The Influence of End Constraints on Smooth Pipe Bends

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With present trends in the power industries towards higher operating temperatures and pressures, problems associated with the design and safety assessment of pipework systems have become increasingly complex. Within such systems, the importance of smooth pipe bends is well established.

Until recently, most of the theoretical work was concerned with what, in the shell theory sense, might be termed "axi-symmetric" solutions where cross-sectional deformations and stresses were assumed to be uniform along the bend. It is known however that the pipe deformation is constrained by the connections between the bend and other parts of the pipework system. The least form of constraint is the so-called 'tangent pipe', while the most severe form of constraint is the rigid flange. Experimental work has confirmed the inadequate predictions of axi-symmetric theories, particularly for flange constraints.

Several analytical solutions have been published in recent years which include the effect of end constraints. The divergence between the results of the various works can only serve to confuse the designer. Further, the difference between these results and present day design codes could lead to inadequate design. Published experimental evidence has to date done little to confirm any set of results as being correct.

The work which will be presented will attempt to clarify the situation and unify the results. An analytical solution of the problem of a linear elastic smooth pipe bend with end constraints under in-plane bending will be presented. The analysis will deal with constraints in the form of flanged tangents of any length.

The analysis employs the theorem of minimum total potential energy with suitable kinematically admissible displacements in the form of Fourier series. The integrations and minimisation were performed numerically, thereby permitting the removal of several of the assumptions made by previous authors.

Typical results for flexibilities will be given along with comparisons with other works. The differences in some earlier theory are clarified and other more recent work using different solution techniques is substantiated. The bend behaviour is shown to be strongly influenced by the pipe bend parameter, the bend angle, the tangent pipe length and the bend/cross-sectional radius ratio.

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1. Introduction

Within the design and safety assessments of high integrity piping systems the importance of the smooth pipe bend is well established. This is particularly so in liquid metal fast breeder reactor pipework. Until relatively recently most of the analytical work on bends neglected any possible variation in behaviour along the length of the bend. However, when a bend forms part of a piping system, its natural cross-sectional deformation, which gives rise to enhanced flexibility and stress levels, are constrained by the connections between it and the other components. Although numerous publications have appeared [1 - 10] on the influence of end constraints on bends, there are no definitive conclusions or guidelines which may be drawn; indeed, some of the results are contradictory particularly for flange constraints. Similar difficulties can be observed for tangent bends of short radius and small angles.

The present paper outlines an analytical solution and gives typical results for both flanged and tangent end constraints. In the process many of the existing difficulties can be clarified.

2. Theoretical Analysis

In what follows the linear elastic in-plane bending behaviour of a smooth pipe bend is examined. The bend is assumed to be part of an assembly which includes straight tangent pipes attached to each end of the bend. For convenience the ends of the tangent are considered to terminate in rigid flanges as shown in Fig.1.

The theorem of minimum total potential energy requires suitable kinematically admissible displacements. For the present problem these are divided into two components, the first termed "rigid section displacements" associated with the tube cross-sections relative to the pipe centreline and the second termed "distortion displacements" associated with the distortion or ovalisation of the tube cross-sections.

The following series is assumed for the rigid section displacements [11] where u_R, v_R and w_R are the thin shell mid surface displacements.

For the bend, $-\alpha/2 < \theta < \alpha/2$

$$u_R = \sum_{j=1} F_j \tfrac{1}{2}\left[\theta - \left(\frac{\alpha}{j\pi}\right)\sin\left(\frac{j\pi\theta}{\alpha}\right)\right]\left(1 + \frac{r}{R}\sin\varphi\right) + \tfrac{1}{2}\sum_{j=1} D_j\left(\frac{j\pi}{\alpha}\right)\sin\left(\frac{j\pi\theta}{\alpha}\right)\frac{r}{R}\sin\varphi$$

$$v_R = -\sum_{j=1} D_j \sin^2\left(\frac{j\pi\theta}{2\alpha}\right)\cos\varphi$$

$$w_R = -\sum_{j=1} D_j \sin^2\left(\frac{j\pi\theta}{\alpha}\right)\sin\varphi \qquad (1a)$$

