Sensitivity Analysis of Actively Controlled Buildings under Seismic Excitation

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ABSTRACT

A sensitivity analysis is performed to show the relative importance of the variation of different parameters of the sliding mode control algorithm used to actively control the building frames under seismic excitation. It also illustrates the robustness of the control strategy against the inherent error in the measurement of the system parameters and feedback state vector. The control parameters include the elements of the diagonal weighing matrix, and the sliding margin variable. The system parameters include the stiffness, damping and mass matrices of the structure. A model of 10-story shear frame building is considered as an example problem in which an actuator placed at the top floor of the building is used to provide the control force. Numerical results are obtained for simulated earthquake records obtained from both narrow-band and broad-band power spectral density functions of earthquake. Results of the analysis show that the control gain of the displacement response of the top story is highly sensitive to the combination of D_{10} and V_{10}, where D_{10} refers to the element of the weighing matrix corresponding to top story displacement and V_{10} refers to the element corresponding to the velocity of top story of the frame. It is also observed that there exists a set of values of the ratio of D_{10}/V_{10} and V_{10}/V_n (n=1 to 9)for which maximum control gain for the top story displacement is achieved. It is also observed that sliding mode control is not so robust as it is claimed to be. Control of response is affected by small errors in the measurement of feedback state vector or in estimating the stiffness, and mass matrix properties of the structure.

INTRODUCTION

Active control of buildings for the reduction of response under earthquake excitation has drawn the attention of many researches in recent years . Two types of control schemes have been investigated namely, diagonal tendon system[1] and active tuned mass damper[2]. Different control algorithms have been developed namely Feedforward, Feedbackward[3], Feedbak-feedforward[4], Instantaneous control[5] and sliding mode control. Efficiency of each control algorithm has been investigated and the maximum control force required to obtain the desired reduction of response has been obtained for different algorithms. In all algorithms, the parameters which influence the reduction of response have been studied to obtain the best combination of their values to achieve the maximum control of the response. However, most of the studies were confined to only a few real earthquake records[6]. The choice of the optimum parameters were not investigated for all kinds of excitations. Further, the robustness of the control algorithms are not thoroughly investigated.

In the present paper, the sensitivity of the Sliding Mode Control (SMC) is investigated in order to study the overall robustness of the SMC strategy. The parameters which are varied include the elements of the weighing matrix, mass and stiffness properties of the building, feedback responses and the ground motion. A model of 10-story shear frame building is considered as an example problem in which a control force is placed at the top of the building. The control of the response quantities which are considered for evaluating the algorithm are displacement and acceleration of the top story and the base shear.

THEORY

For the building frame shown in figure 1, the equation of motion corresponding to the structural sway degrees-of freedom of the frame under seismic excitation $\ddot{x}_g$ (t) and the control actions u_s can be written as

$$M\ddot{X} + C\dot{X} + KX = -MI\ddot{x}_g + Hu_s \tag{1}$$

where M, C and K are, respectively, the mass, damping, and stiffness matrices corresponding to the sway degrees of freedom , X is a vector of displacements of the frame with respect to the ground, overdots indicate time derivatives; I is unit vector of size n ; $u_s(t)$ is a vector consisting of r control forces; H is a $(n\times r)$ matrix denoting location of r controllers.

In modal co-ordinates of the structure, the equation can be written as

$$\overline{M}\ddot{Z} + \overline{C}\dot{Z} + \overline{K}Z = -\phi^T MI\ddot{x}_g + \phi^T Hu_s \tag{2}$$

in which $\overline{M}$, $\overline{C}$, and $\overline{K}$ are $m \times m$ diagonal matrices; $\overline{M} = \phi^T M \phi$; $\overline{C} = \phi^T C \phi$; $\overline{K} = \phi^T K \phi$; ϕ is undamped mode shape matrix of size $n \times m$; Z is a vector of generalised co-ordinates and m is the number of mode shapes considered.

In the state space form, equation (2) can be written as

$$\dot{V} = AV + B_S u'_S + F_S \ddot{x}_g \tag{3}$$

in which $V^T = [Z\dot{Z}]$ and A is $[2(m+1) \times 2(m+1)]$ matrix; B_s is a coefficient matrix of size $2(m+1) \times r$; F_s is the input excitation of size 2(m+1).

