Probabilistic Assessment of NPP Building Structures Strength under Aircraft Impact

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ABSTRACT

The methodology of the probabilistic assessment of NPP safety under aircraft impact is described. The probability of an aircraft fall onto the NPP building, as well as other random parameters, such as the aircraft class, velocity and mass, the place and angle of its impact on the building structure are taken into account. The proposed probabilistic analysis can justify the reduction of the required structure strength. It can also be useful when assessing the safety of existing NPP.

1. INTRODUCTION

An accidental aircraft impact is one of the heaviest external loads considered in NPP designing. Whether it should be taken into account is decided on the probabilistic grounds which are either the immediate determination of aircraft impact probability, or "screening distance value" approach, the distance being determined on the probabilistic basis too (Refs. [1], [2]). As a result of this evaluation, one of the following two alternative resolutions are accepted today: either this load is not taken into account at all, or the most unfavorable case of impact is considered. The latter means that aircraft's mass and velocity are assumed to be maximal, the impact is applied at the critical point of the structure and at the most hazardous angle.

But in reality the aircraft velocity and mass, as well as its impact point and the collision angle are random parameters, and the probability of simultaneous realization of theirs the most unfavorable values is very small. Because of this, a more accurate probabilistic assessment of the event can be made taking into account not only the fact of aircraft fall onto the NPP building, but other random parameters too. On the basis of this analysis the reduction of design impact load and required structure strength can be justified. It can be especially useful when assessing the safety of existing NPP.

2. RANDOM PARAMETERS CONSIDERED

The probabilistic analysis is carried out taking into consideration the following random factors: aircraft fall recurrence; load vector direction in space; the impact load value depending on the aircraft class, mass and velocity. In principle, probability characteristics of these should be determined by means of the airspace situation analysis in the vicinity of the NPP. To explain the described methodology, these characteristics are specified on the basis of the world aircraft accident statistics and technical publication data.

2.1. Aircraft Fall Recurrence

The aircraft fall recurrence is specified as the number μ of aircraft falls per year on the standard horizontal area A_0. It depends on the air traffic intensity and the NPP position with respect to the dangerous flight zones (airports, take-off and landing routs, air traffic corridors, training flight zones, etc.). For example, the recurrence can be calculated using formulae given in the Ref. [2]. Below the aircraft fall recurrence is taken as that in Germany, namely: $\mu = 10^{-6}$ 1/year on area $A_0 = 10^4$ m^2. It should be noted that this fall recurrence is typical for the center of Western Europe, where air traffic is very intensive. In most other regions it provides for the fall probability with some reserve.

2.2. Impact Direction

The load vector $\boldsymbol{R}$ direction is specified by two angles, namely $0 \le \alpha \le \pi/2$ and $0 \le \beta \le 2\pi$ (fig. 1). They are assumed to be independent random variables.

The density function of the angle β depends in principle on the NPP location with respect to air traffic routs. For lack of other information it is assumed below that the probability of an aircraft approach to NPP from any side is the same, i.e. the angle β is uniformly distributed in $(0, 2\pi)$ interval. So, its density function is:

$$p_1(\beta) = \frac{1}{2\pi}. \qquad (1)$$

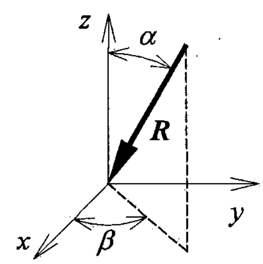

Fig. 1. The load vector direction in space

The angle α density function has been determined by analyzing the aircraft accident data published in Ref. [4]. The density function obtained is:

$$p_2(\alpha) = 0.072\exp(2.2\alpha). \tag{2}$$

Functions $p_1(\beta)$ and $p_2(\alpha)$ are shown in fig. 2.

