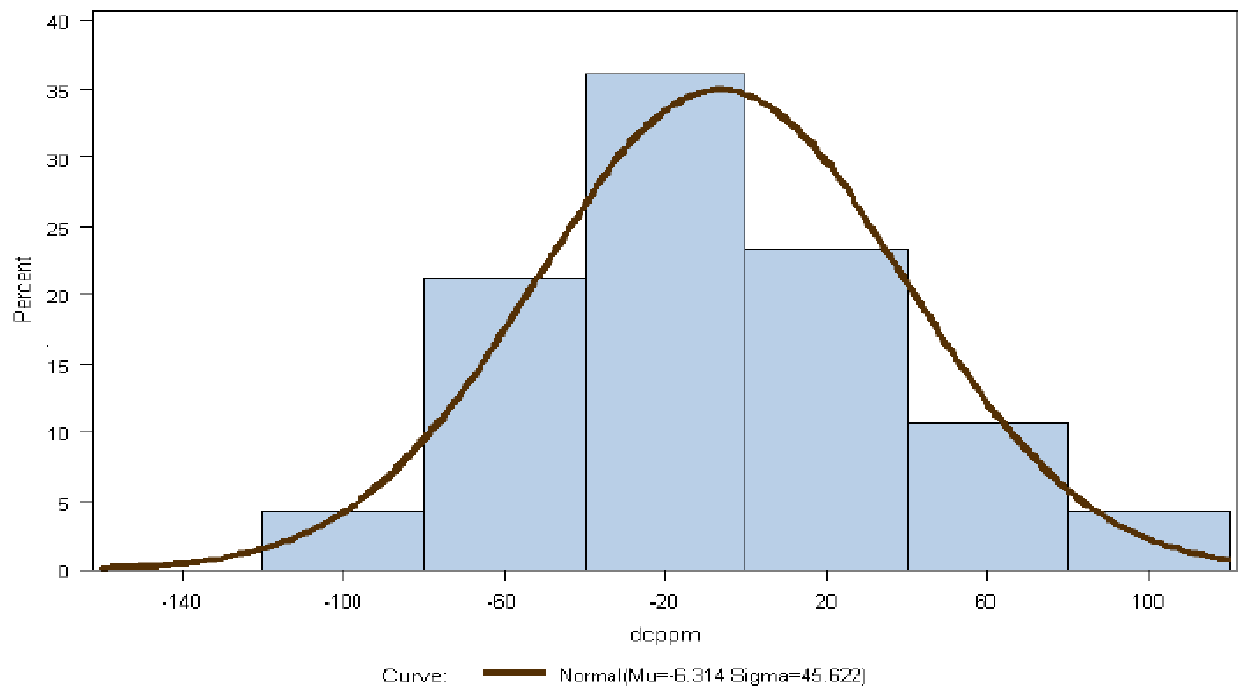


Figure 2.2.1: Histogram of Corn Futures Prices - Returns of Level Prices and their Normal distribution.

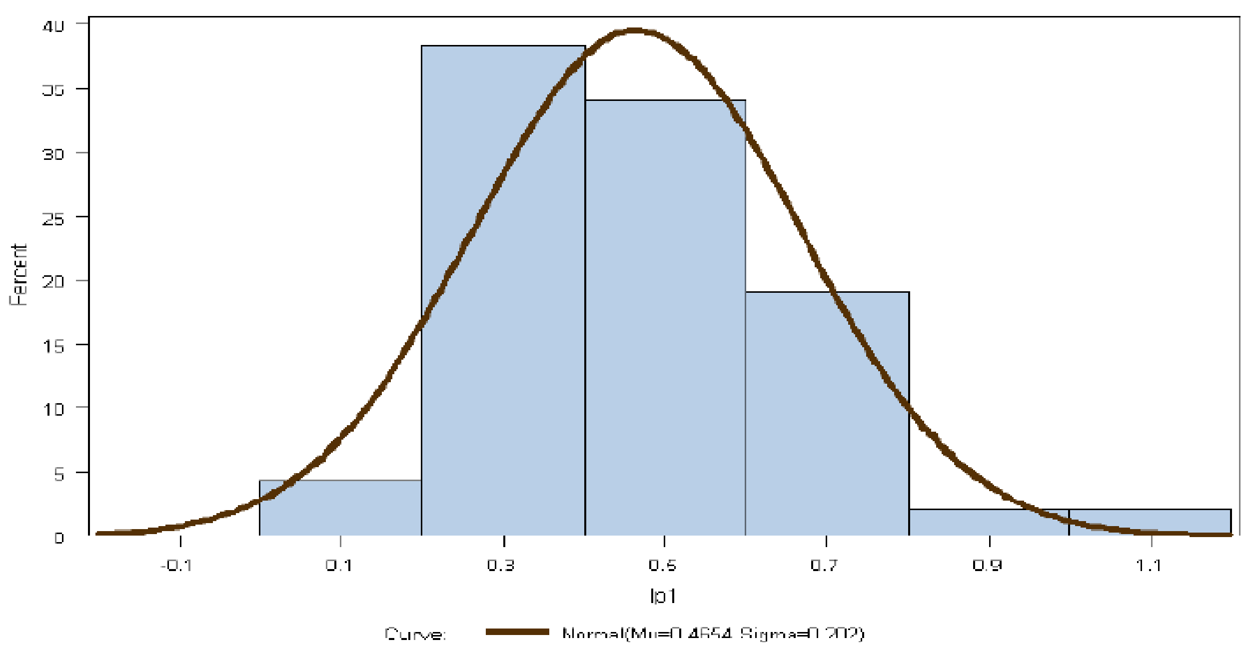


Figure 2.2.2: Histogram of Corn Futures Prices - Returns of Log Prices and their Log-Normal distribution.

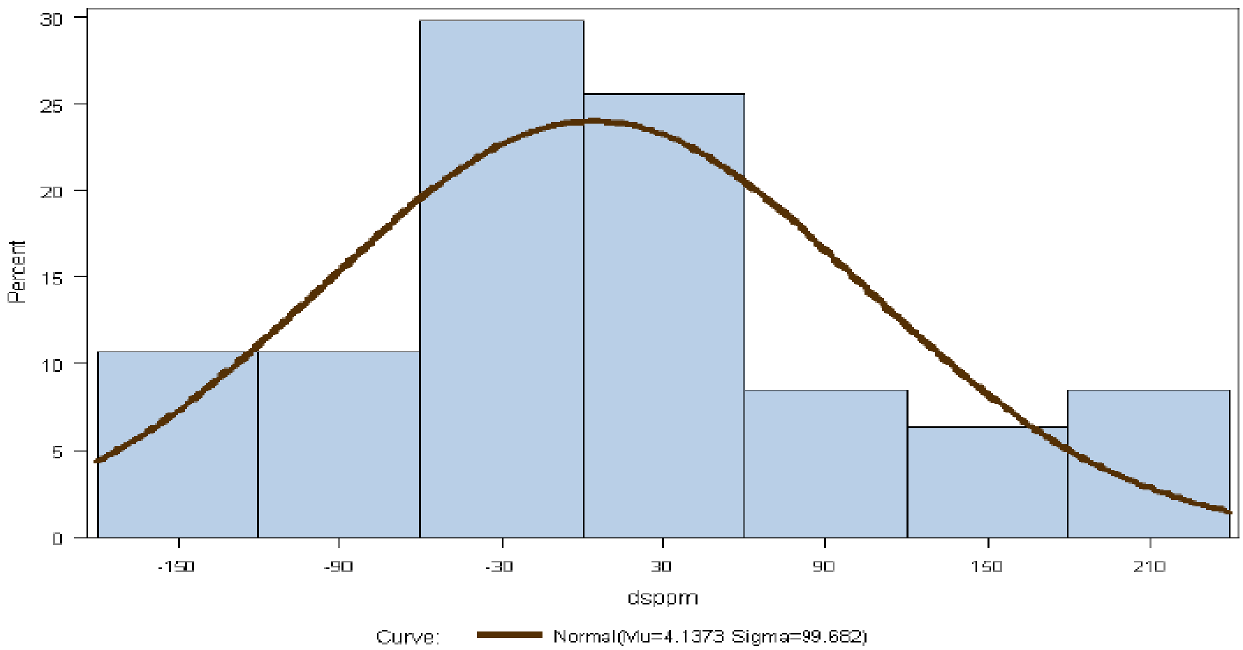


Figure 2.3.1: Histogram of Soybean Futures Prices - Returns of Level Prices and their Normal distribution.

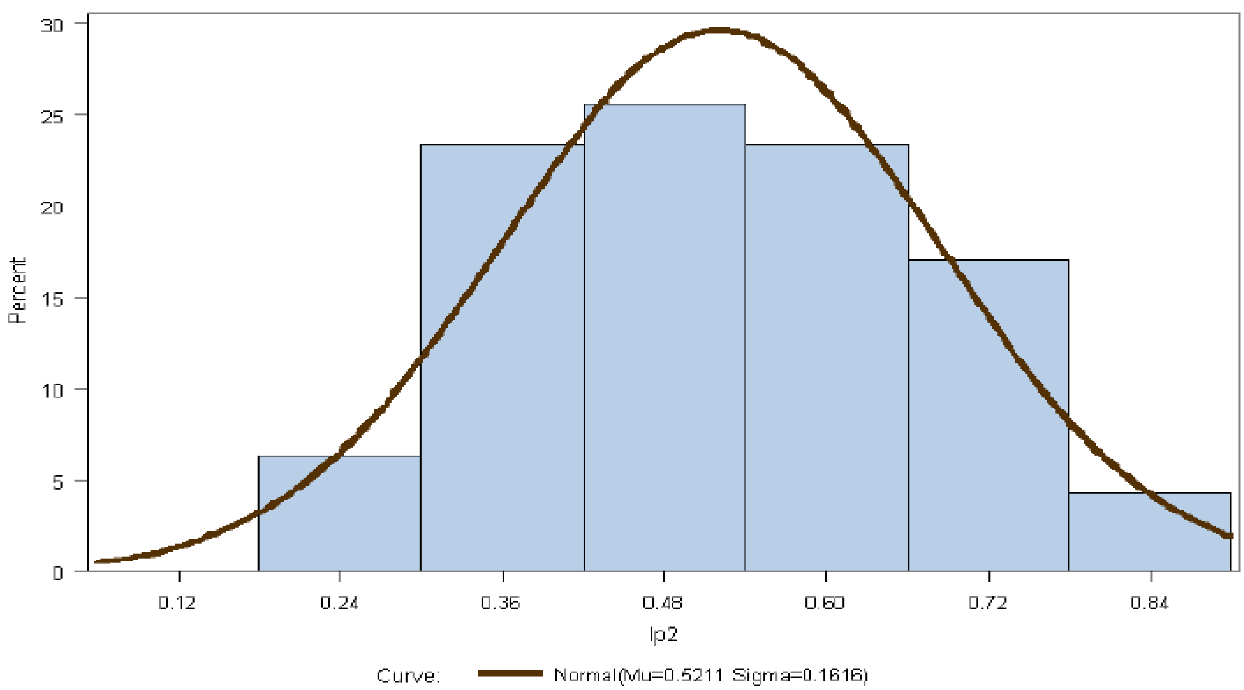


Figure 2.3.2: Histogram of Soybean Future Prices - Returns of Log Prices and their Log-Normal distribution.

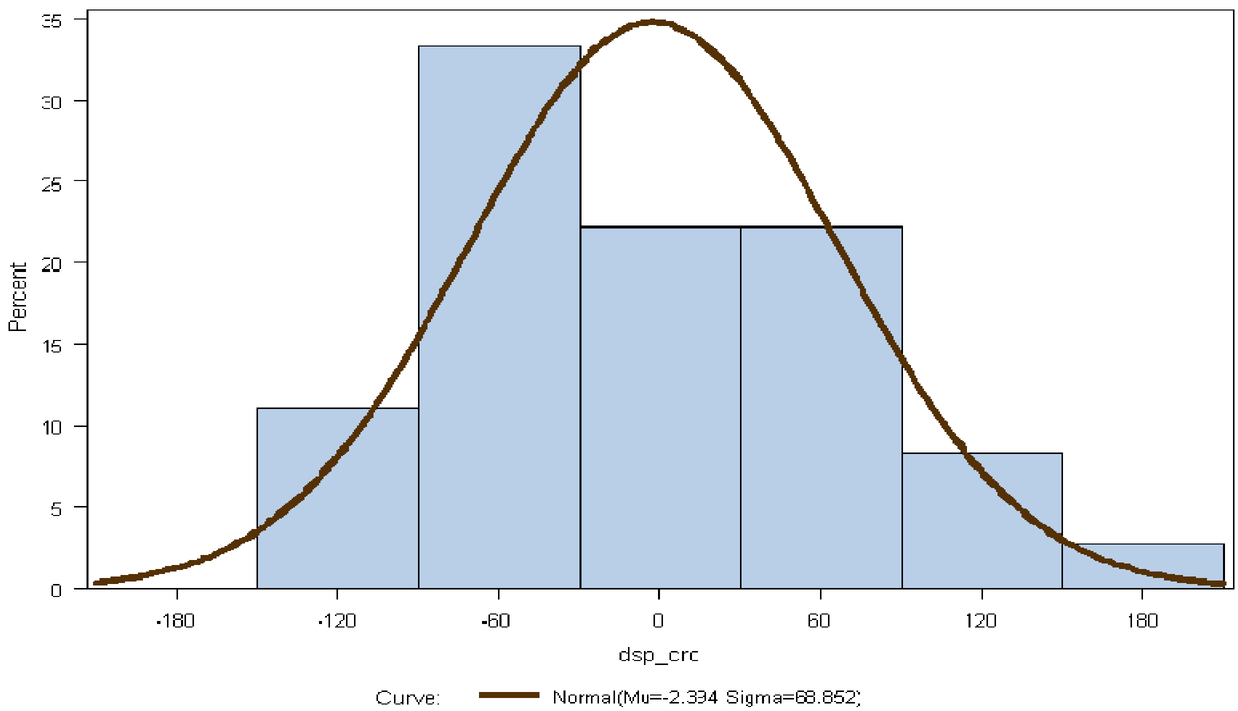


Figure 2.4.1: Histogram of Wheat Futures Prices - Returns of Level Prices and their Normal distribution.

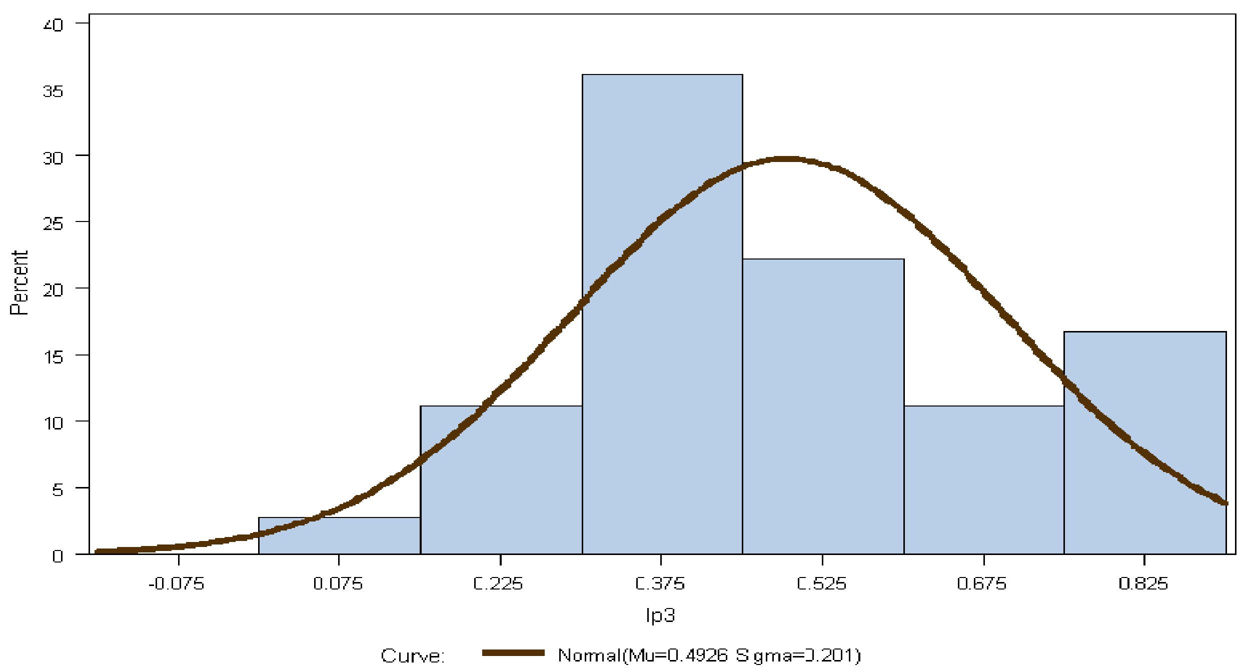


Figure 2.4.2: Histogram of Wheat Future Prices - Returns of Log Prices and their Log-Normal distribution.

In order to validate the existing inverse relationship between crop prices and their yields, the case of corn and soybean production for the entire state of Iowa was taken into account. The fact that the U.S. is the world's leading corn and soybean producer, and Iowa is the largest corn and soybean producing state was considered. These yields were calculated over the acres planted, and not harvested, so as to obtain a realistic view of the ex-ante conditions during planting. The same estimations were made for wheat considering the state of Kansas, which is the largest U.S. producer of that crop.

Only the corn crop had the same recorded years (1960 - 2006) for acres planted in the case of the counties. Therefore, a proxy obtained from these planted acres for corn, in combination with the acres harvested for soybean, was used to estimate the missing records of planted acres for soybean (1960-1969). That is, the ratio of acres planted over acres harvested during each year for the whole period from 1960 through 2006 was computed for corn. Then this ratio was taken and used for soybeans, such that each year with missing planted acres of soybeans was computed as the product of that year's corn ratio times the acres of harvested soybeans. In order to evaluate the precision of this proxy, the product of each year's corn ratio with the corresponding soybean harvested acres for the remaining years was again computed, and these results were compared to the true values of the yearly planted acres for soybeans (1970-2006). Percentage differences between these two values in general did not exceed five %. This procedure was not necessary for the crop yields from the whole state.

Yearly crops, from 1960 through 2006 (from 1971 to 2006 for wheat), was de-trended using a common procedure which consists of applying a linear regression to the yields with a time variable, and then each de-trended observation is afterwards transformed relative to the predicted value of the last data observation, here 2006 (Goodwin and Mahul, 2004). The reason for the error term 'adjustment' is that larger deviations occur at higher yields (Goodwin and Ker 1998). $\tilde{y}_t = \hat{y}_T * (1 + \frac{e_t}{\hat{y}_t})$; with t = 1960,……2006; T = 2006. Summary measurements of these crops for county and state, including de-trended values as per the previous procedure (7), are shown in tables 2.3 and 2.4.

Table 2.3: Crop County Yields - Summary Statistics

	County Yields[21]					
	Kossuth County				Sumner County	
	Corn		*Soybean*		*Wheat*	
	Regular	De-trended	Regular	De-trended	Regular	De-trended
Mean	121.08	165.82	36.46	46.67	31.73	34.44
Std Dev	31.63	22.07	7.77	5.68	6.98	7.29
Max	185.68	198.22	54.41	55.13	49.95	52.05
Min	61.141	77.335	20.74	23.6756	9.73	10.19

Table 2.4: Crop State Yields - Summary Statistics

	State Yields[22]					
	Iowa State				Kansas State	
	Corn		*Soybean*		*Wheat*	
	Regular	De-trended	Regular	De-trended	Regular	De-trended
Mean	110.65	155.51	37.09	47.50	32.08	35.80
Std Dev	30.65	20.42	7.38	4.72	6.49	6.64
Max	176.72	189.90	52.24	56.82	46.25	48.56
Min	61.032	87.55	25.34	33.30	17.23	19.16

[21] Bushels per planted acre.
[22] Bushels per planted acre.

2.5 Results

By using a method of Maximum Likelihood estimation as per Watkins (1999) and Johnson (2003), estimates were obtained and contrasted to Normal and Log-Normal distributions as seen in table 2.5.

Table 2.5: Parameters Estimates for All Three Distributions

Burr Distribution (standard errors in parenthesis):

Parameters:	Corn	Soybeans	Wheat
$\tau\ (shape1) =$	3.53975 (1.6240)	3.66880 (1.8706)	2.80657 (1.3458)
$\alpha(shape2) =$	2.01176 (1.5172)	8.42270 (4.8579)	11.3706 (6.8962)
$\phi\ (scale) =$	0.57102 (0.2322)	0.99306 (0.2564)	1.28633 (0.5458)

Normal Distribution:

Parameters:	Corn	Soybeans	Wheat
$\mu\ (mean) =$	-6.3142	4.137315	-2.3936
$\sigma^2(variance) =$	2081.35	9936.425	4740.574

Log-Normal Distribution:

Parameters:	Corn	Soybeans	Wheat
$\mu\ (location) =$	0.46735	0.510934	0.499256
$\sigma(scale) =$	0.18989	0.16458	0.21045

The Vuong test (Vuong 1989) was used to compare these non-nested models, by computing the log of their likelihood ratio (m_i).[23] The statistic computed for testing the non-nested hypothesis of Model 1 vs. Model 2 is denoted as v.[24] The v statistic distributes as a Standard Normal,[25] and resulting estimates are in table 2.6.

Table 2.6: Estimates for v Statistic Between Two Distributions

$v - statistic$:	Corn	Soybeans	Wheat
$v - Burr\ vs. Normal$	120.353	173.473	35.806
$v - Burr\ vs. LogNormal$	1.2826	1.7809	1.1570

Results confirm there is significant improvement in using the Burr distribution – considering the annual returns as differences in log prices, versus the Normal distribution – considering the difference in level prices. This latter case is the method used for the CRC insurance program. Comparison between the Burr distribution versus the Log-Normal, considering the difference in log prices (these distribute Normal for the Log-Normal case), results in a small improvement in the Burr distribution, yet only statistically significant for the soybeans at the 10% level. In other words, there is no considerable improvement, besides a 10% level in the case of soybeans, when comparing prices with a Burr XII distribution over prices with a Log-Normal.

[23] The log ratio is: $m_i = \log(\frac{f_1(x_i)}{f_2(x_i)})$

[24] $v = \frac{\sqrt{n}[\frac{\sum_{i=1}^{n} m_i}{n}]}{\sqrt{\frac{\sum_{i=1}^{n}(m_i-\overline{m})^2}{n}}}$

[25] The critical values for statistical significance is for v greater (less) than 1.65 (-1.65) which corresponds to significance at the 10% level or less, and for v greater (less) than 1.96 (-1.96) which corresponds to significance at the 5% level or less.

There may be an issue of deflation in the prices that have been presented and how these would be affected by a price index over the period considered. In the case of RA-BP, the prices follow a lognormal distribution, so that including an inflation index rate for each price would not alter the difference in log prices, as these inflation index rates would cancel out. However, for the case of CRC, the difference in level prices would be modified by an inflation rate, as these index rates would not cancel out for each period. For this reason, additional parameter estimations for the CRC method with a Normal distribution were made by incorporating a Producer Price Index (PPI) rate in line with Light and Shevlin (1998) and contrasted with a Burr XII distribution. Results obtained are similar to the ones presented previously. The Burr XII distribution is preferred to the Normal distribution according to the Vuong criteria. Regarding crop yields, results in table 2.7 have the estimated parameters obtained via maximum likelihood for corn and soybeans from both Kossuth County (IA) and from Iowa, and likewise for wheat from Sumner County (KS), and from Kansas.

Table 2.7: Parameter Estimates for Crop Yields with Beta Distribution

Beta Distribution:	County			State		
	Kossuth		Sumner	Iowa		Kansas
Parameters:	Corn	Soybean	Wheat	Corn	Soybean	Wheat
α $(shape1)$	8.5045	8.6012	8.0247	12.4930	19.9925	9.3003
$\beta(shape2)$	1.6818	1.5671	5.5212	3.2506	4.6458	4.7235
ϕ $(scale)$	198.5	55.2	58.0	195.7	58.5	54.0

The two different Copula models estimated are one from the Elliptic Copula class – the Normal, and one from the Archimedean Copula class – The Frank Copula. Estimation was made by two different maximum likelihood methods according to Yan (2007), and appendix 1 has details of both the one-step and the two-step estimation. In addition, similar results of copula estimations were obtained by using lognormal prices, and non-parametrically, detailed in appendix 2. Results are in table 2.8 and were corroborated by comparable maximum likelihood estimation methods from Patton (2004).

Table 2.8: Estimations for Normal and Frank Copulas

Elliptic (Normal) Copula:	*County*			*State*		
	Kossuth		*Sumner*	*Iowa*		*Kansas*
	Corn[+]	Soybeans	Wheat	Corn[*]	Soybeans[*]	Wheat[*]
Kendall's Tau	**-0.1473**	0.0517	-0.1088	**-0.2015**	-0.1676	-0.2117
Log-Likelihood	**1.31972**	0.15938	0.57025	**2.28376**	1.69742	2.1304

Archimedean – Frank Copula:	*County*			*State*		
	Kossuth		*Sumner*	*Iowa*		*Kansas*
	Corn	Soybeans	Wheat	Corn[*]	Soybeans	Wheat[*]
Kendall's Tau	-0.1421	0.0544	-0.0634	-0.2346	-0.1670	**-0.281**
Log-Likelihood	0.86711	0.12183	0.15525	2.23788	0.85220	**2.40132**

* Significant at the 5% level or less. + Significant at the 10% level or less.

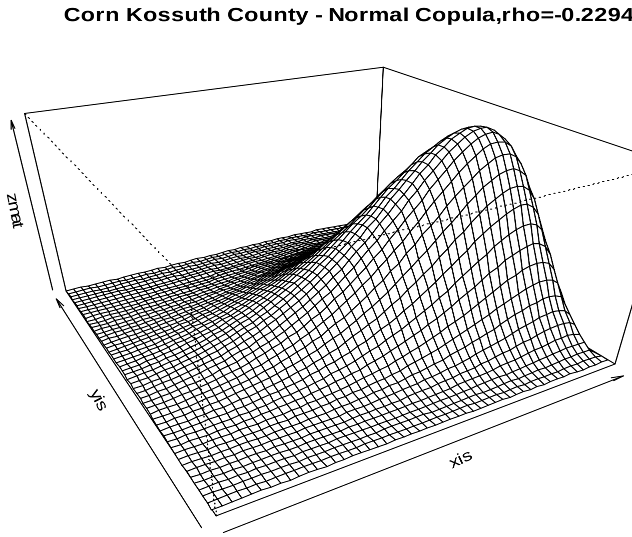
As may be noted, there is a distinctive significant inverse relationship between crop prices and yields, characterized through the particular copula model. The co-dependence (or correlation) factor estimated for each copula – such as Spearman's Rho or Kendall's Tau – may vary for different types of copulas (Nelsen, 1999). Kossuth county corn crops had a larger inverse relationship with the Normal copula given by its Tau correlation factor of -0.1470 over the Tau factor of -0.1212 obtained by the Frank copula. The Normal copula here is significant at the 10%

level or less and the Frank copula is non-significant. In addition, the Vuong test is applied for comparison of the Normal copula and the Frank copula. Test results determine a Vuong statistic of 1.65 (i.e., a significance level of 10% or less). Hence, the Normal copula provides a better characterization than the Frank copula for the inverse relationship between Kossuth county corn yields and the corn prices.

For soybean crops at the county level, both copulas resulted in a positive relationship between prices and yields; however, neither of these relations was statistically significant so this may be a spurious result. For the case of wheat from Sumner County in Kansas, negative correlations are obtained for both copulas, yet likewise insignificant. This occurs despite Sumner being the largest wheat- producing county in Kansas, and Kansas being the largest wheat producing state in the U.S. It may be that these wheat yields do not have enough market power to influence wheat prices, such that the natural hedge between these county crops and prices is not significantly captured by the Copula method.

Considering the total crop yields of corn and soybeans from the state of Iowa and for wheat from Kansas, it is both corn and wheat yields that obtained a significant inverse relationship with respect to their prices for the two copulas considered. In both corn and wheat, the significance was at the 5% level or less of committing a type I error. Corn had Kendall's Tau values of -0.2015 and -0.2346 in the case of Normal and Frank copulas, respectively. The Vuong test application resulted in a coefficient of 0.358, determining no significant preference of one copula model over the other. On the other hand, the Kansas wheat had a higher Kendall's Tau of inverse relation for the Frank copula at -0.2817 over the Normal copula at -0.2117. Yet once again the Voung test resulted in no clear preference of one copula over the other, as the statistic had a score of -0.294. Regarding the soybeans, only the Normal Copula had statistically significant Kendall's Tau coefficient of -0.1676, since the Frank copula is not able to capture with significance the inverse relation between prices and yields. The Voung test statistic of 1.374 obtains a mild preference of the Normal copula over the Frank copula. Charts denoting the inverse relation obtained by the copulas in a multivariate function are presented in figures 2.5, 2.6, and 2.7. These are for corn crop prices and their yields – both state and county – as well as for wheat at the state level, all denoted in bold in table 2.8. These inverse relations capture the natural hedge between prices and yields.

The previous results for state crops corroborate the natural hedge between crop yields and prices given the market power that corn and wheat have. Iowa and Kansas are the largest producing states of corn and wheat in the U.S, respectively; and the U.S. is the world leader in corn production, and the third largest wheat producer trailing closely China and India. In addition, the U.S. is wheat's number one world exporter by far and almost doubles the second place country- Canada- in annual tonnage. Different is the case for soybeans where despite Iowa and the U.S. being the leading producers of soybeans in the country and the world, respectively, this crop does not have such dominance over the rest of global volumes being traded, as there are other very close world competitors like Brazil, Argentina, and China.



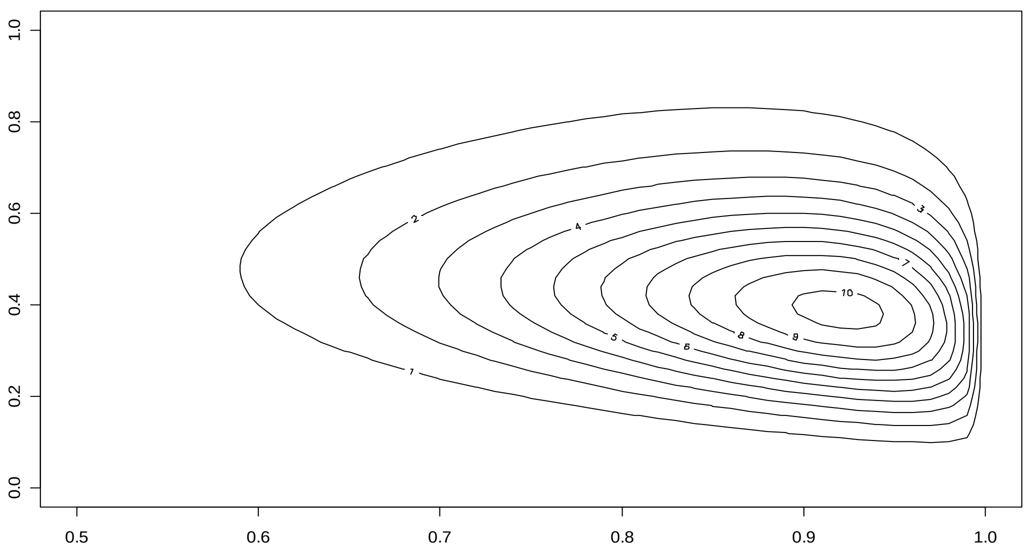


Figure 2.5: Multivariate distribution with *Normal copula* - corn from Kossuth County. Perspective and contour plots

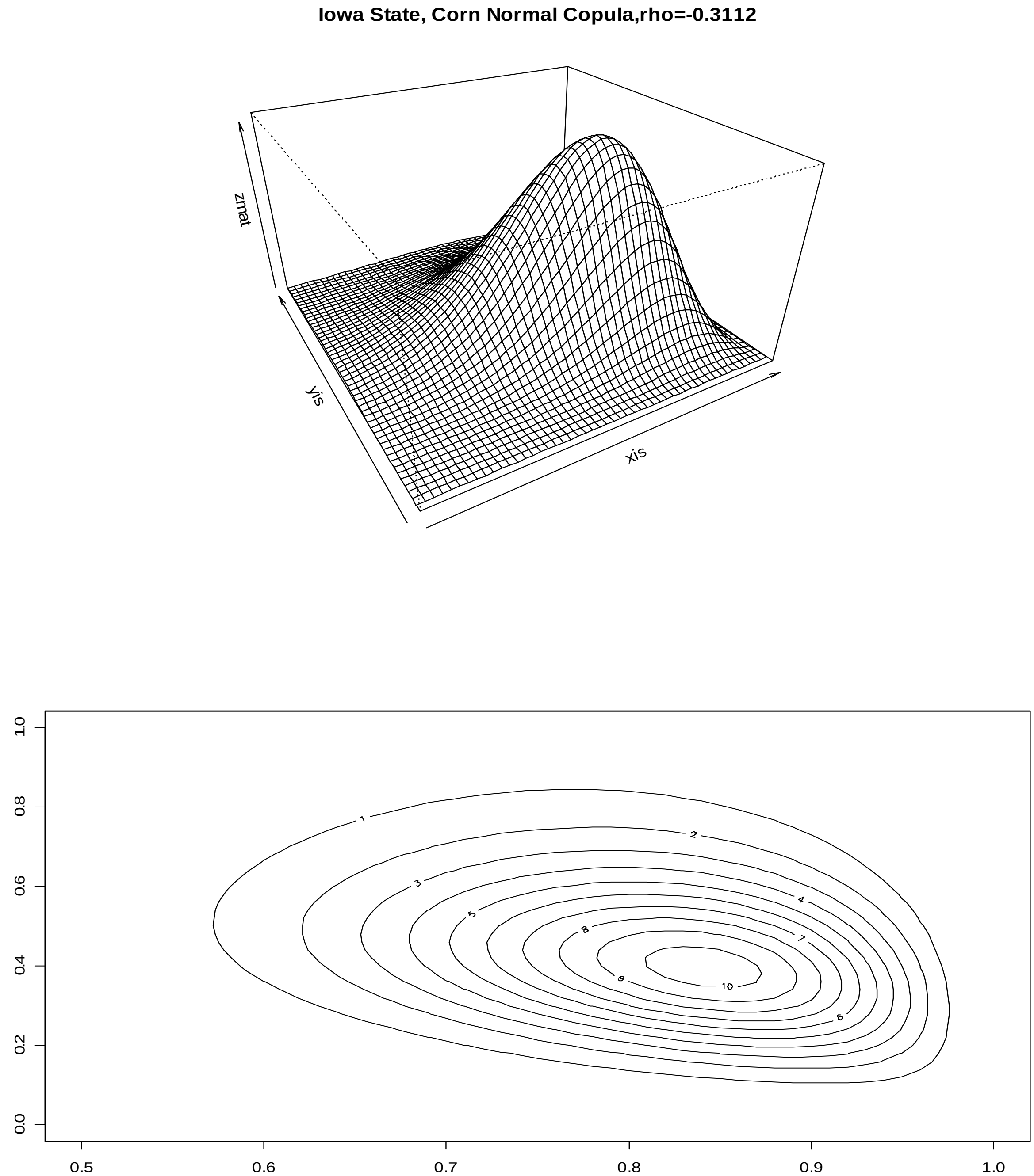
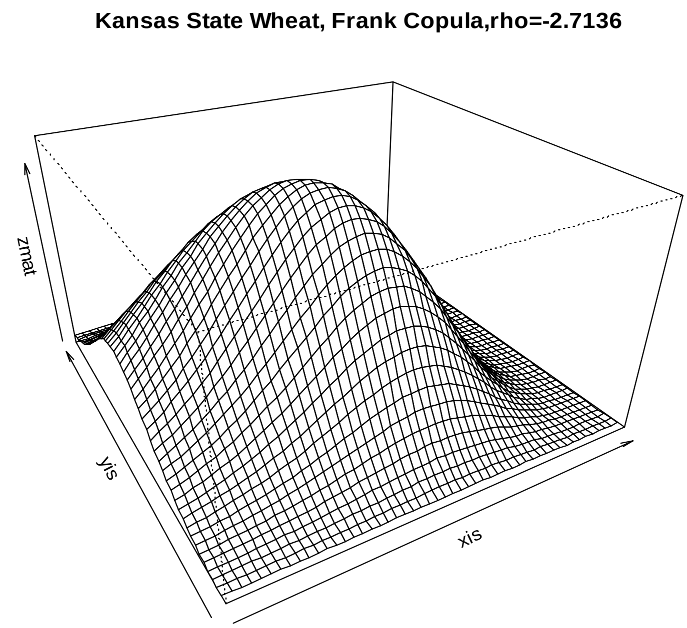


Figure 2.6: Multivariate distribution with *Normal copula* - corn from Iowa. Perspective and Contour plots



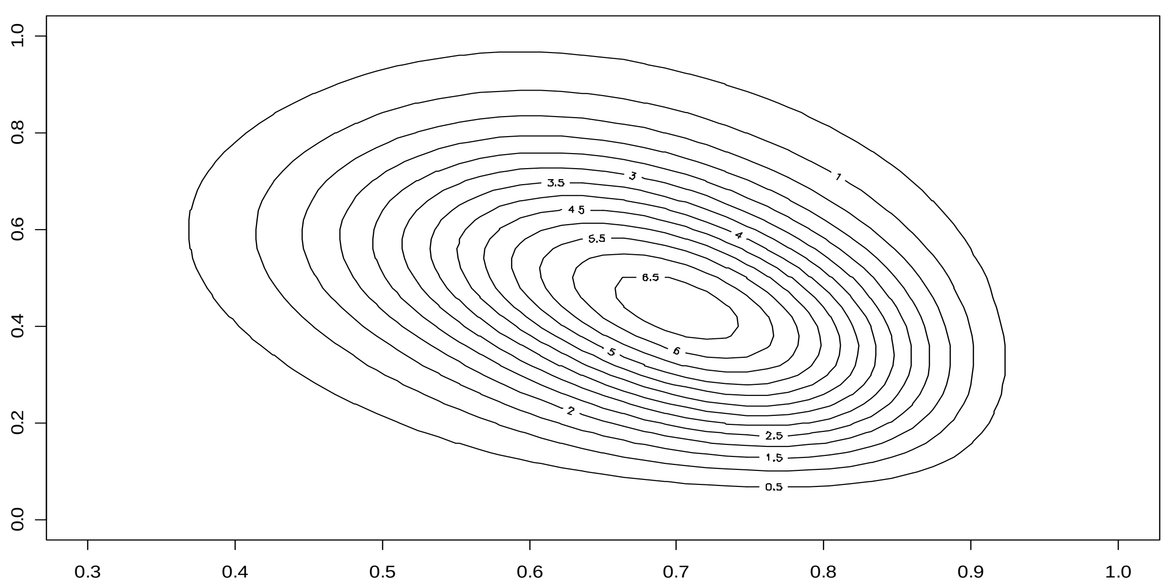


Figure 2.7: Multivariate distribution with *Frank Copula* - Wheat from Kansas. Perspective and Contour plots

2.6 Discussion

The repeatedly higher payouts compared to the premiums paid, or conversely, recent annual cases of much higher premiums paid than payouts received require further study of the rating methods being used (Coble and Knight, 2002). The aim of this chapter is to provide means that result in improved actuarial rate estimations for use in crop revenue insurance programs, and is addressed by testing a different method for estimating these premiums through simulated expected payouts. In the first place, crop prices have been better characterized by using a Burr XII distribution over the Normal distribution utilized in the CRC method; yet there is no considerable improvement over the Log-normal distribution used in the RA-BP method. Secondly, the inverse relationship between crop prices and yields has been gauged by applying copula methods to the marginal distributions of crop prices and yields, making use of the Burr distribution for prices and the Beta distribution for yields (likewise by using the log-normal distribution for prices or by considering the prices and yields in non-parametric form).

In order to test an application of the estimated copula models to the case of crop revenue insurance, a simulation was made assuming a situation in which an insurance payout may be necessary. This is the case when the crop price has fallen below a certain level, and/or the crop yield has fallen below a certain level, such that the product combination of these cases – despite their inverse relation – results in a revenue level which is below the minimum insured.

The expected payout or loss function may be represented by the function below (Embrechts et al., 2003 and Goodwin and Mahul, 2004), assuming X: *yield* and Y: *price* and noting that revenue is equal to the product of the crop's price and yield ($R = X*Y$), there is a minimum insured revenue level r_{min} which is a function of x and y ($r_{min} = f(x,y)$).

(15) $$E\{f(R,r_{min})\} = prob(R \leq r_{min}) *[r_{min} - E(R|R \leq r_{min})]$$

This expected payout or loss function can incorporate the copula model for the probability term on the right, specifically through the application of the pair-wise rank correlation determined from the marginal distributions of X and Y. The second term on the right is the difference between the minimum-insured revenue level and the expected-revenue level obtained, given that it is below the minimum-insured revenue level (i.e., the amount corresponding to the payout).

Hence, the probability for a payout can be estimated by substituting the copula function, i.e.; $prob(R \leq r_{min}) = H(x,y) = C(F_1(x), F_2(y))$, in (15) resulting in the following expression:

(16) $$E\{f(R,r_{min}, x, y)\} = C(F_1(x), F_2(y)) *[r_{min} - E(R|R \leq r_{min}]$$

From our previous copulas we have for Normal, $C_\theta(u,v) = \Phi_\theta[\Phi^{-1}(u), \Phi^{-1}(v)]$, and for Frank, $C_\theta(u,v) = -\frac{1}{\theta}\ln\left(1 + \frac{(e^{-\theta u}-1)(e^{-\theta v}-1)}{e^{-\theta}-1}\right)$ with u being the c.d.f. of Beta distribution for crop yields and v being the c.d.f. of Burr XII distribution for crop prices.

Simulations of yields (x), and prices (y), and the average of their product results was determined for an expected or average revenue that may be insured at different levels (70%, 75% or 80%). The probabilities of being below that insured point were then calculated with the use of the Copula function. Each significant copula previously estimated for corn, soybean, and wheat was used to simulate a 100,000 pairs of observations of covariates that represented estimated crop yields and crop prices. Thus, starting from a particular copula, observations of both marginal variables were created – both crop prices and their yields. Once these simulated prices and yields are obtained, the average of their product – which is equivalent to the revenue[2619] – was calculated, and particular minimum revenue levels for which the insured agents would receive an expected payout for loss in case their revenue fell below it, were likewise computed.

Minimum revenue levels were estimated at 70%, 75%, and 80% of the average revenue, from each significant copula of a specific crop. Thus, there are three different minimum revenue scenarios for each different previously estimated significant copula. These scenarios were compared to the case in which prices and yields are assumed to be unrelated or independent. As mentioned previously, this latter is the case when both copulas have the theta parameter being equal to zero.

Estimations were made for corn, comparing the efficiency gains at both the Kossuth county level and for the state of Iowa. In addition, simulations were made for soybean and wheat considering crop yield results for the state of Iowa and Kansas respectively, since the inverse relationship obtained from the copula is also significant for them. Results from these simulations can be seen in table 2.9, where the % differences are presented for expected payout in the cases of employing a copula method versus the case of unrelated or independent yields and prices.

Simulation results for both cases of Normal and Frank copulas present a smaller expected payout (or loss) when compared to the case of the crop prices and yields not being related or independent. Specifically, when insuring 70% of the average revenue for corn at Kossuth County, there is a difference of 1.74% of less expected payout by using the inverse relationship obtained from the Normal copula than when avoiding this inverse relationship (i.e. assuming independence) between prices and yields.

[26] Prices are ex-post and normalized by ex-ante prices; hence, revenue is in terms of bushels/acre from crop yields.

For corn at the Iowa state level, 70% percent insurance of the average revenue results in a 2.03% lower expected payout by considering the Normal copula than when considering price and yields being unrelated. That is, there is a smaller expected payout when considering the state's corn yields than that of the county's corn yields, as may be anticipated since the production from the state has a larger impact on the prices by way of a natural hedge. For the case of Group Risk Income Plan under the Risk Management Agency (RMA) guidelines, state crop yields may be considered in the calculation of premium rates for county level revenues; hence the previous results are favorable to obtain a lower expected payout, leading to lower premium rates.

Table 2.9: Simulation of Expected Revenue Payouts with a Copula Method versus Independent

Corn		Normal Copula				Normal Copula		
	Kossuth County		Independent		Iowa State		Independent
Average Revenue:	*156.5*		*156.5*	Average Revenue:	*146.4*		*146.4*
Insured Level	*Expected payout*	*%Δ wrt Independent*	Expected payout	Insured Level	*Expected payout*	*%Δ wrt Independent*	Expected payout
70%	23.76	-1.74%	24.18	70%	21.65	-2.03%	22.10
75%	26.86	-0.65%	27.03	75%	24.63	-1.13%	24.92
80%	30.15	0.24%	30.08	80%	27.84	-0.65%	28.02

Soybean		Normal Copula		**Wheat**	Frank Copula		
	Iowa State		Independent		Kansas State		Independent
Average Revenue:	*44.3*		*44.3*	Average Revenue:	*35.6*		*35.6*
Insured Level	*Expected payout*	*%Δ wrt Independent*	Expected payout	Insured Level	*Expected payout*	*%Δ wrt Independent*	Expected payout
70%	6.62	-4.21%	6.91	70%	4.74	-3.64%	4.92
75%	7.51	-2.32%	7.69	75%	5.44	-1.95%	5.55
80%	8.38	-2.55%	8.60	80%	6.19	0.44%	6.17

Likewise, both soybean and wheat have expected payouts for different insured levels of the average revenue, that are smaller than the expected payouts ignoring the inverse relationship between prices and yields captured through the copula. Specifically for the case of wheat in Kansas, there is a 3.64% lower expected payout when being insured at 70% of the average revenue. The smaller expected payouts or losses lead to smaller premium rates being estimated. Hence with the use of a copula method, better efficiency may be obtained in the application of crop revenue insurance under a Group Risk Income Plan.