The theory of Sliding Mode Control is to design controllers to drive the response trajectory into the sliding surface and maintain it there, whereas the motion on the sliding surface is stable. In the design of sliding surface, the external excitation is neglected. However, it is taken into account in the design of controllers. For **r** number of controllers, one needs to define exactly r sliding surfaces. They are defined as a linear combination of the state vector as

$$S = \lambda V = 0 \tag{4}$$

in which S is a vector consisting of **r** sliding variables i.e. $S = [S_{1,} S_{2,} S_{3,} ...., S_r]^T$, **r** being the total number of controllers at the floors and λ is $r \times 2(m+1)$ matrix to be determined such that the motion on the sliding surface is stable. For the response trajectory to stay on the sliding surface, once it reaches there, $\dot{S} = 0$. It follows from equations (3) and (4) that

$$\dot{S} = \left(\frac{\partial S}{\partial V}\right)\dot{V} = \lambda(AV + B_s u'_s) = 0 \tag{5}$$

The solution of equation (5) for u'_s yields the so-called equivalent control on the sliding surface

$$u'_s = -(\lambda B_s)^{-1} \lambda AV \tag{6}$$

Substituting Eq.(6) into Eq.(3), with the external excitation being neglected, the following equation is achieved.

$$\dot{V} = \left[A - B_s (\lambda B_s)^{-1} \lambda A\right] V \tag{7}$$

To determine λ matrix, the state equation of motion, Eq.(3) is transformed into so called regular form by the following transformation[8]

$$\overline{Y} = TV \tag{8}$$

where **T** is a transformation matrix defined in terms of the submatrices B_1 and B_2 , of the input matrix B_s as follows:

$$T = \begin{bmatrix} I_{2(m+1)-r} & -B_1 B_2^{-1} \\ 0 & I_r \end{bmatrix} ; \qquad T^{-1} = \begin{bmatrix} I_{2(m+1)-r} & B_1 B_2^{-1} \\ 0 & I_r \end{bmatrix} \tag{9}$$

where $I_{2(m+1)-r}$ and I_r are $[\{2(m+1)-r\} \times \{2(m+1)-r\}]$ and $(r \times r)$ identity matrices respectively, and B_1 and B_2 are $[\{2(m+1)-r\} \times r]$ and $(r \times r)$ submatrices of the matrix B_s, respectively, as

$$B_s = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \tag{10}$$

The design of sliding surface $S = \lambda V = 0$ is obtained by minimizing the integral of the quadratic function of the state vector

$$J_s = \int_0^\infty (V^T Q_s V) dt \tag{11}$$

in which Q_s , a $2(m+1) \times 2(m+1)$ positive semidefinite matrix, is a weighing matrix associated with the components of V.
The controllers are designed to drive the state trajectory into the sliding surface S=0. To achieve this goal a Lyapunov function Γ is considered[9].

$$\Gamma = \frac{1}{2} S^T S = \frac{1}{2} V^T \lambda^T \lambda V \tag{12}$$

The sufficient condition for the sliding mode S=0 to occur as $t \Rightarrow \infty$ is

$$\dot{\Gamma} = S^T \dot{S} \le 0 \tag{13}$$

Using the state equation of motion, Eq.(3), with the above condition, the following equation can be written[9]

$$\dot{\Gamma} = \Lambda\left(u_s' - \Delta\right) = \sum_{i=1}^{r} \dot{\Gamma}_i \tag{14}$$

in which Λ^T and Δ are r vectors and can be expressed as

$$\Lambda = S^T \lambda B_s \,; \quad \Delta = -(\lambda B_s)^{-1} \lambda\left(AV + F_s \ddot{x}_g\right) \tag{15}$$

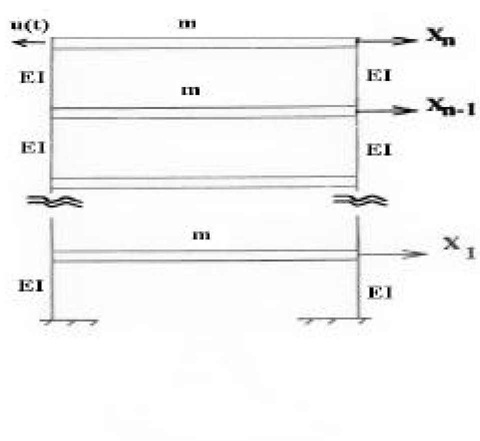
For $\dot{\Gamma} \le 0$, a possible continuous controller is given by[9]

$$u_s' = \Delta - \delta A = -(\lambda B_s)^{-1} \lambda F_s \ddot{x}_g - \left[(\lambda B_s)^{-1} \lambda A + \delta B_s^T \lambda^T \lambda\right] V \tag{16}$$

in which δ is a diagonal matrix with diagonal elements $\delta_i \ge 0$, and is referred as the matrix showing the sliding margin. Here u_s' contains not only a function of state variables Z and $\dot{Z}$ but also the external excitation $\ddot{x}_g$, i.e. knowledge of the external excitation upto the point of time at which control force applied is measured.