To obtain the joint density function $p(\alpha, \beta)$ the probability of load vector $\boldsymbol{R}$ application within the "pyramid" (α, $\alpha+d\alpha$, β, $\beta+d\beta$) has been calculated (fig. 3). The joint density function is:

$$p(\alpha, \beta) = 1.315 \cdot 10^{-2} \exp(2.2\alpha)\sin\alpha. \tag{3}$$

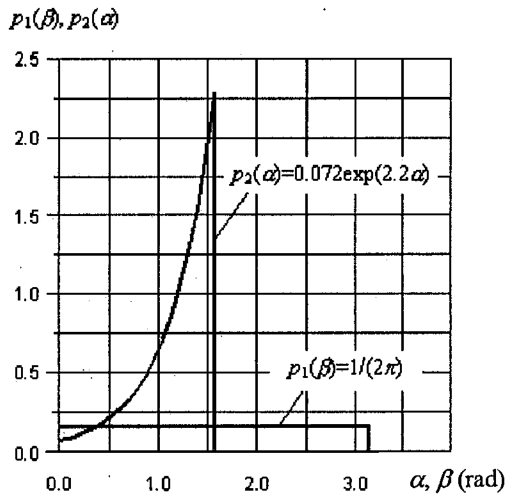

Fig.2. Density functions of α and β

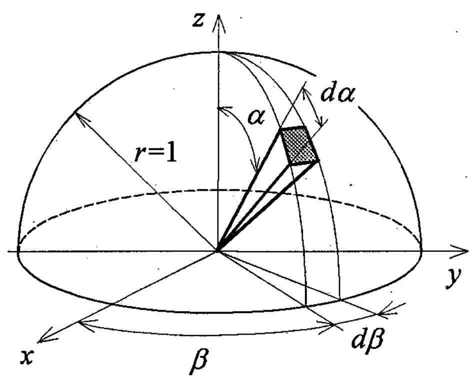

Fig.3. Scheme for calculation of α and β joint density function

2.3. Aircraft Class, Mass and Velocity

An impact of Phantom RF-4E aircraft is considered. Variation ranges of its velocity v and mass m are taken approximately in accordance with W.F. Zorn and G.F. Shuëller data (Ref. [3]) as $70 \le v \le 250$ m/s, and $13500 \le m \le 20000$ kg. Their probability distributions adopted from Ref. [3] are shown in fig.4.

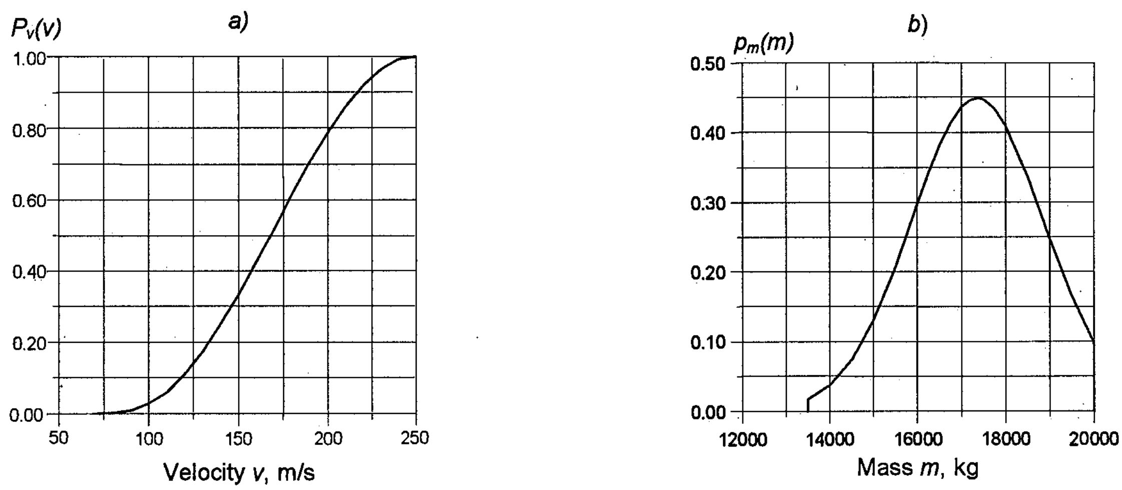

Fig.4. Probability distributions of aircraft Phantom RF-4E velocity and mass (adopted from Ref. [3]): *a*) velocity cumulative distribution function $P_v(v)$; *b*) mass density function $p_m(m)$

3. BUILDING STRUCTURES FAILURE PROBABILITY CALCULATION

3.1. Probability of Aircraft Fall Whithin a Given Solid Angle

To carry out the probabilistic analysis of aircraft impact hazard, the probability of impact at a given angle to the structure surface normal should be calculated. For this purpose an auxiliary problem on the probability of aircraft fall within a solid angle S_0 should first be solved. It can be done using joint density function of angles α and β.

