Sequential Cumulative Fatigue Reliability of a Redundant Peripheral Connector Array Including Depletion Effects

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SUMMARY

For many applications involving a symmetrical peripheral array of identical closure studs or bolts, the loading history begins with a static pressure test to a level considerably above the normal peak of fluctuating cyclic pressure expected during the subsequent operational life but such as to preclude inelastic extension of any of the connectors. The latter precaution is necessary to avoid relaxation of the bolt preload and to maintain a leaktight joint. For such static testing structural reliability values may be readily assessment for the closure system on an active parallel redundancy basis.

The second phase of loading for such a system of connectors is that occurring during (and perhaps continuing for some time after) commissioning proper. This is the first fatigue inducing period of life and the level of the pressure loading peaks in the fluctuating pressure cycle will generally be set somewhat lower than that to be finally achieved under full load operation. The structural reliability of the connector system needs to be evaluated at the conclusion of this first fatigue inducing period.

For the second fatigue period the multiple connector system will be brought up to full operating conditions, the level of the mean fluctuating pressure being raised appreciably from that previously experienced. Withstanding specified total life cycles at full plant operation load is of paramount importance to the component and the structural reliability of the undepleted system is therefore assessed at the end of designed life employing an established method.

Having thus completed the assessment of the structural reliability (plus, for comparison, the cumulative usage factor) over the life of the undepleted system, the second part of the paper examines the effect of connector unit depletion on the hypothesis that one or more of the connector units in the peripheral system fails during the main operational life of the component. Such a failure causes an eccentricity of loading on the remaining connectors so that the two units adjacent to the failed unit (or group) will have their mean fluctuating load level appreciably increased. Should this situation not be detected and remedied, the increase in the loading is certain to reduce the structural reliability and safe life of the system. On the assumption that normal full operational loading is inadvertently allowed to continue, possibly due to operational inaccessability, the analysis indicates instances when the fatigue life of the system can be substantially curtailed due to sequential failure of pairs of connector units adjacent to the failed group leading in the direction of a cascade failure mode.

The methodology used for this sequential depletion of the peripheral connector system has been already described by the first author in a previous paper for cases of static loading, the structural reliability being evaluated at each depletion stage. This degradation assessment process when applied to sequential cumulative fatigue problems is the principal aim of the work described in the present paper. It also correlates the unit structural reliability at any point in life with the current, most generally applied, cumulative usage factor.

1. Introduction

Cumulative fatigue damage of components subjected to varying cyclic load fluctuations is often based on Miner's[1] hypothesis which can provide an estimate of the residual life at any particular life stage. It is now becoming necessary, however, to evaluate not only this empirical "usage factor" at various stages in life, but also the failure probability of each component. Probabilistic risk analysis (PRA) is an increasing requirement for nuclear and conventional plant having any serious accident potential to the surrounding community. The main input to such risk assessments comes from historical data on failure rates particularly in the case of electronic and electrical components. Structural failure rates of mechanical components are much more difficult to assess. This structural reliability assessment problem becomes particularly evident when fatigue loading is involved.

The authors are members of a sub-section of the Mechanical Reliability Research Group associated with the UK National Centre of Systems Reliability. This sub-section's interests have been delineated recently in a review paper [2] on the broad spectrum of mechanical reliability modelling in a relation to failure mechanisms and the present paper is a further extension of this area of work. A demountable flanged joint has been chosen for an attempt to correlate the Miner type approach with a structural reliability assessment of the joint subjected to varying cyclic fatigue loading. The same methodology may be equally well applied to any system of load carrying involving a number of identical connectors. The assessment of structural reliability of a single connector during cyclicly applied loading of differing intensities has been solved by Gardner[3] and Kececioglu[4] under the assumption of log-normal distributions of load and strengths. These two researchers also attempted to correlate their reliability findings with Miner's hypothesis but without much success, in fact the comparison suggesting it inadvisable to apply the "usage factor" approach. It is hoped that the application and comparison of the two methods given in the present paper will assist in resolving the problem.