DISCUSSION OF RESULTS

A 10-storey shear-building frame subjected to earthquake-induced ground motions is considered as an example problem. The structure represents a typical medium sized multistorey building. Each storey has the same mass, stiffness and damping parameters. The structural properties and natural frequencies of this example structure are shown in Figure 1. The modal damping is taken as 2% of the critical damping.



Mass of Each
Floor = 1.15×10^5 Kg

Stiffness of each
Floor = 2.057×10^8 N/m

Modal Damping = 2%

Floor	Natural Frequency (rad/sec)
1	6.3209
2	18.8216
3	30.9018
4	42.2917
5	52.7369
6	62.0040
7	69.8861
8	76.2070
9	80.8264
10	83.6387

Fig.1. Schematic of the Example Problem: 10-Storey Building with Actuator at the Top

The control actions are applied through an actuator placed at the top of the building. To evaluate the effectiveness of the control system for different disturbances, three different excitations are used in numerical simulations, which are synthetically generated from the three power spectral density functions (psdf) taken as the Clough & Penzien double filter psdf [7] with following combinations of filter parameters:

i) Soft Soil (Narrow Band) : $\omega_1=2\pi$, $\omega_2=0.1\omega_1$, $\xi_1=\xi_2=0.4$
ii) Firm Soil : $\omega_1=5\pi$, $\omega_2=0.1\omega_1$, $\xi_1=\xi_2=0.6$
iii) Extremely Firm Soil (Broad Band) : $\omega_1=10\pi$, $\omega_2=0.1\omega_1$, $\xi_1=\xi_2=0.8$

From the perspective of the practical implementation of any active structural control scheme, the maximum value of the required control force is an important operational parameter. This maximum value, in general, depends upon several factors.

As a general thought, one can think that in a well-designed control algorithm, the control requirements are directly related to the reduction in the response achieved. If a larger reduction is desired then a correspondingly larger level of control fore may be required. But, in the sliding mode control approach this relationship between the magnitude of the control force and the level of control achieved is not so simple and straightforward. That is, it is not so simple to regulate the amount of response reduction to a desired predefined level. For a given controller design, the level of control achieved and the corresponding control requirements are determined, to a large extent, by the sliding surface used. The choice of the weighing matrix Q_s determines the sliding surface. The weighing matrix adopted here is a diagonal matrix with the following structure:

$$Q_s = \begin{bmatrix} D & 0 \\ 0 & V \end{bmatrix} \tag{17}$$

The diagonal matrices D and V contain the weights associated with the relative displacements and the relative velocities, respectively. The elements of these matrices can be shown as follows:

$D = \text{diag}(D_1, D_2, D_3, D_4, D_5, D_6, D_7, D_8, D_9, D_{10})$; and $V = \text{diag}(V_1, V_2, V_3, V_4, V_5, V_6, V_7, V_8, V_9, V_{10})$

A change in any element of Q_s will result into a different sliding surface. For a given controller and for a given input ground motion, once a sliding surface is fixed, it determines the time history of the required control force and the reduction in response quantities.

Selection of Elements of Q_s Matrix

The search for the elements of Q_s which predominantly governs the relationship between the level of control action and the magnitude of reduction in response requires extensive parametric study. To see the effect of various elements of the weighing matrix Q_s on the controlled responses and the required maximum control force, each element of Q_s was varied one by one and the percentage reduction in response quantities and the gain {=(% reduction in maximum displacement/maximum control force)×multiplying factor(1.0E+06)} is noted. The study shows that the last element associated with the relative displacements (D_{10}) is more dominant in governing the control as compared to the first nine elements of diagonal matrix D. Similarly, in case of elements associated with the relative velocities (elements of diagonal matrix V) the last element (V_{10}) of the diagonal matrix V is more dominant in governing the control as compared to the rest of the nine elements of V. Note that the control force is applied at the top storey i.e. the 10th floor. Table 1 shows the effect of variation of some of the elements of Q_s matrix on the reduction of the response quantities.