Let the solid angle be bounded by a cone (fig. 5) with an apex angle γ ($0 \le \gamma \le \pi/2$) and inclination of the axis n to the vertical equal φ ($0 \le \varphi \le \pi/2$). Then the cumulative distribution function is

$$P_\gamma(\gamma|\varphi) = f(\varphi,\gamma) + e(\gamma - \pi/2), \tag{4}$$

where $e(\gamma - \pi/2)$ is the unit step-function;

$$f(\varphi,\gamma) = \iint_{(S_0)} p(\alpha,\beta)\, d\alpha\, d\beta; \tag{5}$$

if part of the solid angle lies under the horizontal plane it is not considered. Function $f(\varphi,\gamma)$ is depicted in fig. 6.

Let the cone axis $\boldsymbol{n}$ be the normal to an inclined plane A. Then $P_\gamma(\gamma|\varphi)$ is the probability of aircraft impact at the angle with the normal, which is not more then γ. As one can see, the maximal probability of impact (corresponding to $\gamma=\pi/2$) on the horizontal plane equals 1, because in this case an aircraft can strike from any point of the upper half-sphere. But with any other slope angle φ maximal probability of impact is less than 1, because the bigger the slope angle the larger the «shadow zone», i.e. the part of the upper half-sphere whence the aircraft can not strike (fig. 7). For example, maximal probability of impact on a vertical plane is 0.5.

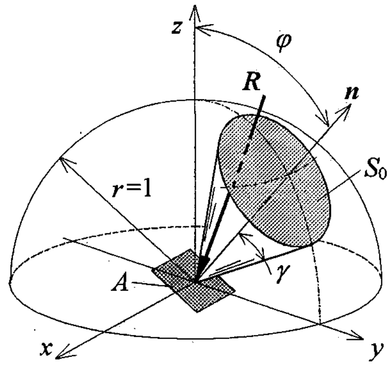

Fig. 5. Scheme for calculation of probability of impact within a solid angle S_0

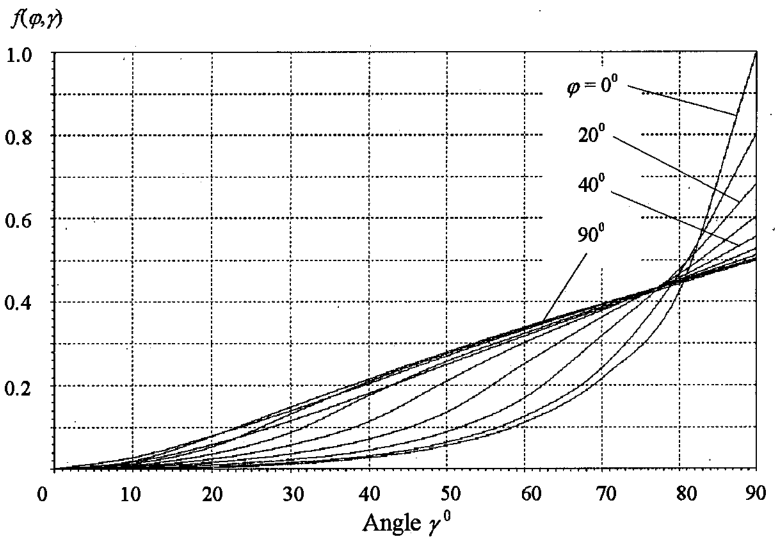

Fig. 6. Function $f(\varphi,\gamma)$

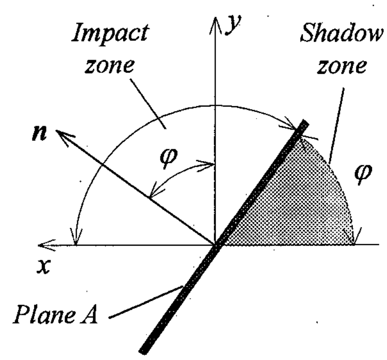

Fig.7. Impact and shadow zones for an inclined plane

3.2. Probability of Aircraft Impact on Plane Element A

The probability of aircraft impact on a plane with the area A for time τ is equal to

$$P = P_\tau P_A, \tag{6}$$

where P_τ is the probability of fall for the specified time; P_A - probability of hit on the plane. It is accepted here, that they both are subordinate to the Poisson distribution, which is usually used for the description of rare occurrences, and are calculated with the use of the aircraft fall recurrence data given in item 2.1.