2.7 Conclusions

A critical issue regarding crop insurance coverage has been the large amounts of indemnity payouts compared to premiums charged, sometimes even exceeding ratios of 2:1; or in more recent cases of much smaller payouts compared to premiums collected; e.g., 1:2 ratio. This factor is compounded by the fact that the resources involved are in the hundreds of millions of dollars, with more than 50% of the premiums being subsidized by the government. In order to gauge a new rating method over current crop revenue insurance programs, two aspects have been studied.

First, a Burr distribution was used to characterize crop prices and its goodness-of-fit was compared to the current normal and lognormal distributions being used in some crop revenue insurance programs. Corn, Soybeans, and Wheat future prices were used, in accordance with the practices of the current CRC and RA-BP programs. This resulted in a better fit of the Burr XII distribution in comparison to the Normal distribution. Yet there was no significant improvement of this distribution when compared to the Log-normal, despite a mild advantage in the case of soybeans.

Second, copulas from two different classes – Normal family (from Elliptical class) and Frank family (from Archimedean class) – were used to determine the correlation between these crop prices and their yields. Crop yields were modeled with a Beta distribution, and the copula method made use of the price and yield distributions to provide a measure of correlation level among them, using Kendall's Tau as the means of correlation coefficient.

Results show that there is a statistically significant negative correlation between the price and yield distributions for corn at both a county level of the state of Iowa and at the state level itself. This result was corroborated by different MLE estimation methods. Conversely, the case of soybeans showed there was a spurious positive relationship between prices and yields at the county level, since these results were not statistically significant. In addition, both soybean and wheat crops had a significant inverse relationship at the state level, for Iowa and Kansas, respectively.

Analysis of the implications of this inverse relationship results between prices and yields for corn, soybean and wheat was made by computing simulated indemnity payouts under a crop revenue insurance context. Following the guidelines of the Group Risk Income Plan (GRIP) in the implementation of a simulated insurance revenue contract, results obtained for indemnity (or loss) payouts show lower amounts of payouts when compared to current programs being offered. This likewise results in lower premium rates, which may lead to gains in efficiency.

Further venues of study may incorporate additional insured crop markets such as rice or likewise cattle markets. Other copula methods may permit an improved estimation of the inverse relation between output and prices.

CHAPTER 3

A State Dependent Regime Switching Model of Dynamic Correlations - Identifying volatility spillover effects from ethanol on corn, soybean and cattle markets

3.1 Introduction

Financial instruments and more specifically commodity prices for agricultural products – such as crops and/or livestock constitute an important source of risk for agricultural producers, consumers, and investors. Fluctuation in these prices, generates risk to producers and consumers called volatility. This volatility is steadily changing through time as new information arrives, which may be directly related to the market(s) or general economic news.

Measurement of this risk gave rise to the Autoregressive Conditional Heteroskedastic (ARCH) and Generalized Autoregressive Conditional Heteroskedastic (GARCH) models of Engle (1982) and Bollerslev (1986), respectively. These models are univariate, permitting the study of single time series. In addition, multivariate GARCH models have been developed to study the variance of multiple series, including the covariances between them. Likewise, multivariate GARCH models permit modeling conditional correlations among multiple time series

These multivariate models have the restriction of requiring a positive semi-definite (PSD) correlation (or covariance) matrix at every period. In addition, some multivariate models can only be used with a few time series, because of a curse of dimensionality. i.e. the parameters of these models can only be estimated non-linearly all at once. An extensive amount of univariate and multivariate ARCH /GARCH models have been developed, with an earlier paper by Bollerslev et al. (1992) addressing broad model developments including empirical financial applications. Recently Bauwens et al. (2006) presents a broad survey of multivariate models in existence.

A simple workhorse for the multivariate GARCH model has been the Constant Conditional Correlation (CCC) model (Bollerslev, 1990). In this model the correlations are constant. However, it has been shown empirically that this condition doesn't hold, since correlations tend to increase in periods of higher volatility (Longin and Solnik, 1995 and Ramchmand and Susmel, 1998). On the other hand, the Dynamic Conditional Correlation (DCC) model considers the case of varying conditional correlations through each period (Engle, 2002). For this model, the correlations between the different assets considered follow a parsimonious parametric model – specifically a GARCH-type dynamic.

A recent model which considers multivariate dynamic correlations is the Regime Switching Dynamic Correlation (RSDC) model from Pelletier (2006). The model considers the correlations within a regime as constant, yet the correlations vary in value from one regime to another. The switch between regimes is governed by a latent Markov chain. In this model the series may remain in a specific regime for ensuing periods before switching to a different regime, and only at this different regime is there a change in the correlation values. This is in contrast to the DCC model, which may have different correlation values for each subsequent period.

This chapter extends and develops a dynamic friction model based on the RSDC model, by modifying the transition probabilities that govern the switching process between regimes - from constant probabilities to state dependent or time-varying probabilities. Weakly exogenous variables were introduced in the probabilities that determine the switch from one regime to another. The new regime switching probabilities, now state dependent or time-varying, incorporate underlying fundamental variables that are directly related to the evolution of the series being studied.

By introducing fundamental related variables to the time series in the regime switching process, the particular effect that these factors may have on the dynamic correlation process is identified. Thus the impact and significance that these fundamental underlying variables have on the evolution of the regimes is determined. The state dependent probabilities that govern the regime switching process are established following Diebold et al. (1994).

These relevant underlying factors may produce asymmetries in the correlations between the series (i.e., the dynamic correlations may result in different values after applying shocks – either positive or negative - to the significant related variables), and they may reveal certain dynamics among the correlations of the markets, which are unaccounted for in the case of regime transitioning with constant probabilities. That is, the underlying fundamental factors may create friction that inhibits switching to a different regime hence the correlation values are maintained by the series staying in a certain regime. Conversely, these variables may prompt switching to a different correlation regime than if they had otherwise not been accounted for. The identification of these variables and their role in the evolution of the correlation values is determined, specifically in grain and cattle markets in lieu of the recent volatility spike of the former, and compared to the case where constant probabilities govern the Markov chain.

Grain market prices sharply increased from 2006 until mid 2008 – approximately two-fold in the case of corn, and also rose steeply for soybeans. This may have impacted livestock markets by increasing prices and leading to significant volatility shocks, since more than half of the corn production is used as animal feed and soybeans remain an important feed source. High price levels and the magnitude of sustained high volatilities raise concerns for many sectors of the economy – consumers may face higher food prices, and producers face unprecedented levels of price uncertainty coupled with higher input prices. Policy makers analyze the interrelationships among these markets and the effects of energy market shocks on agricultural markets. Figures 3.1.1, 3.1.2, 3.1.3 and 3.1.4 include time series charts of futures prices for corn, soybeans, feeder cattle and live or fed cattle, respectively. Likewise, figures 3.2.1, 3.2.2, 3.2.3 and 3.2.4 include times series charts of historical volatilities for these same commodities. Finally, figures, 3.3.1, 3.3.2., 3.3.3, 3.3.4 include times series charts of implied volatilities[1] for these same commodities.

Increasing grain commodity prices coupled with changes in their volatility, has implications for many decision makers. Agents that have a direct relationship with grain markets - specifically agents dealing with corn and also with soybeans (oilseeds) are particularly affected by these price variations. Crop producers are influenced in their planting decision making. They must decide between growing either corn and/or soybean seeking to obtain higher profitability, considering that corn production involves higher input costs than soybeans. At the same time, livestock producers require these crops as inputs - hence their costs and profitability are directly affected by the level and volatility in these input prices. These agents may benefit from appropriate determination of the dynamic interrelationships among these markets, as this may lead to efficiency gains in their operation. Likewise, policy makers need to determine the impact that recent energy policies, in this case directly affecting corn consumption, are having on the prices and markets related to this grain.

[1] Data obtained from Commodity Resource Bureau (CRB) data

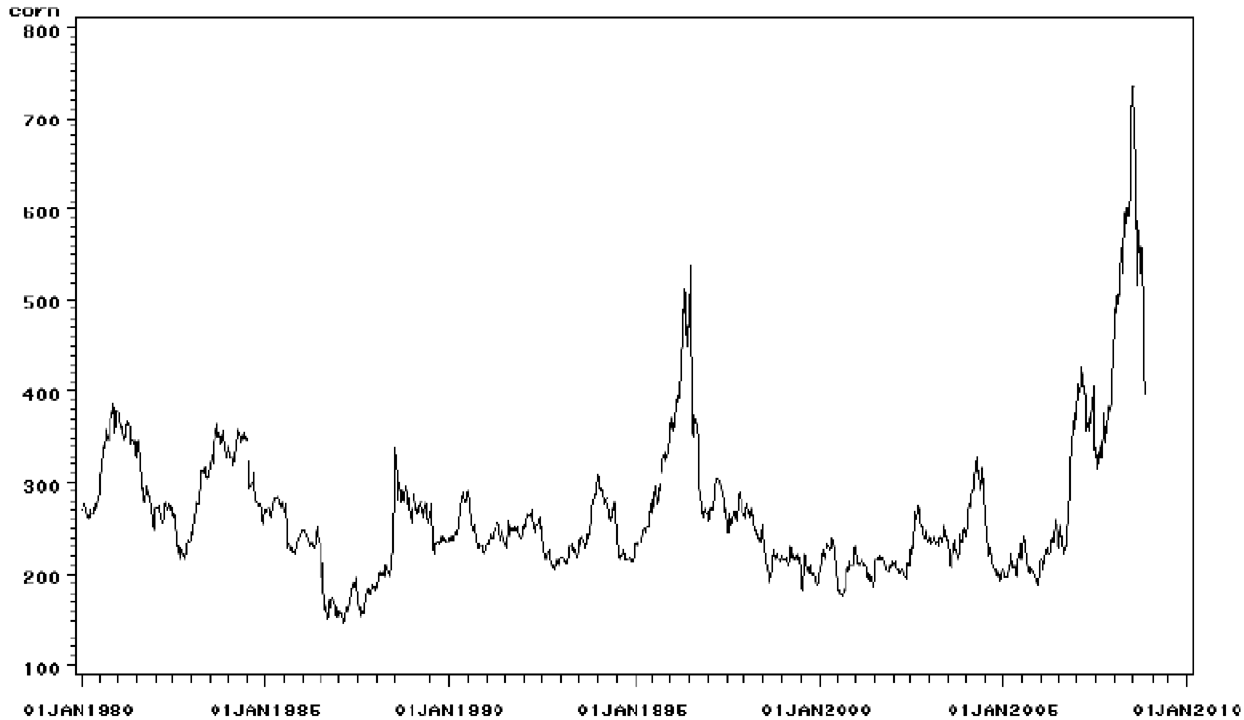


Figure 3.1.1: Corn Futures Prices (cents/bushel).

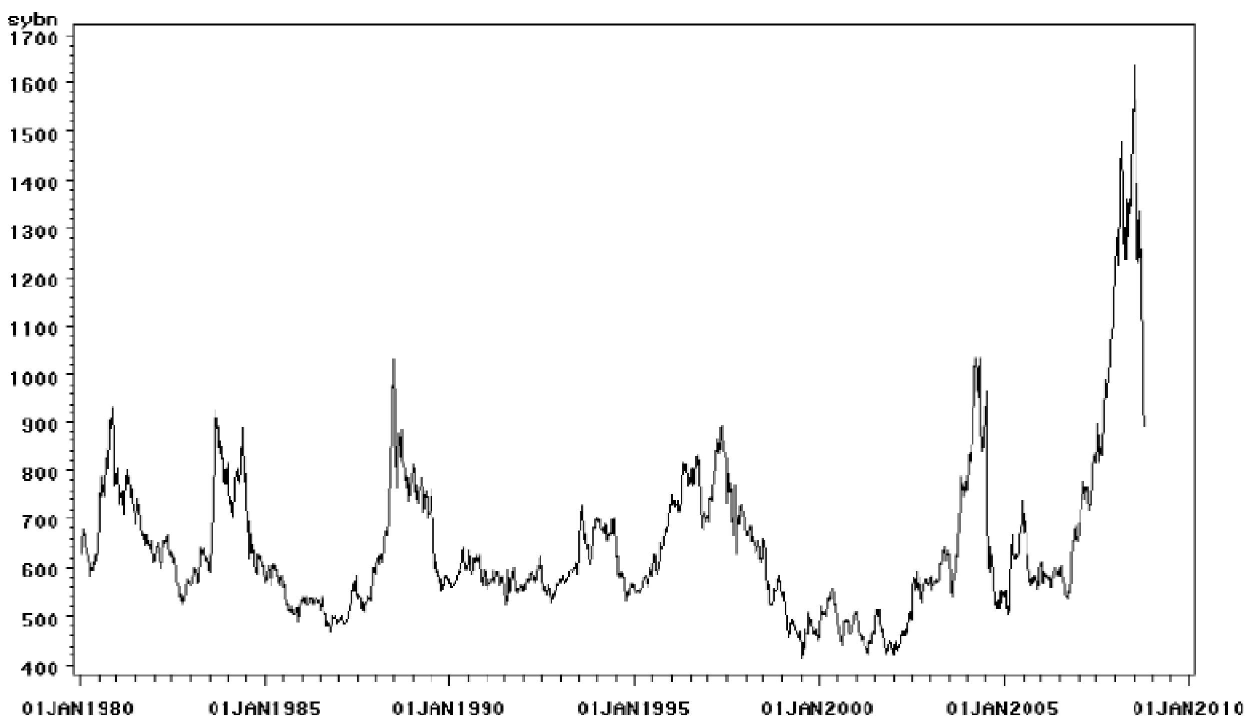


Figure 3.1.2: Soybeans Futures Prices (cents/bushel).

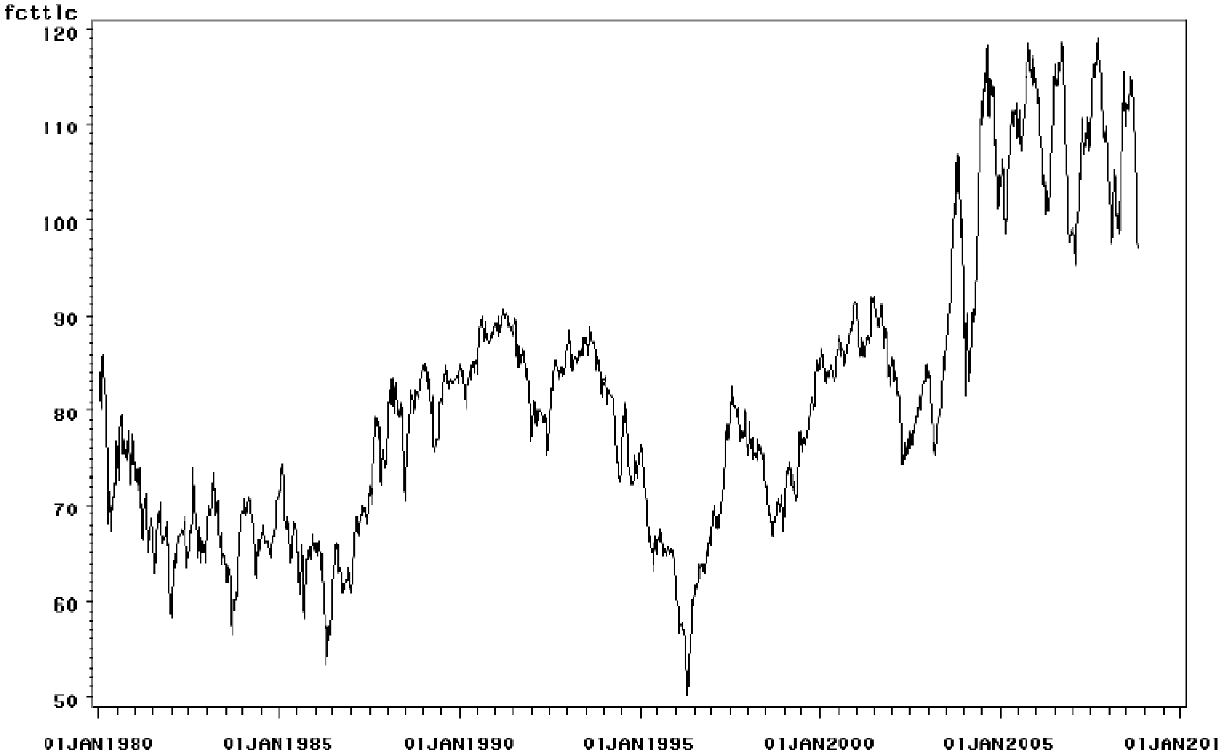


Figure 3.1.3: Feeder Cattle Futures Prices (cents/hundred weight - cwt).

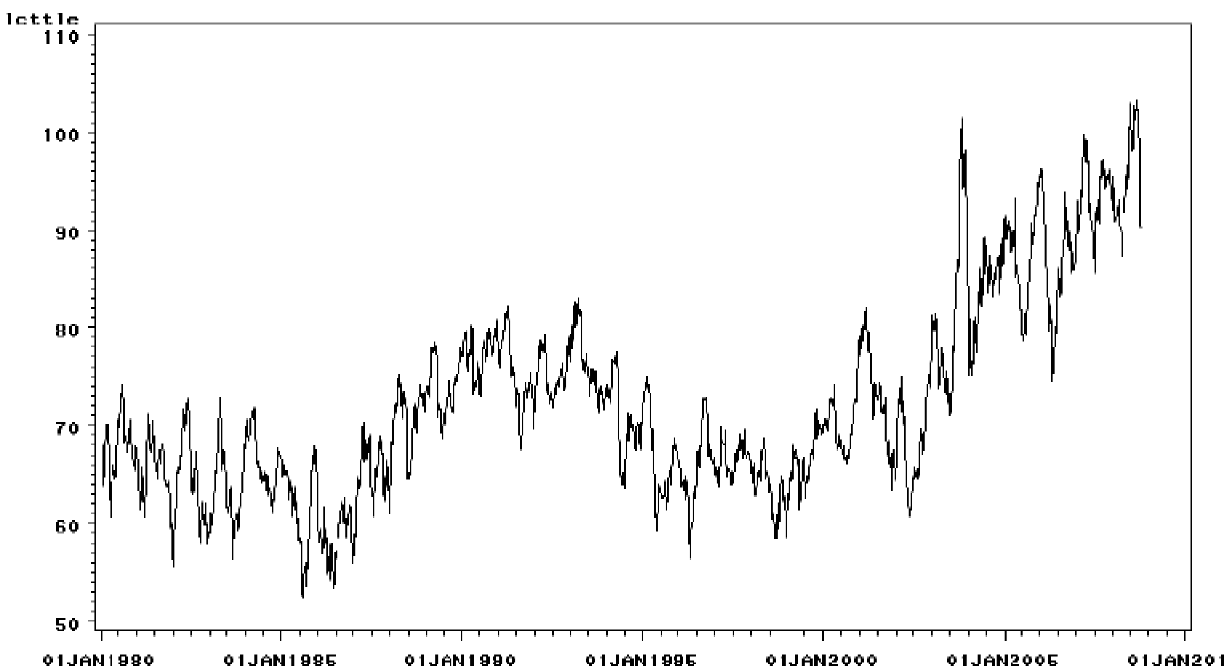


Figure 3.1.4: Live Cattle Futures Prices (cents/hundred weight - cwt).

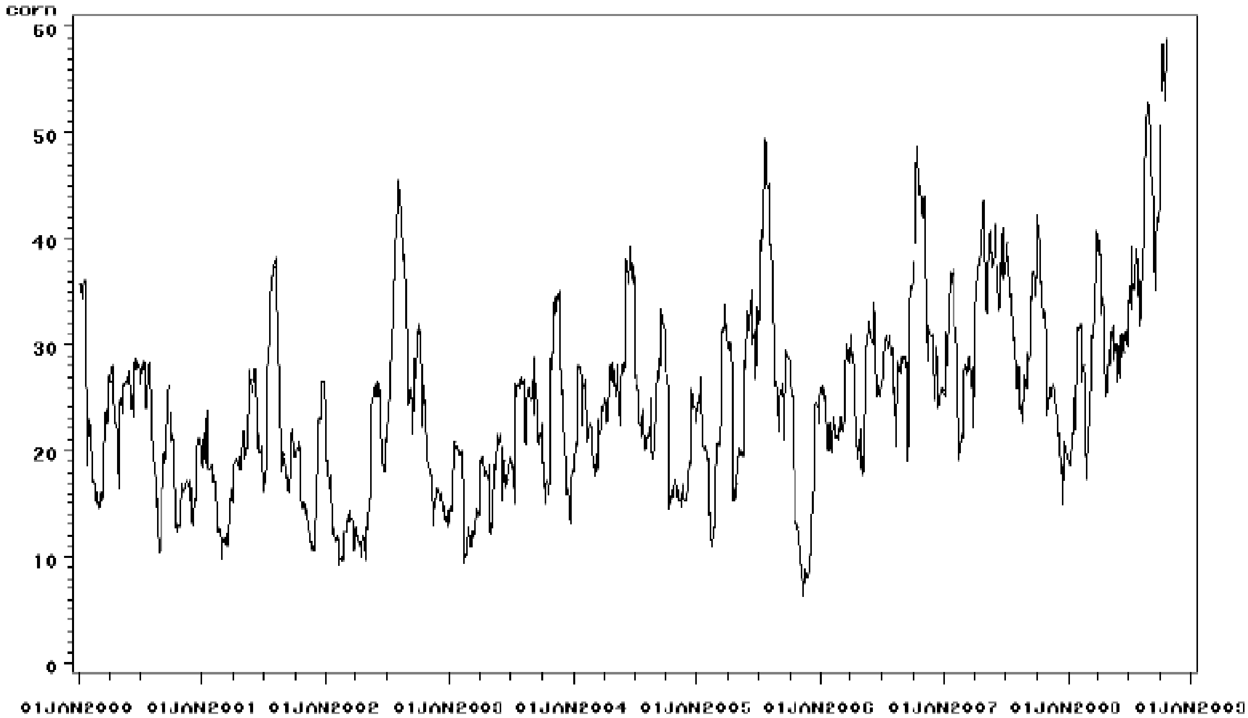
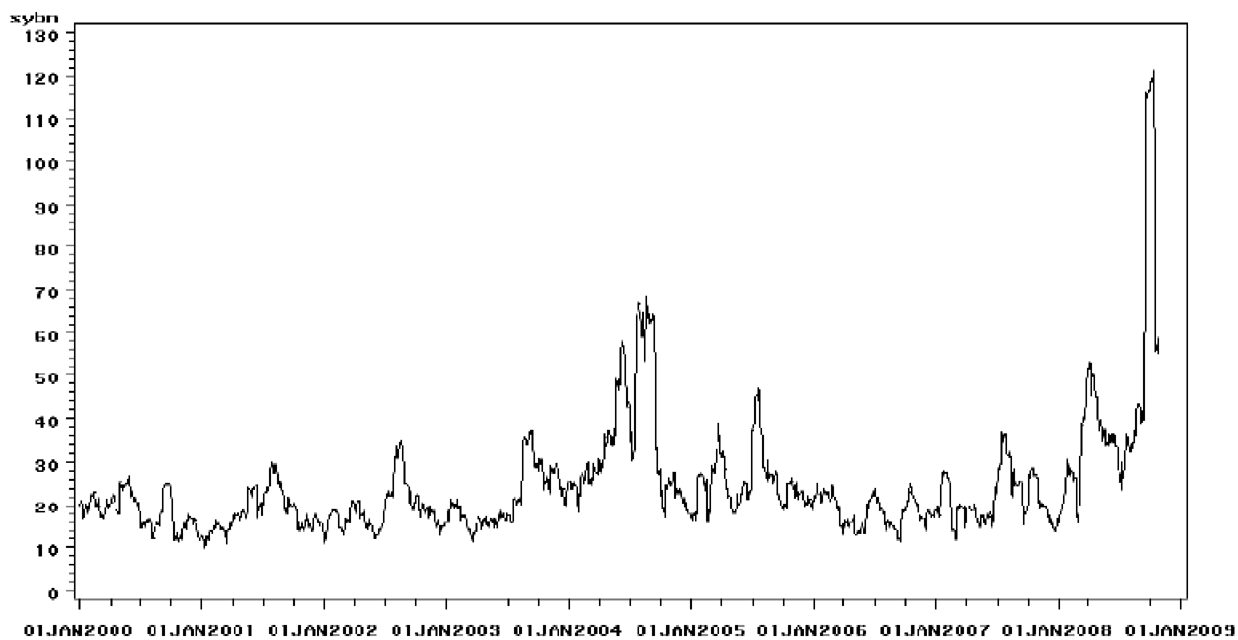


Figure 3.2.1: Corn Historical Futures Price volatility.



September 12 2008 - Soybean Volatility spike due to last day of trade of September contracts, with many 'shorts' having to deliver yet facing a smaller harvest supply due to delay in year planting. Price spiked a record of 2.74 $/bu. i.e. a case of 'short' squeeze.

Figure 3.2.2: Soybean Historical Futures Price volatility.

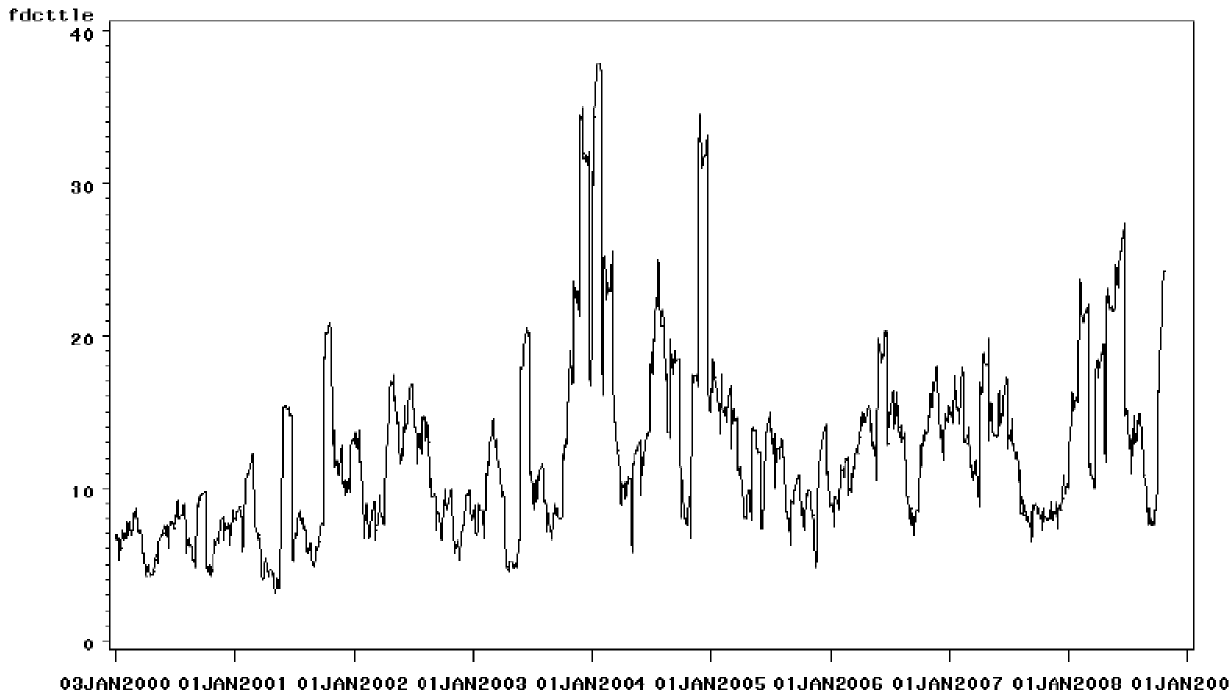


Figure 3.2.3: Feeder Cattle Historical Futures Price volatility.

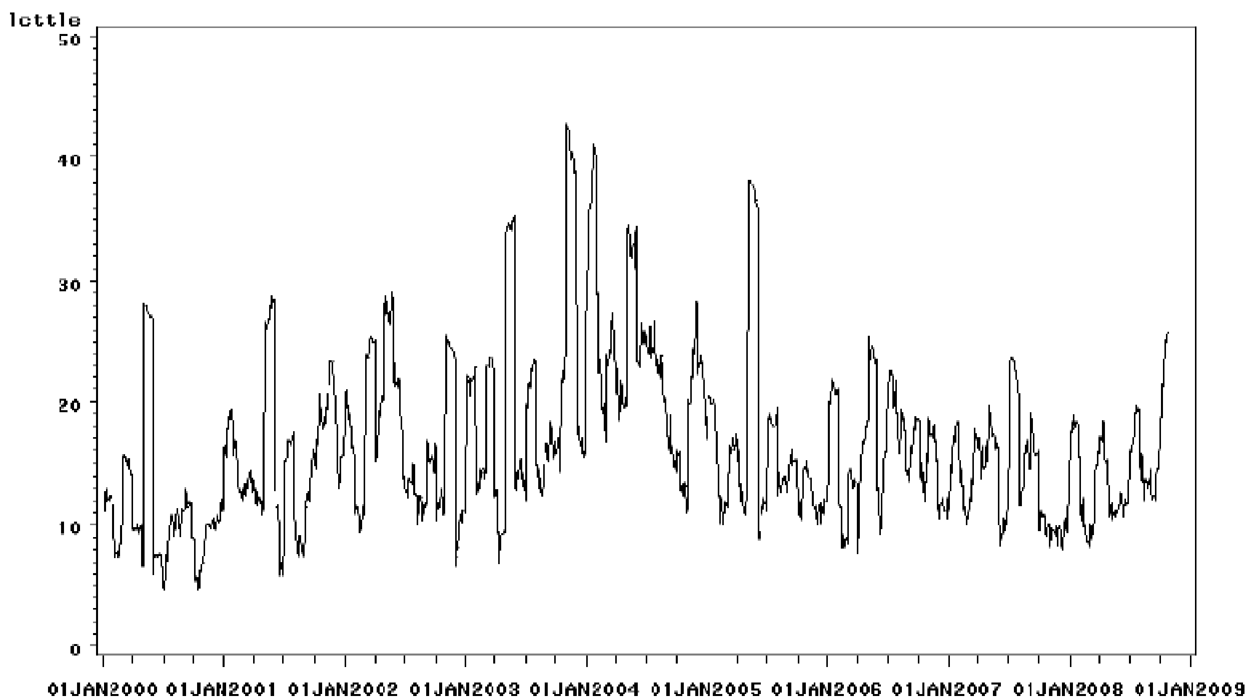


Figure 3.2.4: Live Cattle Historical Futures Price volatility.

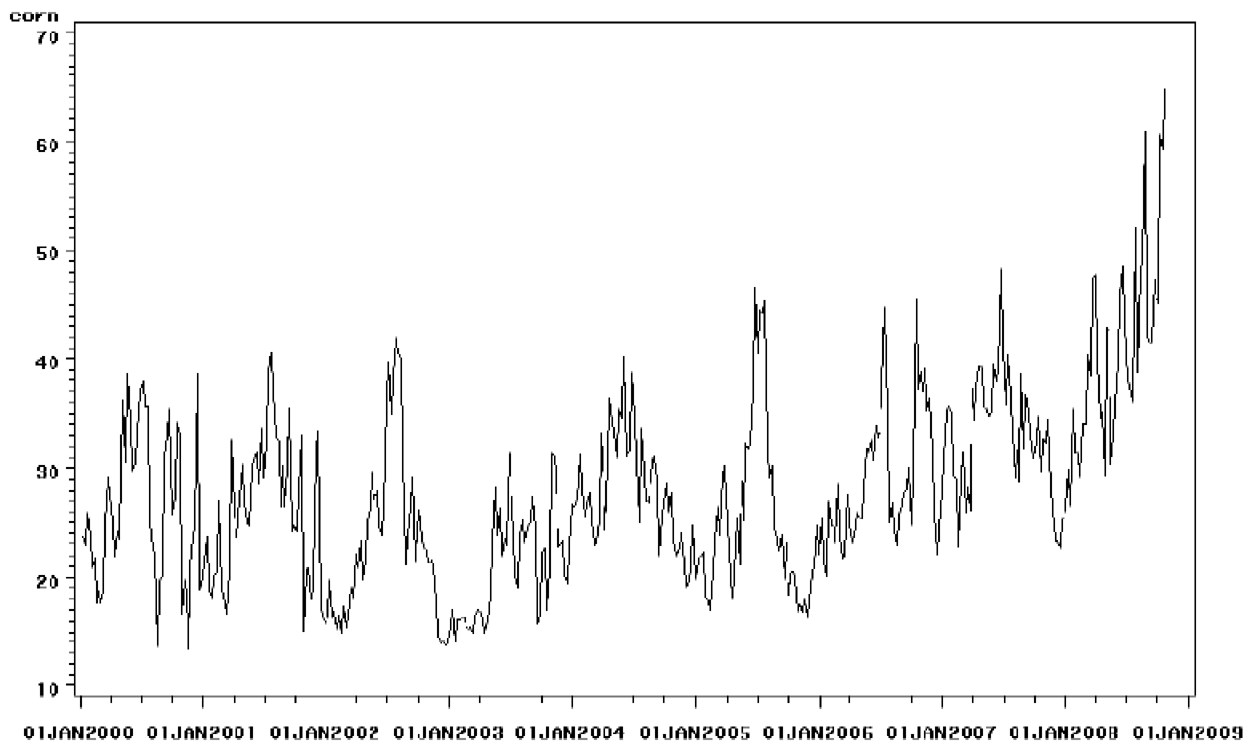


Figure 3.3.1: Corn Futures Prices Implied volatility.

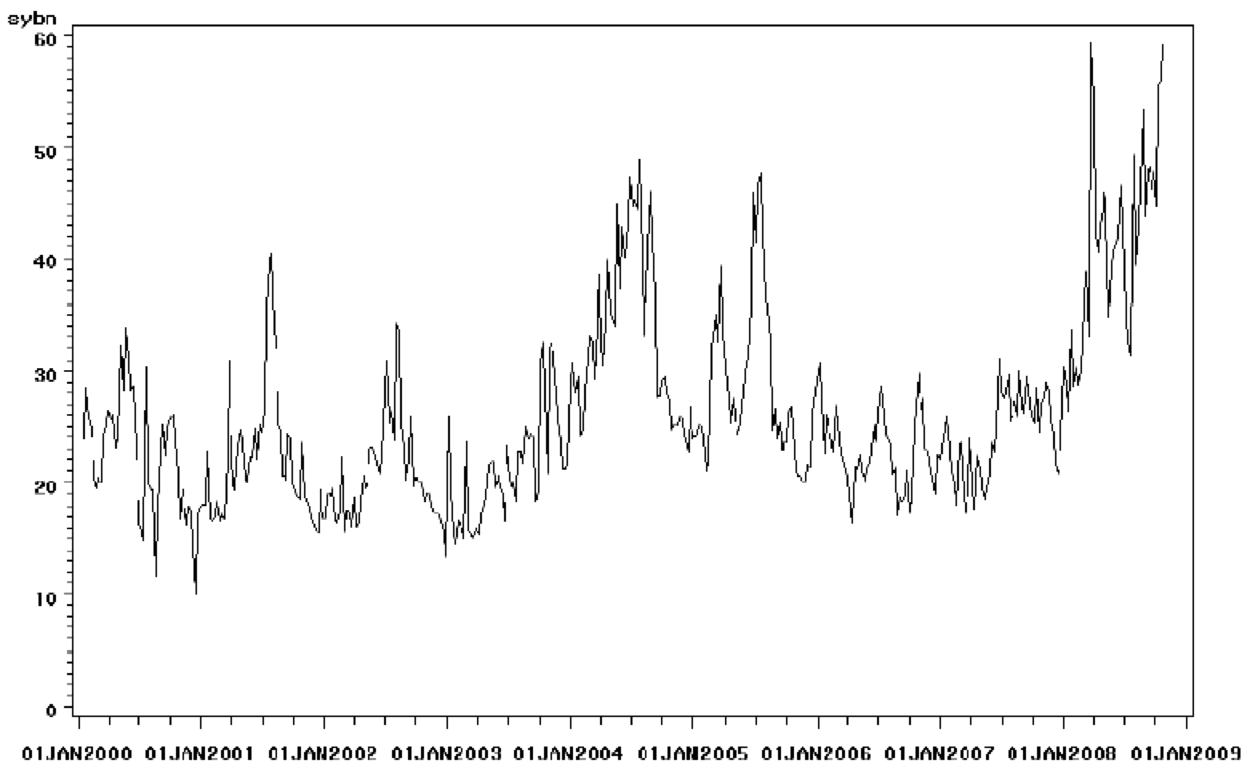


Figure 3.3.2: Soybean Futures Prices Implied volatility.

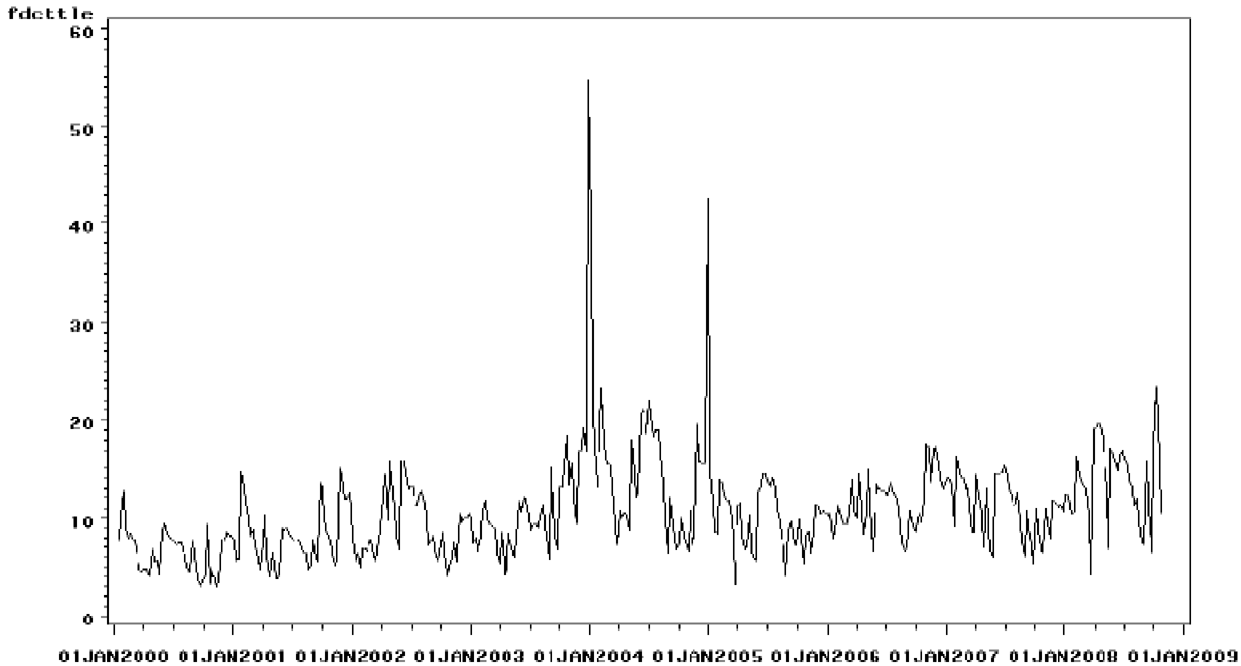


Figure 3.3.3: Feeder Cattle Futures Prices Implied volatility.

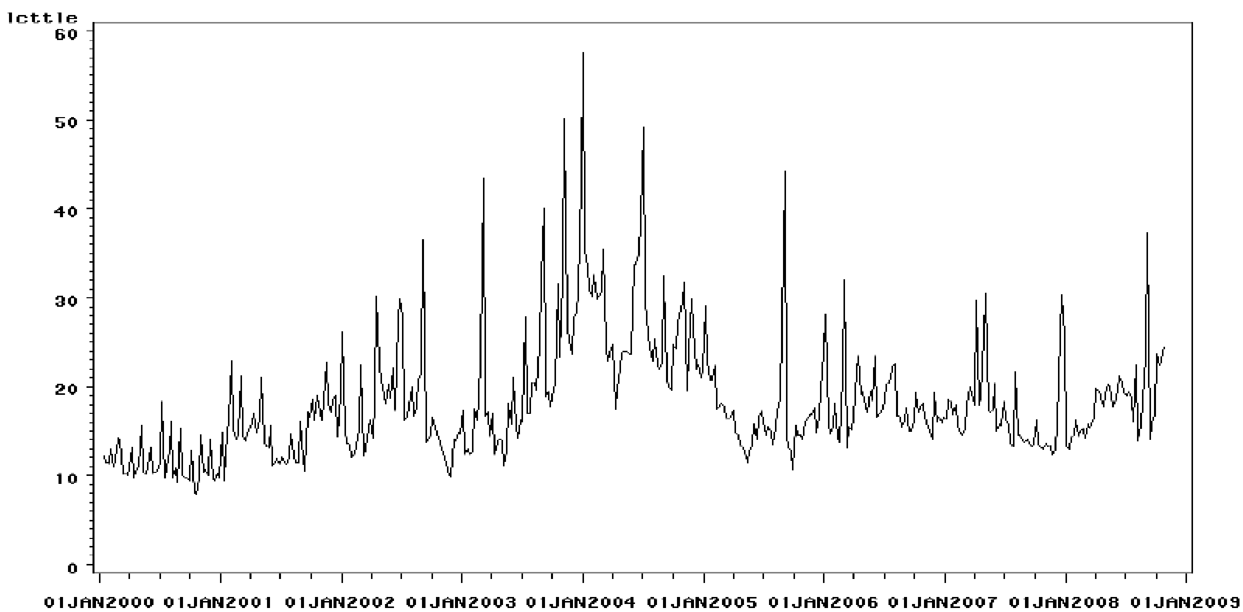
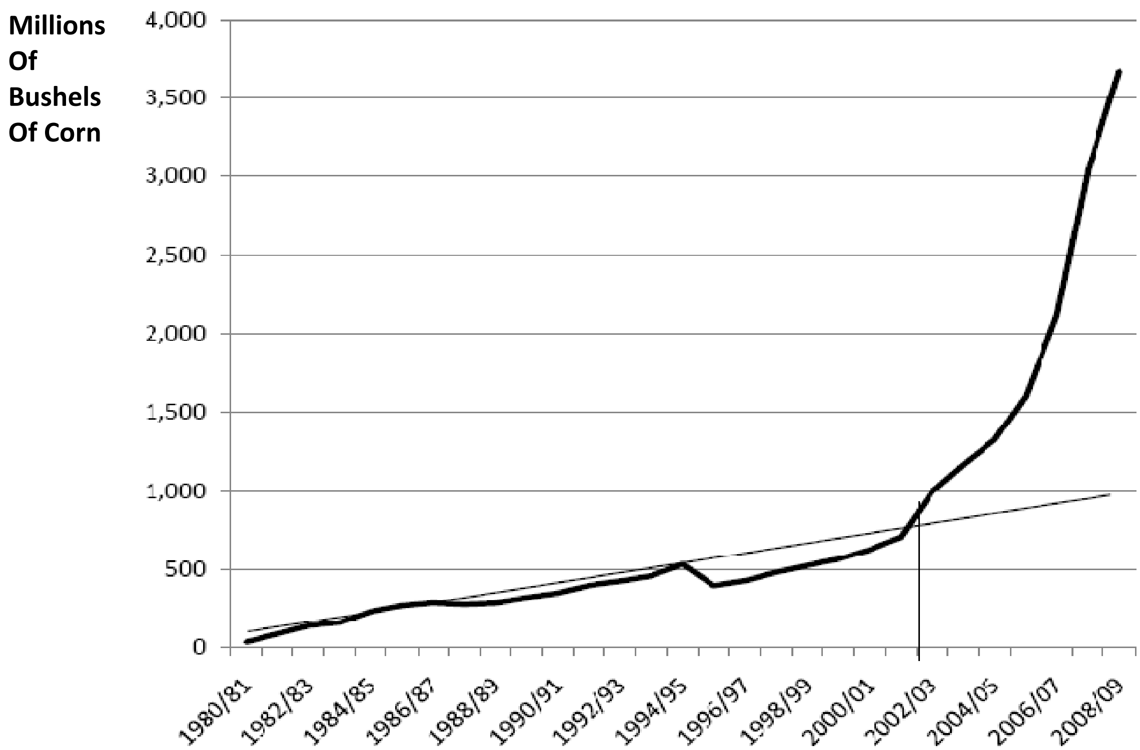


Figure 3.3.4: Live Cattle Futures Prices Implied volatility.

This chapter makes use of a newly developed extension of a time-series model, and determines market linkages between corn, soybeans and feeder and fed cattle prices. The impact of the surge in corn usage for ethanol production in the evolution of these prices is empirically analyzed. By using a friction structure in a multivariate time series model – the regime switching dynamic correlations model of Pelletier (2006), asymmetric correlations between grains and livestock prices are captured, including volatility spillovers. These volatility spillovers are characterized by the resulting persistence of markets remaining at certain correlation levels instead of switching to a different correlation regime. In addition, the potential inhibition in the transmission of prices between markets is identified, which may occur as a result of adjustment costs between these markets.

The dynamic correlations among corn, soybeans and feeder and fed cattle markets are estimated for two separate periods - including the latest period when corn faced a consumption boost from ethanol production. Results of this latter period are compared to the previous period where corn, soybean and cattle production did not face the increased corn demand from ethanol production – which was mandated by the Energy Policy Acts of 2005 and 2007. Figure 3.4 below depicts the corn consumption used in ethanol production through the past years. Implications for how these commodity markets are linked to one another are discussed. Risk spillovers from one market to another are identified, and their impacts on market interrelationships are discussed.

