Probabilistic Safety Assessment of Structures Sequentially Failing

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ABSTRACT

A method for the probabilistic safety assessment of structures with sequential failure is presented. It makes use of specific representation tools and of a Branch & Bound procedure involving discrete probability distributions.

1 INTRODUCTION

Reliability analysis of structures sequentially failing, based on Branch & Bound (B & B) algorithms, has been developed rather recently (Murotsu et al. 1983, 1986, 1989). The authors have also proposed such an algorithm at the mechanism level (Vulpe and Cărăuşu 1987). Economic representation tools for the sequential structural failure based on directed acyclic graphs (dags) have been introduced (Vulpe and Cărăuşu 1988, 1989).

In the present paper the authors develop another PSA technique for sequentially failing structures which involves a B & B algorithm and compound discrete probability distributions ; a representation dag with the associated matrices is also used. The technique allows to identify the stochastically dominant failure modes and gives an interval evaluation for the system probability of failure.

2 SEQUENTIAL STRUCTURAL FAILURE AND ITS REPRESENTATION

The system failure event F_S is assumed to consist of several failure modes FM_i $(i \in I)$, and every failure mode is itself a finite set of elementary failures which occur progressively, i.e., these "atomic" failures develop one by one in a temporally ordered sequence. The typical case is the one of the redundant frame structures which fail by formation of mailure mechanisms. Therefore an instance of the system failure is not a simple set of elementary failures but an ordered sequence of the form

$$q = (r_1, r_2, \ldots, r_k, \ldots, r_p) \quad . \tag{1}$$

A sequence of the form (1) is called (Murotsu et el. 1983, 1986) a complete failure path which corresponds, in terms of structural reliability theory, to a system failure mode. To express it probabilistically, let us consider the set

$$EF \underset{not}{=} \{(f_1), (f_2), \ldots, (f_n)\} \tag{2}$$

of the elementary failures. Then a failure mode is given by

$$FM_i = (f_{i1}) \cap (f_{i2}) \cap \ldots (f_{ik}) \cap \ldots \cap (f_{ip}) \quad , \tag{3}$$

in which $(f_{ik}) \in EF$ represents the failure of element r_k as the k-th stage of the failure mode FM_i. It should be emphasized that the event intersection in Eq. (3) is an ordered one.

A mathematical model for the set of possible failure modes of a sequentially failing structures can be formulated in terms of the labeled directed acyclic graphs (ℓ-dags). A labeled dag is a 4-uple

$$D = (N, U, J, \ell) \tag{4}$$

where: N is a finite set of nodes, in 1-to-1 correspondence with EF of (2), $J \subset \mathbb{N} = \{0, 1, 2, \ldots\}$ is the set of labels, U is the set of the arcs such that

$$U \subset N \times N \times J \quad \text{and, for} \quad u = (x_j, x_k; l) \in U, \quad \ell(u) = l \; . \tag{5}$$

It follows from (5) that the labeling function $\ell : U \longrightarrow J$ assigns a label to each arc of the dag, which is (in fact) its 3rd component. Here appears a slight difference as regards our former definitions for the ℓ-dags (Vulpe and Cărăuşu 1988, 1989), and it is necessary since a dag – unlike the usual graphs – may have several arcs between the same two nodes, and they should be distinguished by the labels they bear. The notion of a path in a dag matches the one of the classical graph theory, and the same holds as regards the acyclicity: all the nodes on a path should be distinct, except the case when D admits loops, i. e., U contains arcs of the form $(x_k, x_k; l)$; in this case, D becomes a quasi-dag. More details on the arcs and paths of a dag may be found in our already quoted papers. We only recall that the labeling function ℓ is extended from arcs to paths, such that

$$\begin{aligned} &u, v \subseteq q = (x_1, \ldots, x_k, \ldots, x_p) \in \text{path}(D) \implies \ell(u) = \ell(v) \\ &\text{and} \quad u, v \in \text{PATH}(D), \quad u \neq v \implies \ell(u) \neq \ell(v) \; . \end{aligned} \tag{6}$$

In (6), path(D) denotes the set of all the paths over D, and PATH(D) stands for the set of all the complete paths. It follows from (6) that all the arcs on the same path are identically labeled, while different complete paths bear distinct labels.

The set J of the labels of D, except the zero label, may be taken as identical to the set I of the subscripts of system's failure modes, when the ℓ-dag D models the sequential failure of a system S. However, the zero label is necessary as it will be seen below. Let the ℓ-dag D_S represent the failure modes of system S, that is, every q_i in PATH(D_S) models a sequential failure mode FM_i as in Eq. (3). D_S may be entirely characterized in terms of a n x n matrix

$$A_D = [a_{jk}] \text{ with } a_{jk} = \{\ell(q) : q \in \text{PATH}(D), (x_j, x_k) \subseteq q\}$$
$$\text{and } a_{jk} = 0 \text{ if } (\nexists q)\ (x_j, x_k) \subseteq q . \quad (7)$$

Conversely, given a matrix A_D with its entries in $2^{\mathbb{Z}_+}$ and certain appropriate constraints on a_{jk}'s, it uniquely defines a dag D together with the set of the complete paths, PATH(D). Moreover, the matrix A_D allows every complete failure path q to be identified by means of a map-matrix M_q (Vulpe & Cărăuşu 1989).

