A TIME DOMAIN ANALYSIS MODEL OF A FLOATING STRUCTURE AND ITS FREQUENCY DEPENDENT ADDED MASS

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SUMMARY

The dynamic response analysis of large floating structures requires appropriate accounting of the interaction effects of the surrounding fluid medium. This is generally done, as in the study of ship motions, by the inclusion of added mass and hydrodynamic damping in the analysis. Many large floating structures, e.g., floating nuclear plants, are enclosed in protective basins, and therefore the major energy dissipation mechanism in the fluid—wave damping—can be conservatively assumed to be absent. The added mass of such structures will thus exhibit a strong dependence on the oscillation frequency of the structure, becoming theoretically inifinite at certain frequencies due to the basin resonance effect. The use of the traditional constant added mass approach for the response analysis of the structure under transient force time histories may, in such cases, lead to large errors. For the same reason, a frequency domain analysis of the structure response, in which the frequency dependence of the added mass is directly incorporated in the model, is also not desirable, since the appropriate accounting of the added mass near the basin resonant frequencies may lead to numerical difficulties.

To circumvent these difficulties in the constant added mass approach and in the frequency domain analysis method, a time domain analysis of the fluid-structure finite element model of the basin provides an answer. However, the size of the basin to be modeled in such an analysis procedure may render the model computationally inefficient, especially if the model is to be repetitively used for response analysis under different force time histories.

This paper presents an alternative approach, wherein limited information on the added mass behavior of the structure is used to synthesize a considerably smaller equivalent time domain model of the structure and the surrounding fluid. Though the construction of this equivalent model is purely mathematical, being defined by stiffness and mass matrices which may not be directly related to the fluid-structure system, it can, however, be physically interpreted as a multi-degree spring-mass system coupled to the floating structure. The model construction is based on the fulfillment of the requirement that the floating structure response under a harmonic excitation in the equivalent model be the same as that implied by its added mass. The model is simple to construct and is insignificantly small in size when compared to a fluid-structure finite element model of the basin.

An example of a two dimensional model of a floating structure under a time history of forces generated by a passing tornado is presented for illustrating the use and the applicability of the method. The behavior of the floating structure in the equivalent model is compared to that predicted by a fluid-structure finite element model of the basin. Excellent agreement in the results is obtained. A comparison of the computation times for the example problem shows that the equivalent model approach cost (inclusive of the cost of added mass determination) is considerably less than the fluid-structure finite element analysis. The model is thus shown to be accurate and computationally efficient.

1. Introduction

Recent years have seen increasing use of finite element methods in the solution of fluid-structure interaction problems. Their decisive advantage over the potential function methods lies in the ease with which complex geometries and different material properties can be modeled. These methods are particularly suited to the problems in which fluid domains are of finite extent. An example of such a problem would be the response analysis of a floating nuclear plant (FNP) in a shallow basin bounded by a protective breakwater structure. The size of the problem, measured by the computational effort needed to solve it, may, however, be an inhibiting factor in the use of these methods. Simplifications with minimum loss of accuracy are, therefore, highly desirable.

One such simplification is the subject of this paper. The method proposed in this paper uses the frequency response characteristics of the fluid-structure system, derived from a finite element model, to establish a simpler equivalent model which can be analyzed in the time domain. Inspite of the use of the finite element model in obtaining the frequency response characteristics of the system, the method is computationally superior to a time domain finite element analysis, especially if the model is to be used repeatedly for different excitation cases.

The various components of the time history analysis process in the equivalent model approach are

i) construction of a finite element model of the fluid-structure system
ii) obtaining frequency response functions for the structure,i.e., its frequency dependent added mass
iii) synthesis of an equivalent fluid-structure model from the knowledge of structure added mass
iv) time domain analysis of the equivalent model

The example of a floating nuclear plant (FNP) will be used to illustrate the various steps. This, in no way, implies limited applications of the method.

2. Finite Element Modeling

In Fig. 1 is shown a two dimensional representation of an FNP and the surrounding fluid medium. The FNP is moored by mooring strut BC to the mooring caisson. A mathematical model of the FNP and the basin was constructed using finite element representation of the fluid and the floating structure. The FNP is modeled as a rigid structure because of the long oscillation periods associated with its motion. The 3 degrees of freedom of the rigid FNP are located at a point A on the FNP which lies on the intersection of the waterline and the vertical through the center of gravity G of the FNP. For seismic response analysis, the added mass characteristics of the basin boundary will also be needed in synthesizing the equivalent model; therefore 2 degrees of freedom associated with horizontal and vertical ground motion have been assigned to an arbitrarily selected point D on the basin boundary which is assumed rigid.

