# An Empirical Approach to Fatigue Crack Growth Data for Ferritic Steels in a Pressurized Water Reactor Environment

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**Abstract**

In a probabilistic assessment of the integrity of PWR pressure vessels the growth of cracks due to fatigue determines how the crack size distribution changes with time. This paper examines an empirical approach to a particular set of crack growth data and describes the growth statistically in such a way that the transfer function relating crack size at two different times is given by a probability distribution. The data used is taken to be illustrative rather than definitive although specific results are given. An example of the way in which such information could be used in probabilistic assessments of vessel failure rates is given briefly.

## 1 Introduction

One way of assessing the reliability of PWR pressure vessels is to use a probabilistic fracture mechanics approach and to calculate the expected failure rate of an ensemble of vessels. In such an analysis the growth of cracks due to fatigue determines how the crack size distribution changes with time. This paper examines an empirical approach to some crack growth data and describes the growth statistically in such a way that the transfer function relating crack size at two different times is given by a probability distribution. The data we use is that used by the American Society of Mechanical Engineers (ASME) in formulating a reference law for fatigue crack growth of PWR steels in a wet environment [ 1 ]. The wet environment gives a growth law which is faster than that obtained in a dry environment and, also, modern steels have slower growth rates. The set of data used is taken to be illustrative rather than definitive. The data have been described by Bamford [ 2 ] in giving the technical justification for the reference law.

In section 2 we consider the empirical approach to this data and in section 3 we present the results of our analysis. In section 4 we show how we intend to make use of the approach in subsequent studies of PWR pressure vessel integrity.

## 2 The empirical approach.

The data are for a number of samples grown in a number of slightly different environments and fatigued on a number of different machines. Each specimen will have seen only one environment and one machine but the fatiguing has been carried out in a number of stages so that for each specimen there are a number of crack growth rates corresponding to different increments in stress intensity $\Delta K$. These data are shown in figure 1. To analyse these data we split the stress intensity increments $\Delta K$ into slices of 2 $MPa\sqrt{m}$ and calculate the mean growth per cycle and the associated standard deviation in each slice. This is depicted schematically in figure 2. Simple functional fits can then be made to the mean and standard deviations found. The analysis is reported in detail elsewhere [ 3 ] and only the results will be considered here.

## 3 The results

Both linear and quadratic fits to the mean and standard deviation of crack growth per cycle were considered and it was concluded that there was little benefit in using the quadratic fit [ 3 ]. The data were also analysed ignoring the effects of $R$ where $R = \Delta K_{min}/\Delta K_{max}$ and then by splitting them into subsets depending on whether $R \leqslant 0.6$ or $R > 0.6$ . Considering first the case in which no allowance is made for the effects of different $R$ values we find that the best linear fit, in the least squares sense, for the mean growth $\langle \frac{da}{dn} \rangle$ in millimetres per cycle is given by [ 3 ] :

$$\langle \frac{da}{dn} \rangle = 6.56\times10^{-5}\Delta K \tag{1}$$

for stress intensity increments $\Delta K$ in $MPa\sqrt{m}$. Associated with this value for the mean crack growth is a standard deviation, in millimetres per cycle, $\sigma$ given by [ 3 ] :

$$\sigma = 2.93\times10^{-5}\Delta K \tag{2}$$

for stress intensity increments $\Delta K$ in $MPa\sqrt{m}$.

Various distributions around the mean were examined [ 3 ] and a normal distribution was found to be the simplest distribution which could be used to adequately describe all the data in the different ranges of $\Delta K$. The main disadvantage of a normal distribution is that it will predict negative values of crack growth, albeit with very small probability. We shall return to this point in the next section. Taking account of the variation of crack growth rate with $R$ gave two additional equations for the mean growth per cycle, in millimetres :

$$\langle \frac{da}{dn} \rangle = -4.70\times10^{-4} + 7.30\times10^{-5}\Delta K \tag{3}$$

for $R<0.6$ whilst for $R \geqslant 0.6$ the best fit obtained is [ 3 ] :

$$\langle \frac{da}{dn} \rangle = -1.02\times10^{-3} + 1.46\times10^{-4}\Delta K \tag{4}$$