As follows from the essence of the problem, a single impact of the aircraft for NPP lifetime should be taken into account. For if this event has occured, NPP should be shutdown and repaired, the conditions again becoming the same, as before the impact of the aircraft.

The probability of a single fall P_τ is

$$P_\tau = \tau\mu \exp(-\tau\mu) = \tau 10^{-6} \exp\left(-\tau 10^{-6}\right). \tag{7}$$

When estimating NPP safety the probability of events within τ=1 year is usually considered. Then

$$P_\tau = 1\cdot10^{-6} \exp\left(-1\cdot10^{-6}\right) \approx 1\cdot10^{-6}. \tag{8}$$

The probability of a single fall on the area A is equal to

$$P_{A,1} = \frac{A}{A_0}\exp\left(-\frac{A}{A_0}\right) = A\cdot10^{-4}\exp\left(-A\cdot10^{-4}\right), \tag{9}$$

and the probability of no fall occurring on this area is

$$4\,P_{A,0} = \exp\left(-\frac{A}{A_0}\right) = \exp\left(-A\cdot10^{-4}\right). \tag{10}$$

Consider that the outside surface of a building A is subdivided on m of planes with the areas A_s. The probability of a single impact on the s-th of building structure equals

$$P_{A_s,1} = A_s 10^{-4}\exp\left(-A\cdot10^{-4}\right). \tag{11}$$

To make this formula clear it can be put like

$$P_{A_s,1} = \underline{A_s 10^{-4}\exp\left(-A_s\cdot10^{-4}\right)}\cdot\exp\left[-10^{-4}\left(A-A_s\right)\right]. \tag{12}$$

The product of the underlined factors represents the probability of a single impact on the s-th of building structure, and the non-underlined exponent is the probability that no impact on other building structures has occured.

3.3. Probability of Building Structure Failure with Aircraft Fall Direction Randomness Consideration

The object of probabilistic analysis is to determine probability of a building structure failure, which means its unability to meet the design requirements. The criterion of failure proceeds from NPP safety conditions. For NPP building structures, depending on their safety relation, the criterion may for example be taken as perforation, or through cracks arising, or inadmissible residual deformations, etc. Failure of a building structure is a random event, which is denoted $\boldsymbol{F}$, and its probability $P(\boldsymbol{F})$.

At the impact of an aircraft at an angle to the building structure normal its strength depends mainly on normal component of load. Therefore when estimating the failure probability the tangent component of load was not taken into account as the first approximation.

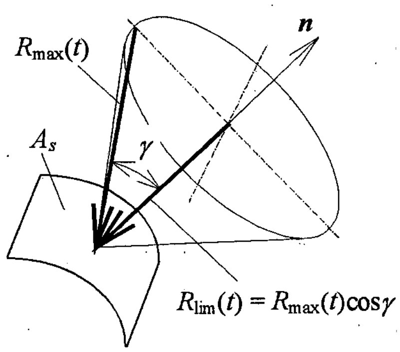

Fig. 8. The cone within which the aircraft impact cause building structure failure

Denote the impact of the aircraft on the s-th building structure as random event $\boldsymbol{B}_s$. Its probability is calculated by Eq. (11). If the randomness of the load direction only without that of the weight and velocity of the aircraft is taken into account, the probability of a building structure failure (the event $\boldsymbol{F}_s$) is determined by means of the following procedure.

First of all, a strength analysis of the ctructure at the maximal load $R_{max}(t)$ should be carried out. If the strength is sufficient, the conditional probability of failure $P_s(\boldsymbol{F}_s|\boldsymbol{B}_s)$ is equal to zero.