Table 1 Effect of Elements of Weighing Matrix on Reduction in Responses

Elements of Q_s Matrix And their Values		Reduction in Top Floor Displacement (%)	Gain	Reduction in Top Floor Abs. Acceleration (%)	Reduction in Base Shear (%)
D_1	1.00E+00	78.78	56.30	83.19	41.03
	1.00E+02	79.27	58.88	83.06	40.95
	1.00E+03	82.31	62.56	82.29	40.39
D_2	1.00E+01	78.96	57.33	83.14	41.01
	1.00E+02	80.59	60.49	82.74	40.73
	1.00E+03	86.12	67.15	81.31	39.50
D_{10}	1.00E+01	82.66	62.77	82.39	40.26
	1.00E+02	90.25	71.00	81.30	37.61
	1.00E+03	96.02	73.09	80.92	36.19
V_1	1.00E+01	76.39	51.42	83.01	41.43
	1.00E+02	67.64	36.52	82.74	43.43
	1.00E+03	54.01	21.02	84.10	46.16
V_2	1.00E+01	73.98	46.83	82.90	42.09
	1.00E+02	60.09	26.91	83.43	45.01
	1.00E+03	52.75	19.58	84.38	47.05
V_{10}	1.00E+01	87.07	64.78	85.30	39.01
	1.00E+02	95.97	57.21	85.07	32.69
	1.00E+03	99.19	50.27	84.46	18.51

Parametric study shows that in addition to the variation of individual elements of Q_s matrix, there exist some ratios between various elements of this matrix which predominantly govern the control. The important ratios are (D_{10}/V_{10}) and (V_{10}/V_n), where n=1 to 9. Thus to obtain the better control, the values of weights associated with the relative velocities should be decided based on these two ratios after selecting the optimum value of the element D_{10}. Table 2 shows the effect of variation of D_{10}/V_{10} and V_{10}/V_n on the reduction of the response quantities.

Table 2 Effect of Variation of Ratios (D_{10}/V_{10}) and V_{10}/V_n

Ratios of Elements And their Values		Reduction in Top Floor Displacement (%)	Gain	Reduction in Top Floor Abs. Acceleration (%)	Reduction in Base Shear (%)
V_{10}/V_n	1	78.78	56.30	83.19	41.03
	10	87.07	64.78	85.30	39.01
	100	95.97	57.21	85.07	32.69
	1000	99.19	50.27	84.46	18.51
Q_{10}/V_{10}	1	88.24	66.40	85.02	38.80
	10	92.20	71.05	84.38	37.85
	100	96.26	74.72	84.27	37.28
	1000	98.68	63.57	84.61	33.72

The optimum values of D_{10} and V_{10} and Optimum values of (D_{10}/V_{10}) and (V_{10}/V_n) are obtained by using univariate method of optimization. It is interesting to note that once the optimum values of the ratios (D_{10}/V_{10}) and (V_{10}/V_n) are selected, the variation of D_{10} and V_{10} does have any significant effect on the controlled responses till their selected ratio remains the same (Table 3).

Table 3 Effect of Variation of D_{10} and V_{10} on Controlled Responses for D_{10}/V_{10}=100 & V_{10}/V_n=10

D_{10}	V_{10}	V_n (n=1-9)	Reduction in Top Floor Displacement (%)	Gain	Reduction in Top Floor Abs. Acceleration (%)	Reduction in Base Shear (%)
1.00E+00	1.00E-02	1.00E-03	95.96	72.86	83.73	36.61
1.00E+03	1.00E+01	1.00E+00	96.26	74.72	84.27	37.28
1.00E+06	1.00E+04	1.00E+03	96.26	74.74	84.27	37.29

Study also shows that the optimum values of these parameters depend on the ground excitation. Optimum values of (D_{10}/V_1) and (V_1/V_n) ratios as obtained for the three different ground excitations are given in Table 4 .

Table 4 Optimum values of (D_{10}/V_{10}) and (V_{10}/V_n)

Type of Ground Excitation	D_{10}/V_{10}	V_{10}/V_n
Narrow Band (Soft Soil)	100	10
Firm Soil	100	1
Broad Band (Very Firm Soil)	10	1

The sensitivity analysis of the control algorithm has been performed with help of perturbation of state-feedback vector and the system parameters.