2. Initial Static Loading

The example considered is a redundant peripheral connector array, as indicated in Fig. 1, subjected to fluctuating cycles of pressure loading but for test requirements loaded to an initially high static load. The structural reliability of the individual connectors during this initial static loading and the total reliability of the system can be assessed by well established methods [5,6]. A very high reliability level need not be expected or insisted upon, since the testing will generally be carried out in readily accessible conditions where replacement and repeat tests may be easily arranged. Should it be desired to cater for the contingency of one or more connectors failing, due to faulty materials or workmanship during this initial testing phase, the reduction in reliability may be assessed by the method described by Kinkead[7]. As will be shown, this same treatment for depletion effects may be applied during the fatigue life of the system.

3. Sequential Fatigue Loading of Connector Unit

In [4] Kececioglu and Gardner make the assumptions that (1) alternating stresses, with or without mean stresses, are applied sequentially, and (2) the cycles-to-failure distribution at each stress level is known. In fact while adopting the same assumptions, instead of taking the probability density function (pdf) of the cycles-to-failure to be in all cases lognormal, it will now be assumed that these pdf's are normally distributed. This is a justifiable assumption for axial loading cases since it was demonstrated by Kececioglu[8] in further experiments and the cases considered here of connectors under tension are all mainly axial loaded. With this change of distribution type, the procedure devised in [4] by Gardner will now be followed.

3.1 Assessment of Equivalent Cycles. Consider the cumulative fatigue stress-cycle sequences indicated in Fig. 2. The reliability R_I of components (connectors) subjected to such fatigue stresses is given by the probability of each connector surviving for N_1 cycles at the combined stress loading of alternating stress s_{a1} plus mean stress s_{m1} and the probability R_{II} of each connector surviving N_2 cycles under alternating stress s_{a2} plus mean stress s_{m2} together with the equivalent of N_1 cycles at the second stressing condition (this equivalent number of cycles being denoted N_{1e}) given that each connector has already survived the first cyclic loading.

The individual connector reliability R_c is then given by:-

$$R_c = R(N_1 + N_2) = R_I \times R_{II} = R_1(N_1) \cdot \frac{R_2(N_{1e} + N_2)}{R_1(N_1)} = R_2(N_{1e} + N_2) \qquad (1)$$

which is also expressible as:-

$$R_c = \int_{(N_{1e} + N_2)}^{\infty} f_2(N)\, dN \qquad (2)$$

Where $f_2(N)$ = pdf of cycles to failure

and if $\overline{N}_1^1$ = mean $f_1(N)$ cycles to failure

and σ_{N_1} = standard deviation of f(N), then, $z_1 = (N_1 - \overline{N}_1)/\sigma_{N_1}$ (3)

The equivalent number of cycles N_{1e} to be run at the second stress loading to provide the same probability of survival as R_I above at the first stress level is obtained by solving for N_{1e} the following equation:-

$$R_I = \int_{N_{1e}}^{\infty} f_2(N)dN = \int_{Z_{1e}}^{\infty} \Phi(z)dz \qquad (4)$$

We will already have calculated R_I for the first cyclic loading and f(N) the cycles to failure pdf at the second stress level will also be known, hence:-

$$z_{1e} = (N_{1e} - \overline{N}_2)/\sigma_{N2}, \text{ from which } N_{1e} = z_{1e}.\sigma_{N_2} + \overline{N}_2 = z_1\sigma_{N_2} + \overline{N}_2 \qquad (5)$$

since it is clear from the above expressions that $z_{1e} = z_1$.

3.2 Assessment of Structural Reliability. Having thus obtained the equivalent number of cycles N_{1e} representing the expended life (or damage) incurred by the first cyclic loading, this equivalent number of cycles is added to the second loading cycles. The reliability at the conclusion of N_2 cycles is given by the probability of surviving the second stress level up to $N_{1e} + N_2$ cycles:-

$$R_2 = \int_{(N_{1e} + N_2)}^{\infty} f_2(N)\, dN = \int_{z_2}^{\infty} \Phi(z)\, dx \qquad (6)$$

where $z_2 = [(N_{1e} + N_2) - \overline{N}_2] / \sigma_{N_2}$

and entering this z_2 value in standardised normal cumulative distribution tables gives the reliability at this stage. The same procedure can be applied for any number of separate sequential cyclic loadings. For three sequential fatigue cycle loadings for example it will be found that:-