If the strength of the building structure is unsufficient, the maximal value of normal load $R_{lim}(t)$ should be determined which does not cause failure of the structure. It means, that the structure failure will take place if the aircraft impact is applied within the limits of a cone with an apex angle γ shown in fig.8, where

$$\gamma = \arccos(R_{\lim}/R_{\max}). \tag{13}$$

Hence the conditional probability of failure is equal to

$$P_s(\boldsymbol{F}_s|\boldsymbol{B}_s) = f(\varphi,\gamma), \tag{14}$$

where φ is an angle between the normal and the vertical. To obtain the total probability of failure $P(\boldsymbol{F}_s)$ for a year, the conditional probability Eq. (14) should be multiplied by the probability of impact $P(\boldsymbol{B}_s)$, Eq. (11), and that of the realization of event P_τ, Eq. (8). For typical shapes of NPP building structures the total probability may be written as

$$P(\boldsymbol{F}_s) = P_s \cdot 10^{-10} \exp(-10^{-4} A), \tag{15}$$

where A is the total area of the building structures accessible to an aircraft impact; P_s is calculated with the use of the following formulae:

- For a horizontal plane with area A_s

$$P_s = f(0,\gamma)A_s. \tag{16}$$

- For a vertical plane with area A_s and a cylinder with a vertical axis and surface area A_s

$$P_s = f(\pi/2,\gamma)A_s. \tag{17}$$

- For a spherical segment (cupola) with a radius r, bounded by a cone with an apex angle δ (fig. 9),

$$P_s = 2\pi r^2 a_{sph}(\delta,\gamma), \tag{18}$$

where $a_{sph}(\delta,\gamma)$ is the function depicted in fig. 10 and calculated by the formula

$$a_{sph}(\delta,\gamma) = \int_0^\delta f(\alpha,\gamma)\sin\alpha \, d\alpha. \tag{19}$$

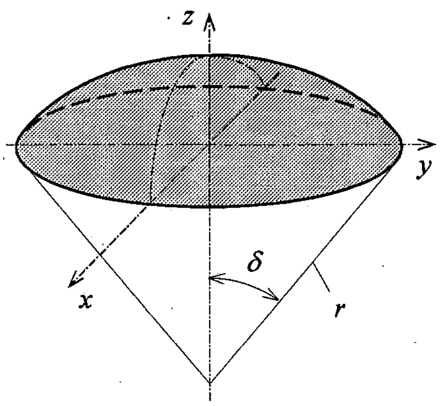

Fig. 9. Spherical segment (cupola)

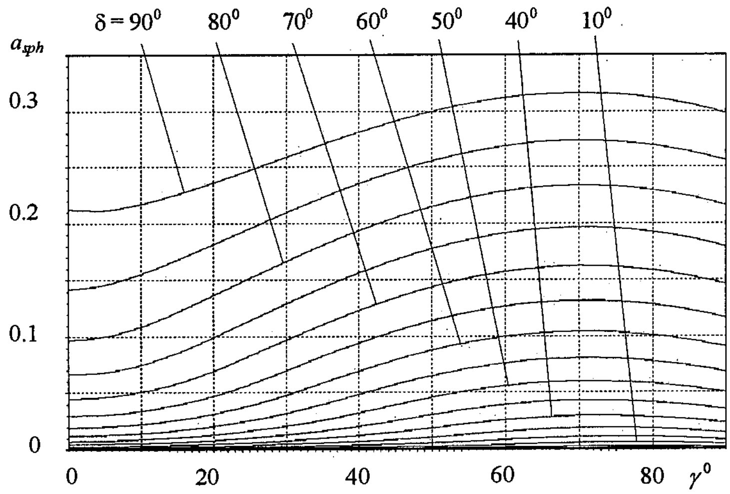

Fig. 10. Function $a_{sph}(\delta,\gamma)$

3.4. Probability of Structure Failure with Aircraft Fall Direction, Mass and Velocity Randomness Consideration

If in addition to the randomness of the aircraft fall direction that of its mass and velocity is considered, the normal component of load on a building structure is equal to:

$$R_n(t) = cR_{\max}(t), \tag{20}$$

where

$$c = k_\gamma k_{mv}; \tag{21}$$

$k_\gamma = \cos\gamma$; as before, γ is an angle between load $R_{max}(t)$ and the structure surface normal; factor k_{mv} is the ratio of the maximum load value corresponding to the given aircraft mass m and velocity v, to the maximum load corresponding to the greatest m and v. The impact load was calculated by Riera's formula, Ref. [5], and thus $0.17 \le k_{mv} \le 1$ was obtained.