3 FAILURE CRITERIA FOR ELEMENTS AND STRUCTURES

3.1 Safety margins and associated DPD's

Let r_i be a failure element (e.g., a member end or potential plastic hinge) of the system S $(1 \leqq i \leqq n)$ and let us consider a sequential failure along a path q as in Eq. (1). The safety margin of the surviving element r_i after the elements $r_1, \ldots, r_{k-1}$ have failed is defined (Murotsu et al. 1986) by

$$Z_i^{(k)} = R_i + \sum_{j=1}^{k-1} a_{ij}^{(k)} R_j - \sum_{m=1}^{31} b_{im}^{(k)} L_m , \quad (8)$$

where R_j's and L_m's are the strengths of the elements and the applied loads respectively, while a_{ij}'s and b_{im}'s are influence coefficients. When r_i is the k-th element to fail on path q of (1), its safety margin should be written

$$Z^{(k)}_{(r_1, \ldots, r_{k-1})} = R_k - L_{k(r_1, \ldots, r_{k-1})} \quad (9)$$

in which $L_{k(\ldots)}$ denotes the load effect on element r_k due both to external loads and to internal forces generated by the previous k-1 elementary failures.

Obviously, the strengths and the loads appearing in Eqs. (8) and (9) are random variates and therefore the safety margins are random too. Hence the failure criterion for element r_k on the failure path q of (1) is given by

$$Z^{(k)}_{(r_1, \ldots, r_{k-1})} \leqq 0 . \quad (10)$$

When path q is complete, the structural failure criterion is obtained from (10) by taking $k = p$.

Let us now consider a discrete probability distribution (Kaplan 1981) associated with the safety margin of element r_k of the form

$$z_k = \{\langle p_j^k , v_j^k \rangle\} . \quad (11)$$

In fact, each elementary DPD z_k of r_k (except the case when r_k is the starting element on a failure path) is dependent of the failure history, that is, it is conditional on the k-1 preceding failures, as the safety margins of Eqs. (8) and (9) are. This condition of path-dependence will be taken into account by considering compound DPD's recursively defined, along a path q, by

$$z_1^c = z_1 \quad \text{and} \quad z_{k+1}^c = z_k^c * z_{k+1} \quad \text{if} \quad (r_k, r_{k+1}) \subseteq q \ ; \qquad (12)$$

details on the DPD composition * may be found in (Kaplan 1981) and (Vulpe and Cărăuşu 1989).

3.2 Failure criteria in terms of DPD's

Specific distributional assumptions on the random safety margins Z_k should be accepted (on the basis of statistical tests and previous engineering experience); they induce correspondding distributions for the associated DPD's z_k as well as for the compound ones z_k^c, according to the composition * of Eq. (12). Then, for the k-th element on a failure path q – with its DPD of the form (11) – the failure criterion will be

$$(v_j^k \leqq 0) \quad \text{with the probability} \quad \sum_{v_j^k \leqq 0} p_j^k \ \overset{\text{not}}{=} \ P(q_k) \ , \qquad (13)$$

$$q_k = (r_1, r_2, \ldots, r_k) \ .$$

The failure criterion for a structural failure mode represented by the complete failure path q as in (1) will therefore be

$$(v_j^p \leqq 0) \quad \text{with the probability} \quad P(q) \ . \qquad (14)$$

Lower and upper bounds on P(q) can be derived, by analogy to those of (Murotsu et al. 1983, 1986). In other words (and using Murotsu's notations)

$$P^L_{fp(q)} \leqq P(q) \leqq P^U_{fp(q)} \ . \qquad (15)$$

With these preliminaries on the representation dags, on the failure criteria and on the interval evaluation of sequential structural failure probability (yet not given in full detail), we proceed to describe a B & B - type algorithm for the probabilistic safety assessment of sequentially failing structures.

4 AN ALGORITHM FOR PSA OF SEQUENTIALLY FAILING STRUCTURES

4.1 Input data

EF = the set of potential failure elements (plastic hinges) deduced from the geometrical and mechanical properties of S; it is stored as the set $N = \{1, 2, \ldots, n\}$, $n = \text{card}\, EF$.

DPD = the set of discrete probability distributions as in. Eq. (11) associated with the failure elements of EF / N ; in fact, DPD is a (3-dimensional) matrix of size $n \times 2 \times m$ $(j=\overline{1,m})$.

$B = [b_{jk}]$ is a $n \times n$ matrix of coefficients involved in the composition of DPD's – Eq. (12).