Results obtained are consistent with past literature, as per the determination of positive dynamic correlations between corn and soybean prices and between prices for feeder cattle and live cattle, for both periods considered - pre and post mandated corn ethanol consumption. Also an inverse or negative dynamic correlation is identified between both corn and feeder cattle and soybean and feeder cattle, for the period of post mandated ethanol consumption when corn and soybean have higher prices. The inverse corn and feeder cattle relationship is also consistent with previous literature, where increases in prices of corn (a main regular feed component for livestock) lead to a decrease in the price of feeder cattle. Similar results are obtained for soybeans, which is another relevant feed component.



Source: Economic Research Service, USDA

Figure 3.4: Corn consumption from Ethanol production (in millions of bushels).

No significant correlation was determined between corn prices (used as feed) and fed or live cattle markets, for both periods estimated. Thus an existing permanent friction level is identified responding to a transaction or adjustment cost which inhibits the transmission of price variations between corn prices and live cattle prices. This transaction or adjustment cost may respond to information and/or negotiation costs from cattle producers selling slaughter cattle (Hobbs 1997), and is materialized in the form of modifications of the feed rations given to cattle for weight gain during cattle production and thus preventing increases in the prices of these crops to be transmitted to the live cattle prices.

Information costs for cattle producers may involve price uncertainty for the case of fed cattle being sold at cash or spot markets,[2] leading producers to switch feed rations with increasing costs to feed components with lower cost in order to maintain costs down during production. Negotiation costs may respond to the limited number of auctions faced by producers when fed cattle are ready to be slaughtered, thus raising the transaction cost for using that channel. For producers selling directly with alternative marketing arrangements[3] to packers, negotiation costs may increase when having only a few different packers to bargain with, thus resulting in the packer exercising marketing power and establishing price and conditions of delivery. This again leads the cattle producer to seek feed rations that do not experience cost increases (i.e., modifying rations when there is a price rise in some of the components).

Corn and soybeans – specifically soybean meal – are main components for carbohydrates and proteins, respectively; of a high energy diet used by weight gaining feeder livestock. Yet there are other feed components for carbohydrates, such as grain sorghum or ‘milo’, and also other feed components for proteins, such as urea or cotton seed meal. In addition, another component of feed ration - the second largest after carbohydrates, is roughage, consisting mainly of corn silage, sorghum silage or alfalfa hay. Hence, there is a usual response from cattle producers to price increases in corn, by changing the contents of the feed rations in favor of different and

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[2] Cash or spot markets here refer to transactions ‘on the spot’. These include auction barn sales; video or electronic auction sales; sales through order buyers, dealers and brokers, and direct trades (RTI 2007). Fed cattle sold to packers by Alternative Marketing Arrangements (AMA) would not face this transaction cost.
[3] This accounted for less than 40 percent of all Fed Cattle volume from October of 2002 to March 2005; and includes forward contracts, marketing agreements, procurement or marketing contracts, production agreements, packer ownership, custom feeding and slaughter (RTI 2007).

potentially lower priced grain(s) in the mix. In particular, the higher priced corn may be lowered in proportion to grain sorghum or 'milo' or wheat. However, some corn content in the feed ration may also be directly substituted by roughage or other lowered price feed components.

The impact of significant underlying economic factor(s) are identified by taking into account the effect of the 'use to stocks' ratio of corn – which comprises the market's demand and supply conditions for corn – and also by considering the ratio of soybeans to corn futures' harvest prices - a measure contemplated by crop producers when deciding their planting acreage. Both of these factors, which will be described in more detail below, have a role on spillover effects among the markets. Results previously mentioned are determined separately by two different models – a restricted or parsimonious version and the full unrestricted version, and are followed by discussion of the findings and implications for dynamic relationships between the markets.

The paper proceeds by providing a general review of the relationship between corn and ethanol, then describes the characteristics of grain markets, and details the particular context of the corn market. A brief literature review of studies that have considered the impact of corn used for livestock feed on cattle profitability is subsequently provided. Likewise, previous studies regarding market linkages and price transmission, including asymmetric price adjustments in different cattle and pork markets are addressed. Subsequently, the two versions of the Regime Switching Dynamic Correlations model from Pelletier (2006) are presented – a parsimonious model and an unrestricted model – which estimate the correlation values at different regime levels for the markets considered.

In this study two different regimes are taken into account, translating into two different correlation levels. The case of the parsimonious model considers each market pair having correlation values between the two regimes that are proportional to each other. e.g. the correlation value between corn and soybean at one regime is proportional to the correlation value at the other regime, similarly the correlation value between corn and feeder cattle at one regime has the same proportion to the value at the other regime, and so forth. This allows for less parameter estimation since the correlation values of the two regimes – for each of the pairs of markets studied, (e.g., feeder cattle and live cattle) – are equally proportional.

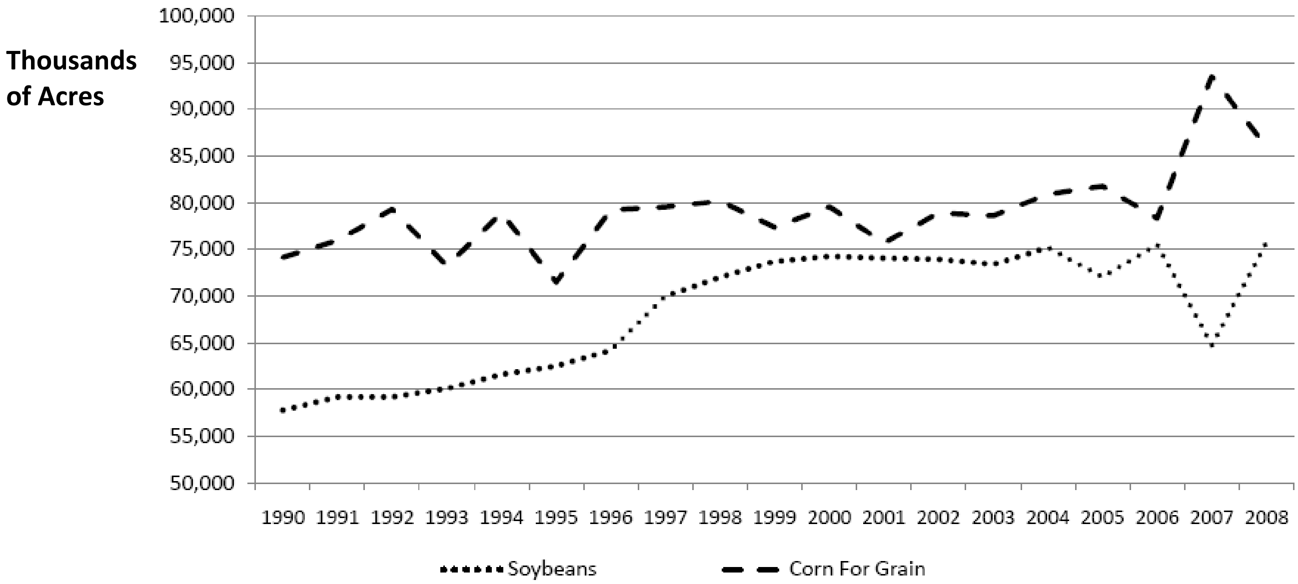
For the case of estimating the unrestricted model, each correlation regime is a function of each particular pair of markets considered. (e.g. the proportion of correlation values between one regime level and another regime level for corn and soybean is different than that of corn and feeder cattle, and so forth). Results are compared between the parsimonious and unrestricted model, which are fairly similar as anticipated, though the unrestricted model has some minor differences that will be discussed.

3.2 Background

The relation between ethanol fuel production and use of agricultural commodities in the U.S., specifically in the case of corn, began regularly in the early 1900's (with the model T from Ford); yet ethanol production was strengthened after Congress passed The Energy Tax Act of 1978. This act stated that for gasoline mixtures that included at least 10% ethanol content, there would be an exemption on the federal excise tax (i.e., subsidy) of 40 cents per gallon of ethanol mixed with gasoline. During the 1980's this tax exemption increased up to 60 cents per gallon of ethanol with the Tax Reform Act of 1984, settling at 54 cents in 1988. Ethanol was mainly used as an oxygenate agent in the mix of gasoline production, yet trailed distantly to MTBE (Methyl

Tertiary Butyl Ether) – which dominated the market – made from natural gas and petroleum. In 2004, this latter component had been banned from almost all states, as the EPA declared in 2000 that its use should be discontinued as it constituted a ground-water pollutant.

Ethanol faced a boost in demand with the previous MTBE ban and also with the Energy Policy Act of 2005, which mandated an increase in the use of renewable source of fuel energy – mainly ethanol. The Act called for a doubling of ethanol use by 2012. Recently, in 2007 Congress passed the Energy Independence and Security Act, which augmented the Renewable Fuels Standard to require that 36 billion gallons of ethanol and other fuels be blended into gasoline, diesel, and jet fuel by 2022. Ethanol production at the end of 2009 was about 10.7 billion gallons per year and is mandated to reach 13 billion gallons by 2012 and 15 billion gallons by 2015. This enormous increase in ethanol production from corn in recent years has led to hikes in corn prices and acreage. Land for corn production has been taken away mainly from soybean production, as these crops have similar production conditions. This has consequently lowered soybean production, causing an increase in soybean prices. A record use of corn acreage leading to a record harvest output was obtained in 2007. In 2008 a slight drop in corn acreage in favor of soybean production occurred. The following figure 3.5 presents time series for corn and soybean acreage in the United States.



Source: NASS – USDA

Figure 3.5: U.S. Yearly All Purpose Planted Crops (thousands of Acres)

Regarding the grain commodities market, there are three characteristics that distinguish this market versus that of other commodities (Schnepf, 2006). There is seasonality inherent in the production period. That is crop producers make their production decisions based on ex-ante information or expectations about their anticipated yield, hence regarding the price of the inputs as well as the harvested crop. A second characteristic is that the demand for these grains is generally of a derived nature. In other words, a majority of the grains may be used as inputs for processing a different final product; in this case - more than half of total corn production is used as a major component of feed for livestock. Finally, the nature of the supply and demand is that they are generally price-inelastic, especially for grains. That is, small movements in supply generate large price swings.

With respect to grains, specifically corn, the U.S., China, and Brazil account for two-thirds of the world's production. Of the three, the U.S. is the largest exporter, covering approximately two-thirds share of the world market with about 18% of its production. Since 2000, approximately 58% of the U.S. corn production has been used as the primary energy source of feed for livestock. The remaining 24% of corn production is used for food and industrial products such as starch, sweetener, fuel ethanol, corn oil and others.

With data from the Foreign Agricultural Service of the USDA, Westhoff (2008) notes that between the marketing years of 2005/2006 and 2007/2008, there was a rise of 35 million tons in U.S. corn consumption attributed to ethanol production. This accounted for approximately 43 percent of the increase in total world grain consumption, which if excluded, would have grown around 2 to 2.5 percent (i.e. being very similar to world population growth). Furthermore, prior to 2005 there has been a regular average increase around 2 percent in total world grain consumption, dating back to 2000. Recent hikes in corn consumption beyond this rate of world population growth may be attributable to use for production of ethanol as can be seen in previous figure 3.4.

Corn is the most broadly produced feed grain in the U.S., encompassing more than 50 percent of the total value and production of all feed grains. Corn feed competes with other feed grains – grain sorghum or 'milo', barley and oats, as well as feed wheat. Other feed ration components are roughage - which may consist of alfalfa, prairie hay, or corn silage among others and the protein component consisting of soybean (soybean meal), and cotton meal among others. In general, the largest feed component is grain, led by roughage and a small amount of protein supplements. However, the choice of rations also depends upon the relative prices among the feed components in line with feed grain markets being sensitive to relative prices among the different feed components.

Cattle producers take feeder or calf cattle and feed them with a high energy diet during a period of four to six months in order to rapidly increase their weight and sell for slaughter meat. These producers operate feedlots, which may be either small home feedlot operations, with less than 1,000 heads, or large commercial feedlot operations with up to 50,000 heads. In 2004, commercial feedlots were only about 11 percent of total number of feedlots in the U.S. However, these commercial feedlots provided about 85 percent of the market's fed cattle. In order to maximize profit, these feedlots are responsive to price changes of feed components.

The 2005 Energy Policy Act, and subsequently the 2007 Energy Independence and Security Act, mandated a significant increase in ethanol production resulting in a substantial rise in the demand for corn (Westhoff, 2008). These policies produced an outward shift in the demand curve of corn – the main input material used for ethanol production, resulting in a higher amount of corn being supplied at a higher price. This higher amount of corn production affected the soybean market, which shares a common geographical production area with corn, by transferring acreage to corn which had been previously used to produce soybeans. This lower supply of soybeans has also resulted in a higher price of soybeans. At the same time, both crops serve as feed for livestock markets, having a possible effect on the price and profitability of these markets.

Charts depicting the relationship between demand and supply for corn and for soybeans may be seen in figures 3.6.1 and 3.6.2, respectively. The price elasticity of supply for corn is anticipated to become more inelastic as there is an outward shift in the demand for corn due to the increase in corn consumption from ethanol production and thus raising the price of corn and its quantity consumed. In addition, the price elasticity of demand for soybean during this period also becomes more inelastic, as there is an inward shift in its supply from transferring acreage from soybeans to corn production; thus also increasing the price yet lowering the consumption.

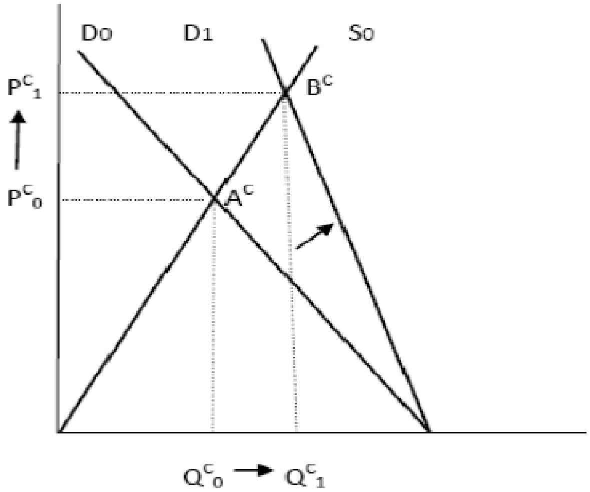


Figure 3.6.1: Corn Demand has an outward shift due to Ethanol corn consumption.

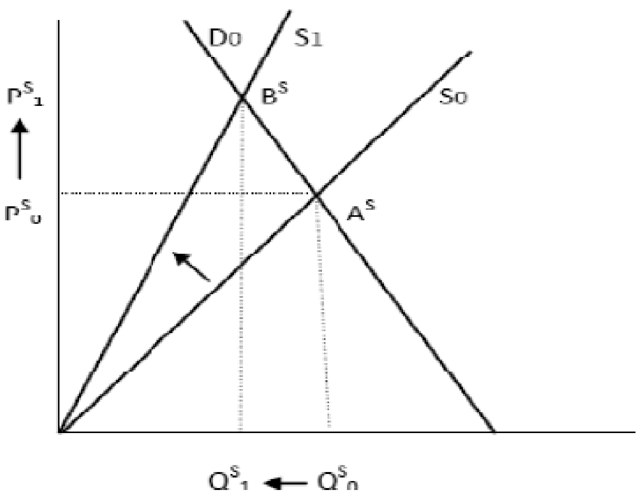


Figure 3.6.2: Soybean Supply has an inward shift due to acreage transferred for corn production.

A study of the effects that these supply and demand factors for corn and soybean have on the price variations of our grain and cattle markets – corn, soybean, feeder cattle and live cattle, is done by specifically taking into account the impact of the global supply and demand conditions for corn. For this objective, the influence of both the demand for corn consumption, as well as the corresponding stock availability of corn is considered. In line with previous studies, including Goodwin and Schnepf (2000), estimates of domestic demand, exports and domestic stocks of the current crop year for corn are used to determine the relevance it has on the evolution of our market prices. A factor with the estimated demand and supply corn data, noted as the corn 'use to stock' ratio is used to evaluate the impact that market forces on corn have on the evolution of the grain and cattle markets. This will be described further below.

In addition, each year crop producers of corn and soybeans begin to assess their acreage decision for the next growing season - following the current harvest period. When analyzing their decision, producers consider a ratio of harvest time prices for soybeans and corn to be around 2.2 to 2.4 - equal to a Break Even Price Ratio (BEPR). [4] The value for this ratio is such that producers do not favor the production of one crop over the other. This BEPR ratio takes into consideration input requirements for each grain, as well as their different yield per acre, and other related expenses (Lin & Riley, 1998). The prices of the ratio are for harvest periods (i.e. December contracts for corn and November contracts for soybean). The ratio is checked by producers prior to the planting season (i.e. during December – March), in order to make a decision about what to plant. Ratio variability within this previous planting period may modify producers' decision to opt for planting corn instead of soybean, when for example the ratio is lower than two. This effect will also be studied.

Another aspect to be addressed is that market linkage and transmission of prices between grains and cattle markets are subject to certain frictions generated by transaction costs. These

[4] BEPR is the ratio of expected soybeans-to-corn price ratio which equates the expected net returns of producing corn and soybeans, given trend yields of corn and soybeans, the expected price of corn, the variable costs of corn and soybean production, the expected program payments and other program expenses. (Lin & Riley 1998).

transaction or adjustment costs may be of information and/or negotiation (Hobbs, 1997), as in the case of cattle producing agents selling slaughter cattle in different market channels (i.e. at auctions or by contract directly to a packer). The adjustment cost here results in price variations not being transmitted at all between the markets. Transaction costs for other markets may result in instances where price variations are not passed on directly between markets, but are likewise subject to certain adjustments. These adjustment costs may be different for the case of applying positive shocks versus negative shocks. A price increase may be not transmitted in an equal inverse form than if it is a price decrease, and thus generating asymmetries between market correlations.

Besides taking into account the effect of the rise in prices and volatilities of these grains in determining the dynamic correlations between corn, soybean, and cattle livestock markets, likewise the effect of this price volatility on cattle feed rations may be inferred from the grain and livestock correlations. It is anticipated that feed rations of livestock change in response to variations in the prices of corn and soybean. Additionally, the resulting higher cost of corn feed is anticipated to have an inverse effect on feeder cattle prices in order to maintain cattle production profitability, which is consistent with the literature reviewed below.

Studies reviewing corn and its implications on livestock feed for cattle profitability is discussed next. Likewise, previous studies regarding market linkages and price transmission, including asymmetric price adjustments in different cattle and pork markets are addressed. Subsequently, the two versions of the regime switching dynamic correlations model are presented, including the introduction of the state dependent transition probabilities between two different regimes. Results are followed by discussion of the findings and implications for the dynamic relationships between the markets and policy analysis.

3.3 Literature Review

Numerous studies have analyzed the relevance of corn price in its interaction to livestock profitability, since it is the main source of feedstock. In cattle production profitability, a study by Langemeier et al. (1992), using monthly average data for 2600 pens from 1980 through 1989, concluded that in addition to cattle feeder and cattle fed i.e. live cattle prices, changes in corn prices had an approximate 22 percent impact in the variability of profits. Another study by Schroeder et al. (1993) using larger individual pen level data from a similar period (1980 to mid 1991), found that corn price changes accounted for between 16 percent and 6 percent of cattle profit risk - decreasing the risk level as cattle placement weight increased (i.e. an increasing weight of the feeder cattle). A study by Mark et al. (2000) for cattle profitability from 1980 to 1998, also considering two feedlots in Kansas showed similar results. That is corn prices are less important in cattle production profit risk than both calf and live cattle prices.

Lawrence et al. (1999) analyzed more than 200 feedlots over a much broader area located in the Corn Belt, including Illinois, Iowa, Minnesota, Nebraska and South Dakota and found that corn prices, while still significant for cattle production profitability, had less impact than feed efficiency and average daily gain. These results were obtained considering corn from 1987 to 1996, where prices fluctuated from slightly under $2 to almost $5 per bushel. Albright et al. (1994) specifically determined that about 60 percent of the variability in the cost of weight gain (i.e. feed efficiency) could be attributed to corn price variability. This analysis of the cost of gain was obtained from data from two Kansas feedlots, considering corn prices from 1980 to 1991.

A different study by Anderson and Trapp (2000) for the feeder cattle market, estimated a dynamic corn price ‘multiplier’ that simultaneously impacts placement weight, slaughter weight and feed-conversion rate as the price of corn changes. This dynamic effect is taken into account in a cost/revenue setting, such that the break-even feeder cattle price may be determined. Results indicate that increases in corn prices are mitigated by changes in feeding programs or composition, producing a smaller decline of calf-feed prices than in a static analysis scenario. In other words, increased corn prices result in a reduction of calf-feed prices to maintain cattle production profitability, yet the reduction in these prices is diminished by changing the feed rations.

A recent paper by Belasco et al. (2009) studied the dynamic relationships between cattle production yield risk factors, including the influence of cattle pen characteristics, in assessing the risks borne by cattle producer’s profits using a dynamic multivariate regression model. The price of feed, including corn, may be indirectly considered as part of a yield risk factor (dry matter feed conversion & average daily gain – i.e. ’cost’ of gain) and the dynamic effect of this factor is estimated. In addition, as mentioned by Lawrence et al. (2008), a monthly survey of commercial cattle feedlots by Kansas State University points out that the cost of gain had risen from an approximate average of 54 cents per lb. in 2006 to 74 cents in 2007, to over 80 cents in 2008.

Regarding the sharp increase in food commodity prices including grains such as corn and soybeans, especially from 2006 to 2008 - an extensive report detailing major factors was presented by Trostle (2008). In the report he states that recent global increases in demand of feedstock for biofuel – mentioned previously with respect to ethanol, along with a decline in the U.S. exchange rate have been relevant demand factors contributing to a hike in commodity prices. In addition supply factors such as increasing energy prices as crude oil prices rise, higher input production costs, and adverse weather have also contributed. Another report by Schnepf (2008) also contends that coarse grains – mainly corn though also barley, sorghum, oats and rye, have faced increased demand due to two major factors. One factor is feed use for livestock due to increased demand for meat from India and China[5] (as these two large countries experience high income growth during this latest period). Another larger factor - which is what we specifically consider, is increased demand through input for biofuel production rising from policy mandates, both here in the U.S. as well as in Europe.

The markets may respond to transmission of price variations by fully passing them along, or by having adjustments according to transaction costs that are present between the markets. In some instances these markets may be related production wise, as in the case of corn and soybeans in the U.S. In other cases, these markets may be vertically related, as in the case of corn used as main feed component for cattle production and soybean being an important protein feed component. In either situation it may be that there are negligible adjustment costs such that market price variations may be passed on concurrently to another market, or it may be that there are significant adjustment costs which delay the transmission of price changes perhaps generating spillover effects.

Several studies have been conducted regarding asymmetric price adjustments, including threshold behavior. A paper by Goodwin and Holt (1999) analyzed the dynamic relationship and

[5] This translates into competition for acreage between corn and soybean.

transmission of market prices among marketing channels in the beef sector using a threshold error correction model accounting for the non-stationary nature of the prices and considered the asymmetric effects produced. From 1981 to early 1998, for weekly price data, they found significance for three different regimes. Threshold behavior occurred, existing mainly two regimes in the 1980's and dropping one of the regimes for a different one during the 1990's. Additionally, and in response to price shocks, lags were found during the adjustment period between each channel. However, the price adjustment of these lags – initially asymmetric – tended to decrease as price shocks occurred during a later time period, and in addition, the price adjustments became symmetric. A subsequent study by Goodwin and Harper (2000) for the pork sector between 1987 and 1998 arrived at similar results.

Earlier papers by Boyd and Brorsen (1985) - studying the pork sector with weekly data from 1974 to 1981, and Hahn (1990) - studying both the pork and beef sector, also found significant lags during price adjustment. Another paper by Boyd and Brorsen (1988) found symmetric responses to price changes, however; Hahn (1990) found asymmetric responses to price changes in the different market channels. Another result from Goodwin and Holt (1999) and Goodwin and Harper (2000) confirmed that price changes within market channels mainly propagated in one direction. i.e. response to price shocks were generally found to produce adjustments when these shocks were applied at the farm markets and from there the adjustments were passed on to the wholesale markets, and then to the retail markets. This result upholds earlier findings by Boyd and Brorsen (1985) and Schroeder (1988).

Goodwin and Piggott (2001) studied market integration in spatially separate regional grain markets, through price linkages. They incorporated in their analysis thresholds that account for transaction costs, which delay price adjustments. The regional markets considered were for corn and soybean in North Carolina. Their results indicated that the markets are well integrated, and also confirm the existence of threshold points for price adjustments. Once these thresholds are accounted for in the model, the speed of adjustment for prices is higher than when they are not considered. For a study that conducted an extensive survey regarding asymmetric price transmission, see Meyer and von Cramon-Taubedel (2004).

3.4 Method Overview

Our empirical analysis uses an extended version of the Regime Switching Dynamics Correlation model (RSDC) of Pelletier (2006). Both models of the original RSDC – a parsimonious version and a full unrestricted model – are extended by modifying the constant transition probabilities for state dependent probabilities in the switch between correlation regimes. The state dependent transition probabilities serve to determine the impact of underlying fundamental economic factors (e.g. indexes and/or prices) in the change of these dynamic correlations. These fundamental underlying related variables may either be weakly exogenous (e.g. a lagged ratio of demand and supply conditions, or price ratio of specific commodities, or other) or exogenous factors, or a mix between them.

Two cases of state dependent transition probabilities with different weakly exogenous variables are contrasted. One case uses the ratio of ‘use to stocks’ for corn, which takes into account the global dynamic effects of supply and demand factors on corn including the increased corn consumption due to ethanol production. The other case of state dependent transition probability considers the ratio of weekly soybeans to corn futures prices, with delivery in

November and December, respectively. This is in line with crop producers' economical break-even point ratio (BEPR) for deciding which crop to produce -either corn or soybean, to obtain higher profitability. Both ratios are anticipated to remain stable during regular market periods (i.e., pre-ethanol corn demand surge), and are anticipated to change as increased demand and volatility of corn impacts the market. The initial case with a constant transition probability for switching between regimes is a nested case within the model.

General results from the model application are as anticipated, as corn and soybean markets have positive correlations since they share a certain geographic relationship of growing conditions in the U.S. At the same time, both these crops are related to livestock, being feed for cattle. A permanent friction level or adjustment cost present in the relationship between corn and live cattle markets is identified. This reveals an adjustment cost which may be the result of cattle producers modifying the feed rations of feeder cattle, when corn prices increase, since a major part of their feed ration is provided by corn.

In addition, the model's ability to determine the changing dynamic correlation among these markets is conducive to potential gains of efficiency in risk management operations. Hedging possibilities may be facilitated by identifying the dynamic correlations among related markets. This may require further analysis, since forecast estimations are not addressed here.

3.5 Data

Weekly averages of future prices for corn, soybean, feeder cattle and live cattle from the Chicago Board of Trade (CBOT) and Chicago Mercantile Exchange (CMEX) respectively are used, obtained through the Commodity Research Bureau (CRB). These weekly average prices of futures are obtained by considering the nearest/closest maturity delivery date. Prices are from September 1998 until August 2008, considering the periods between these months (from September to the following August) as a crop year in conformity with USDA guidelines.

The product of one hundred times the difference of the log values of these prices[6] is considered. Three different scenarios are estimated to determine the effect of the spike in corn consumption due to ethanol mandated production. The first base scenario considers the entire data series previously mentioned, that is from September 1998 till end of August 2008. The next two scenarios considered are partitioned such that one series runs up till previous the energy acts, that is from September 1998 to August 2004. The other series considers data from September 2003 till August 2008. The two partitioned periods have an overlap of the crop year 2004, which experienced unusual volatility, in order to minimize potential different seasonality effects that may be present in one of the partitioned series versus the other. Summary statistics for the three scenarios of futures prices are in table 3.1, and their returns are in table 3.2 below.

For the weakly exogenous variables being considered, in addition to the nested case of this variable being equal to zero, two different cases are estimated. One case considers lagged values of the use to stock ratio for corn. The other case considers the lagged ratio of the soybean to corn price, with delivery in November and December, respectively. In figures 3.7.1 and 3.7.2 below these variables can be seen for the entire period from 1998 to 2008. Data for the 'use to stock' ratio for corn was obtained from the monthly World Agricultural Supply and Demand Estimates (WASDE) reports at the USDA, using a similar method as that by Goodwin and Schnepf (2000). Thus monthly data from the report according to USDA guidelines is taken into account, and a

[6] 100*ln(P_t/P_{t-1})

cubic spline interpolation method is used, converting the monthly data into weekly data and thus enabling the subsequent computation of a weekly 'use to stock' ratio for corn.

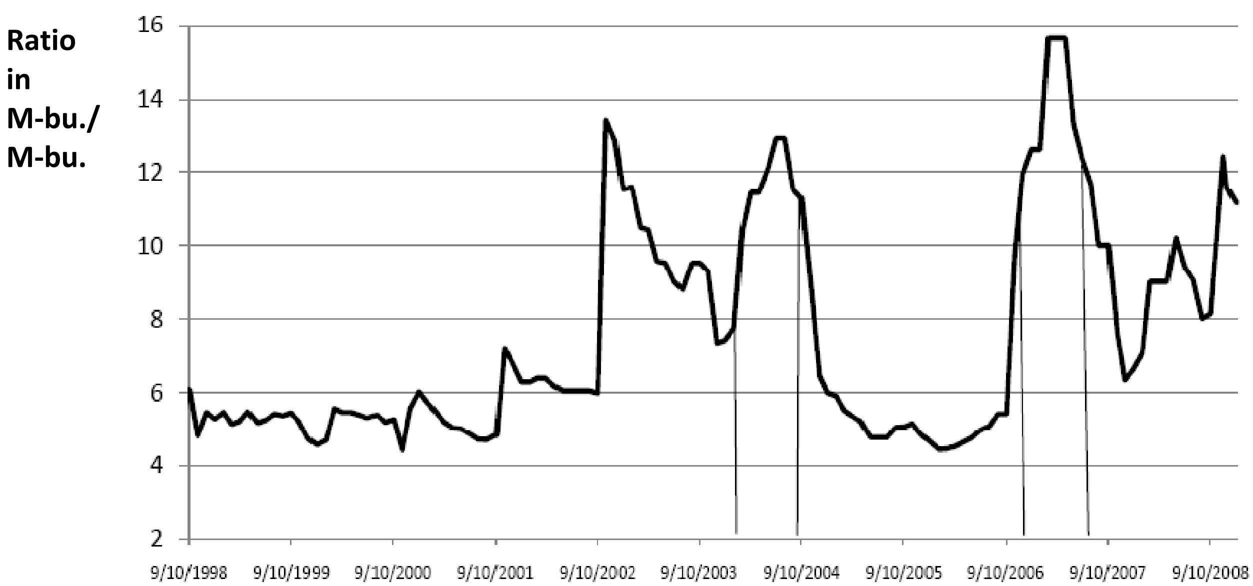
Table 3.1: Summary Statistics for Futures Prices – Weekly Average

	Corn[7]	Soybean	Feeder Cattle[8]	Live Cattle
September 1998 to August 2008				
Mean	264.09	649.19	94.25	79.45
Std Devtn.	102.07	242.90	14.39	11.12
Max	735.05	1634.19	119.07	102.93
Min	175.90	412.94	66.76	58.40
September 1998 to August 2004				
Mean	223.97	556.45	84.71	72.58
Std Devtn.	27.86	136.09	9.73	8.22
Max	329.19	1036.55	118.35	101.42
Min	175.90	412.94	66.76	58.40
September 2003 to August 2008				
Mean	312.00	793.49	106.75	88.98
Std Devtn.	126.21	271.21	7.80	6.01
Max	735.05	1634.19	119.07	102.93
Min	188.10	505.70	81.62	74.66

[7] Corn and Soybean Price units are in Cents per bushel (bu.). Minimum contract is for 5,000 bushels
[8] Feeder Cattle and Live Cattle Price units are in Cents per hundred weight (cwt.). Minimum contract for Feeder Cattle is for 50,000 lbs. Minimum contract for Live Cattle is for 40,000 lbs.

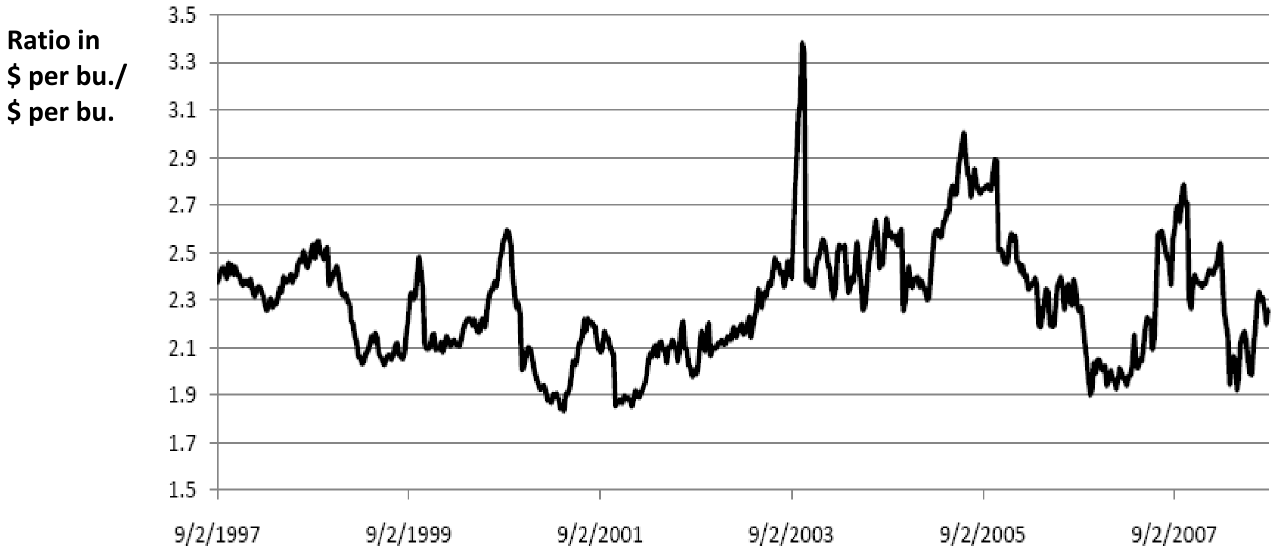
Table 3.2: Summary Statistics for Futures Log Returns

	Corn	Soybean	Feeder Cattle	Live Cattle
September 1998 to August 2008				
Mean	0.21	0.18	0.10	0.11
Std Devtn.	3.22	3.10	1.76	2.25
Max	10.37	9.61	5.58	6.31
Min	-12.59	-18.44	-13.82	-15.13
September 1998 to August 2004				
Mean	0.05	0.05	0.18	0.11
Std Devtn.	2.81	3.01	1.72	2.41
Max	10.10	8.56	5.58	6.31
Min	-11.47	-18.44	-13.82	-15.13
September 2003 to August 2008				
Mean	0.35	0.32	0.06	0.07
Std Devtn.	3.67	3.72	2.12	2.45
Max	10.37	9.61	5.55	6.31
Min	-12.59	-18.44	-13.82	-15.13



Source: WASDE – USDA

Figure 3.7.1: Corn – Use to Stock Ratio



Source: WASDE - USDA

Figure 3.7.2: Ratio of Soybeans Futures Prices with delivery November (X) to Corn Futures Prices with delivery in December (Z)

In addition, lagged values of these previous two weakly exogenous variables are considered for two weeks, one month, three months, six months, one year; yet these did not present significant changes in results. (i.e., each weakly exogenous variable for further lagged periods was statistically insignificant).

Summarizing, three different time period scenarios are estimated for the following three different *cases*:

i. the *nested case* (i.e. $x_{t-1} = 0$) for a constant transition probability.

ii. the *case with lagged 'use to stock' ratio for corn* as a weakly exogenous variable in the state dependent transition probability.

iii. the *case with lagged ratio of soybeans harvest price to corn harvest price* as weakly exogenous variable in the state dependent transition probability.

3.6 Econometric Model:

3.6.1 RSDC model

The dynamic covariances between series are partitioned into standard deviations and correlations, with different correlation values switching between different regimes through a Markov chain.

Considering a K - multivariate time series process,

$$Y_t = H_t^{1/2} U_t \tag{1}$$

where $U_t \sim i.i.d.(0, I_K)$, Y_t may be a filtered process and I_K is an identity matrix. The time varying covariance matrix H_t is decomposed into standard deviations and correlations:

$$H_t \equiv S_t \Gamma_t S_t \tag{2}$$

where S_t is a Diagonal matrix with standard deviations: $s_{k,t}$ $k = 1 \ldots\ldots K$, and Γ_t is the correlations matrix. This decomposition of covariance matrix has been previously used by Bollerslev (1990), Tse & Tsui (2002), Engle (2002), and Pelletier (2006).

The standard deviations $s_{k,t}$ for each time series k - from the diagonal matrix S_t, are assumed to follow an ARMACH model (Taylor, 1986) defined below. The correlation matrix Γ_t follows a Markov chain, with different values for different regimes.

3.6.1.1 Markov Process for Regime Switching between Correlations:

The RSDC model considers multiple series transitioning between regimes of different correlation levels according to a latent Markov chain process. The switch from one regime to another is governed by transition probabilities. The time-varying correlation matrix Γ_t is defined as follows, similar to Pelletier (2006):

$$\Gamma_t = \sum_{n=1}^{N} \mathbf{1}_{\{\Delta_t = n\}} \Gamma_n \tag{2.1}$$

where Δ_t is an unobserved Markov chain process independent of U_t, taking *N* possible regimes or values ($\Delta_t = 1,2, \ldots N$). And **1** is an indicator function for each regime. The $K\ x\ K$ matrices Γ_n are correlation matrices - symmetric, positive semi-definite (PSD), ones on the diagonal, off-diagonal elements between values of -1 and 1, with $\Gamma_n \neq \Gamma_{n'}$ for $n \neq n'$. The 'probability law' governing the Markov chain process Δ_t is defined by its state dependent transition probability matrix: Π_t. The probability of going from regime i in period $t-1$ to regime j in period t is denoted by $\pi_t^{i,j}$, i.e. $\pi_t^{i,j} = P(\Delta_t = j | \Delta_{t-1} = i)$. The matrix Π_t has elements of row i and column j. The Markov chain satisfies the standard assumptions. That is - aperiodic[9], irreducible[10] and ergodic[11] (Ross, 1993). The limiting probability of being in regime n is π^n, as per the ergodic property of a Markov chain.

In equation (2.1) there is a need to impose Γ_n to be a correlation matrix, and thus enabling Γ_t to also be a correlation matrix. In Γ_n the diagonal elements are 1 and the off-diagonal elements are between $[-1, 1]$. Yet this does not imply that the matrix is PSD. Hence the Choleski decomposition is used in helping to impose PSD for Γ_n, such that $\Gamma_n = P_n P_n^{'}$ with P_n as the lower triangular matrix, with its first diagonal value equal to 1. This secures off-diagonal elements between [-1, 1].

[9] An aperiodic process is one that does not return to a certain state i within a definite, known, number of time steps (i.e. returns to a certain state i can occur at any irregular time steps).
[10] An irreducible process is one that has the possibility to arrive at any state from previously being at any state, i.e. there is no limitation to access any certain state when coming from any specific state.
[11] An ergodic state is one that is aperiodic and positive recurrent. This latter refers to a state having a finite expected return time to state i, from previously being at state i., i.e. it will return at some point to state i with a finite expected return time.

Despite an increasing number of parameters coming from each Γ_n that are to be estimated, the curse of dimensionality may be avoided by using the Expectation Maximization (EM) algorithm (Dempster et al. 1977) to estimate the model presented in Diebold et al. (1994) and Hamilton (1994 – Chapter 22). Also, as mentioned before, this model is linear due to the Markov chain, and therefore able to compute directly multi-step ahead conditional expectations of the correlation matrix. This is in contrast to the DCC model, which introduces non-linearities by rescaling the co-variances to obtain correlations between -1 and 1; hence direct conditional expectation computation is not possible.

Another relevant factor is that if persistence exists in the Markov chain (staying in the same regime in subsequent time periods), it will lead to smoother time-varying correlations. This could influence the Value at Risk (VaR)[12] computations and dynamic portfolio allocation, as the benefits of portfolio diversification would be less volatile (Pelletier 2006).

3.6.1.2 Restricted Model

The parsimonious or restricted model for the time-varying correlation matrix Γ_t is as follows and similar to Pelletier (2006):

$$\Gamma_t = \Gamma\lambda(\Delta_t) + I_K\big(1 - \lambda(\Delta_t)\big) \qquad (2.2)$$

with Γ being a fixed KxK correlation matrix – for every state or regime considered, I_K is a KxK identity matrix, and $\lambda(\Delta_t) \in [0,1]$ is a univariate random process governed by an unobserved Markov chain process Δ_t that takes N possible values ($\Delta_t = 1,2 \dots N$) being independent of U_t.

[12] Value at Risk is a certain value in which the probability of a loss of an investment portfolio exceeding this value, is the given probability level, e.g. VaR of $ 1million at 5%, i.e. a 5% chance of losing over $1 million of the portfolio.

The correlation matrix at time t (i.e., Γ_t) is a weighted average of two extreme states or regimes – uncorrelated returns at $\lambda(\Delta_t) = 0$, or highly correlated returns at $\lambda(\Delta_t) = 1$. Changes among correlations of different regimes are strictly proportional to $\lambda(\Delta_t)$, allowing for regimes of higher or lower correlations, and with the diagonals (own-correlations) being left at one.

This model enables calculation of dynamic correlations in the context of time-varying transition probabilities without requiring expectation maximization since fewer parameters are to be estimated. In other words, through maximum likelihood and using the correlation targeting method described in the estimation following section below, we are able to estimate dynamic correlations between regimes when considering state dependent transition probabilities.

3.6.1.3 Univariate Model

The time-varying standard deviations are modeled directly by using the ARMACH process from Taylor (1986). In ARMACH, the conditional standard deviation (s_t) follows:

$$s_t{}^{13} = \omega + \sum_{i=1}^{q} \tilde{\alpha}_i \, |y_{t-i}| + \sum_{j=1}^{p} \beta_j \, s_{t-j} \qquad (2.3)$$

with $\tilde{\alpha}_i{}^{14} = \alpha_i / E|\tilde{u}_t|$, for stationary purposes.

3.6.2 State Dependent, Time-varying Probabilities

State dependent time-varying transition probabilities within regimes are introduced into the dynamic correlations model with a procedure from Diebold et al. (1994). As noted, there are two

[13] Conditional Standard Deviation: s_t, where $\tilde{\alpha}_i$ is the parameter for the series' previous $(t-i)$ observations (or innovation), and β_j is the parameter for the previous $(t-j)$ standard deviations. As mentioned previously since the expectation enters as a linear operator, ARMACH provides ease of use for multi-step ahead computation of conditional expectations.

[14] $\tilde{\alpha}_i = \alpha_i / E|\tilde{u}_t|$ comes from computing the Un-conditional s_t, i.e. $E(s_t)$, and thus assuring s_t of being stationary.

regime switching models that will be extended. The first is the parsimonious (restricted) model (2.2) and the second is the unrestricted model (2.1).

The state dependent probability matrix Π_t has elements of row i and column j:

$$\pi_t^{i,i} = P(\Delta_t = i \mid \Delta_{t-1} = i\,, x_{t-1}; \beta_i) = \frac{\exp(x_{t-1}^{'}\beta_i)}{1+\exp(x_{t-1}^{'}\beta_i)}$$

$$\pi_t^{i,j} = P(\Delta_t = j \mid \Delta_{t-1} = i\,, x_{t-1}; \beta_i) = 1 - \frac{\exp(x_{t-1}^{'}\beta_i)}{1+\exp(x_{t-1}^{'}\beta_i)}$$

For the specific case of considering two regimes or two states: e.g. $\Delta_t = 1$, i.e. being at regime number 1 or $\Delta_t = 2$, i.e. being at regime number 2; and for variables x_{t-1} being weakly exogenous variables, the transition probability matrix Π_t (3.1) is presented below.

Time t

Time t-1	**State 1**	**State 2**
State 1	π_t^{11} $P(\Delta_t = 1 \mid \Delta_{t-1} = 1, x_{t-1}; \gamma_1)$ $= \frac{\exp(x_{t-1}^{'}\gamma_1)}{1+\exp(x_{t-1}^{'}\gamma_1)}$	$\pi_t^{12} = (1 - \pi_t^{11})$ $P(\Delta_t = 2 \mid \Delta_{t-1} = 1, x_{t-1}; \gamma_1)$ $= 1 - \frac{\exp(x_{t-1}^{'}\gamma_1)}{1+\exp(x_{t-1}^{'}\gamma_1)}$
State 2	$\pi_t^{21} = (1 - \pi_t^{22})$ $P(\Delta_t = 1 \mid \Delta_{t-1} = 2, x_{t-1}; \gamma_2)$ $= 1 - \frac{\exp(x_{t-1}^{'}\gamma_2)}{1+\exp(x_{t-1}^{'}\gamma_2)}$	π_t^{22} $P(\Delta_t = 2 \mid \Delta_{t-1} = 2, x_{t-1}; \gamma_2)$ $= \frac{\exp(x_{t-1}^{'}\gamma_2)}{1+\exp(x_{t-1}^{'}\gamma_2)}$

where $x_{t-1} = (1, x_{1,t-1}, \ldots.., x_{(m-1),t-1})'$ & γ_i[15] $= (\gamma_{i1}, \gamma_{i2}, \ldots\ldots\ldots, \gamma_{i(m-1)}), i = 1\ or\ 2$;

[15] Here we use $\gamma_i$the same as β_i of previous general probability definition in order to differentiate from the β_j parameter for the lagged standard deviation (s_{t-j}) to be estimated for the ARMACH model (2.3).