For the tangent pipes, $\frac{\alpha}{2} \leq \theta \leq \frac{L}{2R}$

$$u_R = \sum_{j=1} F_j\tfrac{1}{2}\left(\frac{\alpha}{2} - \frac{\alpha}{j\pi}\sin\frac{j\pi}{2}\right)\left(1 + \frac{r}{R}\sin\varphi\right) + \sum_{j=1} D_j\tfrac{1}{2}\left(\frac{j\pi}{\alpha}\right)\sin\left(\frac{j\pi}{\alpha}\right)\frac{r}{R}\sin\varphi +$$

$$\sum_{j=1} G_j(j+1)\left(\theta - \frac{\alpha}{2}\right)^j \frac{r}{R}\sin\varphi$$

$$v_R = \left[-\sum_{j=1} D_j\sin^2\left(\frac{j\pi}{4}\right) - \left(\sum_{j=1} F_j\tfrac{1}{2}\left(\frac{\alpha}{2} - \left(\frac{\alpha}{j\pi}\right)\sin\left(\frac{j\pi}{2}\right)\right) + \right.\right.$$

$$+ \sum_{j=1} D_j(\frac{j\pi}{\alpha}) \tfrac{1}{2}\sin(\frac{j\pi}{2}) (\theta - \frac{\alpha}{2}) - \sum_{j=1} G_j(\theta - \frac{\alpha}{2})^{j+1}] \cos\varphi$$

$$w_R = [- \sum_{j=1} D_j \sin^2(\frac{j\pi}{4}) - (\sum_{j=1} F_j \tfrac{1}{2}(\frac{\alpha}{2} - (\frac{\alpha}{j\pi})\sin(\frac{j\pi}{2})) +$$

$$+ \sum_{j=1} D_j(\frac{j\pi}{\alpha})\tfrac{1}{2}\sin(\frac{j\pi}{2}) (\theta - \frac{\alpha}{2}) - \sum_{j=1} G_j (\theta - \frac{\alpha}{2})^{j+1}] \sin\varphi \tag{1}$$

For the bend, θ is the angle along the bend from the bend centre and for the tangent pipes $\theta = x/R$ where x is the distance along the pipe assembly centreline, $\varphi = 0$ from the bend centre, $\theta = 0$ (see Fig.1). The overall length of the bend tangent assembly, L, is $L = 2\ell + R\alpha$, where ℓ is the length of one tangent pipe and α is the bend angle. The above displacement series satisfy the boundary conditions for a bend tangent assembly with rigid flanges at the ends of the tangent pipes. When $\ell = 0$ then the above series, eq.1a satisfy the conditions for a flanged bend.

The distortion displacements, u_D, v_D and w_D are taken as, [11]

$$u_D = \frac{r}{R} \sum_{m=1} \sum_{n=2} A_{mn} (\psi_{en} \frac{1}{n^2}\cos n\varphi + \psi_{on} \frac{1}{n^2}\sin n\varphi)(m\eta)\sin(2m\eta\theta)$$

$$v_D = \sum_{m=1} \sum_{n=2} B_{mn} (- \psi_{en} \frac{1}{n} \sin n\varphi + \psi_{on} \frac{1}{n}\cos n\varphi)(\psi_{om}\cos^2(m\eta\theta) - \psi_{en}\sin^2(m\eta\theta))$$

$$w_D = \sum_{m=1} \sum_{n=2} C_{mn} (\psi_{en} \cos n\varphi + \psi_{on} \sin n\varphi)(\psi_{om}\cos^2(m\eta\theta) - \psi_{en}\sin^2(m\eta\theta)) +$$

$$\sum_{j=1} H_j (\psi_{oj} \cos^2(j\eta\theta) - \psi_{ej} \sin^2(j\eta\theta)$$

$$\text{where} \quad \psi_{ek} = \begin{cases} 1 - k \text{ even} \\ 0 - k \text{ odd} \end{cases} \qquad \psi_{ok} = \begin{cases} 0 - k \text{ even} \\ 1 - k \text{ odd} \end{cases} \tag{2}$$

These displacements satisfy the symmetry boundary conditions at $\theta = 0$ and the flange condition at $\theta = \pm L/2R$. They are continuous at the bend tangent connection but slightly violate the slope continuity requirement. The unknown coefficients which have to be determined are A_{mn}, B_{mn}, C_{mn}, D_j, F_j, C_j and H_j.