The standard deviations that go with these means are given by:

$$\sigma = 4.23\times10^{-5} + 2.72\times10^{-5}\Delta K \tag{5}$$

for $R<0.6$ whilst for $R \geqslant 0.6$ the best fit obtained is

$$\sigma = -2.95\times10^{-4} + 6.21\times10^{-5}\Delta K \tag{6}$$

for stress intensity increments $\Delta K$ in $MPa\sqrt{m}$. The results quoted in equations 3 to 6 are for fits which are not constrained to pass through the origin. The straight lines given by these equations are given as solid lines in figure 3. The dotted lines in figure 3 are for fits which have been constrained to pass through the origin. Experimentally a threshold value of $\Delta K$ is found [ 6 ] which is typically about the same magnitude as this analysis reveals, that is about 7 $MPa\sqrt{m}$. However, in practice the threshold value decreases with increasing R–value whereas that predicted by the equations above increases with R–value slightly. This suggests that the particular threshold values found here are probably artefacts of the particular database used in the analysis. On figure 3, the constrained fit, equation 1, for the entire database ignoring R–value effects is not shown since it is almost identical to the constrained value for $R<0.6$.

Now consider what happens as the cracks grow in a reactor. In this environment there are a number of transient stresses (tabulations can be found, for example in [ 4,5 ] ). In most probabilistic assessments of vessel integrity some assumptions are made regarding the type of crack and its location in the vessel wall.These assumptions are necessary in order to translate the applied stresses into stress intensities at the cracks. This stress intensity is then compared with some fracture criterion, such as is given by linear elastic fracture mechanics,

$$K_I \geqslant K_{IC} \tag{7}$$

in order to test whether crack growth initiates. Typical assumptions made about the cracks are that they are extended line cracks, or surface breaking semi-elliptical cracks with a given aspect ratio. Suppose we want to calculate the growth that occurs after $n_1$ cycles which produce stress intensity increments $\Delta K_1$ and $n_2$ cycles which produce $\Delta K_2$ on our chosen crack. The new mean crack size $a$ will be given by:

$$a = a_0 + n_1 \langle \frac{da}{dn} \rangle_1 + n_2 \langle \frac{da}{dn} \rangle_2 \tag{8}$$

where $a_0$ is the original crack size. For convenience in the probabilistic calculations, the final size is taken to be independent of the order in which the cycles are applied. In reactor operations the cycles of different stress levels are assumed to be spread uniformly over a 40 year life which will tend to average out any history effects.

For the standard deviation $\sigma$ about this mean value we will obtain:

$$\sigma^2 = [n_1^2 \sigma_1^2 + n_2^2 \sigma_2^2] \tag{9}$$

The distribution of growth is taken to be normal with mean given by equations (3) or (4) together with the generalization of (8) and standard deviation $\sigma$ given by equations (5) or (6) together with the generalization of equation (9).

**4 Outline of the probabilistic approach using these results.**

Consider a vessel with $Q$ regions that can be considered as distinct on account of their thickness, or levels of stress experienced, or because they are made of material with different fracture toughness, or are more likely to contain cracks, or are inspected to different standards. In each of these regions there is a distribution of fracture toughness $f_Q(K_{IC})$ and a distribution of cracks $N_Q(x_t)dx_t$ with through-wall dimension at time $t$ lying between $x_t$ and $x_t+dx_t$. For an ensemble of reactor vessels, the fraction of the population which fails under the application of a stress S is given by $P_f$ where [ 7,8,9 ]:

$$P_f = \sum_{i=1}^{Q} \int_0^{*} f_i(K_{IC}) \int_{a_c}^{h_i} N_i(x_t)dx_t dK_{IC} \tag{10}$$

where $h_i$ is the thickness of region $i$ and the lower limit of the second integral is related to the variable of integration through:

$$K_{IC} = K_I^i(\mathrm{S},a_c) \tag{11}$$

where $K_I^i$ is the stress intensity function for the stress S applied at location $i$ and crack geometry assumed. The probability of having a crack at time $t$ of size lying in the range $x_t$ to $x_t+dx_t$ is given by $N_i(x_t)dx_t$ where:

$$N_i(x_t) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{\frac{-1}{2}\left[\frac{x_t-\mu}{\sigma}\right]^2\right\} \tag{12}$$

and where $\mu$ is given by a sum over stress intensity increments $\Delta K$ of the mean rate of growth $\langle \frac{da}{dn} \rangle$:

$$\mu = \sum_{p=1}^{d} n_p \langle \frac{da}{dn} \rangle \qquad (13)$$

Here there are taken to be $d$ design basis transient stresses of $n_p$ cycles in the period of time considered and $\langle \frac{da}{dn} \rangle$ is taken from equation (3) for $R<0.6$ and from equation (4) for $R \geqslant 0.6$ and $\sigma$ is given by:

$$\sigma^2 = \sum_{p=1}^{d} n_p^2 \sigma_p^2 \qquad (14)$$

with $\sigma_p$ given by equation (5) for $R<0.6$ and from equation (6) for $R \geqslant 0.6$.

**5 Conclusion**

The main result is given by the equations 3 to 6 and 8 and 9 in which fatigue crack growth is described as a statistical distribution about a mean value. The distribution is taken to be normal and the mean and standard deviation depend linearly on the stress intensity increment on each cycle with differing results for $R<0.6$ or $R \geqslant 0.6$. An explanation of the way in which the data, analysed in the manner described, could be used in probabilistic assessments of PWR pressure vessel failure rates.


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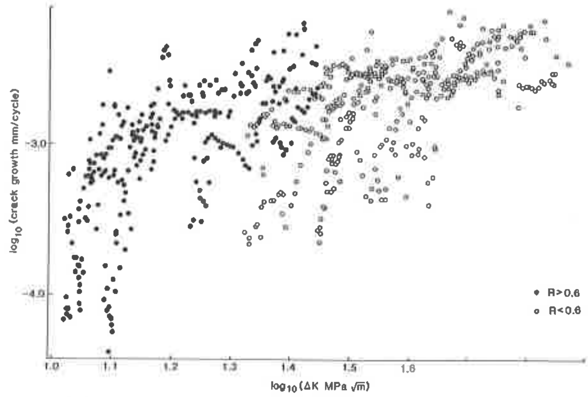


FIG.1. CRACK GROWTH AS A FUNCTION OF STRESS INTENSITY INCREMENT. FROM REFERENCE 2.

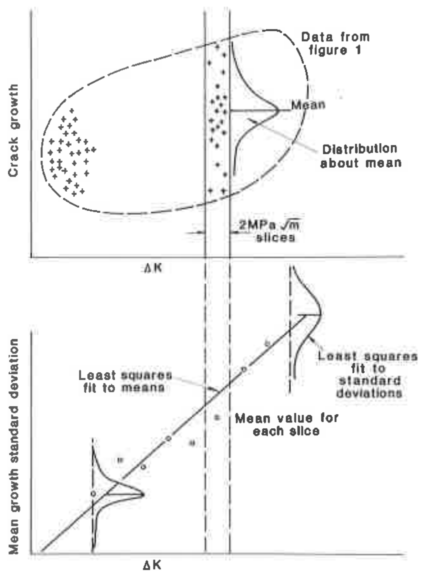


FIG.2. SCHEMATIC OF DATA ANALYSIS

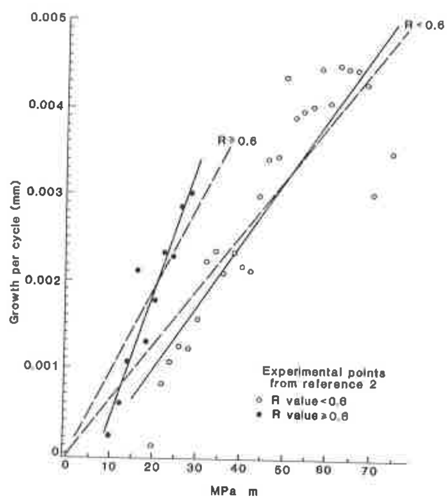


Fig.3. Comparison of the best linear fits to the mean crack growth constrained to pass through the origin.