3.6.3 Estimation:

From equations (1) and (2), the log-likelihood can be written as:

$$L = -\frac{1}{2}\sum_{t=1}^{T}[Klog(2\pi) + \log(|H_t|) + Y_t' H_t^{-1} Y_t]$$

$$= -\frac{1}{2}\sum_{t=1}^{T}[Klog(2\pi) + \log(|S_t \Gamma_t S_t|) + Y_t' S_t^{-1} \Gamma_t^{-1} S_t^{-1} Y_t]$$

$$L = -\frac{1}{2}\sum_{t=1}^{T}[Klog(2\pi) + 2\log(|S_t|) + \log(|\Gamma_t|) + \widetilde{U}_t' \Gamma_t^{-1} \widetilde{U}_t] \qquad (4.1.)$$

where $\widetilde{U}_t = S_t^{-1} Y_t$ and $\widetilde{U}_t = [\tilde{u}_{1,t} \dots\dots\dots\dots. \tilde{u}_{K,t}]'$ is a zero mean process with covariance matrix Γ_t; also $|H_t| = \det(H_t)$.

Estimation is made in two steps, with the assurance that the variance/covariance matrix is PSD (positive semi-definite). First the standard deviations are obtained, and then the correlations are calculated. Standard deviations are obtained directly via the ARMACH model in (2.3), permitting a smooth linear calculation for correlations versus the use of a GARCH type model, which introduces nonlinearities when going from covariances to correlations since it requires the square root of the variances. This arrangement also enables the possibility of calculating analytically multi-step ahead conditional expectations, due to its linear properties; yet using a GARCH type model does not permit this.

The estimation method computes both filtered and smoothed transition probabilities between regimes, and subsequently makes use of an Expectation Maximization (EM) algorithm for the unrestricted model (i.e., the model estimation involves two main parts). The first part is the expectation step, which estimates the expectation of the complete data log-likelihood. This involves estimating the filtered probabilities for the complete empirical log-likelihood conditional on data observed, and then computing 'back' the smoothed probabilities. The second part is the maximization step, which considers the use of these smoothed probabilities in our expected complete empirical log likelihood function and maximizes directly with respect to the parameters.

When there are a large number of parameters being estimated we can use a two step estimation procedure as in Engle (2002) and Pelletier (2006). In the first step, univariate volatility models are estimated independently for each series. In the second step, the correlation matrix is estimated conditional on the first step estimates. This procedure helps in avoiding the dimensionality curse. However, if the number of parameters is not too large, then only a one step procedure may be necessary for the estimation of all the parameters. Both procedures are described below.

3.6.3.1 **One-step estimate**:

For maximum likelihood, we evaluate:

$$QL(\theta; Y) = \sum_{t=1}^{T} \log\, f(Y_t | \overrightarrow{Y_{t-1}}) \qquad (4.1.1)$$

with $\overrightarrow{Y_{t-1}} = \{Y_{t-1}, Y_{t-2}, \dots\}$ and θ is the vector of unknown parameters. This equation comes from (4.1.).

The method of Diebold et al. (1994) is used which considers a filter for time-varying probabilities - creating smoothed probabilities, and then estimating through maximum likelihood for the restricted model and through expected maximization for the general model. General steps for this procedure follow below, with full details of the process in Appendix 1.

To simplify as before, let $\{\Delta_t\}_{t=1}^{T}$ be the sample path of a first order, two state Markov chain process (i.e., $\Delta_t = 1$ or $\Delta_t = 2$), with time varying transition probabilities according to the matrix

Π_t (3.1). In general, the matrix Π_t considers a x_{t-1} vector (mx1) for the underlying related variable(s), i.e. a set of weakly exogenous variables that affect the state dependent transition probabilities. Therefore parameters γ_i with $i = 1, 2$ constitute the two regimes i.e. γ_i is a $2m$x1 vector such that $\gamma = (\gamma_1', \gamma_2')'$, and the constant transition probabilities are nested within these state dependent probabilities (the first element of both mx1 is a constant).

For the unrestricted model (2.1), we estimate the case of two regimes considering state dependent transition probabilities[16], by:

$$\widehat{\Gamma}_t = \widehat{\Gamma}_1 * P(\Delta_t = 1 | \bar{y}_T, x_{t-1}; \theta) + \widehat{\Gamma}_2 * P(\Delta_t = 2 | \bar{y}_T, x_{t-1}; \theta) \qquad (4.1.2)$$

In the restricted or parsimonious model from (2.2), the case of two regimes dynamic correlations is estimated by:

$$\widehat{\Gamma}_t = \widehat{\Gamma} * P(\Delta_t = 1 | \bar{y}_T, x_{t-1}; \theta) + \widehat{\Gamma} * \hat{\lambda}(1) * P(\Delta_t = 2 | \bar{y}_T, x_{t-1}; \theta) \qquad (4.1.3)$$

Let $\{y_t\}_{t=1}^{T}$ be a time series that evolves according to the Markov chain $\{\Delta_t\}_{t=1}^{T}$. In this case, the series are previously standardized with respect to their volatility obtained through the ARMACH model (2.3). Then the following:

$$(y_t | \Delta_t = i; \alpha_i) \sim N(\mu_i, \sigma_i^2), \text{ with } \alpha_i = (\mu_i, \sigma_i^2)' \text{ and } i = 1, 2$$

with the conditional density given by:

$$f(y_t | \Delta_t = i; \alpha_i) = \frac{1}{\sqrt{2\pi}\sigma_i} \exp\left(\frac{-(y_t - \mu_i)^2}{2\sigma_i^2}\right) \quad \text{with } \alpha = (\alpha_1', \alpha_2')' \qquad (4.2)$$

The restricted or parsimonious model enables calculation of dynamic correlations within a time-varying transition probabilities context by maximum likelihood. Hence, with the use of

[16] Here $P(\Delta_t = 1 | x_{t-1}; \theta) = P(\Delta_t = 1, \Delta_{t-1} = 1 | x_{t-1}; \theta) + P(\Delta_t = 1, \Delta_{t-1} = 2 | x_{t-1}; \theta)$, further details - Appendix 3.B

correlation targeting described next, dynamic correlations between regimes are estimated when taking into account state dependent transition probabilities.

Because in (2.2) only the product of Γ and λ is identifiable for off-diagonal constraints (diagonal elements are equal to 1), a similar correlation targeting method from Pelletier (2006) is applied. That is by using either of the two following sets of constraints during our estimation:

i. $\lambda(1) = 1, \lambda(1) > \lambda(2) \ldots \lambda(N-1) > \lambda(N)$ (4.2.1)

which fixes the first λ value at its highest i.e.1, obtaining directly Γ from $\Gamma\lambda$; also the rest of $\lambda's$ are in diminishing order to eliminate the possibility of relabeling regime i as j and viceversa.

ii. $\max_{i \neq j} |\Gamma_{ij}| = 1$; with $1 > \lambda(1) > \lambda(2) \ldots, \lambda(N-1) > \lambda(N)$ (4.2.2)

this fixes an off-diagonal element at 1; but it does not mean the correlation may be 1 (or -1), since the correlation is actually the product given by $\Gamma_{ij}\lambda(\Delta_t)$, and $\lambda(1) < 1$.

For the general model (2.1), the Expectation Maximization (EM) algorithm is a robust and stable method to maximize the incomplete data log-likelihood, via iterative maximization of the expected complete-data log likelihood, conditional upon the observed data set. For the long-run probability of the first (starting) state at $t = 1$, being at state 1, $P(\Delta_1 = 1) = \rho$ is used. In the case of the restricted model, instead of having to estimate this extra parameter ρ, the limiting probabilities for the Markov process: π^1, π^2 are considered and solve for:

$\begin{pmatrix} \pi^1 \\ \pi^2 \end{pmatrix} = \Pi'_t \begin{pmatrix} \pi^1 \\ \pi^2 \end{pmatrix}$; $\pi^1 + \pi^2 = 1$ [17]

Such that:

[17] Ross (1993) – pg. 204

$$\hat{\pi}^1 = \hat{\pi}_t^{21} \Big/ (\hat{\pi}_t^{12} + \hat{\pi}_t^{21}) \qquad \text{and} \qquad \hat{\pi}^2 = \hat{\pi}_t^{12} \Big/ (\hat{\pi}_t^{12} + \hat{\pi}_t^{21})$$

Now the complete-data likelihood – assuming $\{y_t\}$ and all regimes Δ_t are observed – and for $\theta = (\alpha', \gamma', \rho)'$ (yet excluding ρ in the case of the restricted model) would be:

$$f(\vec{y}_T, \vec{\Delta}_T | \vec{x}_T;\ \theta) = f(y_1, \Delta_1 | \vec{x}_T;\ \theta) \prod_{t=2}^{T} f(\ y_t, \Delta_t |\ \vec{y}_{t-1}, \vec{\Delta}_{t-1}, \vec{x}_T; \theta)$$

$$= f(y_1 | \Delta_1, \vec{x}_T;\ \theta) P(\Delta_1) \prod_{t=2}^{T} f(y_t | \Delta_t, \vec{y}_{t-1}, \vec{\Delta}_{t-1}, \vec{x}_T; \theta) P(\Delta_t | \vec{y}_{t-1}, \vec{\Delta}_{t-1}, \vec{x}_t; \theta)$$

$$= f(y_1 | \Delta_1;\ \alpha) P(\Delta_1) \prod_{t=2}^{T} f(y_t | \Delta_t;\ \alpha) P(\Delta_t |\ \Delta_{t-1}, x_{t-1}; \beta)$$

Introducing indicator functions for convenience yields:

$$f(\vec{y}_T, \vec{\Delta}_T | \vec{x}_T;\ \theta) = [\ \mathrm{I}(\Delta_1 = 1) f(\mathrm{y}_1 | \Delta_1 = 1;\ \alpha_1)\rho + \mathrm{I}(\Delta_1 = 0) f(\mathrm{y}_1 | \Delta_1 = 0;\ \alpha_0)(1-\rho)]\ \mathrm{x}$$

$$\mathrm{x} \prod_{t=2}^{T} \{\ \mathrm{I}(\Delta_t = 1, \Delta_{t-1} = 1) f(\mathrm{y}_t | \Delta_t = 1;\ \alpha_1)\pi_t^{11} +$$

$$+\ \ \mathrm{I}(\Delta_t = 0, \Delta_{t-1} = 1) f(\mathrm{y}_t | \Delta_t = 0;\ \alpha_0)(1-\pi_t^{11}) +$$

$$+\ \ \mathrm{I}(\Delta_t = 1, \Delta_{t-1} = 0) f(\mathrm{y}_t | \Delta_t = 1;\ \alpha_1)(1-\pi_t^{00}) +$$

$$+\ \ \mathrm{I}(\Delta_t = 0, \Delta_{t-1} = 0) f(\mathrm{y}_t | \Delta_t = 0;\ \alpha_0)\pi_t^{00}\ \}$$

Taking logs:

$$\log f\left((\vec{y}_T, \vec{\Delta}_T | \vec{x}_T;\ \theta\right) = \mathrm{I}(\Delta_1{=}\ 1)[\ \log f(y_1 | \Delta_1{=}\ 1;\ \propto_1) + \log \rho] +$$

$$+ \mathrm{I}(\Delta_1{=}\ 0)[\log f(y_1 | \Delta_1{=}\ 0;\ \propto_1) + \log(1-\ \rho)] +$$

$$+ \sum_{t=2}^{T} \{\ \mathrm{I}(\Delta_t{=}\ 1) \log f(y_t | \Delta_t = 1; \alpha_1)\ +\ \mathrm{I}(\Delta_t{=}\ 0) \log f((y_t | \Delta_t = 0; \alpha_0) +$$

$$+ \mathrm{I}(\Delta_t = 1, \Delta_{t-1} = 1) \log(\pi_t^{11})\ +\ \ \mathrm{I}(\Delta_t = 0, \Delta_{t-1} = 1) \log(1-\pi_t^{11}) +$$

$$+ \mathrm{I}(\Delta_t = 1, \Delta_{t-1} = 0) \log(1-\pi_t^{00})\ +\ \mathrm{I}(\Delta_t = 0, \Delta_{t-1} = 0) \log(\pi_t^{00}) \qquad (4.3)$$

Therefore a basic algorithm procedure for parameter estimation consists of the following sequence. For $\theta = (\alpha', \gamma', \rho)'$ and once again excluding ρ in the case of the restricted model,

1. Pick $\theta^{(0)}$

2. Obtain filtered and smoothed probabilities for the following – see Appendix 1.2 for details regarding the marginal and joint probabilities, conditional on $\theta^{(0)}$:

$$P(\Delta_t = 1|\vec{y}_T, \vec{x}_T;\ \theta^{(0)}) \quad \forall_t,\ \ P(\Delta_t = 0|\vec{y}_T, \vec{x}_T;\ \theta^{(0)}) \quad \forall_t,$$

$$P(\Delta_t = 1, \Delta_{t-1} = 1|\ \vec{y}_T, \vec{x}_T;\ \theta^{(0)}) \quad \forall_t, \quad P(\Delta_t = 0, \Delta_{t-1} = 1|\vec{y}_T, \vec{x}_T;\ \theta^{(0)}) \quad \forall_t$$

$$P(\Delta_t = 1, \Delta_{t-1} = 0|\ \vec{y}_T, \vec{x}_T;\ \theta^{(0)}) \quad \forall_t, \quad P(\Delta_t = 0, \Delta_{t-1} = 0|\vec{y}_T, \vec{x}_T;\ \theta^{(0)}) \quad \forall_t$$

3. Construct: E log $f\left(\vec{y}_t, \vec{\Delta}_t \middle|\ \vec{x}_t;\ \theta^{(0)}\right)$ - the hypothetical (i.e. assuming all $\{y_t\}$ and $\{\Delta_t\}$ are observed) complete data likelihood (see Appendix 1.1 below) by replacing the indicator functions (I's) from (4.3) with smoothed probabilities (*P*'s) obtained in previous step 2, arriving at the following:

$$\begin{aligned}
\mathrm{E}[\ \log f\left(\vec{y}_t, \vec{\Delta}_t \middle|\ \vec{x}_t;\ \theta^{(0)}\right) = {} & \rho^{(0)}[\log f\left(y_1 \middle| \Delta_1 = 1;\ \alpha_1^{(0)}\right) + \log\rho^{(0)}] + \\
& (1-\rho^{(0)})[\log f\left(y_1 \middle| \Delta_1 = 0;\ \alpha_0^{(0)}\right) + \log(1-\rho^{(0)}) + \\
& \textstyle\sum_{t=2}^{T} \{\ \mathrm{P}(\Delta_t = 1|\vec{y}_T, \vec{x}_T;\ \theta^{(0)})[\log f\left(y_t \middle| \Delta_t = 1;\ \alpha_1^{(0)}\right) + \\
& \quad + \mathrm{P}(\Delta_t = 0|\vec{y}_T, \vec{x}_T;\ \theta^{(0)})[\log f\left(y_t \middle| \Delta_t = 0;\ \alpha_0^{(0)}\right) + \\
& \quad + \mathrm{P}(\Delta_t = 1, \Delta_{t-1} = 1|\vec{y}_T, \vec{x}_T;\ \theta^{(0)})\log(\pi_t^{11}) + \\
& \quad + \mathrm{P}(\Delta_t = 0, \Delta_{t-1} = 1|\vec{y}_T, \vec{x}_T;\ \theta^{(0)})\log(1-\pi_t^{11}) + \\
& \quad + \mathrm{P}(\Delta_t = 1, \Delta_{t-1} = 0|\vec{y}_T, \vec{x}_T;\ \theta^{(0)})\log(1-\pi_t^{00}) + \\
& \quad + \mathrm{P}(\Delta_t = 0, \Delta_{t-1} = 0|\vec{y}_T, \vec{x}_T;\ \theta^{(0)})\log(\pi_t^{00})\ \}
\end{aligned}$$

4. Set $\theta^{(1)} = \arg\max_{\theta} \mathrm{E}[\log f((\vec{y}_t, \vec{\Delta}_t | \vec{x}_t;\ \theta^{(0)})]$

5. Iterate to convergence.

This last convergence criterion may be obtained by computing the change of likelihood from one iteration to the next, or for a change in the gradient vector, or for a difference in estimated parameters, i.e. $||\theta^{(j)} - \theta^{(j-1)}||$, that is smaller than a certain minimum value. For the correlation targeting method used in the restricted model, we just obtain the smoothed probabilities as per previous step 2. i.e., once these smoothed probabilities are estimated, they are used directly for parameter estimation in the restricted model (2.2), via maximum likelihood.

3.6.3.2 Two-step estimate:

A similar procedure from Pelletier (2006) is used, yet the necessary modifications required for considering state dependent transition probabilities instead of having constant transition probabilities are implemented. The parameter space θ is partitioned into θ_1 (parameters from the univariate volatility models), and θ_2(parameters from the correlation model).

The first step from (4.1), the likelihood assumes the correlation matrix is an identity matrix. (i.e. $\Gamma_t = \mathrm{I}_t$):

$$QL_1(\theta_1; Y) = -\frac{1}{2}\sum_{t=1}^{T}(Klog(2\pi) + 2\log(|S_t|) + U'_t U_t)$$

This does not require the use of a filter since the series' volatility is not governed by a Markov chain, and the parameters for each series are estimated separately. For the second step, the likelihood from (4.1) is estimated given θ_1 for which S_t was previously estimated, which becomes:

$$QL_2(\theta_2; Y, \theta_1) = -\frac{1}{2}\sum_{t=1}^{T}(Klog(2\pi) + \log(|\Gamma_t|) + \widehat{U}'_t\Gamma_T^{-1}\widehat{U}_t)$$

This second estimation does require the use of filtering because Γ_t is a function of Δ_t (a Markov chain), which is not observed. The filtering has been obtained previously from step 2 for the one-step estimation case described before.

When considering the non-restricted model (2.1), estimation is done through Expectation Maximization (EM) following steps 3. and 4., from the algorithm detailed previously in the one-step estimation. Iteration of the maximization process is continued until the computed new vectors $\theta^{(j)}$ have a difference with subsequent vectors that becomes sufficiently small.

As mentioned before, the case of the restricted model (2.2) considers correlation targeting. This involves first the calculation of unconditional expectation of the correlation matrix. i.e.

$$E[\Gamma_t] = \Gamma\sum_{n=1}^{N}\lambda(n)\pi^n + I_K\sum_{n=1}^{N}(1-\lambda(n))\,\pi^n$$

The off-diagonal terms are part of the first term (i.e. the matrix Γ is multiplied by the scalar $\sum_{n=1}^{N}\lambda(n)\pi^n$). This means that an estimate of Γ $(\hat{\Gamma})$, computed with the standardized residuals from the first step, will have off-diagonal elements 'scaled' by $\sum_{n=1}^{N}\lambda(n)\pi^n$. Thus these values will be re-scaled according to the natural constraint posed in (2.2.2). In other words, divide the off-diagonal elements of $\hat{\Gamma}$ by the highest absolute value among them, to obtain -1 or 1. This secures $\lambda(1) < 1$ and the number of parameters (λ) to be non-linearly estimated increases with the number of different regimes, and not with the number of time series.

In this case of two regimes, 1 > $\lambda(1) > \lambda(2)$, therefore the unconditional expectation of the correlation matrix is:

$$E[\Gamma_t] = \Gamma(\lambda(1)\pi^1 + \lambda(2)\pi^2) + I_k[(1-\lambda(1))\pi^1 + (1-\lambda(2))\pi^2]$$

These two-step estimates are consistent, and their asymptotic distribution follows a Normal (0, V) distribution:

$$\sqrt{T}\left(\begin{bmatrix}\hat{\theta}_1\\ \hat{\theta}_2\end{bmatrix} - \begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix}\right) \to N(0;V) \qquad \text{with}$$

$$V = \begin{bmatrix} G_{\theta_1}^{-1} & -G_{\theta_1}^{-1}G_{\theta_2}M^{-1} \\ 0 & M^{-1}\end{bmatrix} E\left[\frac{\partial lnf}{\partial\theta}\frac{\partial lnf}{\partial\theta'}\right]\begin{bmatrix} G_{\theta_1}^{-1} & -G_{\theta_1}^{-1}G_{\theta_2}M^{-1} \\ 0 & M^{-1}\end{bmatrix}'$$

such that:

$$G_{\theta_1} = E\left[\frac{\partial g(Y,\boldsymbol{\theta_1},\boldsymbol{\theta_2})}{\partial\boldsymbol{\theta}_1'}\right],\ G_{\theta_2} = E\left[\frac{\partial g(Y,\boldsymbol{\theta_1},\boldsymbol{\theta_2})}{\partial\boldsymbol{\theta}_2'}\right],\quad M = E\left[\frac{\partial m(Y,\boldsymbol{\theta_2})}{\partial\boldsymbol{\theta}_2'}\right],$$

$$g(Y,\theta_1,\theta_2) = \frac{\partial lnf(Y_t|\overrightarrow{Y_{t-1}})}{\partial\boldsymbol{\theta_1}},\quad m(Y,\theta_2) = \frac{\partial lnf(Y_t|\overrightarrow{Y_{t-1}})}{\partial\boldsymbol{\theta_2}}.$$

The proof is in Pelletier (2006).

3.7 Friction level

Persistence of the dynamic correlations in the Markov chain may vary as a function of the underlying fundamental economic variables. These weakly exogenous variables may produce persistence at a specific regime's correlation value along the Markov chain. Thus friction is identified when it holds the series in a particular regime instead of switching it to a different correlation regime, had the related factor not been taken into account. This type of friction may appear for example as an effect or consequence of the steady increasing or decreasing level of the fundamental underlying related market factors such as prices, price ratios or industry specific indices or ratios – which are explicitly included as a weakly exogenous variable in the transition probabilities between regimes.

The scenario may be the case of continuous rising or decreasing price levels in certain commodities or the case of a steady rise or decline of stock levels, with either of these cases responding to market fundamentals and thus producing periods of higher or lower correlation levels among related markets. On the other hand, there may be the case of continuous decreasing prices of financial assets, leading to anticipated increases in correlation levels among markets, in accordance with the literature.

Conversely, the related variable may favor the switch from one regime to another regime at a particular period, versus the case when not specifically considering the related variable in the evolution of the series. This regime change occurring in response to the effect of the related variable is in contrast to the case in which the factor had not been accounted for because of constant transition probabilities, and thus this latter case may not have a regime switch take place at that period. Thus the new or 'switched to' correlation level may be more prevalent when taking into account the related variable, in comparison to the case of unaccounted factors. This identified related factor then becomes the opposite of a friction and may be considered a facilitator. This case where the underlying factor prompts a regime switch, producing a larger switch than if the factor had not previously been considered, may respond to explicitly considering the related market factor(s) or fundamental variables, which had previously not been directly accounted for.

The friction levels or degrees are a function of weakly exogenous variables x_{t-1} and their coefficients β_i (or γ_i) for the different regimes considered. We consider the case of two regimes in our dynamic correlations model and for simplicity we also consider only one weakly exogenous variable, besides the constant factor. This results in the following coefficient(s) β_1 and β_2 in the previous transition probabilities being denoted as:

Coefficients – b_{11} for the constant and $\boldsymbol{b_{12}}$ for the weakly exogenous variable – at Regime ***1***.

Coefficients – b_{21} for the constant and $\boldsymbol{b_{22}}$ for the weakly exogenous variable – at Regime ***2***. The nested case of constant transition probabilities considers $\boldsymbol{b_{12}} = 0$ and $\boldsymbol{b_{22}} = 0$.

To assess the impact of a significant coefficient of a weakly exogenous variable (i.e., considered a friction level), a first order Taylor approximation for the transition probability is made for a small value around our weakly exogenous variable x_{t-1} valued at the mean ($\bar{x}$).

For example being in regime 1 at $t-1$, i.e. $\Delta_{t-1} = 1$, and remaining in regime 1 for the next period, i.e. $\Delta_t = 1$; and for a small value of x_{t-1} around the mean $\bar{x}$:

$$P(\Delta_t = 1 \mid \Delta_{t-1} = 1\, , x_{t-1}; \beta_0) =$$

$$= \quad P(\Delta_t = 1 \mid \Delta_{t-1} = 1\, , x_{t-1}; \beta_0)|_{x_{t-1} = \bar{x}} + \frac{\partial P(\%)}{\partial x}|_{x_{t-1} = \bar{x}} * (x_{t-1} - \bar{x})$$

$$= \quad P(\Delta_t = 1 \mid \Delta_{t-1} = 1, \bar{x}\, ; \beta_0) + \boldsymbol{b}_{12} \frac{\exp(x'_{t-1}\beta_0)}{[1+\exp(x'_{t-1}\beta_0)]^2}|_{x_{t-1} = \bar{x}} * (x_{t-1} - \bar{x})$$

results in:

$$P(\Delta_t = 1 \mid \Delta_{t-1} = 1, x_{t-1}; \beta_0) - P(\Delta_t = 1 | \Delta_{t-1} = 1, \bar{x}\, ; \beta_0) = \boldsymbol{b}_{12} \frac{\exp(\mathrm{b_{11}} + \mathrm{b_{12}} * \bar{\mathrm{x}})}{[1+\exp(\mathrm{b_{11}} + \mathrm{b_{12}} * \bar{\mathrm{x}})]^2} * (x_{t-1} - \bar{x})$$

Or

$$\Delta\, P(\Delta_t = 1 \mid \Delta_{t-1} = 1\, , \boldsymbol{x_{t-1}} - \bar{\boldsymbol{x}}; \beta_0) = \boldsymbol{b}_{12} \frac{\exp(\mathrm{b_{11}} + \mathrm{b_{12}} * \bar{\mathrm{x}})}{[1+\exp(\mathrm{b_{11}} + \mathrm{b_{12}} * \bar{\mathrm{x}})]^2} * (x_{t-1} - \bar{x}) \qquad (4.3)$$

That is, a small change (e.g. an increase in the probability of remaining in regime 1 - spillover effect) resulting from a small change in the weakly exogenous variable $\boldsymbol{x_{t-1}}$ around the mean $\bar{x}$**,** is equal to the product of three terms. These terms are - the coefficient ($\boldsymbol{b_{12}}$) of the weakly exogenous variable, the logarithmic expression of the constant coefficient ($\boldsymbol{b_{11}}$) and the product of the variable's coefficient $(\boldsymbol{b_{12}})$ with the mean of the variable $(\bar{x})$, and the small change of this variable around the mean $(\boldsymbol{x_{t-1}} - \overline{\boldsymbol{x}})$. There are three basic cases which may occur regarding a small change in the weakly exogenous variable around the mean $(\boldsymbol{x_{t-1}} - \overline{\boldsymbol{x}})$.

If the coefficient $\boldsymbol{b_{12}}$ of our weakly exogenous variable is insignificant, then this underlying related variable would not form a friction level for the correlations among markets due to price variations. Changes in this weakly exogenous variable would not make a difference in the evolution of our correlation values, and these market correlations would evolve unaffected by this particular factor.

A different situation takes place if this coefficient $\boldsymbol{b_{12}}$ is significant and positive, from where two cases emerge. One case occurs when the product of $\boldsymbol{b_{12}}$ with the constant coefficient $\boldsymbol{b_{11}}$ and the product of the variable's coefficient $(\boldsymbol{b_{12}})$ with the mean of the variable $(\bar{x})$ as a function of the logarithmic expression is large. Then positive variations of our weakly exogenous variable with respect to the mean $(\boldsymbol{x_{t-1}} - \overline{\boldsymbol{x}})$ will lead to a higher probability of remaining at regime 1 (i.e., a longer spillover effect). The other case considers this former product being zero or very small, resulting in no further effect from positive (or negative) variations of our related variable with respect to the mean.

In other words for this second case – the coefficient $\boldsymbol{b_{12}}$ being positive and significant – determines an existing friction level from this related variable. Yet the effect of increases in this weakly exogenous variable around its mean are dampened because the terms multiplied by this variable, specifically the product of $\boldsymbol{b_{12}}$ and the logarithmic expressions of $\boldsymbol{b_{11}}$ and the product of the variable's coefficient $(\boldsymbol{b_{12}})$ with the mean of the variable $(\bar{x})$, are very small or negligible. Therefore, the friction level identified may produce spillover effects; yet additional spillover effects from increases in the related variable with respect to its mean – coming from shocks – would not be produced.

Conversely, it may be that the coefficient $\boldsymbol{b_{12}}$ is significant and negative at (4.3), and its product with the logarithmic expression of the constant coefficient $\boldsymbol{b_{11}}$ and the product of the variable's coefficient $(\boldsymbol{b_{12}})$ with the mean of the variable $(\bar{x})$ is large. Then positive variations of our weakly exogenous variable $(\boldsymbol{x_{t-1}})$ around its mean $(\bar{x})$ will lead to a lower probability of remaining at regime 1 or a higher probability of switching to a different regime – in this case switching to regime 2. This is in contrast to the possible case of the product of the two former factors being negative, yet negligible. Thus, positive shocks from our related variable will increase the probability of regime switching, resulting in a different correlation level at the new regime, versus positive shocks from the related variables with respect to its mean with no effect.

The previous description may be summarized as follows - If $\boldsymbol{b_{12}}$ is *insignificant*, then there is *no friction* formed by the related variable. Yet if $\boldsymbol{b_{12}}$ is *significant,* there is friction and we use equation (4.3) to determine its effect. Considering the case of $\boldsymbol{b_{12}}$ being *significant,* i.e., an existing friction, and for a small change in $\boldsymbol{x_{t-1}}$ about its mean $(\bar{x})$, if $\boldsymbol{b}_{12} * \frac{\exp(b_{11}+b_{12}*\bar{x})}{[1+\exp(b_{11}+b_{12}*\bar{x})]^2}$ is small, then there is a small response to shocks from the related variable $\boldsymbol{x_{t-1}}$ about its mean $(\bar{x})$. Yet if $\boldsymbol{b}_{12} * \frac{\exp(b_{11}+b_{12}*\bar{x})}{[1+\exp(b_{11}+b_{12}*\bar{x})]^2}$ is large, then there is a larger response to shocks from the related variable with respect to its mean $(\boldsymbol{x_{t-1}} - \bar{x})$, in the form of increasing probabilities of switching or remaining in a certain correlation regime.

This analysis also applies for the case of being in regime 2 during period *t-1* and subsequently remaining in regime 2 for next period *t* or switching to regime 1. In this case, the coefficient $\boldsymbol{b_{22}}$ corresponds to the relevant related variable, and the analysis is analogous the one above.

3.8 Results:

The correlations among the markets for the different cases in each scenario described at the end of the Data section are presented, leaving out the conditional volatilities for a later analysis. For each different time period scenario, model selection may be assessed using either the Likelihood Ratio Test(LR),[18] which is for the case of comparing the nested model with each of the state dependent models, or by using the Rivers-Vuong[19] test criteria. The Rivers-Vuong test serves to compare between the two state dependent transition probabilities, as they are non-nested models. Estimation results are presented below for the three different scenarios or time periods with the unrestricted general model, and these results can be corroborated by those results estimated with the restricted model which are shown in Appendix 3.C.

Table 3.3 below presents the results during the first time period scenario for the three cases of transition probabilities - (i) constant transition probabilities, (ii) state dependent transition probabilities considering the ratio of soybeans to corn futures prices with delivery in November and December, respectively, as a fundamental variable, and (iii) state dependent transition

[18] LR (λ) = $2^*(\mathcal{L}_{ur} - \mathcal{L}_r) \sim X_q^2$, with q: number of constraints.
[19] Rivers-Vuong test from Rivers-Vuong (1991) and (2002):
$V_{12} = \{L_1 - L_2\}/\sqrt{\hat{S}T}$, with L_1 & L_2 maximum likelihood of each model, T: # periods considered, $\hat{S}$: Newey-West estimate for the variance of the time series of likelihoods differences. $V_{12} \sim N(0,1)$ with positive values favoring model 1 and negative values favor model 2.
Denoting the sum of sectoral likelihoods for model i (i = 1,2) at time t by $L_{i,t}$, and $d_t = L_{1,t} - L_{2,t}$, we have:
$\hat{S}(T, q) = \gamma_0 + 2\sum_{j=1}^{q}[1 - \frac{j}{q+1}]\gamma_j$, where γ_j denotes the sample autocorrelation of order j of the d_t time series.

probabilities considering the 'use to stock' ratio for corn as a fundamental variable. That is, results for three cases of transition probabilities are presented for the entire period being considered from September 1998 to August 2008.

Applying the LR test between the case of constant transition probability (i.e. nested model) versus the two state dependent probability models results in a clear preference of the state dependent probability cases. This is inferred since the LR (λ) statistic is clearly above the critical value for the X_2^2 of 13.82, for significance at the 1% or lower. Applying the Rivers-Vuong test for the two latter cases results in a minor, but insignificant, preference of the model considering corn's 'use to stock' ratio as weakly exogenous variable, since the RV_{12} statistic is less than 0.5.

The case of 'use to stocks' ratio has a very minor preference, yet with very similar correlation results to the case of the ratio of soybeans to corn's future prices. The dynamic correlations between corn and soybeans are significant at both regime levels with values of 0.758 and 0.338, respectively. These are both positive values as anticipated, since these crops share common production conditions. Likewise, positive and significant values of 0.8188 and 0.2492 are determined, respectively, for the correlations between feeder cattle and live cattle (fed cattle) at both regimes as expected again by the literature, since these two markets are directly related in the marketing chain.

A negative significant correlation value of -0.275 is determined between corn and feeder cattle prices at the high correlation level. This may be anticipated by the literature (Anderson and Trapp, 2000), responding from the effect of an increase in the price of corn previously noted in figure 3.1.1, since corn is a main component of the feed ration. This inverse significant result is also obtained for the soybean and feeder cattle prices at -0.229. Soybeans are also an important

Table 3.3: Estimated Betas - Unrestricted Model between September1998 and August 2008.

UN_Restricted Model	Constant Transition Probability (by DP)	Standard Error	State Dependent Probability Ratio Sybn/Corn Price	Standard Error	State Dependent Probability Ratio USE_Stock	Standard Error
Likelihood	-4528.1		-4518.3		-4510.0	
Γ1 - Correlation Regime 1.						
Corn –Soybean	0.4274*	0.1483	0.7261*	0.0709	0.7584*	0.0328
Corn -Feeder Cattle	-0.3651*	0.0758	-0.3371*	0.0672	-0.2750*	0.0726
Corn - Live Cattle	0.0161	0.1125	-0.0326	0.0750	-0.0292	0.0700
Soybean - Feeder Cattle	-0.0786	0.0774	-0.2396*	0.0782	-0.2291*	0.0717
Soybean - Live Cattle	0.1200	0.0830	-0.0283	0.0870	-0.0347	0.0753
Feeder Cattle -Live Cattle	0.8121*	0.0738	0.8205*	0.0252	0.8189*	0.0278
Γ2 - Correlation Regime 2.						
Corn –Soybean	0.6694*	0.0969	0.3872*	0.0858	0.3382*	0.0671
Corn -Feeder Cattle	0.0195	0.0749	0.0054	0.0653	-0.0356	0.0766
Corn - Live Cattle	-0.0197	0.0984	0.0148	0.0753	0.0239	0.0738
Soybean - Feeder Cattle	0.0359	0.0788	0.1485*	0.0616	0.1595*	0.0609
Soybean - Live Cattle	0.0005	0.0705	0.0994	0.0750	0.1126+	0.0650
Feeder Cattle -Live Cattle	0.3038*	0.0940	0.2708*	0.0524	0.2492*	0.0537
γ or β - Betas						
probability beta - b11	0.5423*	0.1523	3.0370	3.4463	-0.5244	1.1144
probability beta - b21	0.5875*	0.2767	4.4614+	2.6006	-6.4689	4.1636
probability beta - b12	0		-1.3165	1.5669	0.0600	0.1583
probability beta - b22	0		-2.0072+	1.1820	0.6963	0.4475

* Significant at 5% level or less
\+ Significant at 10% level or less

source of protein feed, and this crop had a similar increase in its price evolution noted in figure 3.1.2, as it is positively correlated to corn during all time periods.

Another finding is that there is no significant correlation between corn prices and live cattle prices at either regime level. This reveals a level of permanent friction as an adjustment cost, as higher feed prices are not passed on to the live cattle prices. This may be a direct consequence of modifying feed rations when faced by increased corn prices, also anticipated by Anderson and Trapp (2000). This permanent friction may be formed by information and/or negotiation costs faced by cattle producers when selling their fed cattle in the market. Information costs may rise by price uncertainty during the selling period from using auction channels, and negotiation costs may come from sellers of fed cattle having comparable lower market power over the buyers. More analysis follows in the discussion section.

In addition, a positive correlation between soybean prices and live cattle prices at the lower correlation regime. It is not clear at this point what this may respond to and may be a spurious result. The dynamic correlations between corn and soybeans for state dependent transition probabilities considering the weakly exogenous variable 'use to stock' ratio, and for the constant transition probabilities, are in figures 3.8.1 and 3.8.2 below.

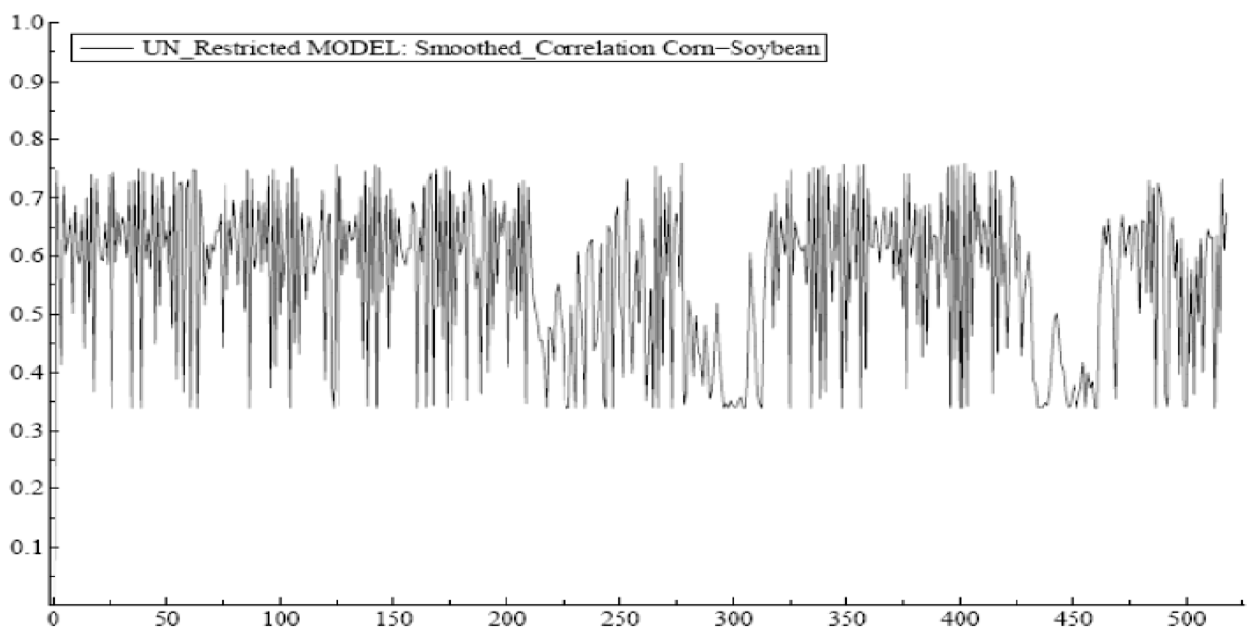


Figure 3.8.1: Smoothed Dynamic Correlations between Corn and Soybeans with *State Dependent transition probabilities* – considering 'use to stock' ratio. Weekly data from September 1998 to August 2008.

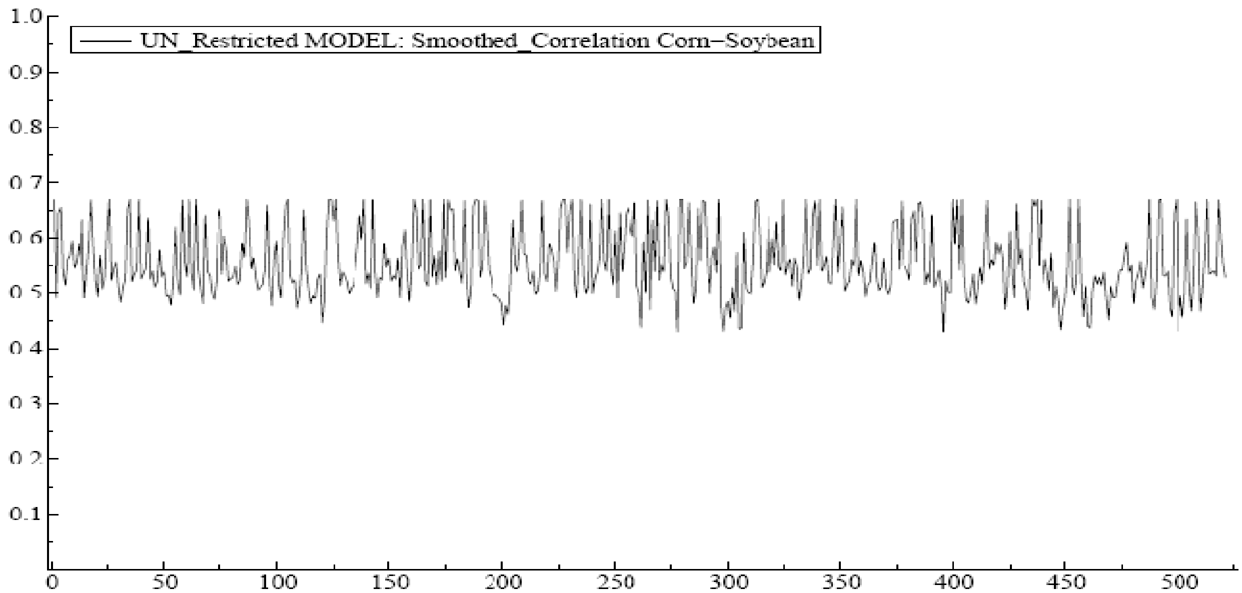


Figure 3.8.2: Smoothed Dynamic Correlations between Corn and Soybeans with *Constant transition probabilities*. Weekly data from September 1998 to August 2008.

In table 3.4 the estimated results for the second scenario are presented; i.e., from September 1998 to August 2004 (the period before the ethanol driven corn consumption). Applying the LR test, the two state dependent probability models are preferred to the constant transition probability model at the 5% level or less. However, upon comparing the two state dependent models by the Rivers-Vuong criteria, it is determined that there is a not a significant preference of the model with the corn 'use to stock' ratio over the soybeans to corn price ratio, since the RV_{12} statistic is less than 0.3

For both these latter models, correlations between corn and soybean prices at both regimes are at odds. Positive values of 0.681 and 0.388 are for the model considering 'use to stock' ratio at regimes 1 and 2, respectively. Yet positive values of 0.4144 and 0.6282 are for the soybean to corn price ratio model at regimes 1 and 2, respectively. The difference of these latter values appears small in magnitude when compared to the magnitude of their standard errors.

Table 3.4: Estimated Betas - Unrestricted Model between September 1998 and August 2004

UN_Restricted Model

	Constant Transition Probability (by DP)	**Standard Error**	***State Dependent Probability Ratio Sybn/Corn Price***	**Standard Error**	***State Dependent Probability Ratio USE_Stock***	**Standard Error**
Likelihood	*-2652.0*		*-2641.4*		*-2639.2*	
Γ1 - Correlation Regime 1.						
Corn -Soybean	0.3647	0.2251	0.4144+	0.2245	*0.6811**	0.0489
Corn -Feeder Cattle	*-0.1986**	0.0766	*-0.2214**	0.0687	-0.2021	0.1544
Corn - Live Cattle	0.1445	0.1206	0.1258	0.1168	0.0419	0.0759
Soybean - Feeder Cattle	-0.0758	0.0993	-0.0690	0.1048	-0.1500	0.1868
Soybean - Live Cattle	0.1004	0.1075	0.1159	0.1027	0.0528	0.0954
Feeder Cattle -Live Cattle	*0.8531**	0.0495	*0.8458**	0.0315	*0.8436**	0.0332
Γ2 - Correlation Regime 2.						
Corn -Soybean	*0.6413**	0.0776	*0.6282**	0.1101	*0.3876**	0.0978
Corn -Feeder Cattle	0.0054	0.0829	0.0343	0.0972	0.0423	0.0962
Corn - Live Cattle	-0.0115	0.0812	-0.0243	0.0797	0.0371	0.0854
Soybean - Feeder Cattle	0.0689	0.0876	0.0740	0.1045	0.1475	0.1057
Soybean - Live Cattle	0.0546	0.0859	0.0390	0.0886	0.0837	0.0834
Feeder Cattle -Live Cattle	*0.3333**	0.0891	*0.3099**	0.0769	*0.2763**	0.1198
γ or β - Betas						
probability beta - b11	*0.4787**	0.1280	5.3203	4.5990	2.7061	2.7526
probability beta - b21	*0.6593**	0.2325	5.7466+	3.1743	-4.3872	7.5435
probability beta - b12	0		-2.3792	2.0615	-0.4660	0.4294
probability beta - b22	0		-2.3603+	1.2650	0.5651	0.7039

* Significant at 5% level
+ Significant at 10% level

Regarding the correlation values between feeder and fed cattle markets - both models have similar positive values at regimes 1 and 2 respectively, at 0.844 or 0.8458 and 0.276 or 0.3099. Furthermore, these values in both regimes are similar to the ones from the previous time period scenarios. Yet in this time period scenario, there is no statistical inverse or negative relationship between corn and feeder calf prices for the preferred model, as the previous scenario. This is an interesting point that may be anticipated, as there was not a substantial increase in corn price during this pre ethanol driven corn consumption period, and hence the corn price had a minor or nil effect on feeder prices. Here the state dependent transition probabilities for corn's 'use to stock' ratio are insignificant and do not have an effect on the dynamics of the correlations among the markets.