Perturbation of the State-Feedback Vector

A control mechanism is designed with the ideal condition that every instrument and machinery of the control device will perform its assigned job perfectly with full accuracy. But there may be an error in its functioning or even a total failure of the instrument. The effect of this possibility of error in measurement of state-feedback vector by the sensors on the reduction in response quantities and the required control force is of great interest. This study is performed by introducing an error in the elements of the state-feedback vector, namely, the ground acceleration, relative displacements and relative velocities of various floors of the structure. The study shows that the control algorithm is more sensitive to the error in the measurement of feedback velocities compared to those of feedback displacement and ground excitation. The error in the feedback displacements does not have any significant effect on controlled responses (Fig. 3). The error in the measurement of ground excitation has some effect on the reductions in displacement and acceleration responses. An error of 20% in the measurement of ground excitation brings down the reduction in displacement from 97% to 95% (Fig. 4). The same error on the positive side brings down the reduction in acceleration from 75% to 70%, whereas on the negative side improves the reduction in acceleration from 75% to 80%. The reduction in base shear remains insensitive to measurement errors. The change in the reduction of response quantities suddenly becomes very large after a certain percentage of error in the measurement of feedback velocities (Fig. 5). An error of 25% (on positive side) in feedback velocity measurement brings down the displacement reduction of top floor from 97% to 7.9% and the absolute acceleration reduction of top floor from 75% to 2.3%. An error of 26% in feedback velocities amplifies the displacement and acceleration of the top floor by about 100%. Error on the negative side does not show much change in the reduction of responses. Further, reduction in base shear is not very much sensitive to the error in the measurements of feedback velocity.

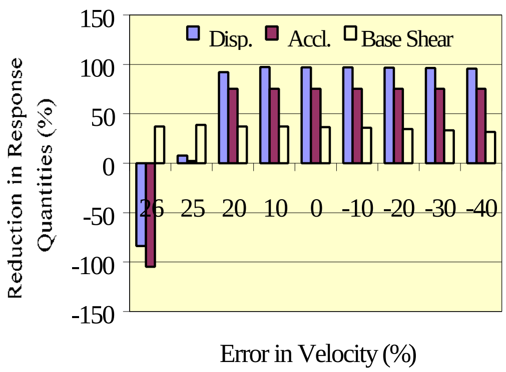


Fig. 5 Effecet of Error in Measurement of Feedback Velocity

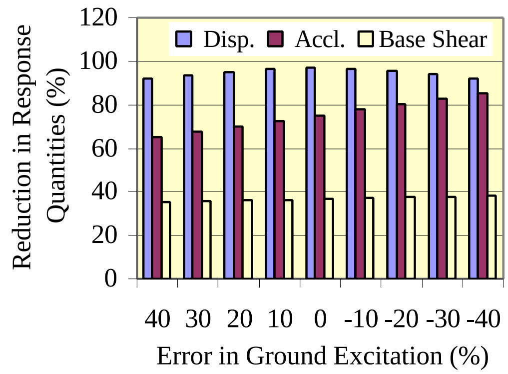


Fig. 4 Effect of Error in Measurement of Ground Excitation

This change in reduction of response quantities occurs even at much less measurement errors if they are introduced simultaneously in the feedback velocities and the ground excitation (Fig. 6). An error of 10% in feedback velocities and the ground acceleration measurements brings down the displacement reduction from 97% to 22% and absolute acceleration reduction from 75% to 61%. An error of 11% in feedback velocities and the ground acceleration measurement amplifies the displacement of the top floor by about 50%.