$$z_3 = [(N_{1,2e} + N_3) - \overline{N}_3]/\sigma_{N_3} \tag{7}$$

in which $N_{1,2e}$ is the equivalent number of cycles occurring at the third loading representing the load fluctuations during the first two loadings in the sequence i.e.,

$$N_{1,2e} = z_2 \, \sigma_{N_3} + \overline{N}_3 \tag{8}$$

3.3 Correlation with Miner's Rule. If now the latter expression is inserted into eq.(7) and then the above value of z_2 is also inserted and the process repeated for N_{1e} and z_1 the following final expression for z_3 is obtained (in which $V_{Ni} = \sigma_{Ni}/\overline{N}_i$):-

$$z_3 = \left[\frac{N_1}{\overline{N}_1} \cdot \frac{1}{V_{N1}} + \frac{N_2}{\overline{N}_2} : \frac{1}{V_{N2}} + \frac{N_3}{\overline{N}_3} \cdot \frac{1}{VN_3} - \frac{1}{VN_1}\right] ; \left(\left[\sum_{i=1}^{k} \frac{N_i}{\overline{N}_i} \cdot \frac{1}{V_{Ni}}\right] V_{N1} - 1\right) \frac{1}{V_{N1}} = z_k \tag{9}$$

From this it is seen that should $V_{N1} = V_{N2} = V_{N3} = V_N$ say then the expression for z_3 reduces to:-

$$z_3 = \left[\frac{N_1}{\overline{N}_1} + \frac{N_2}{\overline{N}_2} + \frac{N_3}{\overline{N}_3} - 1\right] / V_N \tag{10}$$

hence

$$z_{i=k} = \left[\sum_{i=1}^{k} \frac{N_i}{\overline{N}_i} - 1\right] / V_N \tag{11}$$

The summated term is recognised as Miner's assembly of damage alias the 'usage factor' for such sequential fatigue. This then gives a direct correlation between Miner's hypothesis and the structural reliability at the same point in life. For any fixed V_N value, the smaller the value of the summated terms below unity the greater the negative z_k and hence the greater the structural reliability. The latter is also incurred if the coefficient of variation V_N of the S-N cycles is small.

From this it may be deduced that a more accurate value of usage or accumulated damage would be obtained were the summation terms multiplied by the ratio of the appropriate coefficients of variation of the cyclic pdf giving the relationship as:-

$$\left[\Sigma \frac{N_i}{\overline{N}_i} \cdot \frac{1}{V_{Ni}}\right] V_{N1} \leqslant 1 \tag{12}$$

instead of the customary:-

$$\Sigma \frac{N_i}{\overline{N}_i} \leqslant 1 \tag{13}$$

This in fact would provide a statistically improved damage assessment because it would take into account the two principal moments of the cycles to failure instead of relying on the mean value $\overline{N}_i$ alone. Here it might be mentioned that Gardner in [4] using the lognormal distributions did not find any measure of correlation with Miner's Rule and arrived at the conclusion that the claim that this rule provides a quantitative measure of useful life was fallacious. From the above comparison, found by adopting normal distributions for both load and strength, it is clear that Miner's Rule does give a meaningful measure of accumulated damage which may, as indicated, be cross referenced to the residual structural reliability at the same point in life.

3.4 Application of Goodman Diagram and Gerber Parabola. While the assessment given above enables rough reliability figures to be obtained over the life of the sequentially fatigued component, more precise values will be obtainable by taking into account the Gerber parabola modification of the Goodman diagram [9]. This more refined and accurate method may be applied in cases where the necessary experimental data exists. It is however most essential to first of all arrive at the equivalent number of cycles of fatigue in the earlier parts of the loading sequence by the method set out in [5] and for convenience repeated in sections 3.1 and 3.2 above under the assumption of normal cycles to failure distributions.

The steps required in applying the Goodman diagram are as follows:-

(i) From the S-N diagram for the material, obtain for the s_{ai} alternating load stress, the number of cycles N_i that are permissible up to the limit of the material strength.