The fluid-solid boundary is appropriately modeled to allow free sliding of the fluid on the boundary. Buoyancy effects are incorporated in the model by connecting springs of appropriate stiffnesses between the FNP and the rigid basin floor. The free surface condition is also modeled by springs which provide resistance to the vertical movement of particles on the free surface. The stiffness of these springs is derived as follows.

The linearized dynamic boundary condition for small amplitude waves on the free surface can be written as

$$p = \gamma \eta \tag{1}$$

where p is the pressure at a point and η is the displacement of a particle at the same point on the undisturbed free surface, and γ is the weight density of the fluid. Thus, if the free surface condition were modeled by uniformly distributed springs on the free surface, their stiffness per unit length of the free surface would be γ. In a finite element analysis the springs would be provided at the nodes on the free surface and their stiffness would be the product of γ and an appropriate tributory area.

The spacing of the surface springs is controlled by the significant wave of the shortest wave length and as such is independent of the refinement in the fluid finite element.

3. Frequency Dependent Added Mass

The finite element model thus constructed is employed to obtain the frequency response functions for the 3 degrees of freedom for the FNP. If the system is also to be analyzed for seismic excitation, frequency response functions for the 2 degrees of freedom of the rigid basin boundary are also obtained. The frequency response functions are defined by the following equation

$$\{P\} = [M^a]\{a\} \tag{2}$$

in which $\{P\}$ is the vector of amplitudes of harmonic forces applied to the FNP and the basin boundary and consists of 5 elements; $\{a\}$ is the vector of resulting harmonic acceleration amplitudes; and matrix

$[M^a]$ is a 5x5 matrix of frequency response functions dependent upon the oscillation frequency p. In general an element m^a_{ij} of this matrix is complex and represents the harmonic force amplitude at the i^{th} degree of freedom when the harmonic acceleration amplitude at the j^{th} degree of freedom is unity and zero elsewhere. Its real component represents the component of force in phase with acceleration and is called added mass. As a result of the free surface condition given by eq. (1) the added mass is frequency dependent.

For a fluid-structure system of finite extent, the predominant source of energy dissipation mechanism, i.e., wave radiation, is absent. Damping in the system, therefore, may be neglected. In such cases $[M^a]$ will have only real components.

A typical frequency dependent added mass curve, which relates the horizontal force on the FNP to its horizontal acceleration is shown in Fig. 2. Note that the added masses can be theoretically inifinite at some frequencies. This is due to the assumption of zero damping in the system. It is not difficult to show that the frequencies p_i at which the added mass becomes inifinitely large are the natural frequencies of the FNP-basin system with the FNP and the basin boundaries held fixed.

4. Equivalent Fluid-Structure Model

Once $[M^a]$ is known as a function of frequency, an equivalent model can be constructed for the system. The requirement for the construction of this model is that the corresponding frequency response functions for the model closely approximate those given by $[M^a]$ over the frequency range of interest. If this frequency range of interest is the frequency interval $(0,p_n)$, then it can be shown that the equivalent system can be described mathematically by the stiffness matrix $[K]$ and the mass matrix $[M]$, where $[K]$ and $[M]$ are of size n+5 x n+5 and are given by the following expressions (see Kaul [1])

$$[K] = \begin{bmatrix} K^o & H^T S \\ S^T H & S^T W S \end{bmatrix} \tag{3}$$

$$[M] = \begin{bmatrix} M^o & 0 \\ 0 & S^T S \end{bmatrix} \tag{4}$$

in which $[S]$ is any arbitrary non-singular n x n matrix, $[W]$ is an n x n diagonal matrix whose elements are the terms p_i^2, $(i = 1,2,..,n)$, and $[K^o]$, $[M^o]$ and $[H]$ are obtained as the solutions of the following matrix equation

$$[M^*] = [M^o] - (1/p^2)\,[K^o] + (1/p^2)\,[H]^T[W - p^2 I]^{-1}[H] \qquad (5)$$

The degrees of freedom associated with the matrices $[K^o]$ and $[M^o]$ are the 3 FNP and the 2 basin boundary degrees of freedom. The remaining n degrees of freedom in matrices $[K]$ and $[M]$ represent, in an indirect way, the surrounding fluid medium. For the FNP-basin example considered here, n was chosen to be 8. This was proved to be sufficient for obtaining the FNP response accurately by a subsequent time history response analysis.

The set of equations given by eq. (5) may be solved for each ij^{th} component by least square fitting. To obtain a fairly accurate description of the model it is essential that the natural frequencies p_i are estimated reasonably well. This may be done by obtaining frequency response functions from the finite element model at relatively more points in the vicinity of these frequencies or by a suitable determinant search technique in the eigenvalue algorithm.