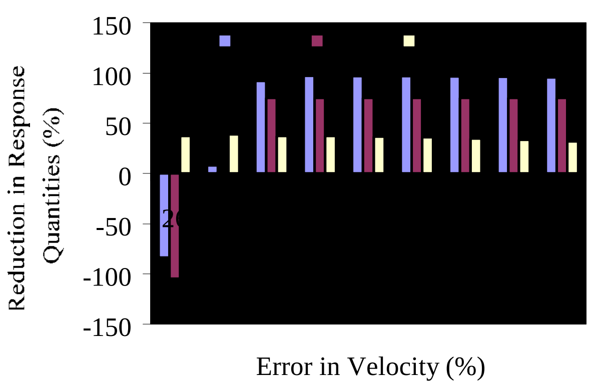


Fig. 5 Effecet of Error in Measurement of Feedback Velocity

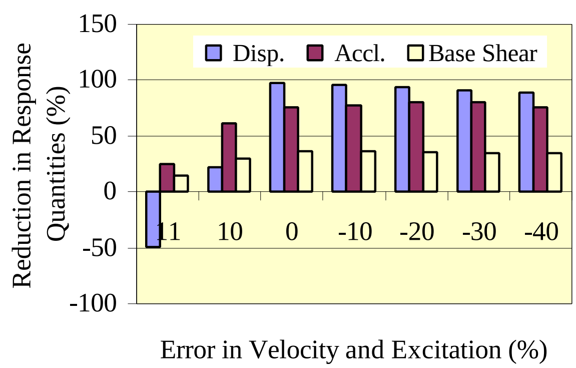


Fig. 6 Effect of Error in Feedback Velocity and Ground Excitation

The study also shows that the sensitivity of the control algorithm is earthquake dependent. In the case of narrow band excitation, an error of 25% (on positive side) in feedback velocity measurement brings down the displacement reduction of top floor from 97% to 7.9%, whereas in the case of broad band excitation an error of 4% (on positive side) in feedback velocity brings down the displacement reduction from 90.7% to 12.8%. An error more than 4% (on positive side) in feedback velocity measurements for broad band excitation amplifies the top floor displacement by about 453%.

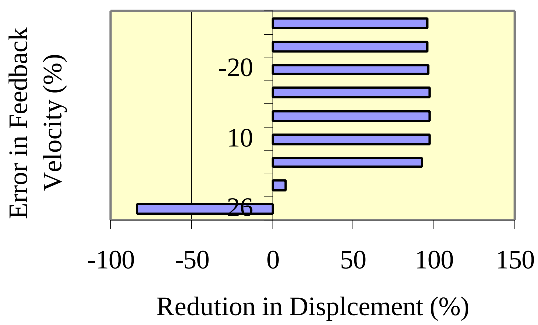


Fig. 7 Effect of Error in Velocity (narrow band excitation)

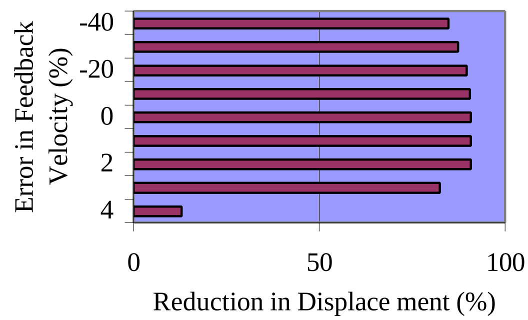


Fig. 8 Effect of Error in Velocity (broad band excitation)

Perturbation of System parameters

To study the effect of error in determining the system parameters (mass and stiffness of the structure) on the controlled responses, an error is introduced in each system parameter one by one. The study shows that control algorithm is more sensitive to the error of measurement in the mass and stiffness of the structure thereby bringing in quite significant changes in the reduction of response quantities after a certain level of error in the measurement of these two system parameters (Fig. 9 & 10). An error of 22% in mass determination (on the positive side) is capable of bringing down the displacement reduction of the top floor from 97% to 8.7% and absolute acceleration of the top floor from 75% to 27.6%. An error of 23% in mass estimate of the system amplifies the top floor displacement and the absolute acceleration.

An error of 18% (on the negative side) in stiffness estimate brings down the displacement reduction from 97% to 13.7% and the reduction in absolute acceleration from 75% to 31.5%. An error of 19% in stiffness estimate amplifies the top floor displacement and the absolute acceleration by about 110% and 75% respectively.

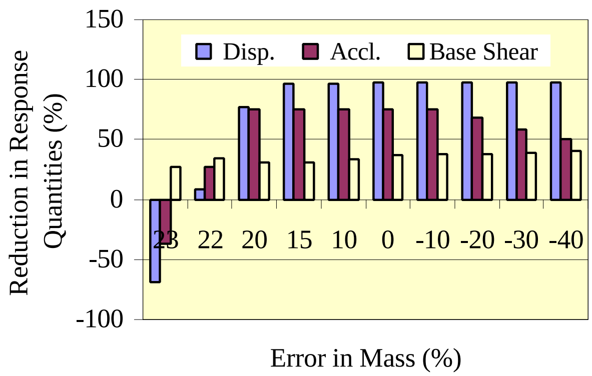


Fig. 9 Effect of Error in Mass of the System (narrow band excitation)