Table 3.5 below is for the last scenario (i.e. from September 2003 to August 2008), which specifically considers the effect of the ethanol driven corn consumption. Application of the LR test results in the two state dependent transition probability models being chosen over the constant transition probability model, since the λ statistic determined is over 25 and 35, respectively. And of these two latter models, the model that uses the corn 'use to stock' ratio as a weakly exogenous variable is mildly preferred by considering the Rivers-Vuong criteria which resulted in a RV_{12} of 1.54 ($p < 0.15$).

Similar positive correlations for corn and soybeans at 0.874 and 0.304 are obtained at both regimes; yet the high regime correlation value is slightly higher considering the standard errors, than for the pre-ethanol corn consumption period described before. i.e. the high correlation regime increased its value as a result of the demand from ethanol driven corn consumption. The feeder and live cattle prices also had positive correlation values of 0.804 and 0.326 at the two regimes, and of similar range than the previous scenario. This scenario has an inverse correlation at the high correlation regime for both corn and feeder prices and for soybean and feeder prices, at -0.378 and -0.369, respectively. This result is similar to the first scenario yet with slightly higher values. The result corroborates the finding that increases in corn price – substantially present during this scenario (figure 3.1.1), lead to a higher correlation with soybean price and both of these affect substantially the feeding composition of cattle production. It is plausible that this increased corn price is stemming from the ethanol driven corn consumption, as mentioned before.

Table 3.5: Estimated Betas - Unrestricted Model between September 2003 and August 2008

UN_Restricted Model

	Constant Transition Probability (by DP)	Standard Error	*State Dependent Probability Ratio Sybn/Corn Price*	Standard Error	*State Dependent Probability Ratio USE_Stock*	Standard Error
Likelihood	*-2413.9*		*-2400.1*		*-2389.8*	
Γ1 - Correlation Regime 1.						
Corn -Soybean	*0.8336**	0.0521	*0.8421**	0.0496	*0.8744**	0.0417
Corn -Feeder Cattle	-0.3123	0.1968	*-0.3483**	0.1013	*-0.3776**	0.0956
Corn - Live Cattle	-0.1206	0.0885	-0.1290	0.0863	-0.1182	0.1130
Soybean - Feeder Cattle	-0.2558	0.1629	*-0.2869**	0.0946	*-0.3691**	0.0912
Soybean - Live Cattle	-0.1553+	0.0846	-0.1558+	0.0821	-0.1286	0.0975
Feeder Cattle -Live Cattle	*0.7895**	0.0815	*0.7858**	0.0458	*0.8044**	0.0368
Γ2 - Correlation Regime 2.						
Corn -Soybean	0.1703	0.1421	*0.2249**	0.0996	*0.3043**	0.0765
Corn -Feeder Cattle	-0.0843	0.2003	-0.0414	0.1194	-0.0774	0.0956
Corn - Live Cattle	0.0826	0.1078	0.1071	0.1037	0.0747	0.0994
Soybean - Feeder Cattle	*0.2249**	0.1028	*0.2306**	0.0681	*0.1943**	0.0582
Soybean - Live Cattle	0.2667*	0.0732	*0.2525**	0.0785	*0.1663**	0.0650
Feeder Cattle -Live Cattle	*0.2574**	0.1071	*0.2568**	0.0970	*0.3256**	0.0963
γ or β - Betas						
probability beta - b11	*0.7018**	0.3219	3.7835	3.4828	-2.8750+	1.6413
probability beta - b21	*0.6152**	0.3330	*11.1565**	3.9360	*-11.5949**	3.1140
probability beta - b12	0		-1.2262	1.4274	0.3431+	0.2031
probability beta - b22	0		*-4.4292**	1.6277	*1.2793**	0.3779

* Significant at 5% level or less
\+ Significant at 10% level or less

Similar to the previous two scenarios, there is no correlation between corn and live cattle markets. Since corn is regarded as the main feeding component for livestock, this finding reveals a permanent friction level or an adjustment cost materialized through the modification of the feeding rations given to the livestock. This transaction cost may be due to information and/or negotiation costs, as mentioned previously, when cattle producers sell fed cattle for slaughtering, as will be discussed further below.

Likewise there is a positive correlation between soybeans and live cattle at the second regime or at the lower correlation regime (where corn and soybean have a lower level of correlation). It is not immediately apparent why these two series would show a relationship, and may be a spurious result and/or require more analysis to determine its plausible causes.

3.9 Discussion

Dynamic correlation values were determined under two different regimes between prices of corn, soybean, feeder- cattle and fed- or live- cattle under three distinct time periods or scenarios. These correlation values corroborate earlier findings with respect to the sign of their values; i.e., positive correlation values between corn and soybean prices as these crops share planting acreage and also positive correlation values for feeder and fed cattle, as these are main price components in the cattle production industry. In addition, an inverse correlation value was determined between prices of corn and feeder cattle during the second period of increasing corn prices, also anticipated as cattle producers are forced to pay less for calf or feeder cattle replacement to maintain profitability.

An unexpected finding regarding the relationship between the market prices of corn and fed or live cattle was determined. Under the three different time scenarios, but especially during the period of increasing corn consumption from ethanol production, the correlation between corn and live cattle markets was always negligible or non-existent. In other words, even for the scenario of higher feed costs due to rising corn prices, the relationship between corn and live cattle prices remained insignificant. This reveals a permanent friction level or adjustment cost which during the last period of increased corn demand from ethanol production, may have been materialized by modifying the feed rations for livestock. This permanent friction responds to the transaction costs faced by cattle producers when selling fed cattle for slaughter in the market. These transaction costs may be due to information and/or negotiation costs and results in inhibiting the transmission of price variations between these two markets (i.e. their dynamic prices behave as unrelated).

Information costs may result from price uncertainty for fed cattle being sold as slaughter cattle, through cash or spot markets. These cash markets refer to transactions being 'on the spot' (RTI 2007), and include auction barn sales, video or electronic auction sales, sales through order from buyers, dealers and brokers and also direct trades. In this sense, cattle producers may need to change feed rations from components with increasing costs, such as corn, in order to keep costs down during production as they face uncertainty in the selling price of their product, assuming they have previously not hedged this price. The negotiation costs faced by cattle producers may respond to a limited number of different auction channels available when the cattle is ready to be slaughtered, thus raising the transaction cost involved in using that selling arrangement.

Producers selling directly with pre-established 'alternative marketing arrangements' (AMA) to packers - which includes forward contracts, marketing agreements, procurement or marketing contracts, production agreements, packer ownership, custom feeding and slaughter - accounted for less than 40 percent of all fed cattle volume sales from October 2002 to March 2005, per RTI

(2007). In this case, negotiation costs may arise due to having only a few packers to bargain with, resulting in the packer exercising marketing power on the cattle producer. Once again, this may lead the cattle producer to seek modifying the feed rations when there is a price increase in some of the feed components, as is the case of corn during the third time period scenario considered. But even more important for the other time period scenarios, these transactions costs reveal the conditions which cattle production faces with respect to sales of slaughter cattle.

Another result in our analysis during this scenario of *post ethanol corn consumption* is the finding of two statistically significant positive coefficients in the case of state dependent transition probability model - considering the corn 'use to stock' ratio as the related variable (our statistically preferred model).[20] These coefficients - b_{12} and b_{22} are for the probability of being in regime 1 and either staying there or switching to regime 2 (i.e., coefficient b_{12}) and for the probability of being in regime 2 and either staying there or switching to regime 1 (i.e., coefficient b_{22}), respectively.

In this sense, as the corn 'use to stock' ratio changes from its mean, the probability of being at regime 1 and remaining there or switching to regime 2 or, conversely, being at regime 2 and remaining there or switching to regime 1 depends on the product of three terms as per previous equation (4.3). One term is the logarithmic function of the sum of the coefficient of the constant variable (i.e. b_{11} for regime 1 or b_{21} for regime 2) and the product of the coefficient of the variable (i.e. b_{12} for regime 1 or b_{22} for regime 2) times the average of the use to stock ratio. The other term is the coefficient of the variable itself (i.e. b_{12} or b_{22}), and the third term is the change

[20] An economic analysis considering the effects of the 'ratio of soybeans to corn prices' as the weakly exogenous variable may provide plausible results, especially considering the statistically significant coefficients obtained in the estimation of the last time period. Yet given the mild preference (at 15% level or less) provided by the Vuong test for the model with the use to stock ratio of corn, the analysis focused more on the effects of this variable.

of the corn's use to stock ratio with respect to the mean. The values obtained for the product of these factors, as well as the impact of shocks from the weakly exogenous variable - corn 'use to stock' ratio, on the probability of remaining at certain regime level of correlation are computed according to our friction equation in (4.3) above , and shown in table 3.6 below.

Table 3.6: Change in Conditional Probabilities of remaining at Regime 1[21] or remaining at Regime 2,for a certain change in corn's use to stock ratio.

Coefficients			Expansion Term		Value	% Δ corn 'use to Stock' ratio….			
						5%	10%	15%	25%
$\bar{x}$	:	8.349				**produces Δ Probability…**			
b11	:	-2.875	$\dfrac{b_{12}*exp(b_{11}+b_{12}*\bar{x})}{[1+exp(b_{11} + b_{12}*\bar{x})]^2}$	:	0.085766	0.036	0.072	0.107	0.179
b12	:	0.343				***increase in prob. of remaining in Reg. 1***			

Coefficients			Expansion Term		Value	% Δ corn 'use to Stock' ratio….			
						5%	10%	15%	25%
$\bar{x}$	:	8.349				**produces Δ Probability…**			
b21	:	-11.595	$\dfrac{b_{22}*exp(b_{21}+b_{22}*\bar{x})}{[1+exp(b_{21} + b_{22}*\bar{x})]^2}$	:	0.261421	0.109	0.218	0.327	0.546
b22	:	1.279				***increase in prob. of remaining in Reg. 2***			

For this scenario of post mandated ethanol corn consumption, there is a small effect in the probability of remaining in the higher correlation levels at regime 1 during an increase in the corn use to stock ratio, and thus produce a spillover effect. In other words, during the period from 2004 to 2008, a 5% or 15% increase in corn's 'use to stock' ratio would produce an increase in the probability of remaining in regime 1 of about 0.036 or 0.107, respectively. This is a small rise in spillover effect driven by shocks to the 'use to stock' corn ratio. Conversely, if the ratio decreased in the same amount, i.e. by either 5% or 15%, then the probability of remaining

[21] i.e. $\Delta P(\Delta_t = 1|\Delta_{t-1} = 1,\ x_{t-1})$ or $\Delta P(\Delta_t = 2|\Delta_{t-1} = 2,\ x_{t-1})$

at regime 1 would lower by 0.036 and 0.107, respectively. This latter is equivalent to a similar increase in the probability of switching to regime 2, from previously being in regime 1, by 0.036 and 0.107, respectively.

More importantly, we identify that shocks to the ‘use to stocks’ ratio of corn would increase or decrease even more the spillover effect present in regime 2. From table 3.6, a 5% or 15% increase in corn’s ‘use to stock’ ratio would produce an increase in the probability of remaining in regime 2 of about 0.109 or 0.327. This larger effect of corn’s use to stock ratio in the probability of staying at regime 2 may be observed from the positive correlations between soybeans and corn during this post ethanol corn consumption period in figures 3.9.1 and 3.9.2 below.

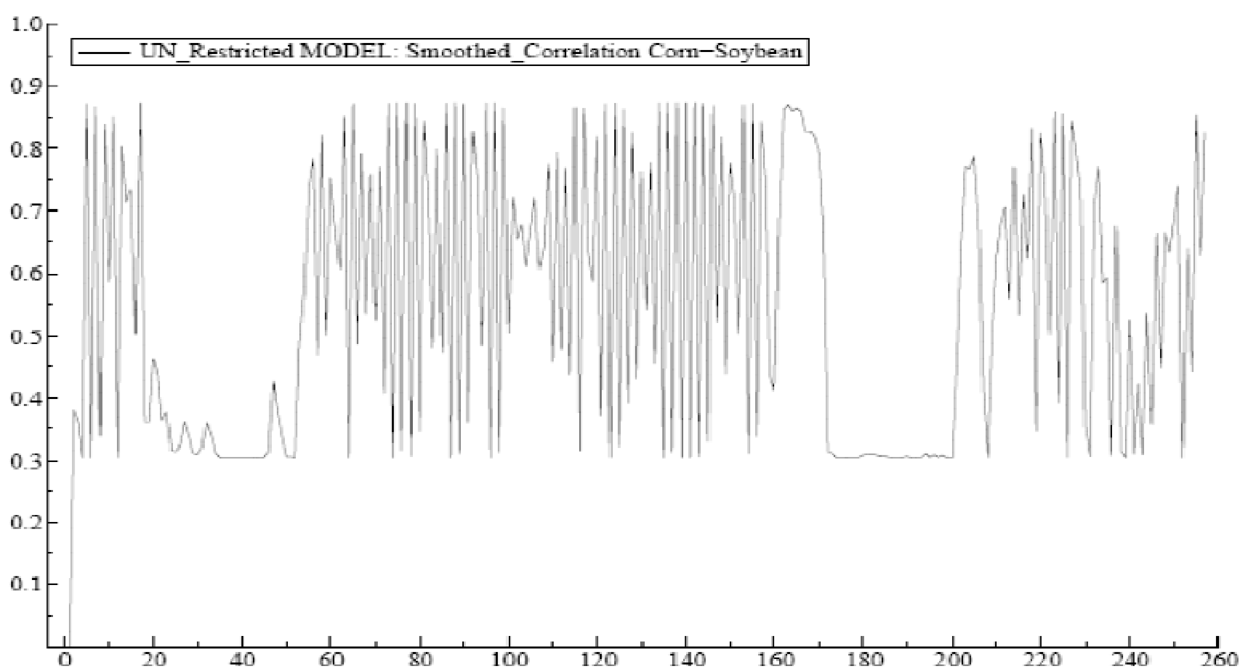


Figure 3.9.1: Smoothed Dynamic Correlations between Corn and Soybeans with *State Dependent transition probabilities* – considering ‘use to stock’ ratio; Sept. 2003 to Aug. 2008.

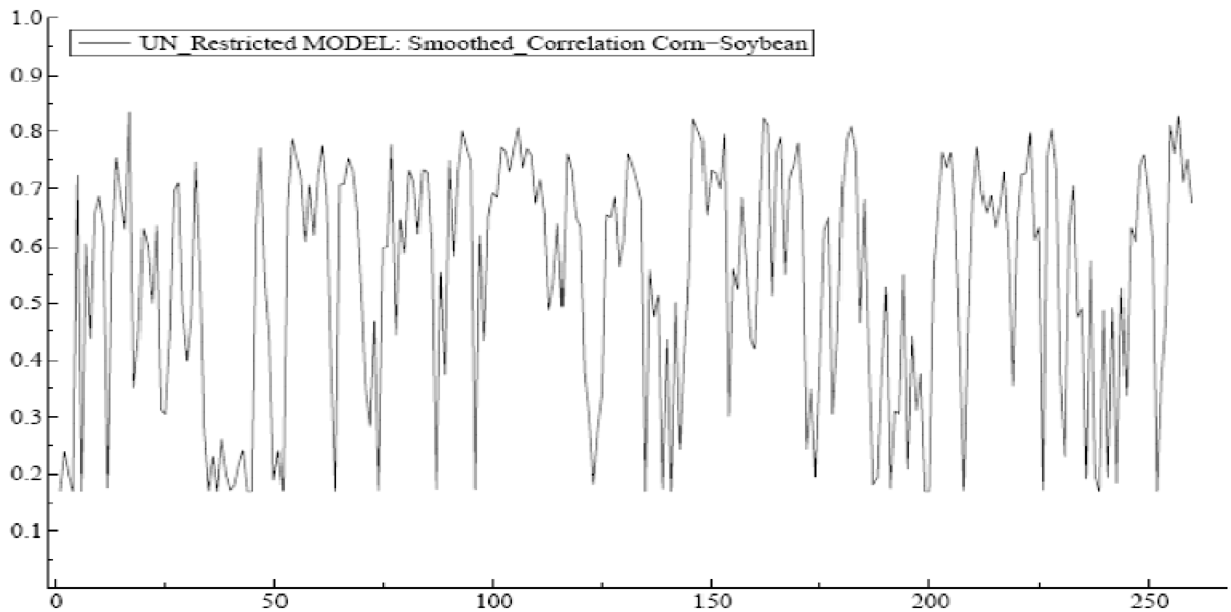


Figure 3.9.2: Smoothed Dynamic Correlations between Corn and Soybeans with *Constant transition probabilities*; from September 2003 to August 2008 (weekly).

As may be noted, there is an evident spillover effect revealed at the lower correlation regime (i.e., regime 2), when considering the inclusion of the 'use to stock' ratio of corn, in comparison to the regular constant transition probability model. The spillover effects identified at this low correlation regime occurs when there is a steady rise in corn consumption, in comparison to the available stock. That is, both persistent or spillover effects occurring at the low correlation levels happen during a continual increase in corn's 'use to stock' ratio to values considerably higher than the usual values that had been prevalent before 2003, as may be noted from figure 3.7.1 above. The first persistent low correlation period occurred from early January 2004 till August of that year, before additional corn production came into the market at the end of the year, to satisfy the increase in demand.

The more noticeable spillover effect occurred from December 2006 till late June 2007, when the ethanol corn consumption had an enormous increase of about 25%, from 1600 million bu. to more than 2,100 million bu. as can be seen from figure 3.4. This demand increase was being satisfied with previous higher production levels from year 2006. However, it was not enough to keep the stock levels unchanged, resulting in a substantial stock decrease. After the harvest period of year 2007 and record corn production, these stocks rose dropping the use to stock ratio to regular levels. The steady increase and record production of corn was achieved in part due to acreage taken from soybean (figure 3.5), but also due to common corn production response to bio-tech industry new varieties and to improved agronomic production practices.

The steady increase in corn consumption during these two above mentioned periods – especially the second one, affected directly the stock of available corn. However, this boost in corn demand did not affect immediately the market for soybeans. Hence these prices for these two crops remained at a low correlation level, since the drop in soybean production or harvest for that period had yet to be taken into account. The 2007 mid-year estimation report from the World Agricultural Supply and Demand Estimates (WASDE)[22] for soybean production confirmed that an extensive amount of acreage had changed from soybean to corn production during that year. Following the mentioned report, the estimated correlation value between prices of both crops increased to the higher regime.

In addition, during the first part of this second estimated period, the prices of both crops – corn and soybeans– were in steady increase responding to the rise in demand and the lower supply, respectively. Subsequently, during the harvest period beginning at the third quarter of 2007, corn had a record yield leading the corn ‘use to stock’ ratio to have a rapid drop from its sharp increase. However, for future periods the ratio still remained at levels above the pre

[22] Obtained from the USDA.

ethanol corn consumption periods, as can be seen from figure 3.7.1 above. The spike in the ratio observed during September 2002 comes from the estimated lowest yield of corn since the mid-nineties, which reduced the estimated stocks sharply. However, the 'use to stock' ratio declined as consumption also declined for the year 2003.

The effect of the significant underlying variable – 'use to stock' ratio, in the state dependent transition probabilities that determine the switch between one regime of correlations and the other; or to gauge the ratio's impact on the chance of persistence at a certain correlation regime, is shown in figures 3.10.1 and 3.10.2 below. The first figure shows the conditional probability of remaining at the lower correlation regime 2, for the case of previously being in that regime. As it can be noted, during the two previous mentioned periods (i.e., 2002 & 2006) of spillover effects at the low correlation level, the probability of remaining at that regime 2 is very close to being 1. This is in contrast to the case of only considering constant transition probabilities between regimes, where the probability of remaining at that regime 2 is only around 0.61. That is, the explicit inclusion of the 'use to stock ratio' in the state dependent transition probability of dynamic correlations, reveals periods where the conditional probability of remaining at the low correlation level permit the identification of spill-over effects; i.e., remaining at a certain correlation regime instead of switching to a different correlation regime.

In other words, by including this underlying economic related variable in the dynamic correlation process between corn and soybean, we are able to identify the significant impact it may have. This variable, the 'use to stock' ratio for corn, explicitly incorporates the markets' dynamic demand and supply forces for corn during the same period as the markets studied, including the effect of higher corn consumption due to ethanol production. In particular, during the post ethanol corn consumption period, the dynamic correlations between corn and soybean, and other markets previously mentioned, reveal spillover effects that were not captured when this underlying economic variable was not accounted for in the dynamic process.

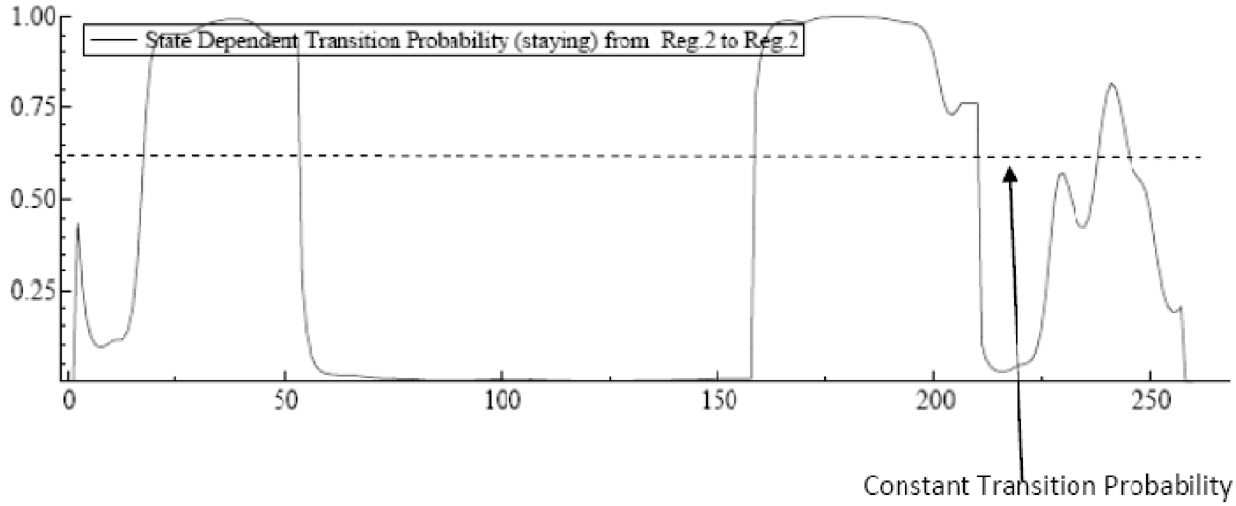


Figure 3.10.1: *Conditional State Dependent Transition Probability* of Remaining at Regime 2 with 'use to stock' ratio as weakly exogenous variable. i.e. $P(\Delta_t = 2 \mid \Delta_{t-1} = 2, \boldsymbol{x_{t-1}}; \beta_1)$ versus *Constant Transition Probability* of Remaining at Regime 2.

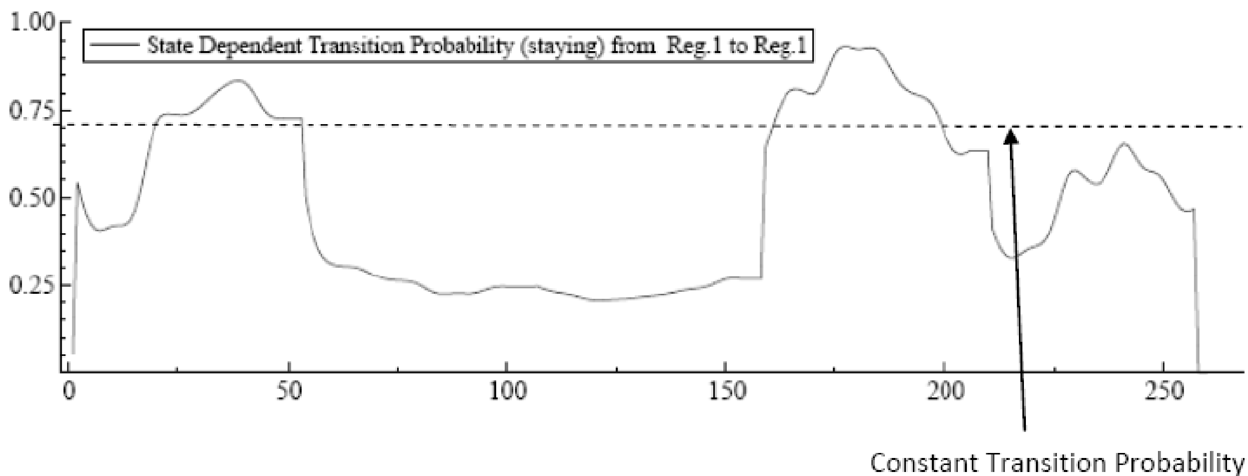


Figure 3.10.2: *Conditional State Dependent Transition Probability* of Remaining at Regime 1 with 'use to stock' ratio as weakly exogenous variable. i.e. $P(\Delta_t = 1 \mid \Delta_{t-1} = 1, \boldsymbol{x_{t-1}}; \beta_1)$ versus *Constant Transition Probability* of Remaining at Regime 1.

These identified spillover effects may be considered for policy analysis or efficiency gains in operations of these markets, though more analysis may be made with additional economic variables such as soybeans' 'use to stock' ratio, exchange rates, loan rates, or speculative variables such as contract volumes or others.

3.10 Conclusions

The effects of the recent increase in corn and soybean prices, and their volatilities, on cattle markets are studied by using a newly developed extension of a multivariate time series model. We specifically determine the dynamic correlations between corn, soybeans, feeder cattle and live cattle markets with an extended regime switching dynamic correlations model. The model extension enables the insertion of underlying economic variables that have an impact in the evolution of the dynamic correlations among the markets. We study the potential effect of recent policies regarding ethanol mandated production, on corn prices and its consumption.

We consider two partitioned scenarios or time periods, where one of them is previous to the mandated ethanol production surge, and the other period is after this policy was installed. In addition, we first consider for parameter estimation the scenario depicting both previous time periods together (i.e. the initial complete time series).

Results obtained for correlation levels between markets such as corn and soybeans, and feeder cattle and live cattle, are positive and consistent with the literature for all three time periods considered. We likewise find there is a significant negative correlation between corn prices and feeder cattle for the last period considered, i.e. during the post ethanol corn consumption period. However, there is not a negative correlation for the period previous to the mandated ethanol production. This significant inverse relationship is consistent with the literature where increases in corn price result in declining calf/feeder prices, responding to cattle producers seeking to maintain profitability by purchasing calf/feeder at lower prices.

We find no transmission of prices between the corn market and the fed or live cattle markets. i.e., there is no correlation for prices between these two markets at any of the scenarios computed. This may respond to transaction costs in both the form of information and negotiation costs. Information costs for cattle producers may involve price uncertainty for the case of fed cattle being sold at cash or spot markets, leading producers to switch from feed rations with increasing costs, i.e. corn in the post ethanol corn consumption scenario, to feed components with lower cost. Negotiation costs may respond to the limited number of auctions faced by producers when fed cattle are ready to be slaughtered, thus raising the transaction cost for using that channel. For producers selling directly with alternative marketing arrangements to packers, negotiation costs may arise for the case of having only a few different packers to bargain with, thus resulting in the packer exercising marketing power and establishing price and conditions of delivery. This again leads the cattle producer to seek feed rations that do not experience cost increases, i.e. modifying rations when there is a price rise in some of the components as in the case of corn.

For the case of introducing underlying economic fundamental variables in the dynamic process, we find that there is a mild effect of the corn 'use to stock' ratio on the dynamic correlations among the previous markets. This ratio contains the dynamic demand and supply conditions for corn, including the increased demand for ethanol production. Specifically for the last scenario estimated, i.e. between 2004 and 2008, there is a substantial spillover effect at a correlation regime, when compared to the previous scenarios. Thus corn and soybean experience effects of spillover that is remaining at a correlation regime, when taking into account this relevant economic variable, compared to the case where this factor is unaccounted for.

A plausible cause for this effect – post end of 2003, may be due to the increased corn consumption for ethanol production, since crop producers were anticipating increase in demand, and producers of ethanol were securing purchases during the year. Spillover effects are unveiled when considering the related corn demand and supply variable in comparison to the case of constant probabilities between regimes. Further analysis may require additional series to be considered as weakly exogenous variables such as the soybean ‘use to stock’ ratio, exchange rates, and contract volumes among others.

CHAPTER 4

Dynamic Price Relationships in the Grain and Cattle Markets

4.1 Introduction

This chapter examines the dynamic interaction between corn, soybean, grain sorghum (milo), wheat, feeder cattle and live cattle (fed) prices considering the recent surge in corn consumption due to a boost in mandated ethanol production, by using a vector autoregressive (VAR) model defined below. The model permits the forecast of these commodity prices and provides insight into the dynamic relationships among these markets, taking into account the recent federal mandated increase in ethanol production that uses corn as a main input. This study differs from the previous study by two main points. First I apply a different model to a broader set of grain prices for the analysis of their dynamic relationships with cattle markets, and second, daily data is taken into account instead of weekly data. The previous chapter considered the log of weekly price returns to determine the dynamic correlations among the markets. This study considers the log of daily data, thus the non-stationary property of each series is accounted for.

The effect of the surge in corn demand and its price on the dynamic relationship with soybeans and other main feed grains such as sorghum and wheat, as well as on the cattle markets of both feeder and fed cattle are investigated through the application of a VAR model. This multivariate model is of a non-structural, reduced form, where all the variables considered are assumed to be jointly endogenous and characterized by autoregressive representations of weakly stationary processes.[1] Hence for a VAR of order K, VAR(K), each variable from the Y vector depends on its own lagged values up to K periods, and likewise on the lagged values of the other variables up to K periods.

It is noteworthy to mention that the general form of studying the interrelationships between non-stationary series in a VAR setting is through a vector error correction (VEC) model defined below. This VEC model is referred to as an "error correction" VAR model and it is similar to the regular VAR model, yet takes into account estimating co-integration factors between the non-stationary data. The co-integration factors identify a common long-run evolution among the series, materialized as a linear combination of these non-stationary variables. Thus the VEC model is a regular VAR model which includes a lag of log prices as a dependent variable for the error correction term.

Similar to the prior chapter the data is partitioned into two different periods, considering separately the pre-mandated and post-mandated ethanol production periods. Results from the application of the "error correction VAR" model identify non-significant co-integration factors for the first period estimated. However, a significant co-integrated factor was determined for the second period.

The dynamic relationships are estimated using daily closing data of cash prices for corn, soybean, grain sorghum (milo), wheat, feeder cattle and live cattle. Two different periods are considered, the first period is from January 1998 to December 2004 and the second period from January 2004 to April 2009. This latter period includes the surge in corn consumption from

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[1] A Stochastic process is weakly stationary if it is (i) Mean Stationary and (ii) Covariance Stationary.
(i) A process is Mean Stationary if E[x_t] = μ_t = μ (constant) for all t.
(ii) A Process is Covariance Stationary if Cov[x_t, x_s] = E[(x_t - u_t)(x_s - u_s)] = γ(|s – t|) is Only function of the time distance between the two random variables and does not depend on the actual point in time t.

ethanol mandated production, as illustrated by Figure 4 in the previous chapter. Prices are from the Chicago Board of Trade (CBOT) and the Chicago Mercantile Exchange (CME), obtained through the Commodity Resource Bureau (CRB). Tests for Granger causality are performed on the data, but more important, dynamic adjustments of the prices in response to exogenous shocks to grain prices are investigated. These analyses serve to draw inferences with respect to the price relationships and linkages among the markets.

Results determined for the second period regarding corn and soybean reveal that each has Granger Causality on grain sorghum. This is in contrast to results obtained from the first period which determined bi-directional Granger Causality between corn and grain sorghum. Likewise for the first period, bi-directional Granger Causality was determined between soybean and grain sorghum. A plausible explanation for corn having Granger Causality on sorghum during the second period may be due to livestock feed rations being modified by substituting grains, especially corn for sorghum, as a consequence of the substantial increase in corn prices. In addition, impulse response functions in the second period corroborate this previous finding, since a shock of corn prices results in a significant increase in grain sorghum prices. The chapter proceeds with a brief literature review, followed by the method overview, empirical methods and data, results and discussion.

4.2 Literature Review

Initial dynamic studies of agricultural commodities incorporating a VAR model include Bessler and Babula (1987), Featherstone and Baker (1987), Goodwin and Schroeder (1991), Schroeder and Goodwin (1992), Goodwin (1992), and Hsu and Goodwin (1995). Recent studies incorporate the non-stationary properties of multiple series by means of a vector error correction model (VEC). This model incorporates a lagged level variable term called an error correction term, within a VAR setting. This error correction term considers long-run relationships between the series, being referred to as co-integration among markets. Vector error correction models have been used in studies by Goodwin and Piggott (2001) and Haigh and Bessler (2004). The application of either of these models permits Granger causality tests among the variables. More importantly, these models allow the use of impulse response functions which analyze the effects from shocks to one variable on the other variables being considered.

4.3 Method Overview

The vector autoregressive (VAR) model was developed by Sims (1980) and permits an analysis of the dynamic relationships between time series of endogenous or interrelated economic variables in a reduced model setting. Thus simultaneous structural equations describing the economic equilibrium between markets being studied is set aside in favor of a specification where all variables are assumed to be jointly endogenous, and simultaneously estimated. This model reduces spurious a priori restrictions on the dynamic relationships among the variables.

The VAR system for n variables may be defined by:

$$Y_t = \sum_{k=1}^{K} \begin{bmatrix} a_{11}(k) & \cdots & a_{1n}(k) \\ \vdots & \ddots & \vdots \\ a_{n1}(k) & \cdots & a_{nn}(k) \end{bmatrix} Y_{t-k} + E_t \tag{1}$$

where t indicates time (t = 1,……,T); Y_t is a n x 1 vector of economic variables (i.e. prices in this case); K is the lag order of the system; $a_{ij}(k)$ are the parameters to be estimated (with i, j =

1,….n); and E_t is a vector of random errors or innovations. Estimation requires choosing the appropriate lag order, K, of the system.

The preceding VAR model is applicable to stationary data. In the case of two or more series with non-stationary data, a co-integrated VAR model referred to as the vector error correction (VEC) model is applied. The idea for this model comes from Engle and Granger (1987) and Johansen (1988). These non-stationary series may be co-integrated (i.e., having a common long-run evolution) and thus have a long-run economic relationship. Hence the model requires a co-integration term which implies the existence of (a) linear combination(s) of these integrated (i.e. non-stationary of order 1 or more) series. In addition, this model takes into account the possibility that the non-stationary elements are not co-integrated by including first differences of the non-stationary series.

Thus, the VEC system for n variables (non-stationary series) is defined as follows:

$$\Delta Y_t = \sum_{i=1}^{K-1} \mathrm{A}_i \Delta Y_{t-i} + \Pi Y_{t-1} + E_t \quad (2)$$

where ΔY_t is a n x 1 vector of the first difference of economic variables (i.e. difference of log prices in this case); $K-1$ is the lag order of the first difference series and A_i (nxn) are its parameters to be estimated. The lagged level variable (Y_{t-1}) is the error correction term and its parameter to be estimated is Π, which may be of order r (with $0 \le r \le K$; for all series integrated of order 1, i.e. I(1)). Lastly, E_t is a vector of random terms or innovations.

4.4 Data

Daily cash prices for corn, soybean, grain sorghum, wheat, feeder cattle and live cattle are taken from the Chicago Board of Trade for the grains and the Chicago Mercantile Exchange

(CMEX) for the livestock. The data is obtained through the Commodity Research Bureau (CRB). Prices are from January 2nd 1998 through April 22nd, 2009 and are partitioned into two periods. The first period is from 1998 to 2004 and considers prices prior to the 2005 Energy Act, which mandated a substantial increase in ethanol consumption. The second period considers prices beginning in 2004 up until April 2009. Below are figures 4.1 and 4.2 with charts of these prices in logarithmic terms.

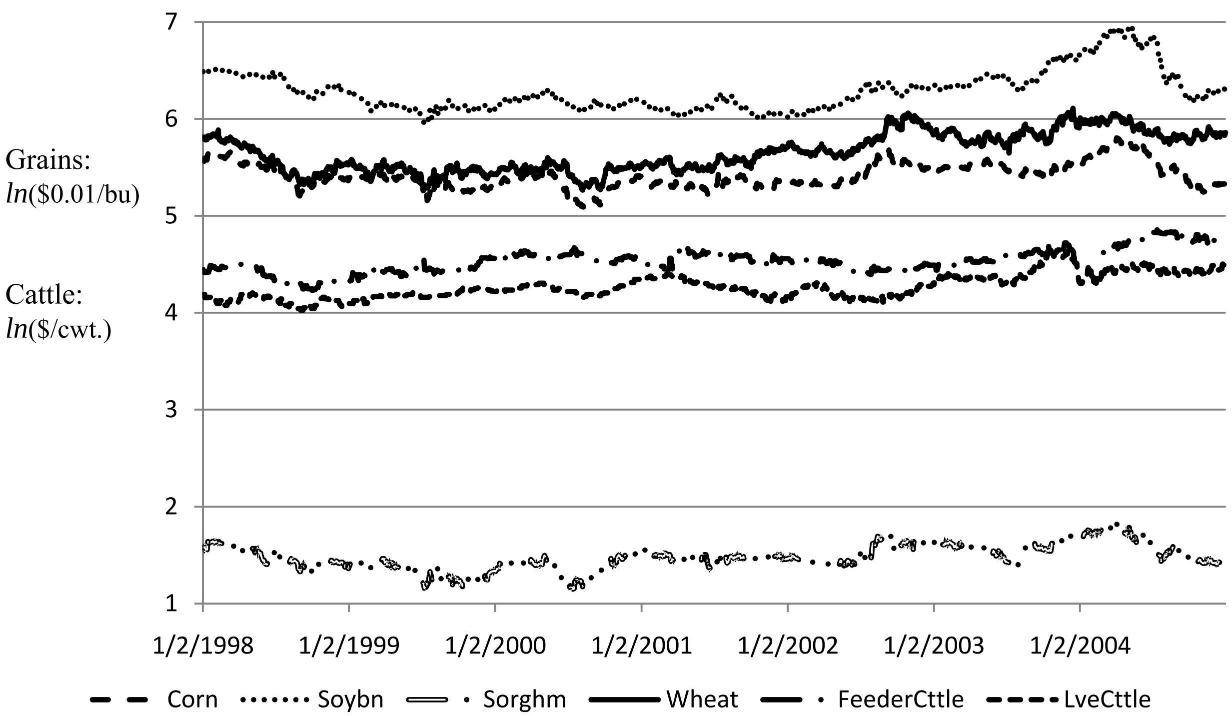


Figure 4.1: Daily Cash Market Prices in logarithmic terms from Jan. 1998 to Dec 2004.

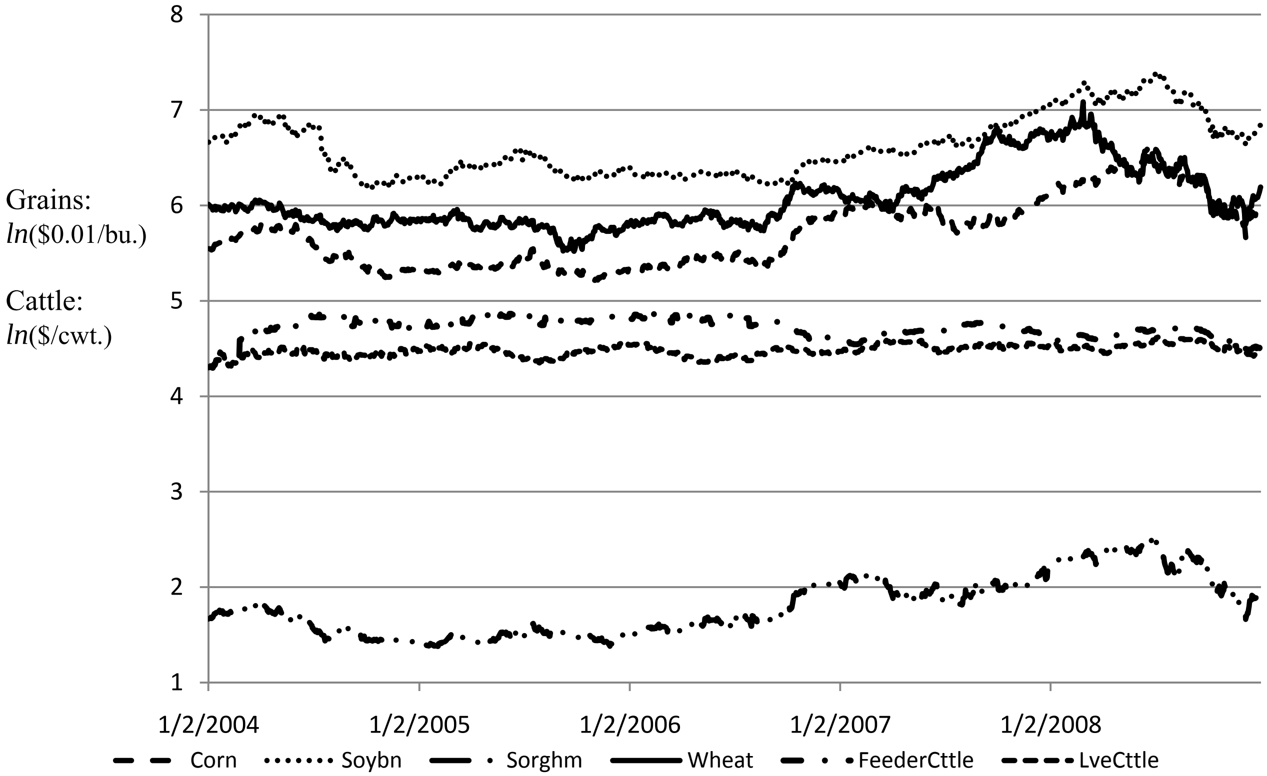


Figure 4.2: Daily Cash Market Prices in logarithmic terms from January 2004 to April 2009.

4.5 Results

Tests of the series being non-stationary for both periods were applied using the Phillips Perron[2] and the KPSS[3] unit root tests. Results for both tests show that the series are non-stationary for the two periods considered, as may be seen in following tables 4.1 and 4.2. Therefore the "co-integrated" VAR or VEC model is applied. Estimation of the proper number of lags and coefficients was done by least squares applying Bayes Information Criteria (BIC), [4]

[2] Unit Root test from Phillips, P.C.B and P. Perron (1988), where the null hypothesis considers the series being non-stationary.

[3] D. Kwiatkowski, P.C.B. Phillips, P. Schmidt, and Y. Shin (1992) Unit root test that considers the null hypothesis for the series as being stationary. Hence it may reject more often the case of a random walk.

[4] Schwartz (1978), BIC = -2*ln*Likelihood* + k*ln(n), k: # of parameters to be estimated and n: # of observations.

and the Portmanteau Test[5] for cross correlations of residuals, as well as the Univariate AR model test diagnostics[6] for the residuals of each series.

The Johansen[7] co-integration test is conducted for both periods studied. Results obtained for the first period indicate that there is no co-integration factor among the variables (i.e., $r = 0$ for the parameter Π in equation 2). However, results for the second period determine that there is a co-integration factor of order 1 among the series. Test results from the co-integration test are in following tables 4.3 and 4.4. From table 4.3 the error correction term in the VEC equation is null in the first period, thus resulting in a VAR of order 3 in ΔY as may be seen from the following table 4.5. This number of lags (3) in the first period responds to a Portmanteau test that does not reject the null hypothesis of correlations of the residuals distributing randomly or as white noise, detailed in table 4.5.

[5] From Hosking (1980), is a test for a group of auto and cross correlations from a model's residuals with the null hypothesis having them distribute as a random walk or white noise.
[6] F test for AR disturbances of Univariate model: Test statistics from the residuals of AR(1), AR(2), AR(3) and AR(4) that test the null hypothesis that residuals are uncorrelated.
[7] Johansen (1991) Co-integration test for many time series. Considers the trace (or the eigenvalues) among the time series and the null hypothesis is that the co-integration vector r is equal to any value between one and the number of time series minus one.

Table 4.1: Non-Stationary Tests for series, from January 1998 to December 2004.