(ii) From the known ratio: $r = \frac{s_{ai}}{s_{mi}} = \frac{S_a}{S_m}$ of the final loading sequence, and from the knowledge of the mean ultimate strength $\overline{S}_u$ of the material, calculate the point on the Gerber parabola appropriate to this loading. Instead of $\overline{S}_e$, the endurance limit, $\overline{S}_{fe}$ = mean finite life stress will be used in the equation:-

$$\left(\frac{S_a}{\overline{S}_{fe}}\right)^1 + \left(\frac{S_m}{\overline{S}_u}\right)^2 = 1 \tag{14}$$

It will be appreciated that $\overline{S}_{fe}$ is obtained from the S-N diagram at $[N_{(1,2...(i-1))e} + N_i]$ cycles, being the alternating strength level for the total equivalent cyclic loading and which must naturally be higher than that at $\overline{N}_i$.

(iii) Eq. (14) may be rewritten as:-

$$S_a^2 + \frac{r^2 \overline{S}_u^2}{\overline{S}_{fe}} \cdot S_a - r^2 \overline{S}_u^2 = 0 \tag{15}$$

from which S_a may be obtained and the actual mean strengths of the material at the desired cumulative cycles is given by:-

$$\overline{S}_f = (S_a^2 + S_m^2)^{\frac{1}{2}} = S_a \left(1 + \frac{1}{r^2}\right)^{\frac{1}{2}} \tag{16}$$

The corresponding standard deviation is obtained from experimental results such as have been listed in [8].

(iv) The loading stress values of the final loading stage s_a and σ_{sa} are known but must be synthesised into failure governing parameters along the stress ratio line of the Goodman diagram as follows:-

$$\left.\begin{aligned} \overline{s}_f &= s_a \left(1 + \frac{1}{r^2}\right)^{\frac{1}{2}} \\ \sigma_{sf} &= \sigma_{sa} \left(1 + \frac{1}{r^2}\right)^{\frac{1}{2}} \end{aligned}\right\} \tag{17}$$

(v) Since both the failure-governing strength distribution $f(S_f)$ and the failure-governing stress distribution $f(s_f)$ are assumed normal, the two distributions can be coupled to calculate the reliability R (of the rod, bolt, stud or connector) as follows:-

$$z(f) = \frac{s_f - S_f}{(\sigma_{sf}^2 + \sigma_{Sf}^2)^{\frac{1}{2}}} \tag{18}$$

$$\text{and } R = \int_{z(f)}^{\infty} \Phi \quad (z)\, dz \tag{19}$$

Where $\phi(z)$ is the standardised normal probability density function. From the above procedure it is possible to evaluate (and hence predict) the reliability of any individual connector unit of a load sharing array at any point of a sequential fatigue loading life.

4. Undepleted System Reliability

Having assessed the reliability of the individual connector unit at any point of interest in its life, that of the system comprising an array of load sharing units may be calculated from the well known active parallel redundancy relationship for n identical units:-

$$R_s = 1 - \prod_{j=1}^{n} P_{uj} \qquad (20)$$

Where P_{uj} is the assessed probability of failure of the individual sub-component.

5. Depleted System Reliability

Having thus completed the methodology for the assessment of a structural reliability (and usage or damage) over the life of the undepleted system, it is next necessary to consider the effect of connector unit degradation on the hypothesis that one or more of the units in the peripheral system may fail during the planned operational life. The case of common mode failure is not contemplated here since this could only occur due to gross human error in design, manufacture or operation. It is however conceivable that for some unanticipated reason (undetected material flaw for example) that a few units could fail under load. The worst effect arising from this is that these few fail more or less simultaneously and that they are grouped together (see Fig. 1) so that the two unfailed units adjacent to this group will have their mean fluctuating load level appreciably increased. It is also, as a worst case, assumed that owing to, for example, operational inaccessibility their failure goes undetected and no remedial action is therefore carried out.

The basic approach to assess the downgraded reliability of such a system has been given in [7] for a static loading case. It utilises the binomial probability distribution treating all the unfailed sub-component connectors as still being equally loaded and applying the k-out-of-n active parallel system expression:-

$$R_s = \sum_{j=k}^{n} \binom{n}{j} R_u^j (1 - R_u)^{n-j} \qquad (21)$$

in which R_u is the unit reliability of the surviving connectors and $j = i + 1$ being the lowest term of the surviving units i of which having failed out of the original total n.