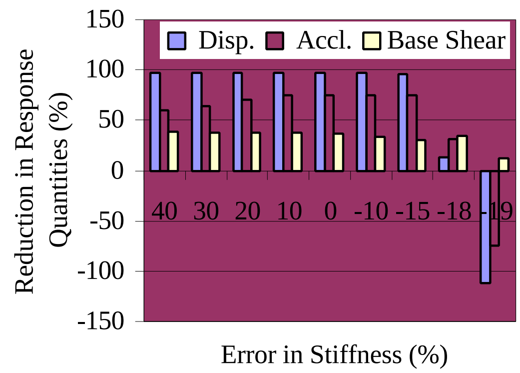


Fig. 10 Effect of Error in Stiffness (narrow band excitation)

The study also shows that the sensitivity of the control algorithm is earthquake dependent. As seen earlier in Fig. 9, in case of narrow band excitation, an error of 22% in mass estimate of the system brings down the displacement reduction of top floor from 97% to 8.7%. In case of broad band excitation (Fig.11), an error of 3% (on the positive side) in mass of the system brings down the displacement reduction of top floor from 91% to 82% and an error of 4% amplifies the top floor displacement. Similarly, in case of broad band excitation (Fig. 12), an error of 3% (on negative side) in the estimation of stiffness brings down the displacement reduction from 91% to 80.3% and an error of 4% amplifies the top floor displacement by 17%.

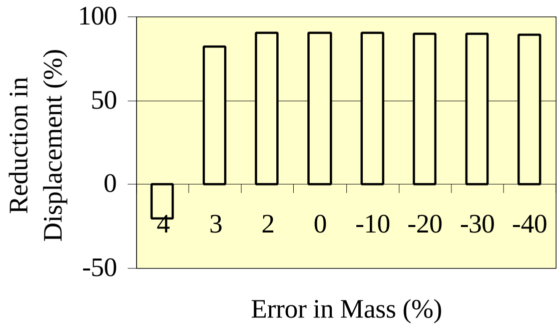


Fig. 11 Effect of Error in Mass of the System (broad band excitation)

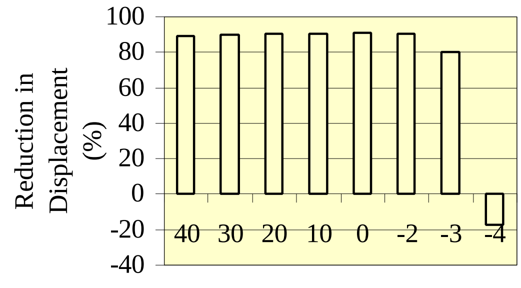


Fig. 12 Effect of Error in Stiffness of the System (broad band excitation)

CONCLUSION

A sensitivity analysis of the sliding mode control strategy for the response reduction of building frames under seismic excitation is performed. The sensitivity analysis shows the performance of the control algorithm with the variation of important control parameters. Also, it demonstrates how the performance of the control strategy may change with inherent errors in the measurement of system parameters and feedback state vector. A 10-storeyed shear building frame is used as an illustrative example which is actively controlled by sliding mode control strategy by employing an actuator at the top story and is analysed for three different types of synthetically generated ground excitations. The results of the numerical study show that:

1) The weighing parameters corresponding to the top story displacements and velocity are more dominant in controlling the response of the building.
2) There exists a ratio between the values of the weighing parameters of the top story to those of the other stories for which performance of control algorithm is best.
3) Performance of the control algorithm is highly sensitive to the percentage error in the measurement of the state feedback vector and ground excitation beyond a certain level of error. The nature (positive or negative) and magnitude deciding the level of error depend upon the response quantity of interest or the ground excitation being measured. A small combined error (within 10%) in the measurement of velocity and ground excitation may drastically reduce the efficiency of the control scheme.
4) The level of error in the measurement of responses and ground excitation influencing the efficiency of the control scheme also depends upon the nature of excitation.
5) Small errors in the estimation of the mass and stiffness of the frame may upset the performance of the control scheme depending upon the type of excitation (for broad band excitation, it is about $\pm$4%).
6) Reduction in base shear is not found to be very sensitive to the measurement/estimation error.
7) Finally, the sliding mode control strategy does not appear to be as robust as it is claimed.

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