Dependent Variable **Corn**

	Phillips-Perron Unit Root Test (Ho: Unit Root)					KPSS Stationary Test (Ho: Stationary series)				
Type	Lags	Rho	Pr < Rho	Tau	Pr < Tau	Lags	Eta	Prob10%	Prob5%	Prob1%
Zero Mean	8	-0.06	*0.670*	-0.490	*0.504*					
Single Mean	8	-12.40	*0.076*	-2.551	*0.105*	8	*4.127*	0.347	0.463	0.739
Trend	8	-14.08	*0.219*	-2.750	*0.217*	8	*1.718*	0.119	0.146	0.216

Dependent Variable **Soybean**

	Phillips-Perron Unit Root Test (Ho: Unit Root)					KPSS Stationary Test (Ho: Stationary series)				
Type	Lags	Rho	Pr < Rho	Tau	Pr < Tau	Lags	Eta	Prob10%	Prob5%	Prob1%
Zero Mean	8	-0.04	*0.675*	-0.347	*0.560*					
Single Mean	8	-5.64	*0.378*	-1.752	*0.405*	8	*5.936*	0.347	0.463	0.739
Trend	8	-7.52	*0.623*	-2.141	*0.523*	8	*2.695*	0.119	0.146	0.216

Dependent Variable **Sorghum**

	Phillips-Perron Unit Root Test (Ho: Unit Root)					KPSS Stationary Test (Ho: Stationary series)				
Type	Lags	Rho	Pr < Rho	Tau	Pr < Tau	Lags	Eta	Prob10%	Prob5%	Prob1%
Zero Mean	8	-0.21	*0.637*	-0.479	*0.508*					
Single Mean	8	-12.23	*0.079*	-2.487	*0.120*	8	*7.156*	0.347	0.463	0.739
Trend	8	-16.74	*0.134*	-2.926	*0.155*	8	*1.297*	0.119	0.146	0.216

Dependent Variable **Wheat**

	Phillips-Perron Unit Root Test (Ho: Unit Root)					KPSS Stationary Test (Ho: Stationary series)				
Type	Lags	Rho	Pr < Rho	Tau	Pr < Tau	Lags	Eta	Prob10%	Prob5%	Prob1%
Zero Mean	8	0.00	*0.683*	-0.004	*0.682*					
Single Mean	8	-7.61	*0.239*	-1.924	*0.322*	8	*12.619*	0.347	0.463	0.739
Trend	8	-20.82	*0.059*	-3.635	*0.028*	8	*2.049*	0.119	0.146	0.216

Dependent Variable **Feeder Cattle**

	Phillips-Perron Unit Root Test (Ho: Unit Root)					KPSS Stationary Test (Ho: Stationary series)				
Type	Lags	Rho	Pr < Rho	Tau	Pr < Tau	Lags	Eta	Prob10%	Prob5%	Prob1%
Zero Mean	8	0.05	*0.696*	0.359	*0.789*					
Single Mean	8	-13.19	*0.063*	-2.477	*0.122*	8	*7.984*	0.347	0.463	0.739
Trend	8	-25.14	*0.024*	-3.561	*0.034*	8	*1.031*	0.119	0.146	0.216

Dependent Variable **Live Cattle**

	Phillips-Perron Unit Root Test (Ho: Unit Root)					KPSS Stationary Test (Ho: Stationary series)				
Type	Lags	Rho	Pr < Rho	Tau	Pr < Tau	Lags	Eta	Prob10%	Prob5%	Prob1%
Zero Mean	8	0.06	*0.697*	0.457	*0.813*					
Single Mean	8	-8.43	*0.197*	-1.934	*0.317*	8	*11.641*	0.347	0.463	0.739
Trend	8	-24.42	*0.028*	-3.543	*0.036*	8	*1.027*	0.119	0.146	0.216

Table 4.2: Non-Stationary Tests for series, from January 2004 to April 2009.

Dependent Variable **Corn**

	Phillips-Perron Unit Root Test (Ho: Unit Root)					*KPSS Stationary Test (Ho: Stationary series)*				
Type	Lags	Rho	Pr < Rho	Tau	Pr < Tau	Lags	Eta	Prob10%	Prob5%	Prob1%
Zero Mean	7	0.06	*0.698*	0.465	*0.815*					
Single Mean	7	-2.37	*0.735*	-1.061	*0.733*	7	*11.179*	0.347	0.463	0.739
Trend	7	-5.76	*0.764*	-1.743	*0.733*	7	*1.772*	0.119	0.146	0.216

Dependent Variable **Soybean**

	Phillips-Perron Unit Root Test (Ho: Unit Root)					*KPSS Stationary Test (Ho: Stationary series)*				
Type	Lags	Rho	Pr < Rho	Tau	Pr < Tau	Lags	Eta	Prob10%	Prob5%	Prob1%
Zero Mean	7	0.03	*0.691*	0.291	*0.771*					
Single Mean	7	-2.68	*0.696*	-1.068	*0.730*	7	*7.815*	0.347	0.463	0.739
Trend	7	-5.48	*0.785*	-1.808	*0.702*	7	*2.288*	0.119	0.146	0.216

Dependent Variable **Sorghum**

	Phillips-Perron Unit Root Test (Ho: Unit Root)					*KPSS Stationary Test (Ho: Stationary series)*				
Type	Lags	Rho	Pr < Rho	Tau	Pr < Tau	Lags	Eta	Prob10%	Prob5%	Prob1%
Zero Mean	7	0.05	*0.695*	0.120	*0.721*					
Single Mean	7	-3.40	*0.609*	-1.309	*0.628*	7	*11.605*	0.347	0.463	0.739
Trend	7	-7.86	*0.596*	-1.967	*0.619*	7	*1.396*	0.119	0.146	0.216

Dependent Variable **Wheat**

	Phillips-Perron Unit Root Test (Ho: Unit Root)					*KPSS Stationary Test (Ho: Stationary series)*				
Type	Lags	Rho	Pr < Rho	Tau	Pr < Tau	Lags	Eta	Prob10%	Prob5%	Prob1%
Zero Mean	7	0.001	*0.684*	0.006	*0.685*					
Single Mean	7	-4.45	*0.491*	-1.495	*0.537*	7	*8.771*	0.347	0.463	0.739
Trend	7	-7.16	*0.651*	-1.868	*0.672*	7	*1.477*	0.119	0.146	0.216

Dependent Variable **Feeder Cattle**

	Phillips-Perron Unit Root Test (Ho: Unit Root)					*KPSS Stationary Test (Ho: Stationary series)*				
Type	Lags	Rho	Pr < Rho	Tau	Pr < Tau	Lags	Eta	Prob10%	Prob5%	Prob1%
Zero Mean	7	0.05	*0.696*	0.506	*0.825*					
Single Mean	7	-17.76	*0.210*	-3.743	*0.004*	7	*4.371*	0.347	0.463	0.739
Trend	7	-24.65	*0.270*	-4.844	*0.001*	7	*1.250*	0.119	0.146	0.216

Dependent Variable **Live Cattle**

	Phillips-Perron Unit Root Test (Ho: Unit Root)					*KPSS Stationary Test (Ho: Stationary series)*				
Type	Lags	Rho	Pr < Rho	Tau	Pr < Tau	Lags	Eta	Prob10%	Prob5%	Prob1%
Zero Mean	7	0.03	*0.691*	0.322	*0.779*					
Single Mean	7	-32.41	*0.020*	-4.334	*0.001*	7	*3.528*	0.347	0.463	0.739
Trend	7	-36.49	*0.021*	-4.451	*0.002*	7	*0.458*	0.119	0.146	0.216

However, the VEC or co-integrated VAR model is applied during the second period estimated since it has an error correction term of order 1 (table 4.4), and thus resulting in a “co-integrated” VAR model of order 5 as indicated in the following table 4.5. In this second period, the univariate AR diagnostic test shows no autocorrelation for residuals from 5 lags.

Table 4.3: Cointegration Test for series, from January 1998 to December 2004.

Johansen's Cointegration Rank Test Using Trace

Ho: Rank = r	H1: Rank > r	Eigenvalue	Trace	5% Critical Value
0	0	0.015	*70.604*	82.61
1	1	0.011	*43.798*	59.24
2	2	0.006	*23.598*	39.71
3	3	0.004	*12.523*	24.08
4	4	0.003	*4.800*	12.21
5	5	0	*0.033*	4.14

Table 4.4: Cointegration Test for series, from January 2004 to April 2009.

Johansen's Cointegration Rank Test Using Trace

Ho: Rank = r	H1: Rank > r	Eigenvalue	Trace	5% Critical Value
0	0	0.023	*87.577*	82.61
1	1	0.016	*56.529*	59.24
2	2	0.015	*35.655*	39.71
3	3	0.006	*15.082*	24.08
4	4	0.005	*6.615*	12.21
5	5	0.001	*0.155*	4.14

Table 4.5: Portmanteau Test of Residuals, from January 1998 to December 2004.

Test for Cross Correlations of Residuals
(Ho: Residuals from # Lags of Series is a random walk)

Up To Lag	DF	Chi-Square	Pr > ChiSq
4	36	44.74	0.1505
5	72	107.06	0.0046
6	108	145.18	0.0099
7	144	182.55	0.0164
8	180	249.86	0.0004
9	216	277.8	0.0029
10	252	324.76	0.0013

Table 4.6: Univariate AR Diagnostic Tests, from January 2004 to April 2009.

Test for Univariate Correlations of Residuals after 5 lags
(Ho: Residuals from AR # Lags of univariate Series are uncorrelated)

	AR1		AR2		AR3		AR4	
Variable	F Value	Pr > F	F Value	Pr > F	F Value	Pr > F	F Value	Pr > F
Corn	0.02	0.8797	0.01	0.9867	0.01	0.9989	0.01	0.9999
Soybean	0.01	0.9245	0.01	0.9875	0.02	0.9967	0.02	0.9994
Sorghum	0.00	0.9877	0.00	0.9980	0.00	0.9998	0.02	0.9995
Wheat	0.00	0.9501	0.01	0.9937	0.01	0.9994	0.03	0.9979
Feeder Cattle	0.00	0.9864	0.06	0.9443	0.05	0.9849	0.03	0.9980
Live Cattle	0.02	0.8825	0.02	0.9836	0.07	0.9741	0.09	0.9841

The coefficients for the estimated models for the series from January 1998 to December 2004 are in the following table 4.7, and from January 2004 to December 2009 are in tables 4.8.1 and 4.8.2. In general, each of the grain and cattle markets has an autoregressive factor of its own with a particular lag, and may include another significant coefficient from its product type with a particular lag (i.e., a specific grain having an autoregressive component from another grain and/or a particular cattle market having an autoregressive component from the other cattle market). Analysis of the impact of these coefficients may be better assessed through Granger Causality tests.

Table 4.7: Parameter Estimates VAR (3): from January 1998 to December 2004.

VAR Coefficient Estimates (p values in parenthesis)

Lag	Variable	Corn	Soybean	Sorghum	Wheat	Feeder Cattle	Live Cattle
1	Corn	-0.0649*	0.0159	0.0767*	0.0297	0.0103	0.0274
		(0.0499)	(0.5873)	(0.0096)	(0.1635)	(0.6315)	(0.3346)
	Soybean	-0.0235	-0.0155	0.0353	-0.0048	0.0009	0.0187
		(0.4607)	(0.5837)	(0.2165)	(0.8159)	(0.9649)	(0.4936)
	Sorghum	0.2126*	0.0065	-0.2435*	0.0232	-0.0095	0.0128
		(0.0001)	(0.8320)	(0.0001)	(0.3004)	(0.6721)	(0.6662)
	Wheat	0.0408	-0.0560	0.0563	-0.0455+	0.0129	0.0375
		(0.3311)	(0.1330)	(0.1344)	(0.0937)	(0.6350)	(0.2981)
	Feeder Cattle	0.0154	-0.0168	-0.0020	0.0081	-0.1413*	0.0322
		(0.6813)	(0.6129)	(0.9527)	(0.7386)	(0.0001)	(0.3175)
	Live Cattle	0.0425	-0.0516	0.0089	0.0173	0.0418*	-0.0627*
		(0.1318)	(0.0395)	(0.7260)	(0.3418)	(0.0221)	(0.0098)
2	Corn	-0.0772*	0.0297	0.0794*	0.0042	-0.0340	0.0728*
		(0.0221)	(0.3119)	(0.0098)	(0.8445)	(0.1143)	(0.0099)
	Soybean	-0.0532	0.0386	0.0734*	0.0003	-0.0112	0.0660*
		(0.1022)	(0.1724)	(0.0134)	(0.9882)	(0.5894)	(0.0154)
	Sorghum	0.0447	-0.0226	-0.0226	0.0164	-0.0313	0.0652*
		(0.2060)	(0.4622)	(0.4833)	(0.4638)	(0.1663)	(0.0277)
	Wheat	0.0137	-0.0108	0.0469	-0.0036	-0.0202	0.0401
		(0.7487)	(0.7717)	(0.2297)	(0.8959)	(0.4604)	(0.2630)
	Feeder Cattle	-0.0058	0.0385	-0.0137	-0.0101	-0.0236	0.1287*
		(0.8794)	(0.2472)	(0.6952)	(0.6780)	(0.3347)	(0.0001)
	Live Cattle	-0.0313	0.0535*	0.0192	-0.0091	0.0723*	-0.0786*
		(0.2763)	(0.0326)	(0.4643)	(0.6203)	(0.0001)	(0.0011)
3	Corn	-0.0928*	0.0208	0.0304	0.0070	-0.0408+	-0.0151
		(0.0051)	(0.4785)	(0.3032)	(0.7437)	(0.0566)	(0.5956)
	Soybean	-0.034	0.0499+	-0.0346	-0.0069	0.0088	0.0377
		(0.2872)	(0.0788)	(0.2237)	(0.7354)	(0.6684)	(0.1688)
	Sorghum	0.0034	0.0199	-0.0331	-0.0066	0.0106	-0.0421
		(0.9212)	(0.5188)	(0.2845)	(0.7676)	(0.6357)	(0.1574)
	Wheat	-0.0660	0.0552	-0.0016	-0.0506+	-0.0255	-0.0382
		(0.1168)	(0.1392)	(0.9655)	(0.0618)	(0.3472)	(0.2895)
	Feeder Cattle	0.0134	0.0333	0.0242	-0.0317	0.0095	0.0369
		(0.7213)	(0.3176)	(0.4687)	(0.1900)	(0.6956)	(0.2514)
	Live Cattle	0.0154	0.0289	-0.0035	0.0135	0.0478*	0.0047
		(0.5855)	(0.2496)	(0.8894)	(0.4580)	(0.0090)	(0.8448)

* Significant at 5 % level or less

\+ Significant at 10 % level or less

Table 4.8.1: Parameter Estimates of Π for VEC (5): from January 2004 to April 2009.

Parameters Π Estimates (Standard Errors in parenthesis, yet of Non-Gaussian distribution)						
Variable	Corn	Soybean	Sorghum	Wheat	Feeder Cattle	Live Cattle
Corn	-0.0119	*0.0052*	0.0083	-0.0022	*-0.0080*	*0.0156*
	(0.0075)	(0.0033)	(0.0052)	(0.0014)	(0.0050)	(0.0098)
Soybean	-0.0006	0.0002	0.0004	-0.0001	-0.0004	0.0007
	(0.0072)	(0.0032)	(0.0051)	(0.0014)	(0.0049)	(0.0095)
Sorghum	0.0127	-0.0056	-0.0089	0.0024	0.0085	-0.0167
	(0.0079)	(0.0034)	(0.0055)	(0.0015)	(0.0053)	(0.0103)
Wheat	-0.0144	0.0063	0.0100	-0.0027	-0.0097	0.0188
	(0.0101)	(0.0044)	(0.0071)	(0.0019)	(0.0068)	(0.0132)
Feeder Cattle	-0.0094	0.0041	0.0066	-0.0018	-0.0063	*0.0123*
	(0.0053)	(0.0023)	(0.0037)	(0.0010)	(0.0036)	(0.0069)
Live Cattle	0.0131	-0.0057	-0.0092	0.0025	0.0088	-0.0172
	(0.0046)	(0.0020)	(0.0032)	(0.0009)	(0.0031)	(0.0061)

For the second period there are the long-run estimates for the relationships between the variables given by the error correction term (Π) from previous table 4.8.1. These long-run estimates are in line with what may be anticipated from the literature, as in the case of corn and soybean having a long run positive (0.0052) relationship due to shared acreage. Similar positive long run relationship is obtained between feeder cattle and live cattle (0.0123) as they are both major components of cattle production profitability. Regarding corn and feeder cattle prices, they have a long run negative relationship (-0.0080) since calf producers tend to sell earlier than usual when corn prices go up, thus driving the calf/feeder prices down (Anderson and Trapp, 2000). It is not clear at this moment the resulting long-run positive relationship between corn and live cattle (0.0156), and it may be a spurious finding yet this requires further study.

Table 4.8.2: Parameter Estimates A_i *(K-1)* for VEC (5): from January 2004 to April 2009.

(p values in parenthesis)

Lag	Variable	Corn	Soybean	Sorghum	Wheat	Feeder Cattle	Live Cattle
1	Corn	0.0226	-0.0261	0.0518	-0.0068	0.0145	-0.0155
		(0.6017)	(0.4690)	(0.1646)	(0.7758)	(0.7136)	(0.7323)
	Soybean	-0.0142	-0.0160	-0.0047	-0.0176	0.0030	0.0163
		(0.7340)	(0.6448)	(0.8956)	(0.4472)	(0.9379)	(0.7083)
	Sorghum	0.3255	-0.0120	-0.2715	0.0032	-0.0043	0.0002
		(0.0001)	(0.7496)	(0.0001)	(0.8979)	(0.9167)	(0.9969)
	Wheat	-0.0792	-0.1261	0.0263	-0.0143	0.0034	-0.0662
		(0.1752)	(0.0095)	(0.5998)	(0.6580)	(0.9497)	(0.2779)
	Feeder Cattle	0.0278	0.0140	-0.0153	0.0078	-0.0963	0.0614
		(0.3618)	(0.5805)	(0.5586)	(0.6465)	(0.0005)	(0.0541)
	Live Cattle	-0.0325	0.0158	0.0448	0.0091	0.0924	-0.1155
		(0.2235)	(0.4757)	(0.0509)	(0.5390)	(0.0001)	(0.0001)
2	Corn	-0.0212	-0.0498	0.0561	0.0025	-0.0140	0.0177
		(0.6359)	(0.1671)	(0.1453)	(0.9174)	(0.7241)	(0.6981)
	Soybean	-0.0261	0.0052	0.0777	-0.0212	0.0183	0.0823
		(0.5452)	(0.8804)	(0.0367)	(0.3555)	(0.6333)	(0.0615)
	Sorghum	0.1063	-0.0480	-0.0578	0.0026	0.0087	0.0343
		(0.0236)	(0.2043)	(0.1525)	(0.9158)	(0.8338)	(0.4723)
	Wheat	-0.0217	-0.0802	0.2081	-0.0192	-0.0254	0.0251
		(0.7190)	(0.0991)	(0.0001)	(0.5492)	(0.6356)	(0.6834)
	Feeder Cattle	0.0258	0.0071	-0.0377	0.0154	-0.0069	0.0890
		(0.4127)	(0.7810)	(0.1642)	(0.3580)	(0.8039)	(0.0056)
	Live Cattle	-0.0295	0.0369	0.0524	0.0047	0.0527	-0.0614
		(0.2849)	(0.0971)	(0.0274)	(0.7469)	(0.0313)	(0.0292)
3	Corn	-0.0395	0.0514	0.0250	-0.0023	-0.0997	0.0636
		(0.3786)	(0.1544)	(0.5193)	(0.9241)	(0.0119)	(0.1615)
	Soybean	-0.0773	0.0546	0.0374	0.0101	-0.0282	0.0597
		(0.0744)	(0.1168)	(0.3181)	(0.6588)	(0.4600)	(0.1727)
	Sorghum	0.0624	0.0556	-0.1341	0.0341	-0.0732	0.0671
		(0.1845)	(0.1409)	(0.0010)	(0.1702)	(0.0778)	(0.1583)
	Wheat	-0.2051	0.0697	0.0865	-0.0185	-0.0827	-0.0468
		(0.0007)	(0.1521)	(0.0985)	(0.5630)	(0.1214)	(0.4448)
	Feeder Cattle	0.0410	0.0051	-0.0303	0.0064	0.0452	0.0591
		(0.1942)	(0.8394)	(0.2667)	(0.7008)	(0.1050)	(0.0647)
	Live Cattle	-0.0025	0.0416	-0.0104	0.0105	0.0424	0.0192
		(0.9281)	(0.0611)	(0.6628)	(0.4743)	(0.0821)	(0.4926)
4	Corn	-0.0379	0.0548	0.0499	-0.0116	0.0054	-0.0346
		(0.3907)	(0.1291)	(0.1792)	(0.6228)	(0.8921)	(0.4403)
	Soybean	0.0117	0.0758	0.0085	0.0042	0.0013	-0.0284
		(0.7845)	(0.0295)	(0.8132)	(0.8542)	(0.9734)	(0.5119)
	Sorghum	-0.0090	0.0221	0.0153	0.0049	-0.0210	-0.0098
		(0.8466)	(0.5581)	(0.6931)	(0.8435)	(0.6107)	(0.8353)
	Wheat	0.0103	0.0747	-0.0104	-0.0196	-0.0212	-0.0162
		(0.8630)	(0.1249)	(0.8347)	(0.5369)	(0.6893)	(0.7884)
	Feeder Cattle	0.0090	0.0118	-0.0014	0.0055	0.0051	0.0522
		(0.7718)	(0.6424)	(0.9579)	(0.7377)	(0.8555)	(0.0984)
	Live Cattle	-0.0023	0.0091	-0.0151	0.0027	0.0249	0.0291
		(0.9330)	(0.6816)	(0.5083)	(0.8517)	(0.3044)	(0.2914)

Results from Granger Causality tests among the commodities, during each estimated period, are in the following tables 4.9 and 4.10.

Table 4.9: Granger-Causality Test: from January 1998 to December 2004.

Granger-Causality Wald Test: p-values

Lag / ***Lead***	Corn	Soybean	Sorghum	Wheat	Feeder Cattle	Live Cattle
Corn	-	0.3470	<.0001*	0.1321	0.6758	0.1265
Soybean	0.3414	-	0.0517+	0.7564	0.5262	0.0369*
Sorghum	0.0029*	0.0190*	-	0.0973+	0.5878	0.4001
Wheat	0.2487	0.8047	0.0191*	-	0.9269	0.3231
Feeder Cattle	0.1698	0.8499	0.7284	0.6143	-	<.0001*
Live Cattle	0.0698+	0.0525+	0.1037	0.3195	0.0006*	-

* Significant at 5 % level or less
\+ Significant at 10 % level or less

Table 4.10: Granger-Causality Test: from January 2004 to April 2009.

Granger-Causality Wald Test: p-values

Lag / ***Lead***	Corn	Soybean	Sorghum	Wheat	Feeder Cattle	Live Cattle
Corn	-	0.3694	<.0001*	0.0070*	0.3684	0.1134
Soybean	0.2567	-	0.0127*	0.0095*	0.1124	0.0069*
Sorghum	0.5394	0.4684	-	0.0022*	0.7304	0.0088*
Wheat	0.0582+	0.0146*	0.0040*	-	0.6030	0.1472
Feeder Cattle	0.3253	0.9750	0.3870	0.4738	-	<.0001*
Live Cattle	0.5401	0.1421	0.6448	0.4334	0.0004*	-

* Significant at 5 % level or less
\+ Significant at 10 % level or less

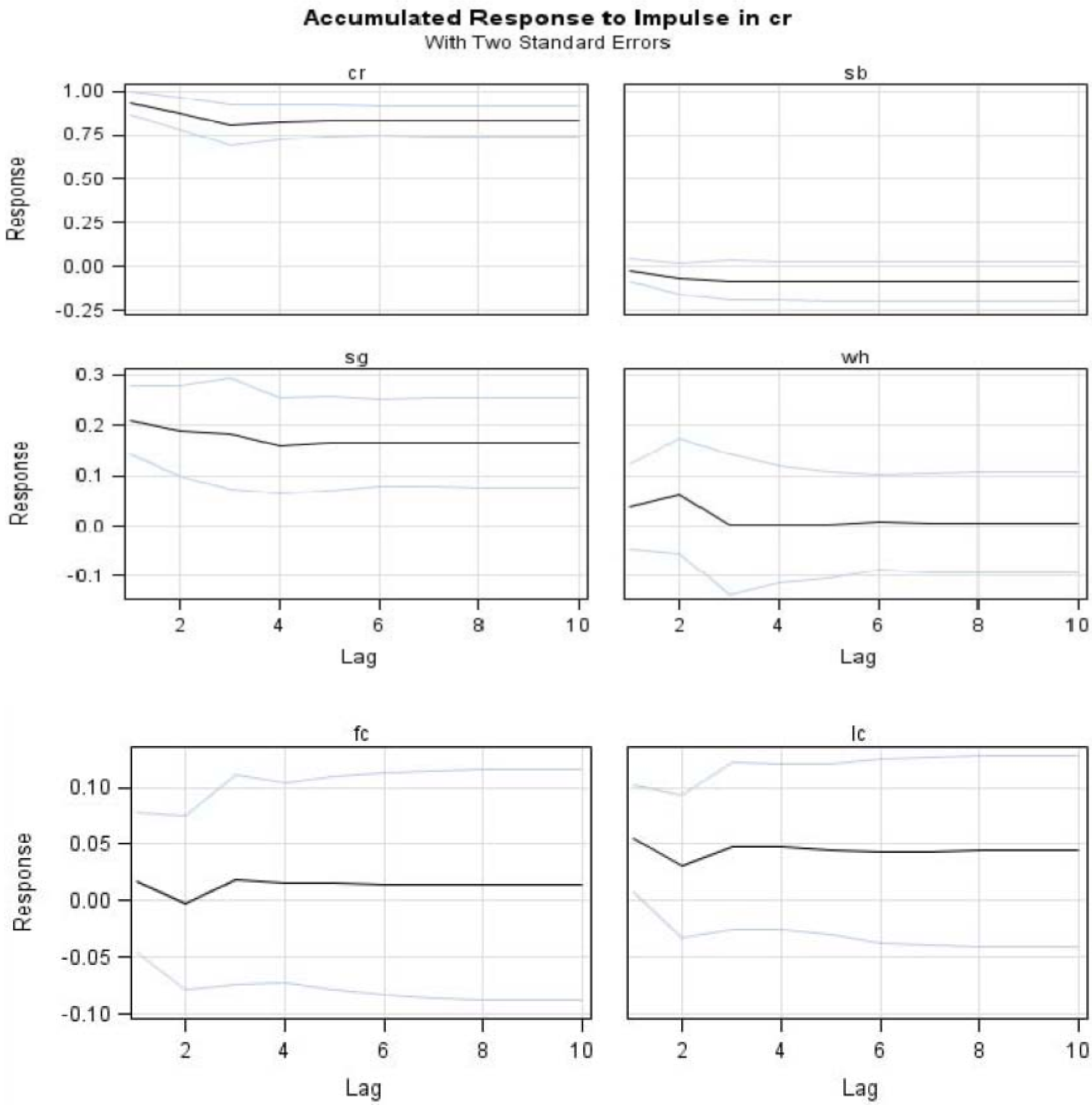
Results for the first period indicate that changes in corn prices Granger cause (or lead) changes in sorghum prices, and conversely changes in sorghum prices also Granger cause changes in corn prices (i.e., there is a bidirectional Granger causality between these two grains). Hence price changes in either of these two grains affect the other one. In the second period, changes in corn price maintain Granger causality on changes in sorghum prices, but the inverse Granger causality relation does not hold anymore (i.e., changes in sorghum prices do not Granger cause changes in corn prices). Thus for the second period, only corn price changes lead changes in sorghum prices and not vice-versa. This result may be plausibly caused by major corn consumption from ethanol production during this period, bringing about a substitution away from corn to sorghum in livestock feeding components. This latter increase in sorghum consumption produces a subsequent increase in sorghum prices as may be seen toward the end of the period in figure 4.2.

During the second period, a change in corn prices likewise Granger causes changes in the price of wheat. The same applies for changes in soybean prices, as it has Granger causality on changes of both sorghum and wheat prices. However, none of these Granger Causalities are obtained during the first period. These causality patterns may again possibly respond to modification of livestock feed rations in the second period, although soybean is a protein component and both sorghum and wheat are carbohydrates. In addition, in neither of the two periods considered are changes in corn prices Granger causing changes in soybean prices or vice-versa. Thus there is no Granger Causality in either direction between corn and soybeans.

A bidirectional Granger Causality is determined for cattle markets prices, during both periods estimated. Hence changes in prices of feeder cattle have granger causality in price changes of live cattle and vice-versa. During the second period, price changes in sorghum and soybean Granger cause changes of live cattle prices. However, in the first period only soybean Granger causes changes of live cattle prices.

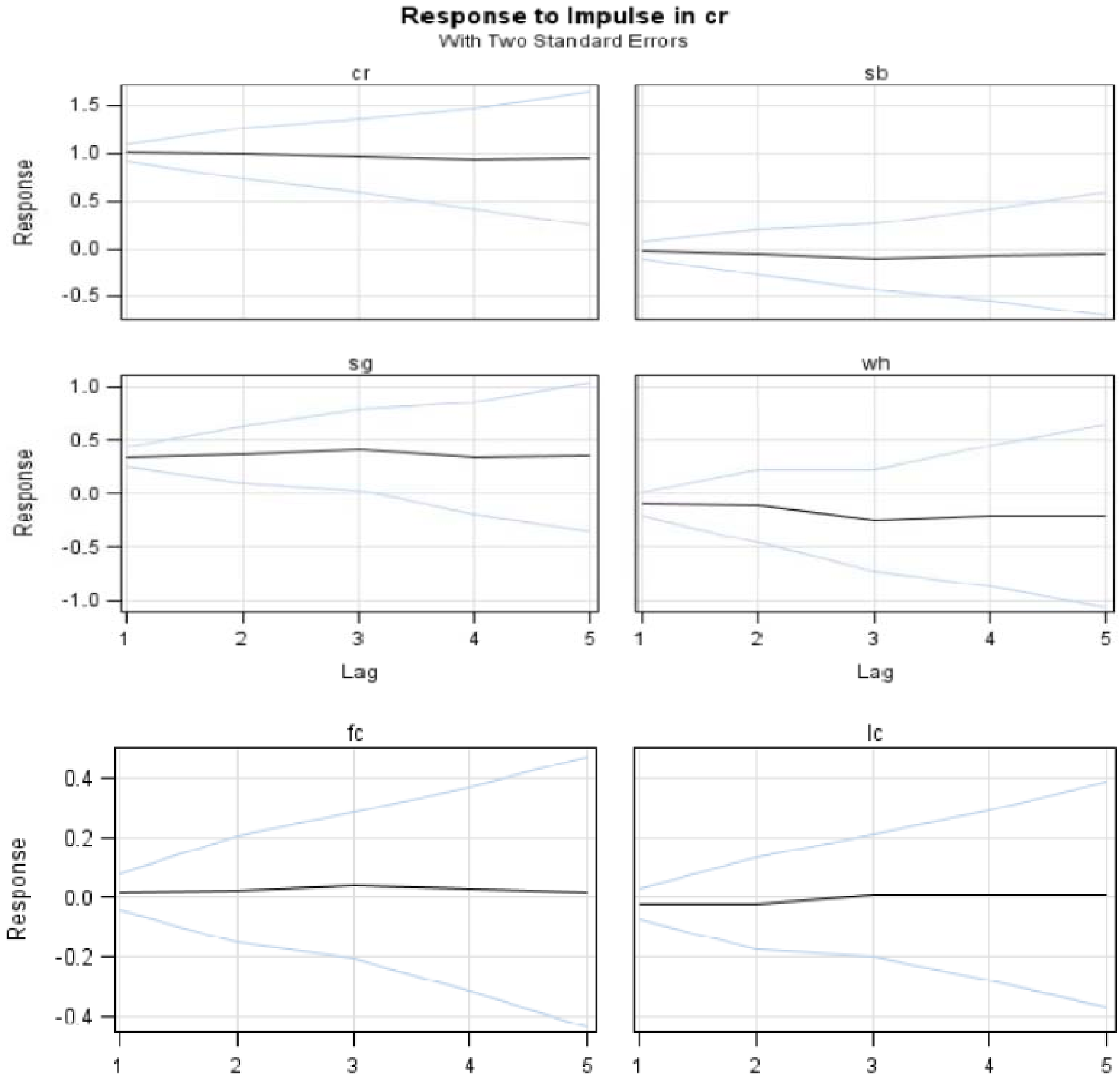
Accumulated impulse responses for the commodities studied were determined for the first period estimated by considering a shock of one unit on a specific commodity, and its corresponding effect on each market over a length of 10 time periods (i.e., days). The result of these accumulated impulse responses includes confidence bands of two standard errors. For the second period estimated through the VEC(5) model, simple impulse responses were computed by considering a one unit shock on a specific variable and its effect on each commodity over a length of five time periods (days). The resulting impulse responses from corn price shocks, for the two periods are in following figures 4.3.1 and 4.3.2.

From figure 4.3.1 it may be noted that after the shock on corn price, only sorghum has an initial increase in its price of 0.2 units, and subsequently lowers to 0.15 units. Yet as seen from figure 4.3.2 below, in the second period the response from corn price shocks on sorghum prices is higher beginning at 0.35 units, it subsequently rises to 0.4 units after three days, before returning to about 0.34 units (i.e., the response on sorghum from the shock effect of corn is larger in this period) where it becomes insignificant by taking into account the confidence band.



cr: corn; **sb**: soybeans; **sg**: sorghum; **wh**: wheat; **fc**: feeder cattle; **lc**: live cattle

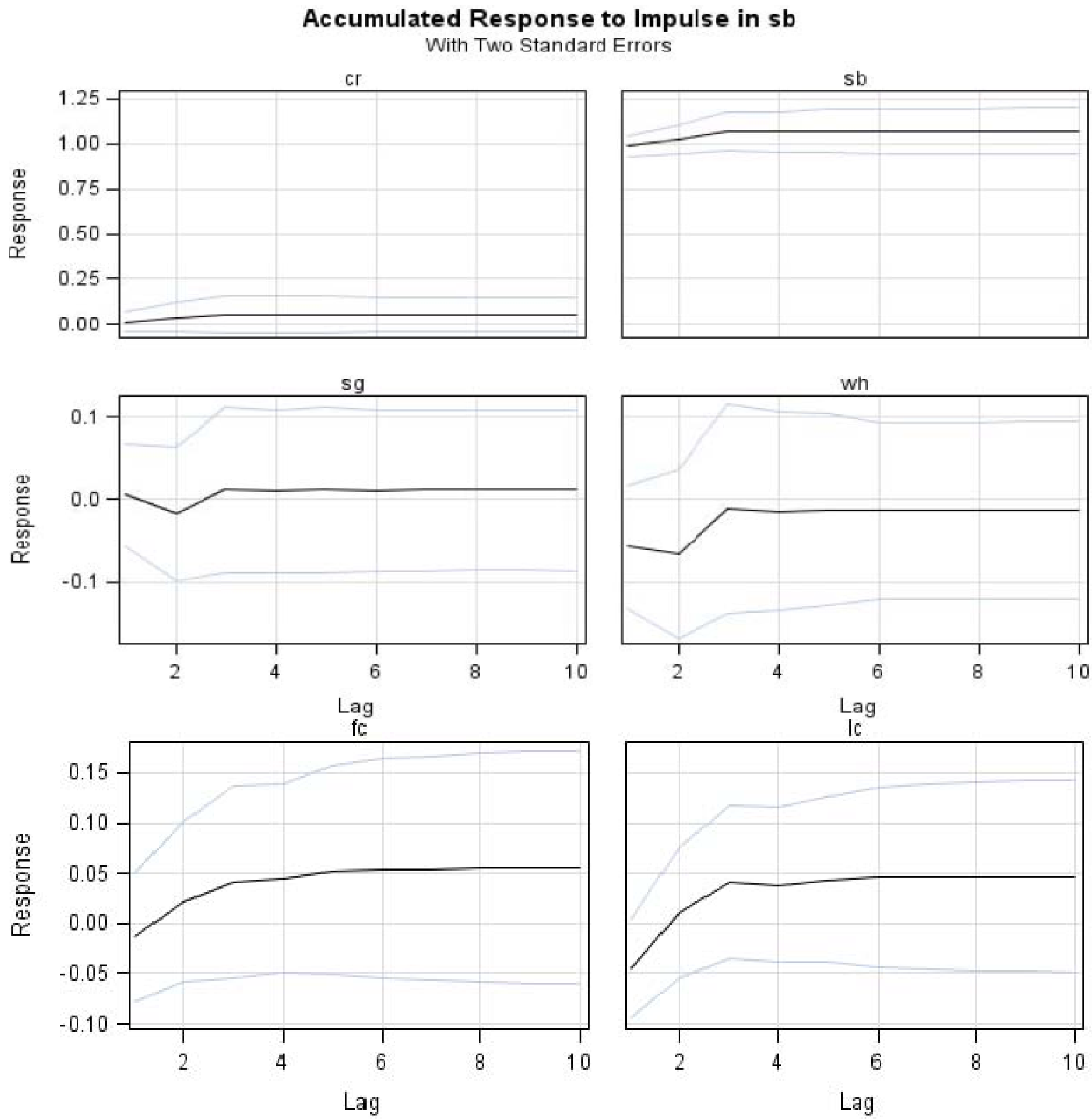
Figure 4.3.1: Accumulated Price Responses of Selected Markets in days, for a one unit shock in Corn prices, during the period from January 1998 to December 2004.



cr: corn; **sb**: soybeans; **sg**: sorghum; **wh**: wheat; **fc**: feeder cattle; **lc**: live cattle

Figure 4.3.2: Simple Price Responses of Selected Markets in days, for one unit shock in Corn price, during the period from January 2004 to April 2009.

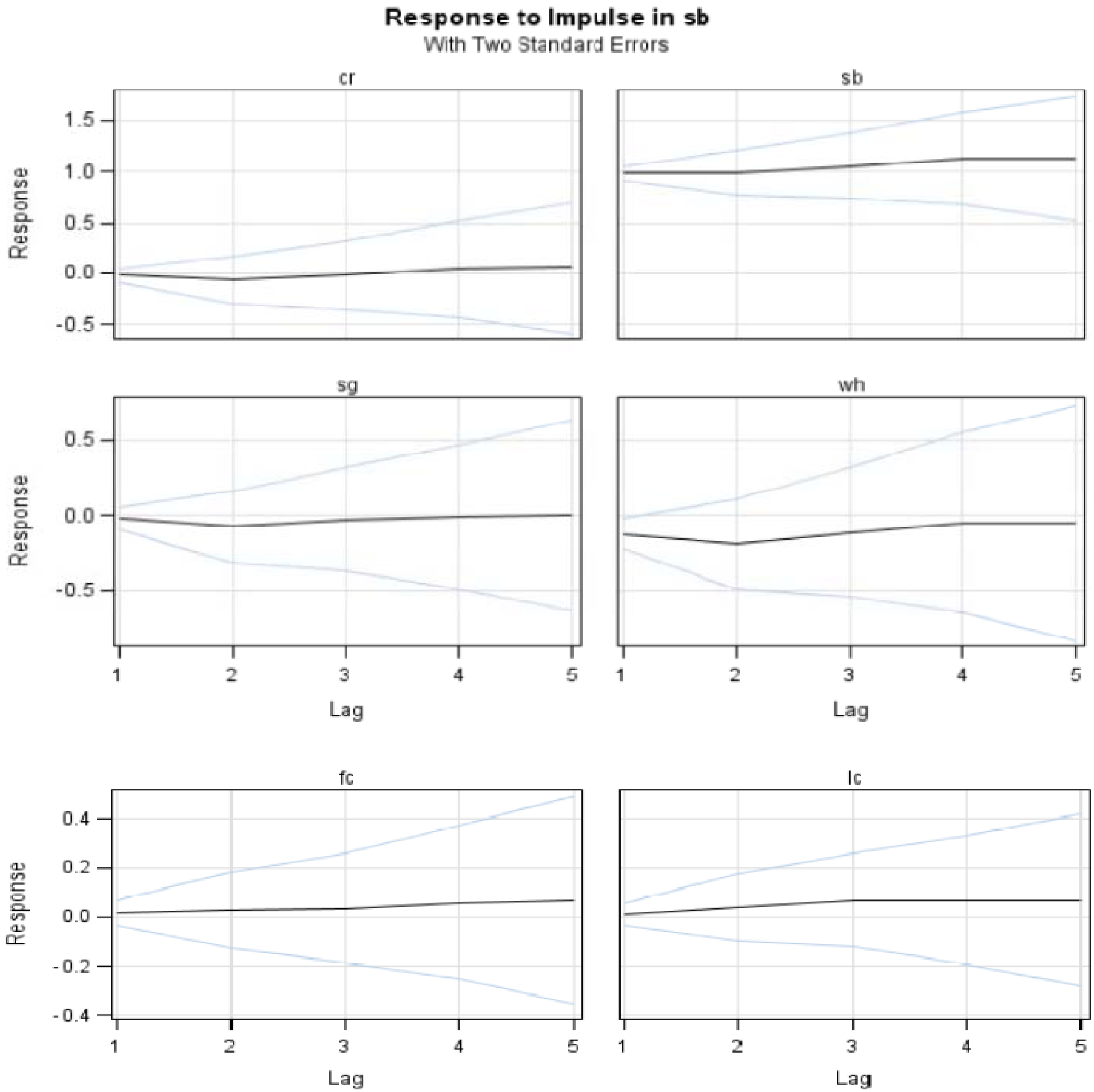
The impulse responses determined from shocks of one unit of soybean prices on each market are in the following figures 4.4.1 and 4.4.2, considering the first and second estimated period, respectively.



cr: corn; **sb**: soybeans; **sg**: sorghum; **wh**: wheat; **fc**: feeder cattle; **lc**: live cattle

Figure 4.4.1: Accumulated Price Responses of Selected Markets in days, for a one unit shock in Soybean prices, for the period from January 1998 to December 2004.

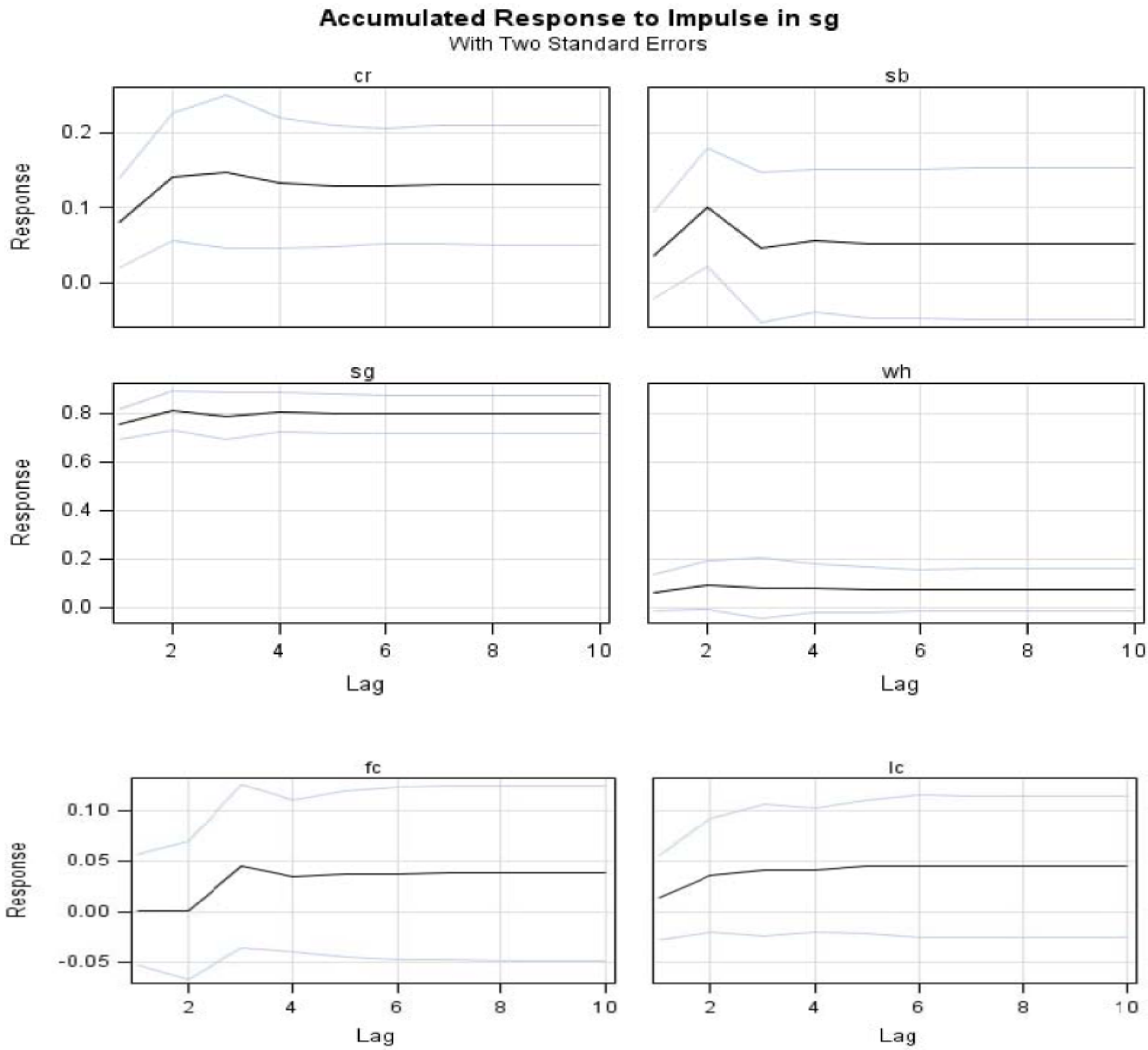
For the first period from figure 4.4.1 above, only soybean has an accumulated price response which is expected, yet no other market has a significant response. However, for the second period in figure 4.4.2 below, wheat prices have a decrease to 0.21 units on the first day before this drop becomes insignificant.



cr: corn; **sb**: soybeans; **sg**: sorghum; **wh**: wheat; **fc**: feeder cattle; **lc**: live cattle

Figure 4.4.2: Simple Price Responses of Selected Markets in days, for a one unit shock in Soybean prices, for the period from January 2004 to April 2009.

The impulse responses from shocks of one unit of grain sorghum prices on each market are in the following figures 4.5.1 and 4.5.2, for both estimated periods.