The density function $p_{mv}(k_{mv})$ of factor k_{mv} was calculated with the use of m and v probability densitiy functions given in item 2.3. The factor k_γ density function $p_{k\gamma}(\gamma|\varphi)$ was obtained from Eq. (4). The cumulative distribution function $P(c|\varphi)$ of factor c corresponding to different plane slope angles φ equals:

$$P_c(c|\varphi) = \int_{0.17}^{c} p_{mv}(k_{mv})dk_{mv} + \int_{c}^{1} p_{mv}(k_{mv}) f(\varphi, \frac{c}{k_{mv}})dk_{mv} + e(c)[1 - f(\varphi, \pi/2)]. \quad (22)$$

Function $[1-P(c|\varphi)]$ used for building structures failure probability calculation is shown in fig. 11

The probability of a building structures failure for one year $P(\boldsymbol{F}_s)$ is calculated by Eq. (15) where P_s for typical shapes of NPP structures equals:

- For a horizontal plane with area A_s

$$P_s = [1 - P_c(c_{\lim}|0)]A_s, \quad (23)$$

where

$$c_{\lim} = R_{\lim}/R_{\max}; \quad (24)$$

- For a vertical plane with area A_s and a cylinder with a vertical axis and surface area A_s

$$P_s = [1 - P_c(c_{\lim}|\pi/2)]A_s; \quad (25)$$

- For a spherical segment (cupola) with a radius r, bounded by a cone with an apex angle δ (fig. 9)

$$P_s = 2\pi r^2 b_{sph}(\delta, \gamma), \quad (26)$$

where $b_{sph}(\delta, c_{\lim})$ is the function shown in fig. 12 and calculated according to the following formula:

$$b_{sph}(\delta, c_{\lim}) = \int_0^{\delta} [1 - P_c(c_{\lim}|\varphi)]\sin\alpha \, d\alpha. \quad (27)$$

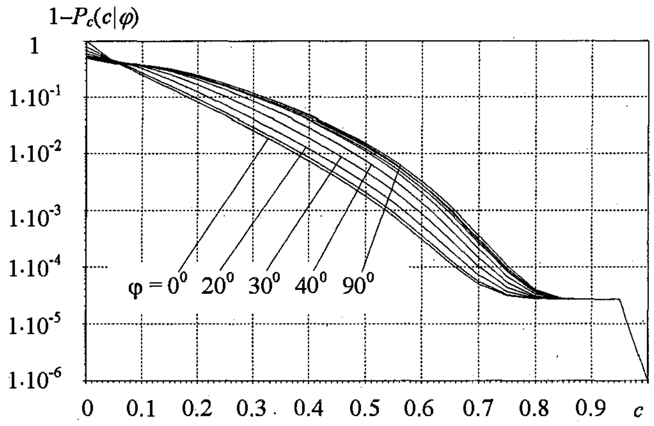

Fig. 11. Function $[1-P(c|\varphi)]$

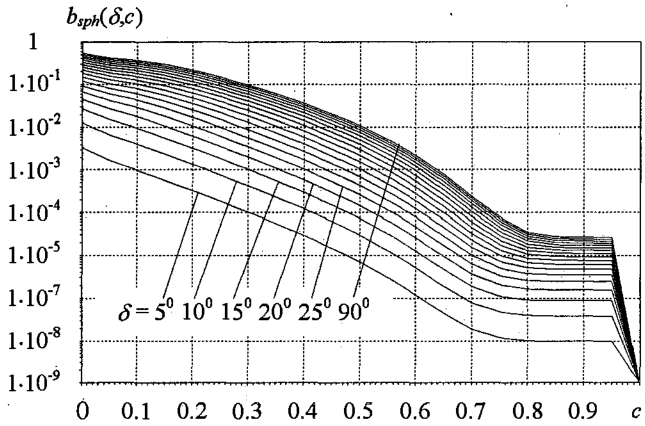

Fig. 12. Function $b_{sph}(\delta,c)$

3.5. Total Probability of Building Structures Failure

As impacts on various structures of a building are independent random events, the total probability $P(\boldsymbol{F})$ of failure of its structures equals to the sum of probabilities of failures of all individual structures:

$$P(\boldsymbol{F}) = \sum_{s=1}^{m} P(\boldsymbol{F}_s) = 10^{-10} \exp(-10^{-4} A) \cdot \sum_{s=1}^{m} P_s, \quad (28)$$

where m is the number of structures, in which an impact is possible. This total probability of failure should not surpass the admitted value $[P]$:

$$P(\boldsymbol{F}) \le [P]. \quad (29)$$

This value should in principle be set considering the safety requirements for NPP as a whole. In the IAEA Safety Guide, Ref. [1] the following value is given as a limit of the probability of occurrence per annum of events eventually leading to radiological consequences:

$$[P] = 10^{-7}\ \text{1/year}. \tag{30}$$

If being assumed as the admitted probability of failure, the following condition is obtained from Eqs. (28) and (29):

$$10^{-3}\exp\left(-10^{-4}A\right)\cdot\sum_{s=1}^{m}P_s < 1. \tag{31}$$

It should be born in mind that NPP building structures failure themselves do not mean the achievement of inadmissible radiological consequences. Therefore the specified admitted probability provides for a margin of NPP reliability.

3.7. Aircraft Class Randomness Consideration

If the fall on NPP of aircraft of m types with probabilities $P(B_k)$ is possible, the conditional probability $P(F_k|B_k)$ of building structures failure corresponding to every of them should be calculated, and then the total probability of failure is calculated:

$$P(F) = \sum_{k=1}^{m}P(F_k|B_k)P(B_k). \tag{32}$$

4. EXAMPLES

Two sequences of probabilistic analysis are possible which may be named as "direct" and "reverse" ones. In the first case the strength of a building structure is known, and the probability of its failure is calculated which is compared with the admitted value. Such approach is applicable, in particular, in the analysis of the reliability of existing NPPs. With the "reverse" sequence the value of a design load, whose realization probability is not more than the admitted one is determined, and then the usual strength analysis of a structure at this load is carried out. Below, as an example, the probabilistic analysis of two containments of different sizes is carried out with the use of a "reverse" sequence.

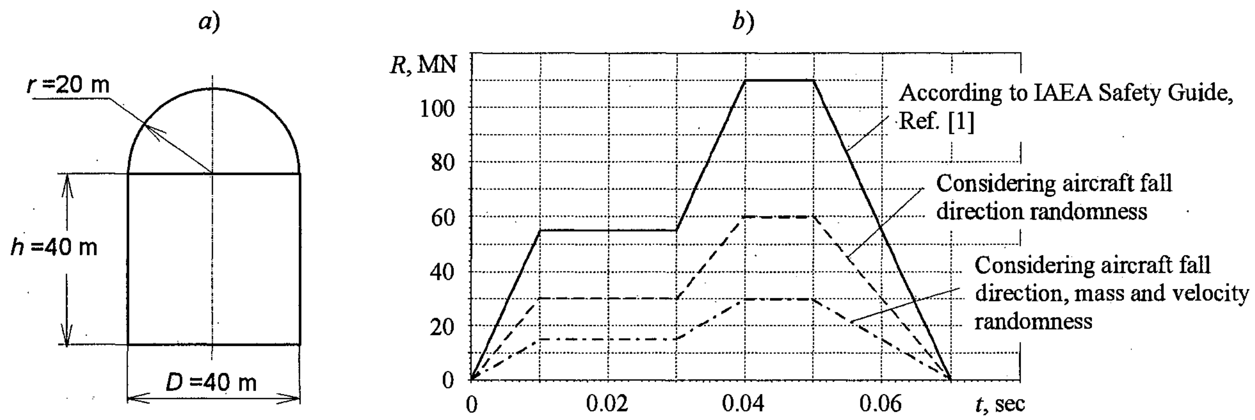

Fig. 13. Design loads on containment caused by aircraft impact:
a – containment dimensions; *b* – design loads

The first of the containments is shown in fig. 13,*a*. When determining the load it was accepted, that the strength of its spherical and cylindrical parts, i.e. the load $R_{\lim}$ is the same, and that 30% of the cylindrical part area is protected from an impact by other structures.