6. Application

Due to space limitations demonstration of numerical values obtained using the above analysis is confined to a single illustrative example.

A low redundancy four bolt demountable flange of the type indicated in Fig. 3 is subjected to a specified cyclic loading. This consists of 3800 fluctuations of pressure from zero to 2000 psi (shared bolt load 6824 lbf) followed by 25000 cycles of pressure fluctuation from zero to 2320 psi (shared bolt load 7916 lbf). Over an approximate 40 year life, twice daily fluctuations in this double sequence are considered as a life involving 28800 fluctuations. Bolt material is AISI 4340 steel with an ultimate strength, $\overline{S}_u = 116400$ psi and an endurance limit, $S_e = 48705$ psi. Bolt thread root area is 0.226 in^2 and thread root SCF = 2.2.

A bolt preload of 5000 lbf per bolt, determined using the method given by Shigley[10] is used, although an equivalent flange volume given by a diameter of 2.36d has been adopted in place of the 3d value considered by Shigley.

6.1 Undepleted System

(i) First Cyclic Phase

A determination of the maximum and minimum tensile stresses followed by the mean and alternating stress components results in a ratio of alternting to mean stress component, r_1 = 5807/27931 = 0.2079. Hence from eq. (16), the actual mean stress at the thread root $\bar{s}_{f1}$ = 62761 psi.

A nominal S-N diagram for the bolt material is given by a straight line on log-log paper joining $\bar{S}_u$ at 10^3 cycles to S_e at 10^6 cycles.

At N_i = 3800 cycles, $\bar{S}_{fe1}$ = 98200 psi and at $\bar{s}_{f1}$ = 62761 psi, N_{f1} = 135000 cycles. Hence from eq. (15), S_{a1} = 21401 and from eq. (16) $\bar{S}_{f1}$ = 105139 psi.

The standard deviation on strength is obtained from an interpretation of data from [8] and the standard deviation on load is taken as 1/15 of $\bar{s}_{f1}$ (a frequently assumed figure). From eq. (18), Z_{f1} = -8.062 and from tables of area under standardised normal distribution, the risk of bolt failure = 0.381×10^{-15}. The individual connector reliability at the conclusion of the first phase $R_{u1} = 0.9_{15}\,619$.

(ii) Second Cyclic Phase

Following the same steps $\bar{s}_{f2}$ = 65198 psi giving $\bar{N}_2$ = 100,000 cycles from S-N diagram.

Equivalent cycles at second phase loading to represent first phase

For this assessment interpolation from Table 2 of [11] gives standard deviation. The use of eqs. (3) and (5) then gives N_{1e} = 2185 cycles.

Evaluation of reliability at end of second phase

At the conclusion of the second fatigue stage the total cycles = 27185 giving S_{fe2} = 77000 psi from S-N diagram. Applying eqs. (15) and (16) again and obtaining a value of standard deviation on strength from Table 2 of [8] and again taking σ_{sf2} as 1/15 of $\bar{s}_{f2}$ allows (18) to be used to give Z_{f2} = -6.481. Hence the risk of bolt failure = 4.585×10^{-11}. The individual connector reliability at the conclusion of the second phase is then $R_{u2} = 0.9_{10}54$.

Although this is a slightly lower reliability figure than that at the end of the first phase, it still gives an extremely high system reliability of the order $R_{s2} = 0.9_{100+}$.

Miner's Law Damage Comparison

It is of interest to compare R_{u2} with the Cumulative Usage Factor (Miner-Palmgren-Manson [12]) which gives:

$$C_{uf} = \Sigma \frac{N_i}{\bar{N}_i} = \frac{3800}{135000} + \frac{25000}{100000} = 0.278$$

6.2 Depleted System. Consider the failure of one of the four bolts during second fatigue phase (at 8815 cycles) and assume that this is undetected thus producing a third fatigue phase. Proceeding as before, it is found that the unit reliability falls from $R_{u22} = 0.9_{11}\,40$ before failure to $R_{u31} = 0.9_2\,25$ immediately after failure. The system reliability obtained from this initial depleted state from eq. (21) is $R_{s31} = 0.9_5\,83$ (i.e., equivalent to a failure probability of 1.7×10^{-6} which is still within the desired standard for nuclear pressure parts).