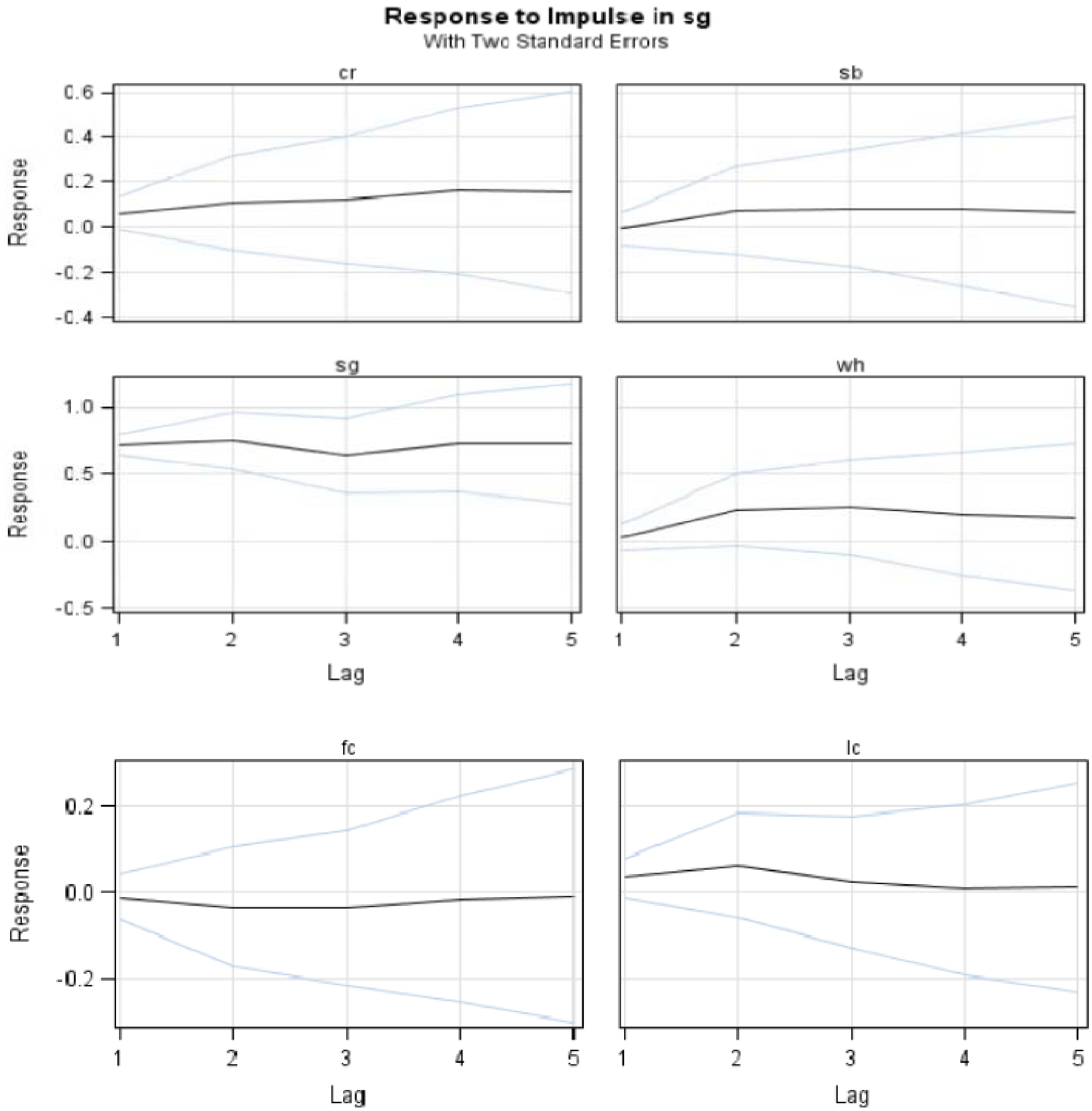


cr: corn; **sb**: soybeans; **sg**: sorghum; **wh**: wheat; **fc**: feeder cattle; **lc**: live cattle

Figure 4.5.1: Accumulated Price Responses of Selected Markets in days, for a one unit shock in Sorghum prices, for the period from January 1998 to December 2004.

During the first period above, corn prices experience an accumulated response that increases to about 0.15 units at the third day, before decreasing to a steady accumulated response of 0.13

units at the fourth day. Likewise, soybean prices experience an increased accumulated response of 0.1 units in the second day before decreasing to a non-significant value afterwards.

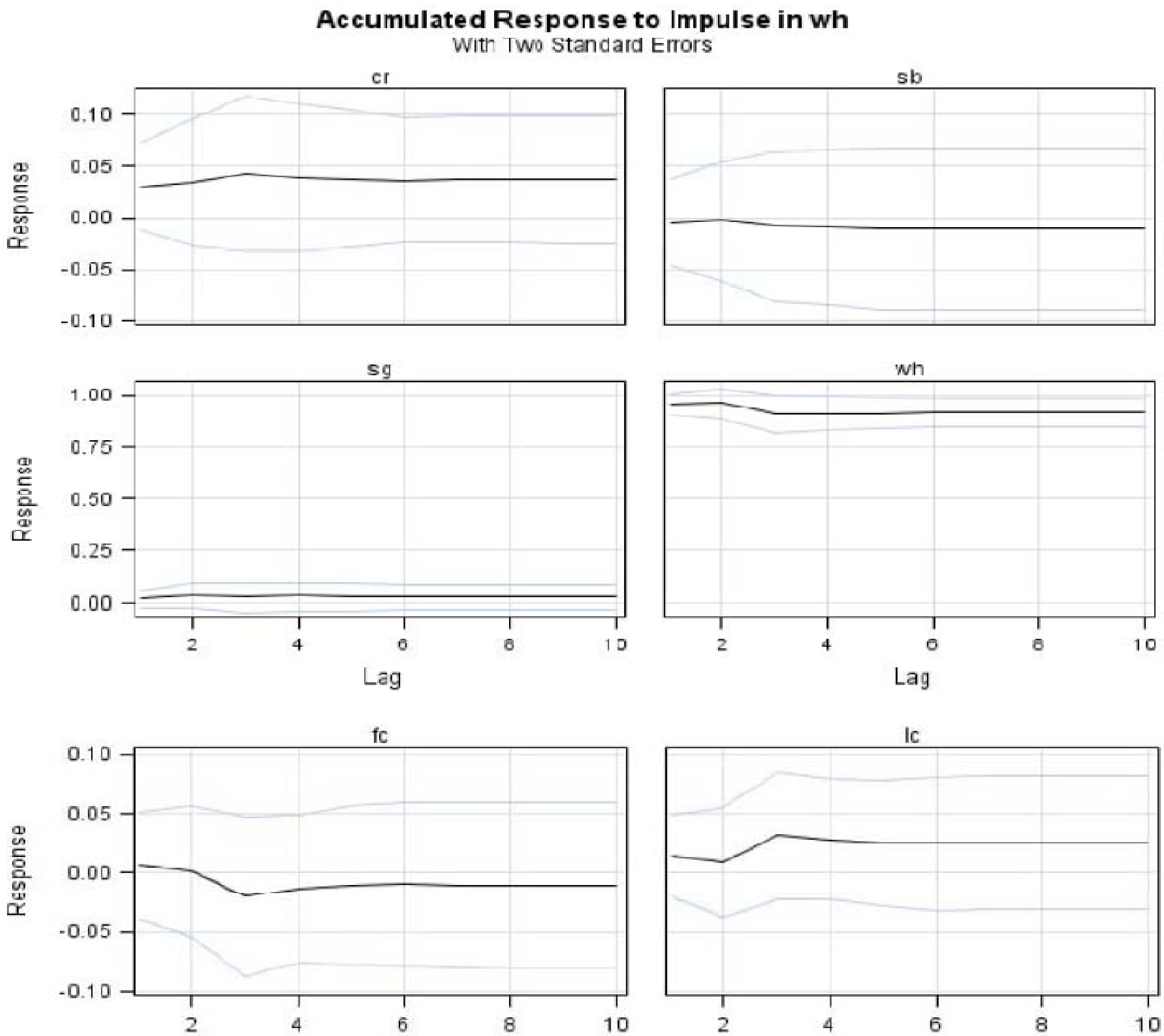


cr: corn; **sb**: soybeans; **sg**: sorghum; **wh**: wheat; **fc**: feeder cattle; **lc**: live cattle

Figure 4.5.2: Simple Price Responses of Selected Markets in days, for a one unit shock in Sorghum prices, for the period from January 2004 to April 2009.

For the second period above however, corn prices do not have a significant response from a shock to sorghum prices. Soybean prices are likewise not significantly affected by a shock to sorghum prices.

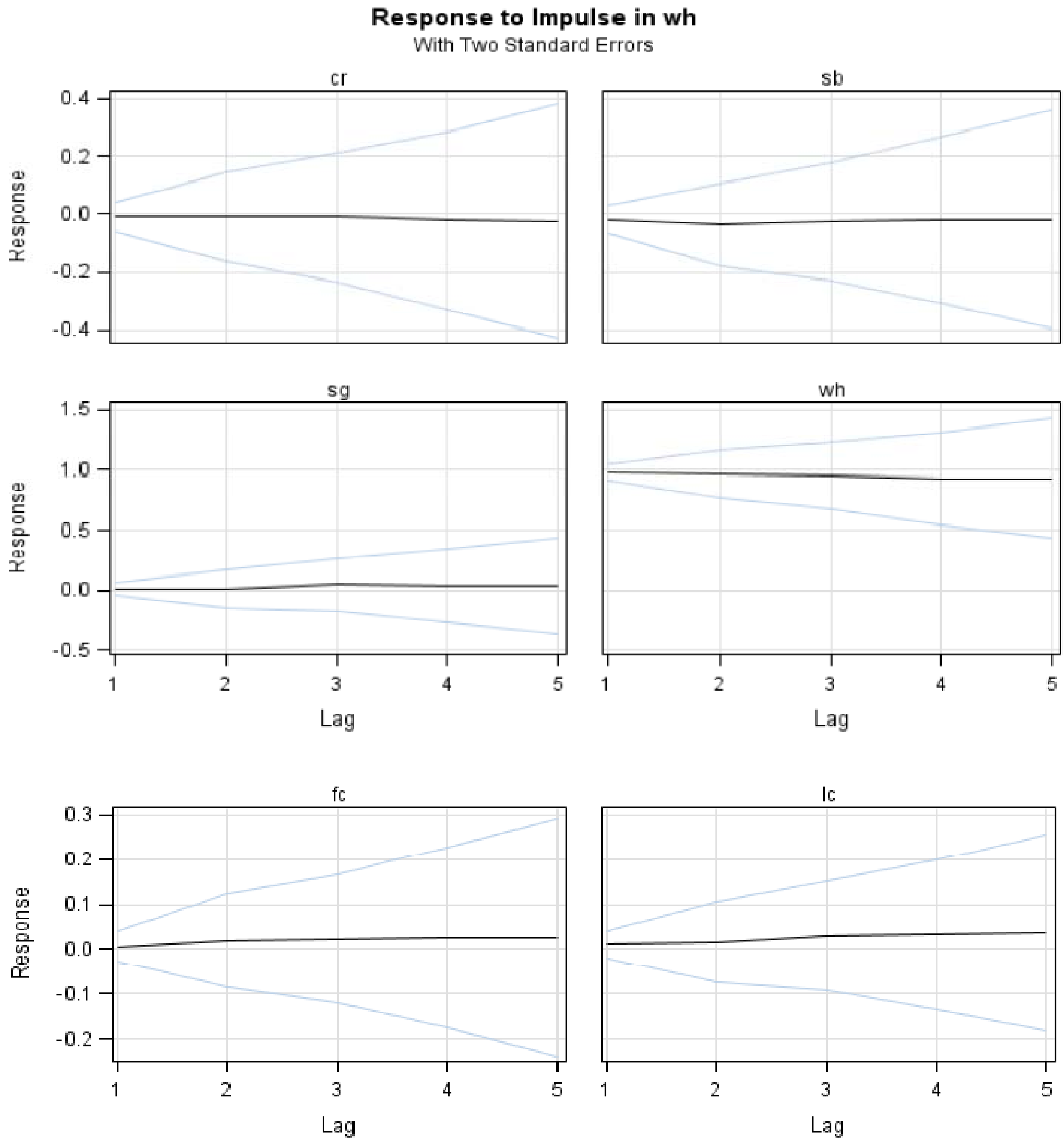
Next, the impulse responses obtained from shocks of one unit of wheat prices on each market, are in the following figures 4.6.1 and 4.6.2, considering both estimated periods.



cr: corn; **sb**: soybeans; **sg**: sorghum; **wh**: wheat; **fc**: feeder cattle; **lc**: live cattle

Figure 4.6.1: Accumulated Price Responses of Selected Markets in days, for a one unit shock in Wheat prices, for the period from January 1998 to December 2004.

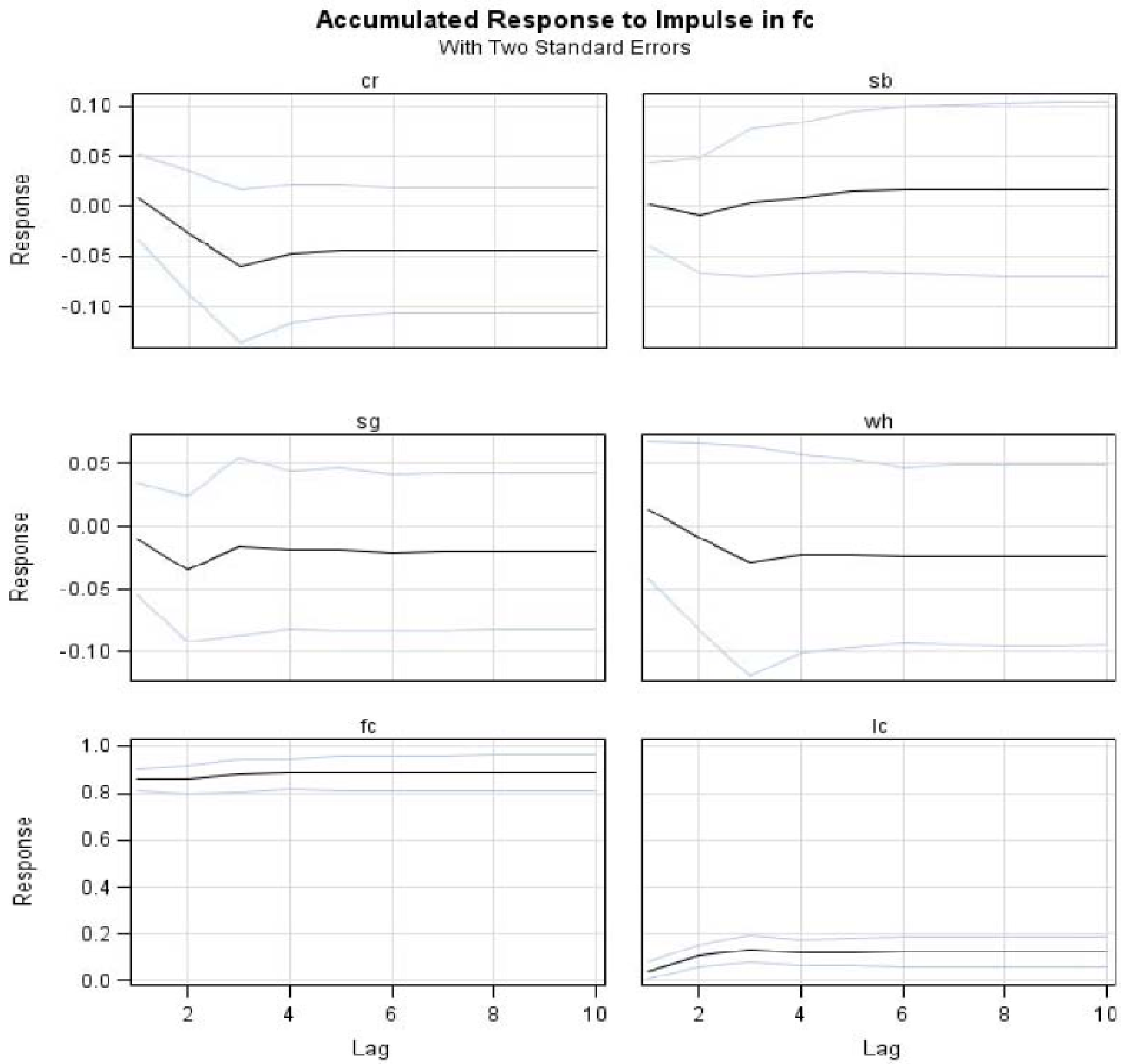
Shocks to wheat prices do not produce a significant response on any other market, for both periods considered.



cr: corn; **sb**: soybeans; **sg**: sorghum; **wh**: wheat; **fc**: feeder cattle; **lc**: live cattle

Figure 4.6.2: Simple Price Responses of Selected Markets in days, for one unit shock in Wheat prices, for the period from January 2004 to April 2009.

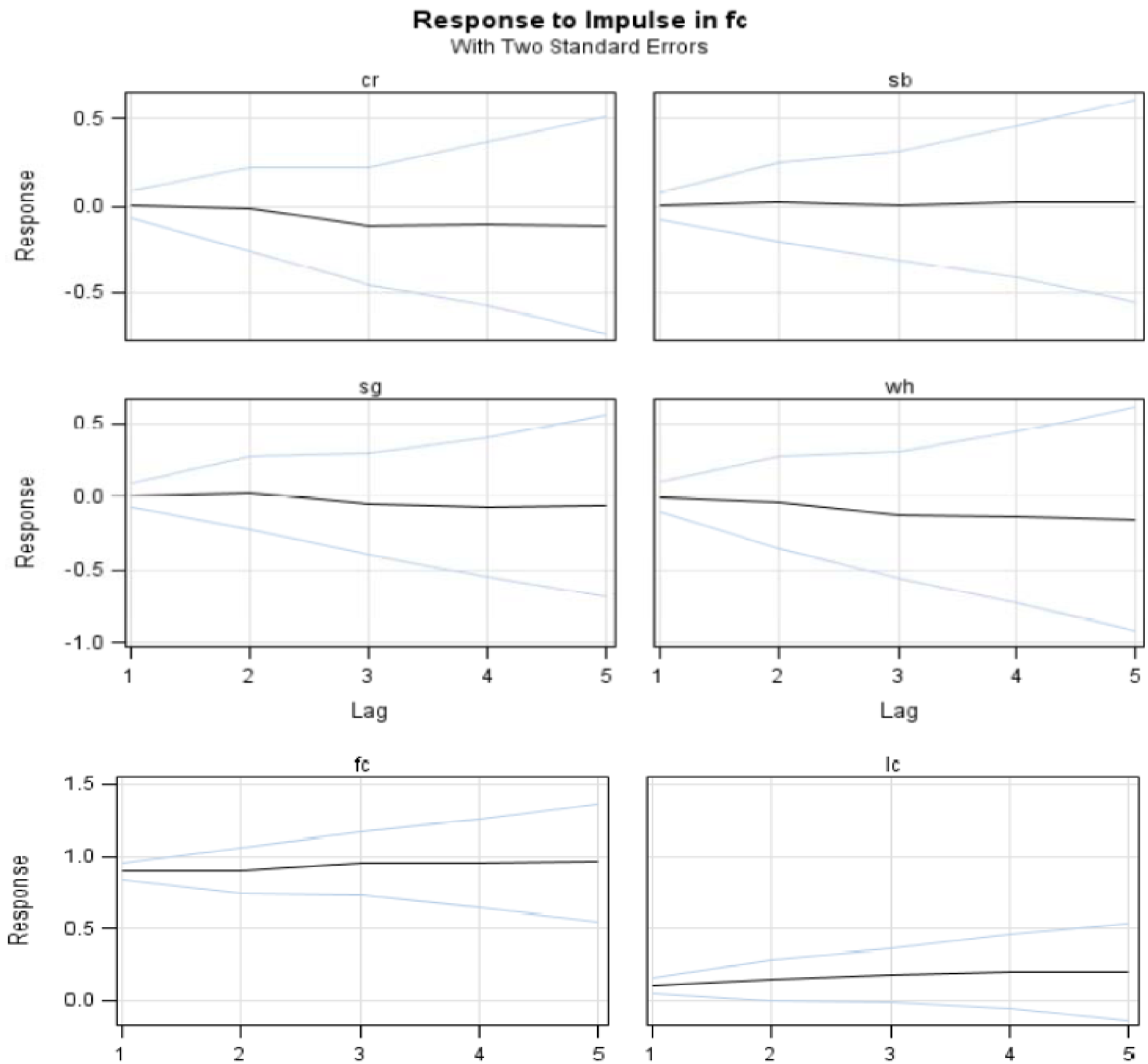
In following figures 4.7.1 and 4.7.2, the impulse responses on each market are determined from shocks of one unit in feeder cattle prices, for both estimated periods.



cr: corn; **sb**: soybeans; **sg**: sorghum; **wh**: wheat; **fc**: feeder cattle; **lc**: live cattle

Figure 4.7.1: Accumulated Price Responses of Selected Markets in days, for a one unit shock in Feeder Cattle prices, for period from January 1998 to December 2004.

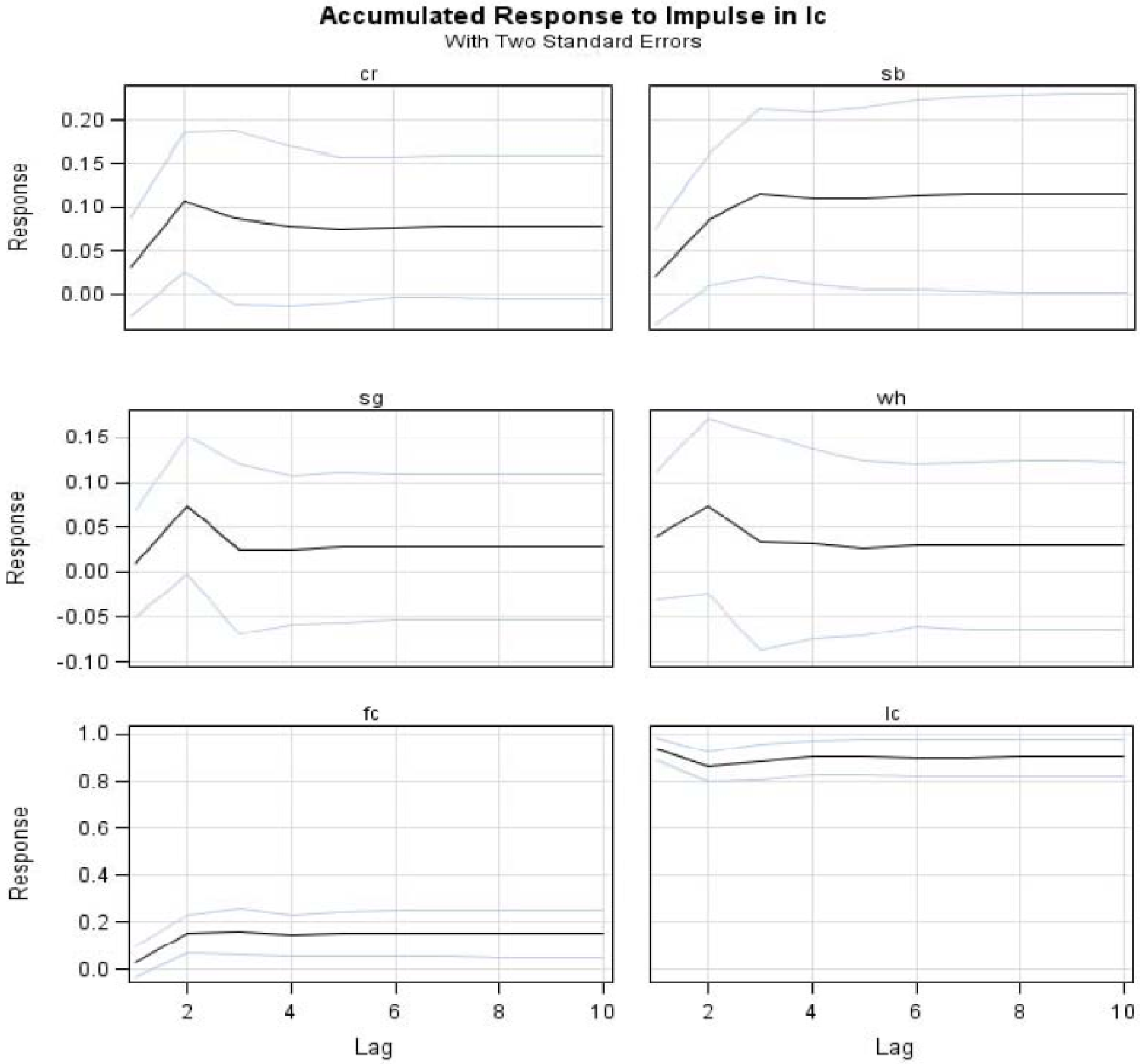
During the first period, an increasing accumulated price response of 0.13 units for live cattle is obtained at the third day, and then maintained through the rest of the days. This price response for live cattle is similar during the second estimated period, rising from 0.13 to 0.16 on the second day and then becoming insignificant, as seen below in figure 4.7.2



cr: corn; **sb**: soybeans; **sg**: sorghum; **wh**: wheat; **fc**: feeder cattle; **lc**: live cattle

Figure 4.7.2: Simple Price Responses of Selected Markets in days, for one unit shock in Feeder Cattle prices, for period from January 2004 to April 2009.

In the following figures 4.8.1 and 4.8.2, the impulse responses from shocks of one unit of live cattle prices on each market are determined for each estimated period.

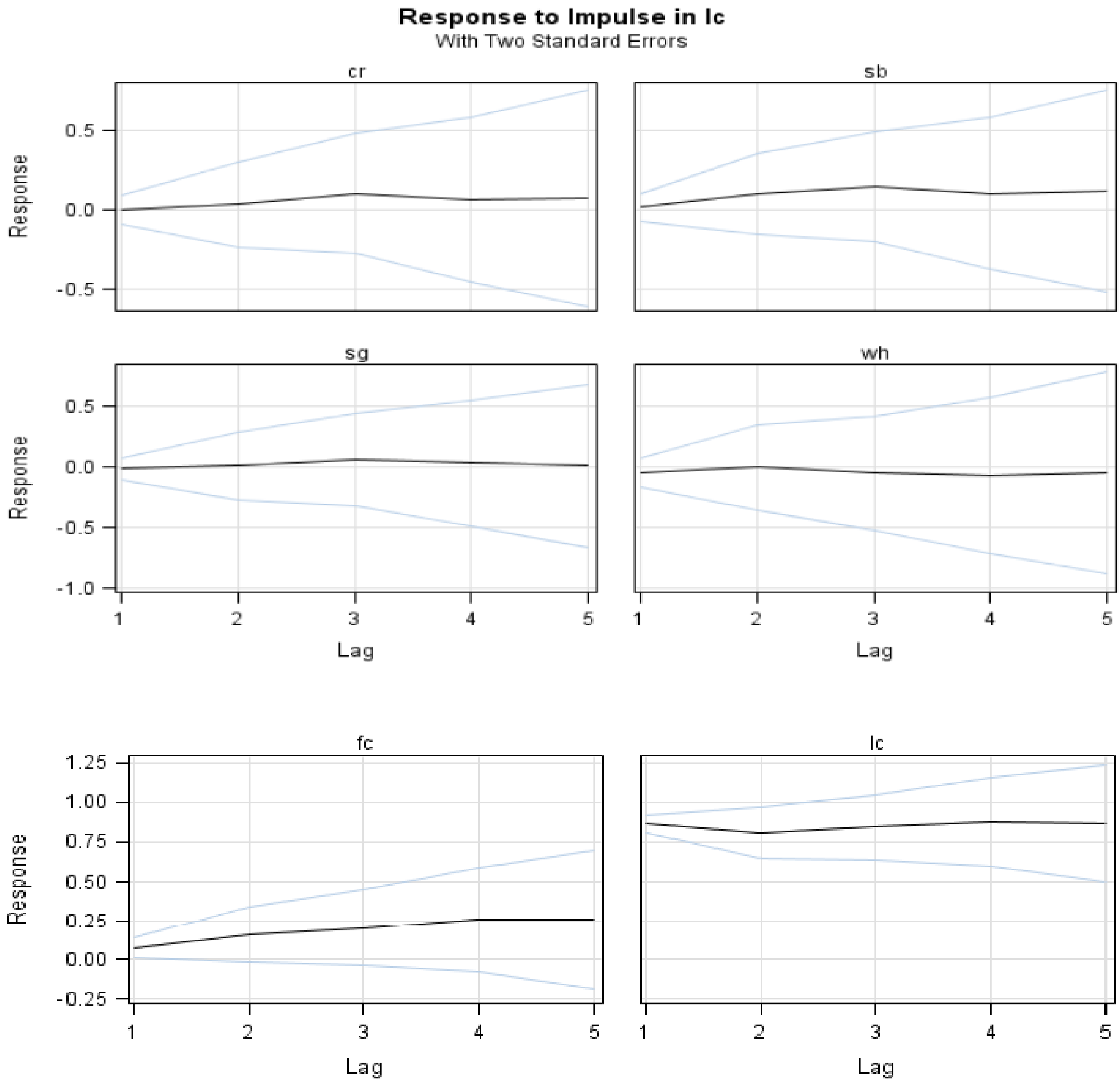


cr: corn; **sb**: soybeans; **sg**: sorghum; **wh**: wheat; **fc**: feeder cattle; **lc**: live cattle

Figure 4.8.1: Accumulated Price Responses of Selected Markets in days, for one unit shock in Live Cattle prices, for period from January 1998 to December 2004.

As seen from the first period above, corn prices have an accumulated price response of 0.10 units in the second day and then it decreases to insignificant value from the third day onwards.

Also soybeans have an accumulated price response of 0.12 units in the third day, before becoming steady in subsequent days; yet this value is almost insignificant after the seventh day. Feeder cattle prices reach a response of 0.16 units at the second day, maintained thereafter.



cr: corn; **sb**: soybeans; **sg**: sorghum; **wh**: wheat; **fc**: feeder cattle; **lc**: live cattle

Figure 4.8.2: Price Responses of Selected Markets in days, for one unit shock in Live Cattle prices, for period from January 2004 to April 2009.

However during the second period from figure 4.8.2 above, only feeder cattle prices experience a significant price response at 0.11 during the first day, but then becomes insignificant for the rest of the days considered.

4.6 Discussion

Relevant differences regarding the dynamic relationships between the commodities were determined after comparing results from the first and second estimated periods. In the first period, corn and sorghum price changes have bi-directional Granger Causality. However, in the second period only corn price changes Granger cause sorghum price changes. This may reflect substitution away from corn to sorghum used in livestock feeding rations[8] during the second period in which corn had higher prices, as both grains are carbohydrate nutrients in the feed diet. The increase in sorghum consumption during the second period estimated, instead of corn, may lead to a rise in the price of sorghum. This may be corroborated through the impulse response functions, where a shock to the corn price during the second period almost doubles the response effect on sorghum when compared to the effect obtained from the first period, as can be seen from figures 4.3.2 and 4.3.1, respectively.

Another result refers to the first period where corn has no Granger Causality with wheat or vice-versa. This lack of relationship is corroborated by a null response from wheat due to a shock from corn prices or likewise from a null response of corn due to a shock from wheat prices. However, for the second estimated period corn price changes and wheat price changes are

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[8] "Higher corn prices provide an incentive to substitute other feed sources, most likely grain sorghum, for animal feed."pg. 3. *Factsheet, Livestock Marketing Information Center* - (Oklahoma) State Extension Services in Cooperation with USDA, EFC-02, Feb. 2007.
"During 2008/09, sorghum has been used as a lower priced substitute to corn." pg. 3. *Feed Outlook,FDS-09d* ERS – USDA, March 13, 2009.

determined to have bi-directional Granger Causality. Nonetheless, for this second period there is no response from shocks on price changes of one commodity on the other. It is not clear if this latter is a spurious result since wheat is also a substitution feed for corn, and thus should have a response from shocks to corn prices during this period. The bi-directional Granger Causality found between them corroborates their direct relationship, since it translates into price changes of one commodity being affected by the price changes of the other commodity and vice-versa.

Regarding the cattle markets, there is a bi-directional Granger Causality for both estimated periods. This bi-direct relationship is corroborated through the impulse response functions obtained from shocks on either the feeder cattle or live cattle markets. These results are anticipated according to the literature, since both feeder and cattle prices are main risk components in the cattle production industry. Thus a shock in feeder cattle prices leads to an increase in live cattle prices (for either period considered), in order to maintain cattle production profitability. Conversely, a shock in live cattle prices likewise leads to an increase in feeder cattle prices.

An unusual result determined for the first period is that a shock in live cattle prices produces a positive response function on both corn and soybean prices. This is corroborated by live cattle prices having Granger Causality on corn and soybean prices. However, these results are not determined during the second period, either by Granger Causality results or impulse response functions. A plausible explanation for live cattle prices leading corn and soybean prices during the first period is that under regular conditions, an increase in live or fed cattle prices may potentially induce corn growers to likewise increase their prices since more than 50% of corn is used for feed rations, and soybean is a relevant protein ration. These mildly rising prices may be seen from figure 4.1, where both corn and soybean follow a rise in the prices of live cattle, especially towards the second half of the first estimated time period.

Two unexpected results regard the anticipated relationship between corn and feeder cattle prices, and corn and soybean prices. In the first case, corn price changes have no Granger Causality on feeder cattle price changes, especially for the second estimated period. This Granger Causality may have been expected according to the literature with reference to cattle production profitability (i.e., rising corn prices, especially in the second period, should lead to lower feeder cattle prices). This lack of a relationship is corroborated by the null response obtained on feeder cattle markets from a shock to corn prices as can be seen from Figure 4.2.2. However, there is a negative long run relationship determined between these two commodities, and it is identified by the negative error correction term during the second estimated period. Thus the anticipated inverse relationship is duly captured in the long-run.

The other unexpected result is that for both estimated periods, price changes in corn have no Granger Causality effect on price changes of soybean, nor vice-versa. These results are likewise determined from the null impulse response functions obtained from shocks on either market. This may be observed in figures 4.2.1 and 4.2.2 from shocks to corn prices and from figures 4.3.1 and 4.3.2 from shocks to soybean prices. These unusual results are obtained despite the known positive relationship between corn and soybeans as they share planting acreage.

Nonetheless, for the second estimated period a long-run positive relationship has been determined between corn and soybean prices through the error correction term from the co-integrated VAR model. This positive value confirms their long-run direct relationship, especially for the second period where much acreage was taken from soybean for corn production.

4.7 Conclusions

A multivariate non-structural model was applied to gauge the interrelationship between grain and cattle daily cash market prices, and to specifically contrast the results determined from two different estimated periods, a pre-ethanol mandated period and a post-ethanol mandated period. The objective of the paper sought to determine the influence that a surge in the demand of corn consumption, with its increased price, may have on its related markets. These six interrelated markets to corn – sorghum being a direct substitute for carbohydrate feed ration of cattle production, soybean being a direct substitute for planted acreage, wheat being a less direct carbohydrate substitute in feed rations for livestock, as well as feeder cattle and live cattle being major components of price risk for cattle production along with corn; are each impacted by corn's change in demand and price in a unique form. The effect of the changes in corn prices on these markets is analyzed within a co-integrated VAR model framework.

The main results are consistent with the literature such that steadily rising corn prices, due to mandated ethanol production, lead to rising sorghum prices. This may be in response to livestock producers modifying the feeding rations from corn to sorghum as anticipated by the literature, since both are carbohydrates. Also feeder cattle and live cattle price changes have bi-directional Granger Causality for both periods estimated. This is in line with cattle production profitability, as both these prices are the main sources of risk for cattle production.

A result which may not have been anticipated was that price changes in corn, specifically in the second period estimated, also led (Granger Causality) price changes in wheat, and vice-versa, the price changes in wheat likewise led price changes in corn. This bi-directional Granger Causality may respond to wheat additionally being used as carbohydrate in the feed ration. Null responses on wheat prices from shocks to corn prices were obtained; however a long-run positive relationship between corn and wheat was determined with the error correction term.

Two unexpected results are that corn price changes had no Granger Causality on feeder cattle prices during the second period estimated (i.e. during rising corn prices) as was anticipated by the literature for increasing corn prices in order to maintain cattle production profitability. It is also unexpected that corn and soybeans had no Granger Causality among them (not even bi-directional) for either period estimated, despite their acreage relationship. However, for both these cases there was an error correction term which accounted for the long-run relationship between these commodities and with the proper sign. That is, there was an inverse long-run relationship established between corn and feeder cattle given by a negative value for the error correction term, and there was a positive long-run relationship determined between corn and soybeans identified by a positive value for the error correction term. These error correction terms corroborate the literature regarding the anticipated relationship between these commodities.

CONCLUSIONS

This study analyzed issues regarding the risk that crop and livestock production faces with respect to price changes and volatility. Commodity prices for agricultural products – such as crops and/or livestock, constitute an important source of risk for farmers, livestock producers, consumers, and investors, among others. Fluctuation in these prices that include sharp spikes and plunges or slight increases and drops, generates risk called volatility. This volatility is steadily changing through time as new information arrives

Producers of crops and livestock generally face an inelastic demand for their products. That is, different consumers tend to purchase regular amounts of these crop and livestock products, despite changes that may occur in their selling price. On the other hand, supply from these crop and livestock producers may encounter different unexpected production shocks. Particular examples of production shocks may be adverse weather conditions or new environmental policies being implemented, thus affecting their production or supply. These shocks affecting production generate a rise in price changes or price variability, bringing about price risk called price volatility. This price risk faced by producers is further increased when it includes varying prices of production inputs, as in the case of feed for livestock which consists of rations of different crops.

The first chapter addressed matters with respect to the reduction of price risk by crop producers through the use of crop insurance. More specifically, this chapter addressed shortfalls concerning crop revenue insurance contracts. These shortfalls referred mainly to the steady average yearly losses[1] that had resulted from crop revenue insurance contracts, most recently due to premium subsidies. The study first discussed distributional assumptions of crop prices being used by the most typical crop revenue insurance contracts – Crop Revenue Coverage (CRC) and Revenue Assurance with Base Price (RA-BP). These distributions are Normal and Log-normal, respectively. Large amount of empirical evidence holds that price returns tend to have a positively skewed distribution and fatter tails (i.e. leptokurtosis) than Log-normal distributions.

With actual price data, the study compared goodness of fit of these assumed distributions with respect to a different, more flexible distribution – the Burr XII distribution, which is able to capture different shapes of the prices (i.e. this distribution identifies via parameters, more than just the mean and variance of the price data). Results indicated a better characterization of the prices with the Burr XII distribution over the Normal distribution, yet there was not considerable significant improvement over the Log-Normal distribution of prices.

The study then introduced a Copula method to determine the inverse relation between crop prices and yields, by considering yields from corn, soybean and wheat from a specific county and also from a specific state[2]. A Copula method is able to determine the relation between many variables without requiring the variables to assume a particular distribution. Subsequently, simulations for the expected payout of a crop revenue insurance coverage level where made by using this copula method, and contrasting different coverage scenarios to the case of using the CRC and RA-BP crop revenue insurance contract. Results indicated potential gains in efficiency for crop insurance with a copula method for state crop yields, as expected payouts decreased.

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[1] In the tens and hundreds of millions of dollars, approximately from 1940 to 1980, and 1981 to 1994, respectively per Goodwin and Smith (1995).
[2] Kossuth County in the Iowa state for corn and soybeans, and Sumner County in the Kansas state for wheat.

The next chapter extended the Regime Switching Dynamic Correlations (RSDC)[3] model. This model is able to measure price risk or price volatility in a multivariate time series setting; yet more importantly, the RSDC model is able to determine the changing or dynamic correlations between the series. The model considers the price series switching from one correlation regime to a different correlation regime according to a Markov chain - with constant transition probabilities determining the change from one regime to another regime.

The study extended and developed a dynamic friction model based on the RSDC model, by modifying the transition probabilities that govern the switching process between regimes - from constant probabilities to state dependent or time-varying probabilities.[4] The model extension introduced underlying fundamental economic variables (i.e. weakly exogenous variables) in the probabilities that determine the switch from one regime to another. The new regime switching probabilities, now state dependent or time-varying, incorporate underlying fundamental economic variables that are directly related to the evolution of the series being studied. The economic related variables have a direct impact on the series remaining at one correlation regime or switching to a different correlation regime. These underlying economic related variables included in the regime switching probabilities, identify specific friction levels which have particular effects in the dynamic process. This friction level becomes proportional to the estimated parameter of the underlying variable - in the state dependent transition probabilities.

During periods of increasing changes in price levels and rising volatilities, it is especially relevant to determine both the dynamic market linkages among related markets or assets and the evolution of the transmission of price changes between these related markets. This model permits

[3] from Pelletier (2006).
[4] following Diebold et al. (1994)

a better portrayal of the price relationships and transmission between markets for operational improvement, as well as providing better information for risk management purposes. This extended model determines the dynamic correlation values between multiple related markets, including the particular effect that underlying fundamental economic variables have in the evolution of these markets. Capturing the impact of these underlying variables enables to also determine the response effect that specific shocks on these variables may produce on the dynamic process of these series. The model is also applicable for managing risk through multiproduct hedging, and may provide better information for potential increases of efficiency in the operation of related markets.

The chapter makes use of this new extended RSDC model - the state dependent regime switching model of dynamic correlations, to determine dynamic price transmissions or linkages between corn, soybean and feeder and fed cattle markets. The study identified the specific impact produced by the recent increase in corn consumption used for ethanol production, on the dynamic price relationships among these markets. This surge in ethanol production using corn as input responded to the Energy Acts of 2005 and 2007, which mandated substantial increases in ethanol consumption.

These market prices, in particular the grain prices, rose sharply from 2006 until mid 2008 – approximately two-fold in the case of corn, and also increased steeply for soybeans. This study analyzed the potential impact livestock markets may have experienced by these increasing prices and significant volatility shocks, given that more than half of the corn production is used as animal feed and soybeans is also an important feed source as proteins. This study encompassed the period between 1998 and 2008 and parameter estimation was partitioned into two separate time periods – from 1998 to 2004 and the latest period beginning late 2003 until 2008, specifically when corn faced a consumption boost from ethanol production. Data for grains and cattle markets are from the Chicago Board of Trade (CBOT) and the Chicago Mercantile Exchange (CMEX), respectively.

Many decision makers are affected by these rising grain commodity prices, coupled with changes in their volatility. Agents with a direct relationship with these grain markets - corn and also soybeans (oilseed), are particularly affected by these price variations. Corn and soybean farmers must decide acreage allocation for either corn and/or soybean in their farm, seeking to obtain higher profitability. Livestock cattle producers require these crops as inputs, affecting their costs and profitability. These agents benefit from an appropriate determination of the dynamic interrelationships among these markets, as it may lead to efficiency gains in their operation through risk management. At the same time, policy makers need to determine the impact that recent energy policies – in this case directly affecting corn consumption, are having on the prices and markets related with this grain.

Our estimation results captured asymmetric correlations between grains and livestock prices, including volatility spillovers. These volatility spillovers are characterized by the resulting persistence of markets remaining at certain correlation levels, instead of switching to a different correlation level. In addition, we find potential inhibition in the transmission of prices between markets, as a response to adjustment costs between these markets.

Results obtained are consistent with past literature. Positive dynamic correlations between corn and soybean prices are obtained, concordant with the two crops sharing planting acreage. Also positive dynamic correlations for prices between feeder cattle and live cattle markets are verified, as these are the main components of cattle production. These results are determined for both periods estimated - pre and post mandated ethanol corn consumption. For the period of post mandated ethanol consumption (i.e. when corn and soybean experience sharply rising prices), an inverse or negative dynamic correlations is determined for both - corn and feeder cattle markets and for soybean and feeder cattle markets. The inverse corn and feeder cattle relation is also consistent with previous literature, where increases in prices of corn i.e. a main regular feed component for livestock, leads to a decrease in the price of feeder cattle in order to maintain cattle production profitability. Similar results are obtained for soybeans, which is another relevant feed component.

No significant correlation is obtained between corn prices (used as feed) and fed or live cattle markets, for both periods considered. Thus an existing permanent friction level is identified - responding to a transaction or adjustment cost which inhibits the transmission of price variations between corn prices and live cattle prices. This transaction or adjustment cost may respond to information and/or negotiation[5] costs from cattle producers selling slaughter cattle, and is materialized in the form of modifications of the feed rations given to cattle for weight gain during cattle production; and thus preventing increases in the price of these crops to be transmitted to the live cattle prices.

Information costs for cattle producers may involve price uncertainty for the case of fed cattle being sold at cash or spot markets[6], leading producers to switch feed rations with increasing costs to feed components with lower costs in order to maintain costs down during production.

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[5] Per Hobbs (1997)

[6] Cash or spot markets here refer to transactions ‘on the spot’. These include auction barn sales; video or electronic auction sales; sales through order buyers, dealers and brokers, and direct trades as per RTI (2007).

Negotiation costs may respond to the limited number of cash or spot markets faced by producers when fed cattle are ready to be slaughtered, thus raising the transaction cost for using that channel. For producers selling directly with alternative marketing arrangements[7] to packers, negotiation costs may arise for the case of having only a few different packers to bargain with, thus resulting in the packer exercising marketing power over the cattle producer. This again leads the cattle producer to seek feed rations that do not experience cost increases, i.e. modifying rations when there is a price rise in some of the components.