The calculations were carried out by Eq.(30), i.e. proceeding from the admitted probability of failure 10^{-7} 1/years. The aircraft impact direction randomness alone being considered, Eqs. (17) and (18) for P_s are substituted in Eq.(31). As a result an equation with respect to angle γ is obtained, out of which $\gamma = 57^0$ is found. Thus, the design load is equal to $R_{\lim} = R_{\max}\cos\gamma = 0.545R_{\max}$, i.e. it makes 54.5% of the maximum load $R_{\max}$. This load is shown in fig. 13, where $R_{\max}$ is accepted according to IAEA Safety Guide, Ref. [1], often used when designing NPPs.

Considering the randomness of the aircraft impact angle, mass and velocity, Eqs. (25) and (26) were substituted in Eq. (31), and from the obtained equation with respect to c the value $c_{\lim}=0.15$ was found. Hence, design load is equal to $R_{\lim}= 0.15R_{\max}$. It should be taken into consideration, that for values of the maximal aircraft mass and velocity specified in item 2.3, which were used when the factor c density function was calculated, the maximum force is $R_{\max} = 200$ MH. The design load obtained with the use of this value is shown in fig. 13,*b*. The additional account of aircraft velocity and mass randomness is seen to allow the reduction of design load by another half.

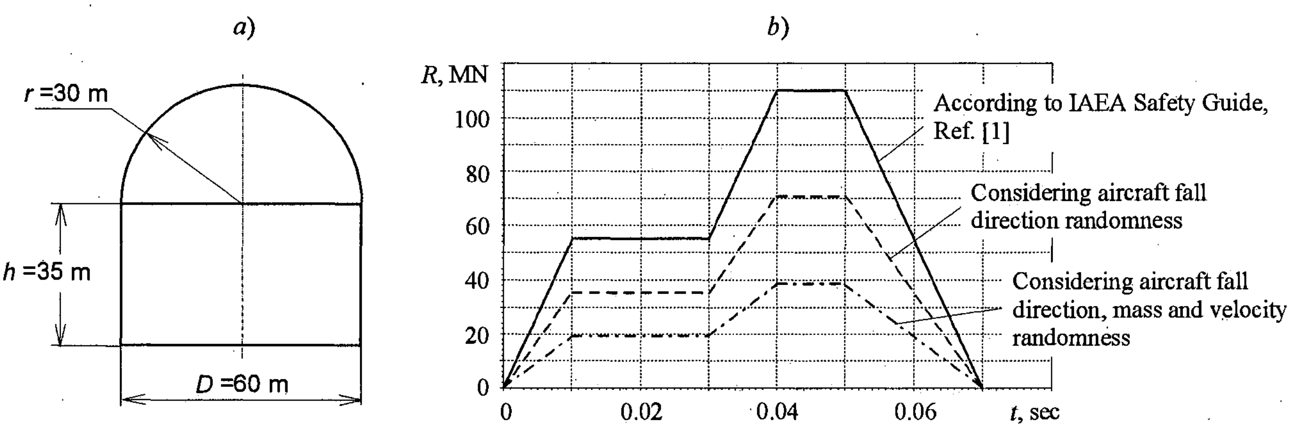

Fig. 14. Design loads on containment caused by aircraft impact:
a – containment dimensions; *b* – design loads

Design loads for the containment shown in fig. 14 were similarly calculated. Its dementions are adopted from Ref. [6] and correspond to the German NPP "Biblis". The obtained reduction of the design load is seen to be a little less significant, than for the first containment, which is smaller in size. This means, that the smaller the building the smaller design load should be applied to provide the same its failure probability, as probability of the aircraft fall on it is less.

CONCLUSION

The fall of the aircraft on NPP is an extremely rare event. The intuition prompts that taking it into account in the NPP design by the principle "of the worst variant of load" is an excessively conservative approach. The above mentioned technique allows quantitatively to estimate the realistic probability of building structures failure at the aircraft impact. The falls recurrence, as well as the randomness of the type, velocity, mass, place and angle of the aircraft impact are taken into consideration. The examples of numerical calculations show, that with the usual NPP building dimensions the design load can be taken several times as little as the maximal one, an extremely small probability of building structures failure being simultaneously guaranteed.

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