The reliability at the conclusion of the third (i.e., depleted) fatigue phase is next evaluated using the same procedure. Owing to the higher stress, the total equivalent cycles at the conclusion of the third phase is 22690 cycles. Proceeding as before, the individual connector reliability at the conclusion of the third phase becomes $R_{u32} = 0.9838$.

The system reliability obtained from eq. (21) for this end of life depleted state is $R_{s32} = 0.9_4 83$. This is equivalent to a failure probability of 1.67×10^{-5} which is marginally within the desired standard for nuclear pressure parts.

Miner's Law Damage Comparison

It is also of interest to compare R_{u32} with the Cumulative Usage Factor (Miner-Palmgren-Manson [12]) for the three phases of the depleted system and this gives $C_{uf} = 1.015$.

Hence, this also suggests that the safe life of the two individual bolts most heavily stressed by the single bolt failure has been exceeded at this point.

Figure 4 gives the history of reliability of this illustrative example on a failure probability scale. It shows the large increase in failure probability caused by the assumed loss of one bolt at 12615 cycles of fatigue.

7. Summary and Conclusion

In the procedures set out in the paper a means of predicting the structural reliability of a redundant peripheral array of identical connectors is proposed and has been demonstrated in a simple example. The method is founded on the assessment of equivalent cycles to represent the effect of all pripr cyclic fatigue loadings in the sequence as deduced by Gardner[4] but now adopting normal distributions of load and strength. The effects of redundancy and its possible partial depletion have utilised the approach set out in [7].

During this analysis a comparison was afforded between structural reliability degradation caused by the sequential fatigue process and the cumulative damage assessment rule currently employed in many applications and originally proposed by Miner[11]. This correlation shows that the 'usage factor' forms part of the expression evolved for structural reliability of any connector unit of such an array at any point in life and that a more statistically correct form of the rule would be as given in eq. (12).

In the emerging need for probabilistic risk assessments of potentially hazardous plant, it is important to know the structural reliability at all stages of life allowing for at least some non-common-mode depletion effects. The procedure described should enable such assessments to be carried out where the necessary materials and loading variability data exists. Such assessments should be correlated with the currently applied cumulative usage factor for applications involving sequential fatigue loading.

8. References

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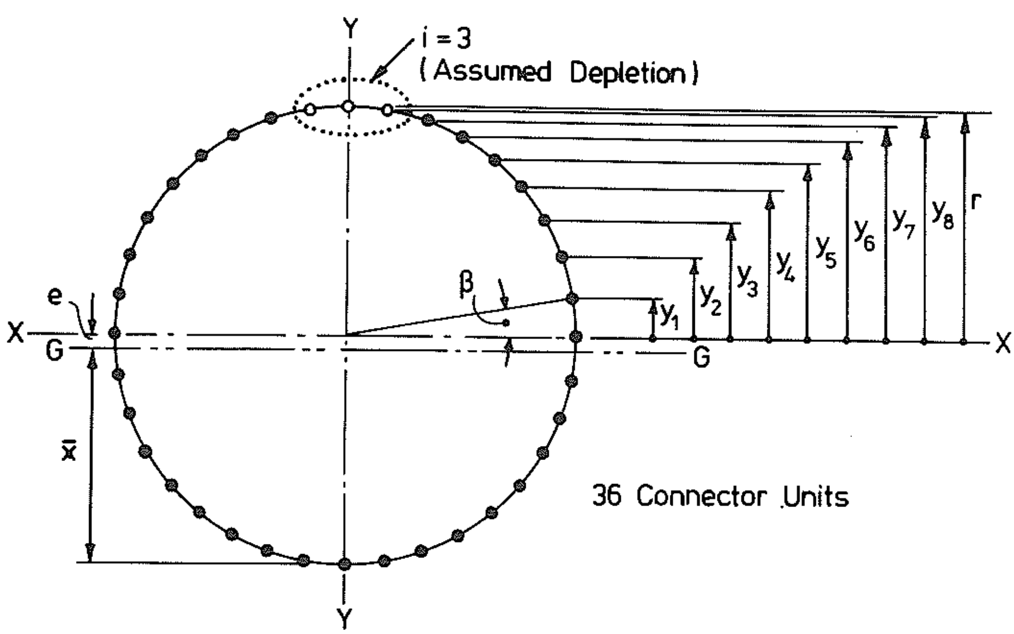

FIG.1 High Redundancy Annular Array of Identical Connectors.