The impact or effect of significant underlying economic factor(s) are identified by taking into account (1.) the effect of the 'use to stocks' ratio of corn – which comprises the market's dynamic demand and supply conditions for corn, and also by considering (2.) the ratio of soybeans to corn futures' harvest prices – which is a measure contemplated by crop producers when deciding the acreage portion to either plant corn or soybeans. Both of these factors have a role on spillover effects among the markets. Results previously mentioned are determined separately by two different models – a restricted or parsimonious version and the full unrestricted version. The model with a mild better fit considers the 'use to stock' ratio from corn as the relevant underlying economic variable.

A relevant point determined during the post ethanol corn consumption scenario is the identification of the significance of corn's "use to stock" ratio in the state dependent transition probabilities - the slightly preferred model. This results in the determination of the volatility spillover effects, or markets (e.g. corn and soybeans, or feeder and fed cattle) remaining at

[7] These arrangements accounted for less than 40 percent of all Fed Cattle volume from October of 2002 to March 2005; and includes forward contracts, marketing agreements, procurement or marketing contracts, production agreements, packer ownership, custom feeding and slaughter; per RTI (2007)

certain correlation regimes instead of switching to a different correlation regime. These spillover effects are identified by explicitly taking into account the variable that includes the dynamic (i.e. up to date) market demand and supply conditions of corn. This variable – the use to stock ratio of corn, has incorporated the impact of the increase in demand of corn resulting from the surge in ethanol production. Thus the effect of ethanol production on corn's rapidly growing consumption is contained in the dynamic use to stock ratio of corn, during the post ethanol corn consumption period. And the explicit impact of this increased corn consumption, from ethanol production, is captured and/or identified by the resulting spillover effects produced among the markets.

Hence the extended model permits the inclusion of economic fundamental variables into the evolution of the dynamic correlations among the markets, indicating an improved fit and characterization of the process than the case of the initial model with constant transition probabilities. Yet more importantly, the impact of significant fundamental economic variable(s) on the evolution of the market prices is identified and captured by the spillover effects, produced among the correlations between the markets. These spillover effects may assist agents involved in these markets with better operational and risk management information. The model also assists in the analysis of the effects of implementing certain energy policies or may also serve to study the effect of implementing related environmental policies.

The final chapter makes use of a Co-Integrated vector auto regressive model (i.e. Co-Integrated VAR), also known as an error correction vector (VEC) model, to further gauge the dynamic relations between grains and cattle markets. The co-integration factor refers to an error correction term among the variables that estimates the long-run relation between the markets considered. This model also permits the determination of Granger Causality among the variables, but more importantly, it allows for the analysis of accumulated responses on each market from the effects of individual shocks done to a market. (i.e. what type of response does a market have when shocking a related variable).

In this study, we consider daily data instead of prior study with weekly data, and include not only the variables considered before (i.e. corn, soybeans, feeder cattle and live cattle), but also grain sorghum (also known as milo) and wheat. It is important to note that these latter grains are main elements of carbohydrate substitution for corn used in feed rations for livestock. The time periods considered are once again for intervals of pre and post mandated ethanol production, which has mostly relied on corn consumption. Estimations were made from January 1998 to December 2004 and from January 2004 to April 2009, with grains data obtained from the CBOT and cattle data obtained from the CMEX. Two different unit roots tests conducted confirm the presence of a unit root for each of the series during both periods considered (i.e. the series are non-stationary for each period).

Some results are consistent with past literature, as in the case of corn and grain sorghum having bi-directional Granger Causality among them during the first or pre-mandated ethanol period (i.e. changes in prices of corn lead the changes in prices of sorghum, and also changes in prices of sorghum lead the changes in prices of corn). Yet for the post-mandated ethanol period, it is only price changes in corn which produce Granger Causality in price changes in sorghum. Thus in the second period corn price changes are leading those of sorghum, being this a plausible result from substituting corn at higher prices, for sorghum in livestock feed rations. These results are corroborated by the response function of sorghum from shocks to corn, and from the response function of corn from shocks to sorghum, for each period estimated. As there is a higher response on sorghum obtained from a corn shock in the second period, when compared to the response during the first period.

Another result anticipated from the literature is that feeder cattle and live cattle price changes have bi-directional Granger Causality during the first period and also during the second period estimated. This result is anticipated as previous studies have shown that both of these prices are the main risk components in cattle production profitability, and hence affect each other. This result is also corroborated by our prior model's estimation, where there is a significant dynamic correlation between feeder cattle and live cattle markets.

Two unexpected results obtained are on the one hand, not determining any Granger Causality relation between changes in prices of corn and feeder cattle. This result is unexpected, especially when taking into account the second period where there is a substantial increase in the price of corn. The higher corn price should lead to lower feeder cattle prices in order to maintain cattle production profitability, as anticipated by the literature. Nonetheless, a long-run inverse relationship is properly determined during this second period, given by the negative error correction term from the VEC model.

Another unexpected result is that there is no Granger Causality in any direction between price changes of corn and soybean, for either period estimated. The lack of relation between these two markets is corroborated by the null accumulated response on a market from price shocks to the other market. The relation between price changes of corn and soybeans is expected, since these crops share planting acreage (i.e. depending on the crop's price and profitability, farmers will regularly choose planting either corn or soybean). However, once again there is a long-run positive relationship between corn and soybean determined by the VEC model during the second period estimated. This corroborates the relationship between these crops, especially during the period where acreage was taken from soybeans for increased corn production and subsequently returned.

In conclusion, this study of certain aspects regarding the effect of price risk in agricultural commodity markets focuses on just a few particular topics. The study aimed at providing some novel tools for improving efficiency for crop insurance policy makers, and also making available better information for, though not limited to, farmers and cattle producers. Further research lines may be in various directions, either by using copulas for insurance contracts with other crops or in livestock, or by using the state dependent regime switching model with the corresponding economic fundamental variables for risk management hedging possibilities, as well as policy analysis, or also by applying the Co-integrated VAR model for analysis of price transmissions in a different setting.

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APPENDICES

Appendix 2.A.

One Step Copula Estimation method:

Take n independent realizations from a multivariate distribution, $\{(X_{i1}, \ldots \ldots \ldots, X_{ip})^T : i = 1, \ldots \ldots, n\}$. Our de-trended yield and price data may still be considered 'sequentially' correlated, hence not formally independent; yet estimation through this method may be considered as a second-best approach with respect to a different method that doesn't make use of this.

Assume the multivariate distribution may be specified by p marginal cumulative distributions cdf F_i & density distributions pdf $f_i, i = 1 \ldots . p$; and a copula with density $\boldsymbol{c}$.

Consider β the vector of marginal parameters and α the vector of copula parameters. The parameter vector to be estimated is $\theta = (\beta^T, \alpha^T)^T$. The loglikelihood function is:

$$l(\theta) = \sum_{i=1}^{n} \log \boldsymbol{c}\{F_1(X_{i1}\,; \beta), \ldots \ldots \ldots, F_p\big(X_{ip}; \beta\big); \alpha\} + \sum_{i=1}^{n} \sum_{j=1}^{p} \log f_i\,(X_{ij}; \beta)$$

Being the ML estimator of θ:

$$\hat{\theta}_{ML} = \arg \max_{\theta \epsilon \Theta} l(\theta) \quad \text{where } \Theta \text{ is the parameter space.}$$

Two Step Copula Estimation method:

Considering a substantial increase in the dimensions (p) of the Multivariate distribution, the previous method may be more difficult. Hence a two step optimization method may be more expeditious and reach similar results, proposed by Joe and Xu, 1996.

This method, called inference functions for margins (IFM) estimates the marginal parameters β, in a first step:

$$\hat{\beta}_{IFM} = \arg\max_{\beta} \sum_{i=1}^{n} \sum_{j=1}^{p} \log f_i \left(X_{ij};\ \beta\right)$$

And then estimates the parameters of association α given by $\hat{\beta}_{IFM}$, by:

$$\hat{\alpha}_{IFM} = \arg\max_{\alpha} \sum_{i=1}^{n} \log \boldsymbol{c} \left(F_1(X_{i1};\ \hat{\beta}_{IFM}), \dots \dots \dots \dots F_p(X_{ip};\ \hat{\beta}_{IFM});\ \alpha\right)$$

When each marginal distribution F_i has its own parameter set β_i , such that $\beta = (\beta_1^T, \dots \dots \dots \beta_p^T)^T$, *then* the first step involves a MLE for each margin $j = 1, \dots \dots . ., p$:

$$\hat{\beta}_{jIFM} = \arg\max_{\beta j} \sum_{i=1}^{n} logf \left(X_{ij};\ \beta_j\right)$$

Appendix 2.B.

CORN - State Iowa

Frank Copula

> fit.cml *w/ Cumulative data for yields & prices*

The ML estimation is based on 47 observations.

	Estimate	Std. Error	z value	Pr(>|z|)
param	**-1.94895**	0.9228	-2.112	0.0347

The maximized loglikelihood is **2.191**

The convergence code is 0

tau_fl = **-0.2088**

> fit.ifl *w/ Beta for yields & Burr 12 for prices*

The ML estimation is based on 47 observations.

	Estimate	Std. Error	z value	Pr(>|z|)
param	**-2.21131**	1.0169	-2.175	0.0297

The maximized loglikelihood is **2.238**

The convergence code is 0

tau_fl = **-0.2346**

> fit.ifl *w/ Beta for yields & LogNormal for prices*

The ML estimation is based on 47 observations.

	Estimate	Std. Error	z value	Pr(>|z|)
param	**-2.34575**	1.0287	-2.280	0.0226

The maximized loglikelihood is **2.446**

The convergence code is 0

tau_fl = **-0.2475**

CORN - Kossuth (IA)

Frank Copula

> fit.cml *w/ Cumulative data for yields & prices*

The ML estimation is based on 47 observations.

	Estimate	Std. Error	z value	Pr(>|z|)
param	**-1.10426**	0.8654	-1.276	0.2020

The maximized loglikelihood is **0.809**

The convergence code is 0

tau_f1k = -0.1212

> fit.ifl *w/ Beta for yields & Burr 12 for prices*

The ML estimation is based on 47 observations.

	Estimate	Std. Error	z value	Pr(>|z|)
param	**-1.29985**	0.9726	-1.336	0.1814

The maximized loglikelihood is **0.867**

The convergence code is 0

tau_f1k = -0.1421

> fit.ifl *w/ Beta for yields & LogNormal for prices*

The ML estimation is based on 47 observations.

	Estimate	Std. Error	z value	Pr(>|z|)
param	**-1.46076**	0.9830	-1.486	0.1373

The maximized loglikelihood is **1.066**

The convergence code is 0

tau_f1k = -0.1590

<u>CORN - State Iowa</u>

<u>Normal Copula</u>

> fit.cml w/ Cumulative data for yields & prices

The ML estimation is based on 47 observations.

	Estimate	Std. Error	z value	Pr(>|z|)
rho.1	**-0.31124**	0.1283	-2.424	0.0154

The maximized loglikelihood is **2.283**

The convergence code is 0

tau_n1 = **-0.2015**

> fit.ifl w/ Beta for yields & Burr 12 for prices

The ML estimation is based on 47 observations.

	Estimate	Std. Error	z value	Pr(>|z|)
rho.1	-0.20913	0.1365	-1.532	0.1256

The maximized loglikelihood is 1.051

The convergence code is 0

tau_n1 = -0.1341

> fit.ifl w/ Beta for yields & LogNormal for prices

The ML estimation is based on 47 observations.

	Estimate	Std. Error	z value	Pr(>|z|)
rho.1	-0.21981	0.1341	-1.638	0.1013

The maximized loglikelihood is 1.191

The convergence code is 0

tau_n1 = -0.1411

<u>CORN - Kossuth (IA)</u>

<u>Normal Copula</u>

> fit.cml w/ Cumulative data for yields & prices

The ML estimation is based on 47 observations.

	Estimate	Std. Error	z value	Pr(>|z|)
rho.1	**-0.22885**	0.1379	-1.659	0.0970

The maximized loglikelihood is **1.202**

The convergence code is 0

tau_n1k = **-0.1470**

> fit.ifl w/ Beta for yields & Burr 12 for prices

The ML estimation is based on 47 observations.

	Estimate	Std. Error	z value	Pr(>|z|)
rho.1	-0.22938	0.1323	-1.733	0.0830

The maximized loglikelihood is 1.319717

The convergence code is 0

tau_n1k = -0.1473

> fit.ifl w/ Beta for yields & LogNormal for prices

The ML estimation is based on 47 observations.

	Estimate	Std. Error	z value	Pr(>|z|)
rho.1	-0.23707	0.1303	-1.819	0.0689

The maximized loglikelihood is 1.442836

The convergence code is 0

tau_n1k = -0.1524

SOYBEAN - State Iowa

Frank Copula

```
>fit.cml     w/ Cumulative data for yields & prices
The ML estimation is based on  47  observations.
          Estimate   Std. Error   z value   Pr(>|z|)
param     -0.67146   0.89793      -0.7477   0.4545
The maximized loglikelihood is  0.277
The convergence code is  0

tau_f2 =    -0.0743
```

```
> fit.ifl    w/ Beta for yields & Burr 12 for prices
The ML estimation is based on  47  observations.
          Estimate   Std. Error   z value   Pr(>|z|)
param     -1.53777   1.127278     -1.3641   0.1725
The maximized loglikelihood is  0.852
The convergence code is  0

tau_f2 =    -0.167
```

```
> fit.ifl    w/ Beta for yields & LogNormal for prices
The ML estimation is based on  47  observations.
          Estimate   Std. Error   z value   Pr(>|z|)
param     -1.62634   1.129924     -1.4393   0.1500
The maximized loglikelihood is  0.952
The convergence code is  0

tau_f2 =      -0.1761
```

SOYBEAN - Kossuth (IA)

Frank Copula

```
> fit.cml    w/ Cumulative data for yields & prices
The ML estimation is based on  47  observations.
          Estimate   Std. Error   z value   Pr(>|z|)
param     0.197245   0.847908     0.2326    0.8160
The maximized loglikelihood is  0.027
The convergence code is  0

tau_f2k =          0.0219
```

```
> fit.ifl    w/ Beta for yields & Burr 12 for prices
The ML estimation is based on  47  observations.
          Estimate   Std. Error   z value   Pr(>|z|)
param     0.490549   0.992243     0.4943    0.6210
The maximized loglikelihood is  0.1218339
The convergence code is  0

tau_f2k =     0.0544
```

```
> fit.ifl    w/ Beta for yields & LogNormal for prices
The ML estimation is based on  47  observations.
          Estimate   Std. Error   z value   Pr(>|z|)
param     0.371561   0.989029     0.3756    0.707
The maximized loglikelihood is  0.07035596
The convergence code is  0

tau_f2k =          0.0412
```

SOYBEAN - State Iowa

Normal Copula

> fit.cml w/ Cumulative data for yields & prices

The ML estimation is based on 47 observations.

	Estimate	Std. Error	z value	Pr(>|z|)
rho.1	-0.17786	0.142569	-1.2475	0.2122

The maximized loglikelihood is 0.71697

The convergence code is 0

tau_n2 = -0.1138

> fit.ifl w/ Beta for yields & Burr 12 for prices

The ML estimation is based on 47 observations.

	Estimate	Std. Error	z value	Pr(>|z|)
rho.1	-0.26024	0.129854	-2.0041	0.04506

The maximized loglikelihood is 1.6974

The convergence code is 0

tau_n2 = -0.1676

> fit.ifl w/ Beta for yields & LogNormal for prices

The ML estimation is based on 47 observations.

	Estimate	Std. Error	z value	Pr(>|z|)
rho.1	-0.24791	0.12964	-1.9123	0.0558

The maximized loglikelihood is 1.573202

The convergence code is 0

tau_n2 = -0.1595

SOYBEAN - Kossuth (IA)

Normal Copula

> fit.cml w/ Cumulative data for yields & prices

The ML estimation is based on 47 observations.

	Estimate	Std. Error	z value	Pr(>|z|)
rho.1	0.02954	0.14971	0.1973	0.8436

The maximized loglikelihood is 0.01942759

The convergence code is 0

tau_n2k = 0.0188

> fit.ifl w/ Beta for yields & Burr 12 for prices

The ML estimation is based on 47 observations.

	Estimate	Std. Error	z value	Pr(>|z|)
rho.1	0.08109	0.14243	0.56937	0.5691

The maximized loglikelihood is 0.15938

The convergence code is 0

tau_n2k = 0.0517

> fit.ifl w/ Beta for yields & LogNormal for prices

The ML estimation is based on 47 observations.

	Estimate	Std. Error	z value	Pr(>|z|)
rho.1	0.078346	0.14068	0.5569	0.57759

The maximized loglikelihood is 0.15277

The convergence code is 0

tau_n2k = 0.0499

WHEAT - State Kansas

Frank Copula

> fit.cml w/ Cumulative data for yields & prices

The ML estimation is based on 36 observations.

	Estimate	Std. Error	z value	Pr(>|z|)
param	-2.2042	1.09264	-2.017	0.043661

The maximized loglikelihood is 2.0215

The convergence code is 0

tau_f3 = -0.2339

> fit.ifl w/ Beta for yields & Burr 12 for prices

The ML estimation is based on 36 observations.

	Estimate	Std. Error	z value	Pr(>|z|)
param	-2.7136	1.1999	-2.26135	0.0237

The maximized loglikelihood is 2.401316

The convergence code is 0

tau_f3 = -0.2817

> fit.ifl w/ Beta for yields & LogNormal for prices

The ML estimation is based on 36 observations.

	Estimate	Std. Error	z value	Pr(>|z|)
param	-3.36079	1.256335	-2.67507	0.00747

The maximized loglikelihood is 3.223018

The convergence code is 0

tau_f3 = -0.3379

WHEAT - Sumner (KS)

Frank Copula

> fit.cml w/ Cumulative data for yields & prices

The ML estimation is based on 36 observations.

	Estimate	Std. Error	z value	Pr(>|z|)
param	-0.57286	1.0270	-0.5577	0.577

The maximized loglikelihood is 0.155246

The convergence code is 0

tau_f3sm = -0.0634

> fit.ifl w/ Beta for yields & Burr 12 for prices

The ML estimation is based on 36 observations.

	Estimate	Std. Error	z value	Pr(>|z|)
param	-0.75552	1.39390	-0.542	0.5878

The maximized loglikelihood is 0.1417135

The convergence code is 0

tau_f3sm = -0.0835

> fit.ifl w/ Beta for yields & LogNormal for prices

The ML estimation is based on 36 observations.

	Estimate	Std. Error	z value	Pr(>|z|)
param	-1.36017	1.45241	-0.9365	0.34902

The maximized loglikelihood is 0.3952323

The convergence code is 0

tau_f3sm = -0.1484

WHEAT - State Kansas

Normal Copula

fit.cml w/ Cumulative data for yields & prices

The ML estimation is based on 36 observations.

	Estimate	Std. Error	z value	Pr(>|z|)
rho.1	-0.3264	0.14528	-2.247	0.0247

The maximized loglikelihood is 1.907639

The convergence code is 0

tau_n3 = -0.2117

> fit.ifl w/ Beta for yields & Burr 12 for prices

The ML estimation is based on 36 observations.

	Estimate	Std. Error	z value	Pr(>|z|)
rho.1	-0.32647	0.138612	-2.3553	0.01851

The maximized loglikelihood is 2.130426

The convergence code is 0

tau_n3 = -0.2117

> fit.ifl w/ Beta for yields & LogNormal for prices

The ML estimation is based on 36 observations.

	Estimate	Std. Error	z value	Pr(>|z|)
rho.1	-0.3132	0.139446	-2.2458	0.02472

The maximized loglikelihood is 1.983208

The convergence code is 0

tau_n3 = -0.2028

WHEAT - Sumner (KS)

Normal Copula

> fit.cml w/ Cumulative data for yields & prices

The ML estimation is based on 36 observations.

	Estimate	Std. Error	z value	Pr(>|z|)
rho.1	-0.1000	0.17000	-0.5883	0.5563

The maximized loglikelihood is 0.1685749

The convergence code is 0

tau_n3sm = -0.0638

> fit.ifl w/ Beta for yields & Burr 12 for prices

The ML estimation is based on 36 observations.

	Estimate	Std. Error	z value	Pr(>|z|)
rho.1	-0.1701	0.15381	-1.1057	0.26887

The maximized loglikelihood is 0.5702493

The convergence code is 0

tau_n3sm = -0.1088

> fit.ifl w/ Beta for yields & LogNormal for prices

The ML estimation is based on 36 observations.

	Estimate	Std. Error	z value	Pr(>|z|)
rho.1	-0.1672	0.15277	-1.0947	0.27362

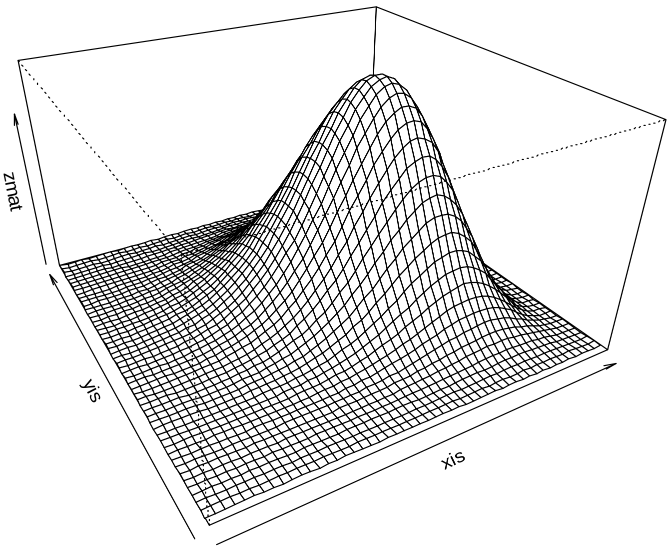
The maximized loglikelihood is 0.560695

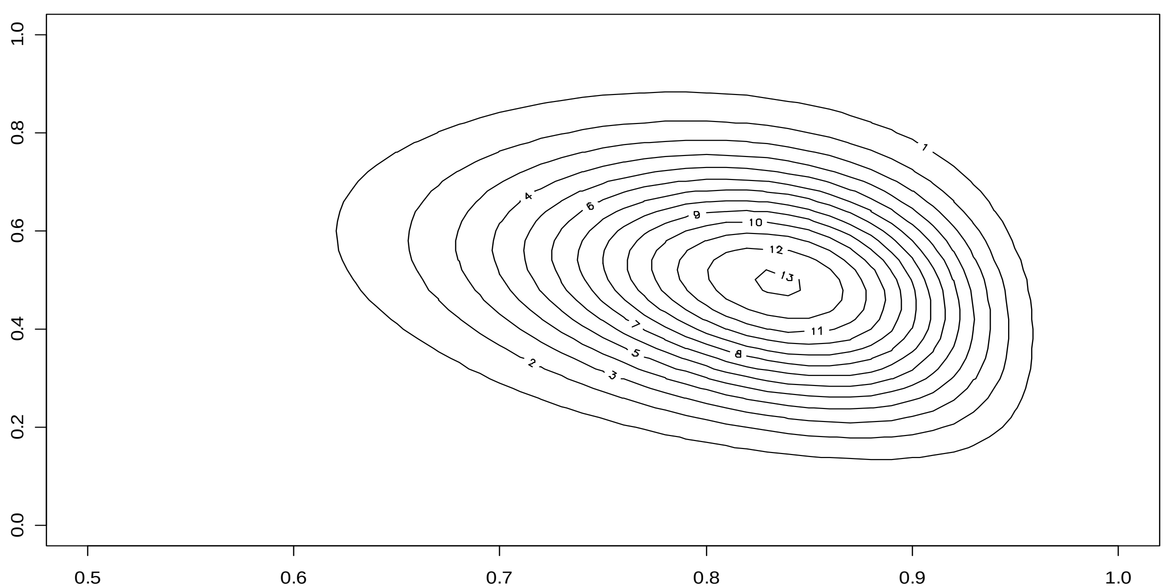
The convergence code is 0

tau_n3sm = -0.1070

Appendix 2.C.

Iowa State, Soybean Normal Copula,rho= -0.2602





Appendix 3.A.

Expectation Step (as per Hamilton 1990):

Substitution of ‘smoothed state’ probabilities for the indicator functions in the complete-data log likelihood (from 3. above):

$$
\begin{aligned}
\mathrm{E}\,[\log f(\vec{y}_t, \vec{\Delta}_t \,|\, \vec{x}_t;\ \theta^{(j-1)})] = \ & \rho^{(j-1)}\,[\,\log f\big(y_1 \big| \Delta_1 = 1;\ \propto_1^{(j-1)}\big) + \log \rho^{(j-1)}\,] + \\
& + (1 - \rho^{(j-1)})\,[\,\log f\big(y_1 \big| \Delta_1 = 0;\ \propto_0^{(j-1)}\big) + \log(1 - \rho^{(j-1)})\,] \\
& + \textstyle\sum_{t=2}^{T} \{ P(\Delta_t = 1 | \vec{y}_T, \vec{x}_T;\ \theta^{(j-1)}) \log f\big(y_t \big| \Delta_t = 1;\ \alpha_1^{(j-1)}\big) + \\
& + P(\Delta_t = 0 | \vec{y}_T, \vec{x}_T;\ \theta^{(j-1)}) \log f\big(y_t \big| \Delta_t = 0;\ \alpha_0^{(j-1)}\big) + \\
& + P(\Delta_t = 1, \Delta_{t-1} = 1 |\ \vec{y}_T, \vec{x}_T;\ \theta^{(j-1)}) \log(p_t^{11}) + \\
& + P(\Delta_t = 0, \Delta_{t-1} = 1 | \vec{y}_T, \vec{x}_T;\ \theta^{(j-1)}) \log(1 - p_t^{11}) + \\
& + P(\Delta_t = 1, \Delta_{t-1} = 0 |\ \vec{y}_T, \vec{x}_T;\ \theta^{(j-1)}) \log(1 - p_t^{00}) + \\
& + P(\Delta_t = 0, \Delta_{t-1} = 0 | \vec{y}_T, \vec{x}_T;\ \theta^{(j-1)}) \log(p_t^{00}) \}
\end{aligned}
$$

The ‘smoothed state’ probabilities are obtained from the optimal nonlinear smoother, conditional upon the current ‘best guess’ of θ, i.e. $\theta^{(j-1)}$.

Appendix 3.B.

Algorithm for calculating the 'smoothed state' probabilities for state *j*, **given** $\theta^{(j-1)}$, $\vec{y}_T$ and $\vec{x}_T$:

1.i. Calculate the Conditional densities of y_t, i.e. $f\left(y_t \middle| \Delta_t;\ \alpha_i^{(j-1)}\right)$ by (3.3): a (T x 2) matrix

ii. Calculate Transition Probabilities matrix Π_t as per pg. 19: which is a (T-1) x 4 matrix

2. Calculate 'filtered' joint state probabilities ((T-1) x 4 matrix) by iterating on steps 2a to 2d below, for t = 2,…,T

2.a. Calculate the joint conditional distribution of $(y_t, \Delta_t, \Delta_{t-1})$ (i.e. FOUR #'s) given $\vec{y}_{t-1}$ and $\vec{x}_{t-1}$:

For t = 2, joint conditional distribution is:

$$f(y_2, \Delta_2, \Delta_1 | y_1, x_1;\ \theta^{(j-1)}) = f\left(y_2 \middle| \Delta_2;\ \alpha^{(j-1)}\right) P(\Delta_2 | \Delta_1, x_1;\ \beta^{(j-1)}) P(\Delta_1)$$

For subsequent time periods t, the joint conditional distribution is:

$$f(y_t, \Delta_t, \Delta_{t-1} | \vec{y}_{t-1}, \vec{x}_{t-1};\ \theta^{(j-1)}) =$$

$$\sum_{\Delta_{t-2}=0}^{1} f(y_t | \Delta_t;\ \alpha^{(j-1)}) P\left(\Delta_t \middle| \Delta_{t-1}, x_{t-1};\ \beta^{(j-1)}\right) P(\Delta_t, \Delta_{t-1} | \vec{y}_{t-1}, \vec{x}_{t-1};\ \theta^{(j-1)})$$

where

$f(y_t | \Delta_t;\ \alpha^{(j-1)})$ *and* $P\left(\Delta_t \middle| \Delta_{t-1}, x_{t-1};\ \beta^{(j-1)}\right)$ are obtained from previous step 1.

and

$P(\Delta_t, \Delta_{t-1} | \vec{y}_{t-1}, \vec{x}_{t-1};\ \theta^{(j-1)})$ is the 'filtered probability' obtained for previous t.

2.b. Calculate conditional likelihood of y_t (ONE #):

(Adds up over All 'states' – in this case two regimes)

$$f\left(y_t \middle| \vec{y}_{t-1}, \vec{x}_{t-1};\ \theta^{(j-1)}\right) = \sum_{\Delta_t=0}^{1} \sum_{\Delta_{t-1}=0}^{1} f((y_t, \Delta_t, \Delta_{t-1} | \vec{y}_{t-1}, \vec{x}_{t-1};\ \theta^{(j-1)})$$

2.c. Calculate the time-t *filtered state* probabilities (FOUR #'s):

$$P(\Delta_t, \Delta_{t-1} | \vec{y}_t, \vec{x}_t;\ \theta^{(j-1)}) = {f(y_t, \Delta_t, \Delta_{t-1} | \vec{y}_t, \vec{x}_t;\ \theta^{(j-1)})} \Big/ {f(y_t | \vec{y}_{t-1}, \vec{x}_{t-1};\ \theta^{(j-1)})}$$

where the numerator is obtained for 2.a. (joint conditional distribution of $y_t, \Delta_t, \Delta_{t-1}$) and the denominator is the conditional likelihood of y_t , from 2.b.

2.d. These previous FOUR filtered probabilities are used as input for step 2.a. to calculate the filtered probabilities for the next time period and steps 2.a. – 2.d. are repeated (T-2) times.

3. The calculation of the 'smoothed' joint state probabilities as follows ((*T*-1) x 6) matrix:

3.a. For t = 2 and given values for (Δ_t, Δ_{t-1}), sequentially calculate the joint probability of $(\Delta_\tau, \Delta_{\tau-1}, \Delta_t, \Delta_{t-1})$ given $\vec{y}_\tau$ and $\vec{x}_\tau$, for τ = t + 2, t + 3, ………..*T:*

$$P(\Delta_\tau, \Delta_{\tau-1}, \Delta_t, \Delta_{t-1} | \vec{y}_\tau, \vec{x}_\tau;\ \theta^{(j-1)}) =$$

$$\frac{\sum_{\Delta_{\tau-2}=0}^{1} f(y_\tau | \Delta_\tau;\ \alpha^{(j-1)}) P\left(\Delta_\tau \middle| \Delta_{\tau-1}, x_{\tau-1};\ \beta^{(j-1)}\right) P(\Delta_{\tau-1} \Delta_{\tau-2}, \Delta_t, \Delta_{t-1} | \vec{y}_{\tau-1}, \vec{x}_{\tau-1};\ \theta^{(j-1)})}{f(y_\tau | \vec{y}_{\tau-1}, \vec{x}_{\tau-1};\ \theta^{(j-1)})}$$

where the first two terms in the numerator are from step 1., the third term in the numerator is from previous computation of step 3.a. , and the denominator by step 2.b.

when τ = t + 2, the numerator's third term is 'initialized' with:

$$P(\Delta_{t+1}, \Delta_t, \Delta_{t-1} | \vec{y}_{t+1}, \vec{x}_{t+1};\ \theta^{(j-1)}) =$$

$$\frac{f(y_{t+1} | \Delta_{t+1}\ ; \alpha^{(j-1)}) P\left(\Delta_{t+1} \middle| \Delta_t, x_t;\ \beta^{(j-1)}\right) P(\Delta_t, \Delta_{t-1} | \vec{y}_t, \vec{x}_t;\ \theta^{(j-1)})}{f(y_{t+1} | \vec{y}_t, \vec{x}_t;\ \theta^{(j-1)})}$$

The last term in the numerator is from 2.c.

For each value τ - a (4 x 1) vector of probabilities is produced corresponding to the four valuations of $(\Delta_\tau, \Delta_{\tau-1})$. Hence upon reaching $\tau = T$, we've calculated and saved a (T – 3) x 4 matrix; <u>in which the Last row is used at step 3.b. below</u>.

3.b. Once at $\tau = T$, then the 'smoothed joint state probability for time t' and the chosen valuation

of (Δ_t, Δ_{t-1}) is calculated as follows:

$$P(\Delta_t, \Delta_{t-1} | \vec{y}_T, \vec{x}_T;\ \theta^{(j-1)}) = \sum_{\Delta_T=0}^{1} \sum_{\Delta_{T-1}=0}^{1} P(\Delta_T, \Delta_{T-1}, \Delta_t, \Delta_{t-1} | \vec{y}_T, \vec{x}_T;\ \theta^{(j-1)})$$

3. c. Steps 3.a. and 3.b. are repeated for all possible time t valuations (Δ_t, Δ_{t-1}) (FOUR in this case), until a smoothed probability has been calculated for *each* of the four possible valuations. Now we have a (1 x 4) vector of 'smoothed joint state probabilities' for (Δ_t, Δ_{t-1}).

3.d. Steps 3.a. – 3.c. are repeated for t = 3, 4, …..T, obtaining a total of T – 1 x 4 smoothed joint state probabilities.

4. Smoothed 'marginal state probabilities' are found by summing over the smoothed joint state probabilities. For example:

$$P\left(\Delta_t = 1 \middle| \vec{y}_T, \vec{x}_T;\ \theta^{(j-1)}\right) = P(\Delta_t = 1, \Delta_{t-1} = 1 | \vec{y}_T, \vec{x}_T;\ \theta^{(j-1)}) +$$
$$P(\Delta_t = 1, \Delta_{t-1} = 0 | \vec{y}_T, \vec{x}_T;\ \theta^{(j-1)})$$

These (T – 1 x 6) 'smoothed state probabilities' (FOUR joint and TWO marginal) are used as input for the maximization step.

3.2. Maximization Step:

Once the smoothed probabilities are obtained, the expected complete-data log likelihood given by (3. from pg. 2) is maximized directly with respect to the model parameters.

The first order conditions for the non-linear (logit) transition probabilities parameter vector β, result in a closed form solution for $\beta_0^{(j)}$ and $\beta_1^{(j)}$:

$$\beta_0^{(j)} = (\sum_{t=2}^{T} P(s_{t-1} = 0|\vec{y}_T, \vec{x}_T;\ \theta^{(j-1)}) \frac{\partial \pi_t^{00}(\beta_0)}{\partial \beta_0})^{-1} \text{ x}$$

$$(\sum_{t=2}^{T} x_{t-1} \Big\{ P\big(s_t = 0, s_{t-1} = 0\big|\vec{y}_T, \vec{x}_T;\ \theta^{(j-1)}\big)$$

$$- P\big(s_{t-1} = 0\big|\vec{y}_T, \vec{x}_T;\ \theta^{(j-1)}\big) \left[\pi_t^{00}\Big(\beta_0^{(-1)}\Big) - \frac{\partial \pi_t^{00}(\beta_0)}{\partial \beta_0} \beta_0^{(j-1)}\right] \Big\})$$

$$\beta_1^{(j)} = (\sum_{t=2}^{T} P(s_{t-1} = 1|\vec{y}_T, \vec{x}_T;\ \theta^{(j-1)}) \frac{\partial \pi_t^{11}(\beta_1)}{\partial \beta_1})^{-1} \text{ x}$$

$$(\sum_{t=2}^{T} x_{t-1} \Big\{ P\big(s_t = 1, s_{t-1} = 1\big|\vec{y}_T, \vec{x}_T;\ \theta^{(j-1)}\big)$$

$$- P\big(s_{t-1} = 1\big|\vec{y}_T, \vec{x}_T;\ \theta^{(j-1)}\big) \left[\pi_t^{11}\Big(\beta_1^{(-1)}\Big) - \frac{\partial \pi_t^{11}(\beta_1)}{\partial \beta_1} \beta_1^{(j-1)}\right] \Big\})$$

Which in the case of 2 regimes, specifically becomes:

$$\beta_0^{(j)} = \begin{pmatrix} \beta_{00}^{(j)} \\ \beta_{01}^{(j)} \end{pmatrix} = (\begin{matrix} \sum_{t=2}^{T} x_{0,t-1} P(s_{t-1} = 0)\pi_{0t}^{00} & \sum_{t=2}^{T} x_{0,t-1} P(s_{t-1} = 0)\pi_{1t}^{00} \\ \sum_{t=2}^{T} x_{1,t-1} P(s_{t-1} = 0)\pi_{0t}^{00} & \sum_{t=2}^{T} x_{1,t-1} P(s_{t-1} = 0)\pi_{1t}^{00} \end{matrix})^{-1} \text{ x}$$

$$\begin{pmatrix} \sum_{t=2}^{T} x_{0,t-1}\{P(s_{t-1}=0, s_{t-1}=0) - P(s_{t-1}=0)\left[\pi_t^{00} - \frac{\partial \pi_t^{00}}{\partial \beta_0}\beta_0^{(j-1}\right]\} \\ \sum_{t=2}^{T} x_{1,t-1}\{P(s_{t-1}=0, s_{t-1}=0) - P(s_{t-1}=0)\left[\pi_t^{00} - \frac{\partial \pi_t^{00}}{\partial \beta_0}\beta_0^{(j-1)}\right]\} \end{pmatrix}$$

$$\beta_1^{(j)} = \begin{pmatrix}\beta_{10}^{(j)} \\ \beta_{11}^{(j)}\end{pmatrix} = \begin{pmatrix} \sum_{t=2}^{T} x_{0,t-1} P(s_{t-1}=1)\pi_{0t}^{11} & \sum_{t=2}^{T} x_{0,t-1} P(s_{t-1}=1)\pi_{1t}^{11} \\ \sum_{t=2}^{T} x_{1,t-1} P(s_{t-1}=1)\pi_{0t}^{11} & \sum_{t=2}^{T} x_{1,t-1} P(s_{t-1}=0)\pi_{1t}^{11} \end{pmatrix}^{-1} \text{x}$$

$$\begin{pmatrix} \sum_{t=2}^{T} x_{0,t-1}\{P(s_{t-1}=1, s_{t-1}=1) - P(s_{t-1}=1)\left[\pi_t^{11} - \frac{\partial \pi_t^{11}}{\partial \beta_1}\beta_1^{(j-1}\right]\} \\ \sum_{t=2}^{T} x_{1,t-1}\{P(s_{t-1}=1, s_{t-1}=1) - P(s_{t-1}=1)\left[\pi_t^{11} - \frac{\partial \pi_t^{11}}{\partial \beta_1}\beta_1^{(j-1}\right]\} \end{pmatrix}$$

Appendix 3.C.

Betas Estimated September 1998 - August 2008

RESTRICTED MODEL

	Constant Transition Probability (by DP)	**Standard Error**	***State Dependent Probability Ratio Sybn/Corn Price***	**Standard Error**	***State Dependent Probability Ratio USE_Stock***	**Standard Error**
Likelihood	***-4535.1***		***-4525.4***		***-4517.0***	
Γ - Correlation Regime 1.						
Corn -Soybean	*0.7374**	0.0457	*0.7424**	0.0440	*0.7569**	0.0304
Corn -Feeder Cattle	*-0.2216**	0.0635	*-0.2291**	0.0641	*-0.1952**	0.0546
Corn - Live Cattle	0.0334	0.0561	0.0275	0.0570	0.0324	0.0565
Soybean - Feeder Cattle	-0.0921	0.0680	-0.0954	0.0644	-0.1032+	0.0592
Soybean - Live Cattle	0.0738	0.0687	0.0742	0.0691	0.0667	0.0611
Feeder Cattle -Live Cattle	*0.8157**	0.0304	*0.8181**	0.0288	*0.8098**	0.0365
λ *Lambda Transition*	*0.3537**	0.0565	*0.3664**	0.0603	*0.3596**	0.0611
Γ*λ - Correlation Regime 2.						
Corn -Soybean	0.2608*	0.0447	0.2720*	0.0476	0.2721*	0.0475
Corn -Feeder Cattle	-0.0784*	0.0257	-0.0839*	0.0273	-0.0702*	0.0230
Corn - Live Cattle	0.0118	0.0199	0.0101	0.0209	0.0117	0.0204
Soybean - Feeder Cattle	-0.0326	0.0246	-0.0350	0.0243	-0.0371+	0.0222
Soybean - Live Cattle	0.0261	0.0247	0.0272	0.0257	0.0240	0.0223
Feeder Cattle -Live Cattle	*0.2885**	0.0473	*0.2998**	0.0505	*0.2912**	0.0512
γ or β - Betas						
probability beta - b11	0.5024*	0.1308	1.585	4.1228	-0.9947	1.8777
probability beta - b21	0.3928	0.2396	4.723	3.5030	-10.919	16.208
probability beta - b12	0		-0.654	1.8333	0.1335	0.2941
probability beta - b22	0		-2.180	1.4755	1.1461	1.7249

* Significant at 5% level or less
+ Significant at 10% level or less
Standard Errors for Correlations in Regime 2 by Delta Method

Betas Estimated September 1998 - August 2004

RESTRICTED MODEL

	Constant Transition Probability (by DP)	Standard Error	*State Dependent Probability Ratio Sybn/Corn Price*	Standard Error	*State Dependent Probability Ratio USE_Stock*	Standard Error
Likelihood	***-2659.3***		***-2649.4***		***-2643.9***	
Γ - Correlation Regime 1.						
Corn -Soybean	*0.6851**	0.0701	*0.7162**	0.0608	*0.7035**	0.0423
Corn -Feeder Cattle	*-0.1186+*	0.0717	*-0.1464**	0.0690	*-0.1109+*	0.0656
Corn - Live Cattle	0.0940	0.0624	0.0952	0.0659	0.0897	0.0626
Soybean - Feeder Cattle	-0.0173	0.0838	-0.0386	0.0847	-0.0309	0.0791
Soybean - Live Cattle	0.1244	0.0925	0.1407	0.0911	0.1196	0.0842
Feeder Cattle -Live Cattle	*0.8309**	0.0386	*0.8515**	0.0331	*0.8418**	0.0290
λ *Lambda Transition*	*0.3693**	0.0650	*0.4149**	0.0572	*0.3589**	0.0628
Γ*λ - Correlation Regime 2.						
Corn –Soybean	0.2530*	0.0515	0.2971*	0.0481	0.2525*	0.0467
Corn -Feeder Cattle	-0.0438	0.0276	-0.0607*	0.0298	-0.0398	0.0246
Corn - Live Cattle	0.0347	0.0239	0.0395	0.0279	0.0322	0.0231
Soybean - Feeder Cattle	-0.0064	0.0310	-0.0160	0.0352	-0.0111	0.0284
Soybean - Live Cattle	0.0459	0.0351	0.0584	0.0386	0.0429	0.0311
Feeder Cattle -Live Cattle	*0.3069**	0.0559	*0.3533**	0.0506	*0.3022**	0.0539
γ or β - Betas						
probability beta - b11	0.4717*	0.1086	9.4484*	4.8165	0.4479	1.5678
probability beta - b21	0.4512+	0.2457	3.1500	3.9300	-5.6545+	3.0890
probability beta - b12	0		-4.3050*	2.1863	-0.1033	0.2742
probability beta - b22	0		-1.1690	1.8239	0.6675*	0.3242

* Significant at 5% level or less

+ Significant at 10% level or less

Standard Errors for Correlations in Regime 2 by Delta Method

Betas Estimated September 2003 - August 2008

RESTRICTED MODEL

	Constant Transition Probability (by DP)	**Standard Error**	***State Dependent Probability Ratio Sybn/Corn Price***	**Standard Error**	***State Dependent Probability Ratio USE_Stock***	**Standard Error**
Likelihood	***-2421.0***		***-2407.9***		***-2397.1***	
Γ - Correlation Regime 1.						
Corn -Soybean	*0.7569**	0.0642	*0.7671**	0.0573	*0.8505**	0.0364
Corn -Feeder Cattle	*-0.2602**	0.0748	*-0.2405**	0.0821	*-0.2450**	0.0770
Corn - Live Cattle	-0.0033	0.0794	-0.0019	0.0764	-0.0018	0.0879
Soybean - Feeder Cattle	-0.0962	0.0861	-0.1111	0.1006	-0.1738*	0.0893
Soybean - Live Cattle	0.0338	0.0828	0.0152	0.0879	0.0309	0.0820
Feeder Cattle -Live Cattle	*0.7928**	0.0380	*0.7927**	0.0398	*0.8065**	0.0392
λ *Lambda Transition*	*0.3750**	0.1084	*0.3217*	0.2081	*0.3566**	0.0726
Γ*λ - Correlation Regime 2.						
Corn -Soybean	0.2838*	0.0855	0.2468*	0.1606	0.3033*	0.0631
Corn -Feeder Cattle	*-0.0976**	0.0398	-0.0774	0.0566	*-0.0874**	0.0327
Corn - Live Cattle	-0.0012	0.0298	-0.0006	0.0246	-0.0006	0.0313
Soybean - Feeder Cattle	-0.0361	0.0339	-0.0357	0.0398	-0.0620*	0.0343
Soybean - Live Cattle	0.0127	0.0313	0.0049	0.0285	0.0110	0.0293
Feeder Cattle -Live Cattle	*0.2973**	0.0871	*0.2550+*	0.1654	*0.2876**	0.0602
γ or β - Betas						
probability beta - b11	0.2062	0.2149	-6.9516	7.617	-2.5461	1.7405
probability beta - b21	0.1396	0.0964	-0.9749	2.082	-15.5098+	8.8850
probability beta - b12	0		2.5834	3.699	0.2816	0.2167
probability beta - b22	0		-0.3921	0.837	1.5514*	0.7245

* Significant at 5% level or less

+ Significant at 10% level or less

Standard Errors for Correlations in Regime 2 by Delta Method