A_D = the matrix associated with the failure representation ℓ-dag D_S; in another version of the algorithm, this matrix may be omitted from the input data since the dag D'_S that represents the most probable structural failure modes FM_i is obtained by assembling the complete failure paths automatically selected by the B & B procedure.

B, δ are two pre-established levels involved in the selection of the most probable initial nodes and in discarding operation, respectively.

4.2 Description of Program SEQ-PSA

Step 1 Select the set $N_0 \subset N$ of the most probable roots (i.e. initial failure elements) of possible failure paths by the criterion

$$P(r) \geqq B \implies r = x_0 \in N_0, \quad P \text{ as in } (14) \quad .$$

Step 2 (Initializing) Set $P_{fpM} = 0$, $X_c = \emptyset$, $X_t = \emptyset$, $X = \{x_0\}$ for a $x_0 \in N_0$ and set $N_0 \longleftarrow N_0 \setminus \{x_0\}$.

Step 3 (Partitioning)

3.1 Extend the current path $q \in X$ by adjoining the possible successors of its final node in D_S, that is, the nodes in $s(x_k)$ with $x_k = fin(q)$; let Q' be the set of paths q' thus obtained.

3.2 Evaluate $P^L_{fp(q')}$ and $P^U_{fp(q')}$ by Subroutine PBOUND (see Eq. (15)).

Step 4 (Branching)

4.1 Select q'_M : $P(q'_M) = \max\{P(q') : q' \in Q'\}$. (16)

4.2 Check whether this q'_M is complete; if YES add it to X_c and go to Step 5 for bounding, if NOT go to Step 3 with q'_M as the current path.

Step 5 (Bounding/ Discarding)

5.1 Update P_{fpM}: $P_{fpM} \longleftarrow \max\{P_{fpM}, P(q'_M)\}$. (17)

5.2 Discard the failure paths $q' \in Q'$: $P(q') \leqq 10^{-\delta} P_{fpM}$, set $X \longleftarrow X \setminus (X_t \cup X_c)$.

Step 6 (Terminating) If $X = \emptyset$ (that is, there are no more q' rooted at x_0 for branching), go to Step 1 if $N_0 \neq \emptyset$; otherwise go to PSAM. If $X \neq \emptyset$ go to Step 4.2 with the the most probable path selected of X.

PSAM (Probabilistic Safety Assessment Module). For every x_0 of N_0 and with the final X_c's, recover the set $PATH_d(D)$ of the stochastically dominant complete failure paths, together with the set $PROB_d(S)$ of the associated probabilities. Rearrange $PATH_d(D)$ and $PROB_d(S)$ in decreasing order of the probabilities.

Evaluate the lower and upper bounds on system failure probabilities

$$P_f^L(S) \leqq P_f(S) \leqq P_f^U(S) \tag{18}$$

by using Ditlevsen's bounds (Ditlevsen 1981).

REFERENCES

Ditlevsen, O. (1981). Uncertainty Modeling, Mc Graw - Hill Book Company, pp.236-237.

Kaplan, S. (1981). On the Method of Discrete Probability Distributions in Risk and Reliability Calculations - Applications to Seismic Risk Assessment, Risk Analysis, Vol.1, No.3, pp.189-196.

Murotsu, Y., Okada, H., Yonezawa, M., Kishi, M. (1983). Identification of Stochastically Dominant Failure Modes in Frame Structure, Proc. Fourth International Conference on Applications of Statistics and Probability in Soil and Structural Engineering, Univ. di Firenze (Italy), Pitagora Editrice, pp. 1325-1338.

Murotsu, Y. (1986). Development in Structural Reliability Theory, Nuclear Engineering and Design, Vol.94, pp.101-114.

Murotsu, Y., Shao, S., Matsui, H., Tong, Q.S. (1989). Contributions to the Systems Reliability of Structures, Transactions of the 10th Int.Conf.on SMiRT, AA SMiRT, Vol.M, pp.205-214.

Vulpe, A. and Cărăuşu, A. (1987). On Some Graph-Theoretic Concepts and Techniques Applicable in the Reliability Analysis of Structural Systems, Reliability and Optimization of Structural Systems (Proc. First IFIP WG 7.5 Working Conf., Aalborg, Denmark), Lecture Notes in Engineering, Vol.33, Springer Verlag, pp.399-416.

Vulpe, A. and Cărăuşu, A. (1988). Fragility Models in Seismic Probabilistic Risk Assessment of Systems with Multiple Failure Modes, 9 WCEE, Tokyo-Kyoto, Paper 6189 (Abstracts); full version in Romanian; Proc. Jubilee Scientific Session, Iasi Polytechnic Institute, November 1988.

Vulpe, A. and Cărăuşu, A. (1989). An Integrated Technique for the Seismic Probabilistic Risk Assessment of Structural Systems, Transactions of the 10th International Conference on SMiRT, AA SMiRT, Vol.P, pp.169-174.