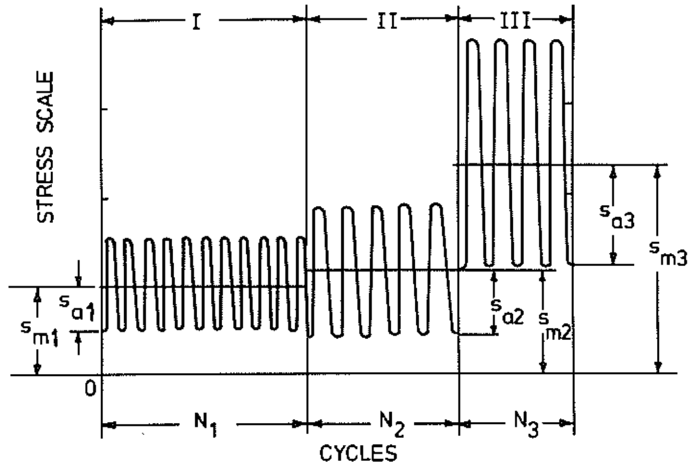

FIG. 2. STRESS HISTORY OF COMPONENT SUBJECTED TO CUMULATIVE FATIGUE LOADS.

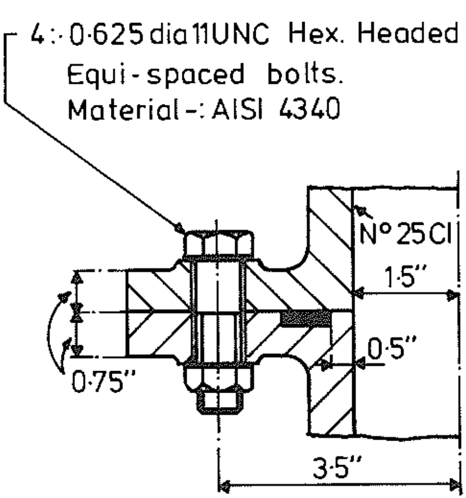

FIG.3 Low redundancy bolted flange connection

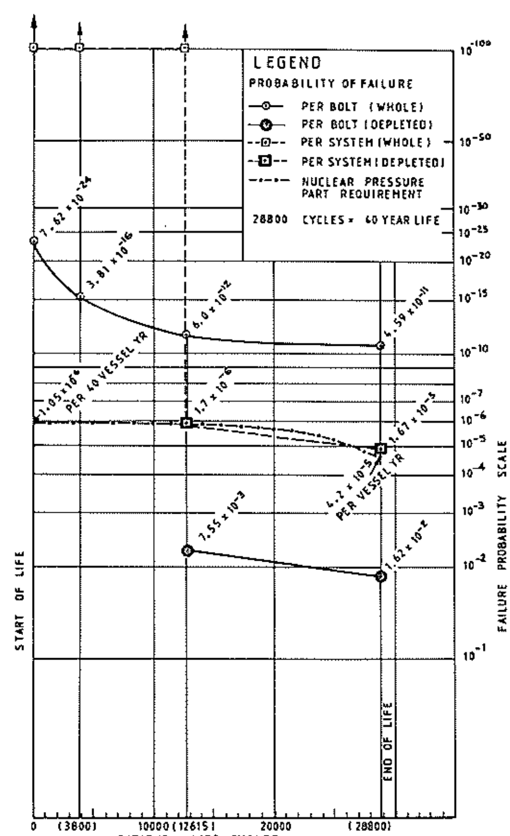

FIG. 4 EFFECT OF DEPLETION IN APPLICATION EXAMPLE