5. Time History Analysis for Structure Response

For forces acting on the structure or for seismic excitation or a combination of both, the equations of motion for the equivalent system can now be written as

$$\begin{bmatrix} K^o & H^T S \\ S^T H & S^T W\, S \end{bmatrix} \begin{Bmatrix} x_s \\ x_f \end{Bmatrix} + \begin{bmatrix} M^o & 0 \\ 0 & S^T S \end{bmatrix} \begin{Bmatrix} \ddot{x}_s \\ \ddot{x}_f \end{Bmatrix} = \begin{Bmatrix} F \\ 0 \end{Bmatrix} \qquad (6)$$

in which x and $\ddot{x}$ are the displacements and the accelerations referred to a stationary reference frame. Subscript s refers to the FNP and the basin boundary and subscript f refers to the "fluid". Vector $\{F\}$ is the vector of force time histories applied to the FNP and the basin boundary.

Equation (6) may be simplified by eliminating the degrees of freedom associated with the basin boundary and rewriting the remaining equations with the displacements and accelerations relative to the basin boundary as unknowns

6. Results and Conclusions

The model shown in Fig. 1 and the equivalent fluid-structure model were analyzed for the tornado generated forces acting on the FNP at point G. The time histories of the three in-plane components of these forces are shown in Fig. 3. The FNP response quantities obtained from the two analyses were practically indistinguishable. Time history of the response for the horizontal FNP displacement is shown in Fig. 4.

A separate analysis was performed for horizontal seismic excitation of the basin boundary. In this case too comparison of response time histories obtained from the two models showed excellent agreement.

A comparison of cost shows the equivalent model approach to be superior to the finite element analysis. A very large portion of the analysis cost for the equivalent model comes from added mass determination, which is much less than the cost of one time history analysis on the finite element model. Repetitive use of the model results in much larger savings in computation time.

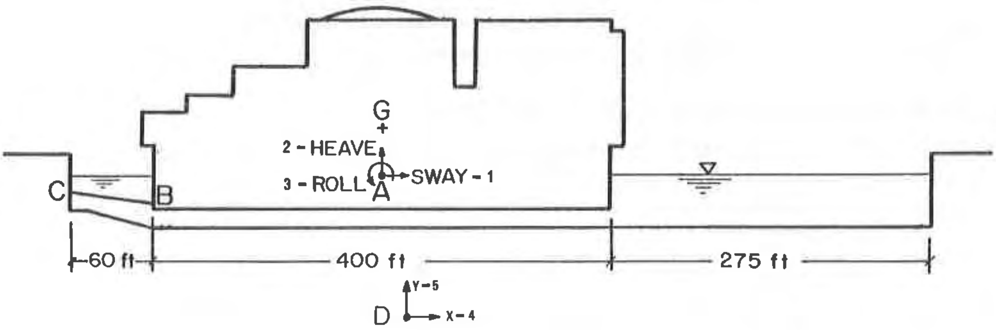

Figure 1 Two Dimensional Model of a Floating Nuclear Plant

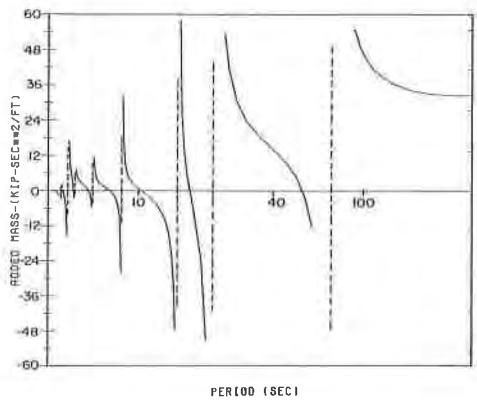

Figure 2 FNP Added Mass in Sway

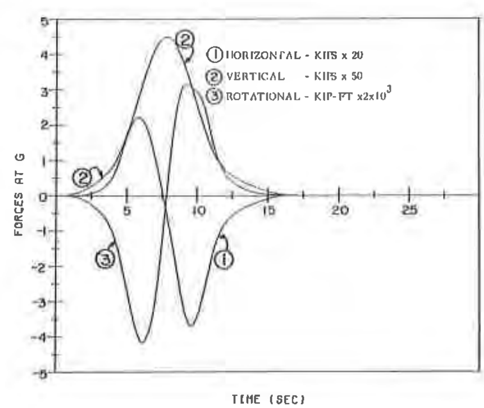

Figure 3 Time Histories of Applied Forces

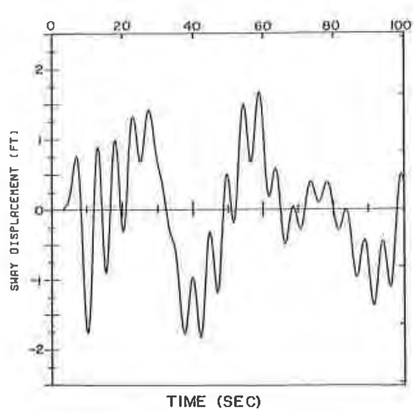

Figure 4 FNP Displacement Response in Sway