The strains for the bend tangent assembly are found by adding the two displacement components and substituting into the following strain displacement relations [12].

For the bend:

$$\varepsilon_\varphi = \frac{1}{r} (\frac{\partial v}{\partial \varphi} + w) \qquad K_\varphi = \frac{1}{r^2} (\frac{\partial v}{\partial \varphi} - \frac{\partial^2 w}{\partial \varphi^2})$$

$$\varepsilon_\theta = \frac{1}{R} [\frac{\partial u}{\partial \theta} + v \sin\varphi + w \cos\varphi]$$

$$K_\theta = \frac{1}{R^{1\,2}} [\frac{\partial u}{\partial \theta} \sin\varphi - \frac{\partial^2 w}{\partial \theta^2} + \frac{R^1\cos\varphi}{r}(v - \frac{\partial w}{\partial \varphi})]$$

$$\gamma_{\theta\varphi} = \frac{1}{R^1} [\frac{\partial v}{\partial \theta} - u \cos\theta + \frac{R^1}{r} \frac{\partial u}{\partial \varphi}]$$

$$K_{\theta\varphi} = \frac{1}{rR^1} [\frac{\partial u}{\partial \varphi} \sin\varphi + \frac{\partial v}{\partial \theta} - \frac{\partial^2 w}{\partial \theta \partial \varphi} + \frac{r\cos\varphi}{R^1} (\frac{\partial w}{\partial \theta} - u \sin\varphi)] \tag{3}$$

For the tangent pipes:

$$\varepsilon_\varphi = \frac{1}{r}\left(\frac{\partial v}{\partial \varphi} + w\right) \qquad K_\varphi = \frac{1}{r^2}\left(\frac{\partial v}{\partial \varphi} - \frac{\partial^2 w}{\partial \varphi^2}\right)$$

$$\varepsilon_x = \frac{\partial u}{\partial x} \qquad K_x = -\frac{\partial^2 w}{\partial x^2}$$

$$\gamma_{x\varphi} = \frac{\partial v}{\partial x} + \frac{1}{r}\frac{\partial u}{\partial \varphi} \qquad K_{x\varphi} = \frac{1}{r}\left(\frac{\partial v}{\partial x} - \frac{\partial^2 w}{\partial x \partial \varphi}\right) \tag{3}$$

The total potential energy (T.P.E.) of the assembly under an in-plane bending moment, M, can then be found from,

$$\begin{aligned} V = {} & 2C \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{\frac{\alpha}{2}} [(\varepsilon_\varphi + \varepsilon_\theta)^2 - 2(1-\nu)(\varepsilon_\theta \varepsilon_\varphi - \tfrac{1}{4}\gamma^2_{\theta\varphi})] rR^1 d\theta d\varphi \\ & + 2D \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{\frac{\alpha}{2}} [(K_\varphi + K_\theta)^2 - 2(1-\nu)(K_\theta K_\varphi - K^2_{\theta\varphi})] rR^1 d\theta d\varphi \\ & + 2C \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{\frac{\alpha}{2}}^{\frac{L}{2R}} [(\varepsilon_\varphi + \varepsilon_x)^2 - 2(1-\nu)(\varepsilon_x \varepsilon_\varphi - \tfrac{1}{4}\gamma^2_{x\varphi})] rR d\theta d\varphi \\ & + 2D \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{\frac{\alpha}{2}}^{\frac{L}{2R}} [(K_\varphi + K_x)^2 - 2(1-\nu)(K_x K_\varphi - K^2_{x\varphi})] rR d\theta d\varphi - M\gamma \end{aligned} \tag{4}$$

where $C = Et/(1 - \nu^2)$ and $D = Et^3/12(1 - \nu^2)$ and $R^1 = R + r\sin\varphi$.

Note that the assembly symmetries have been used in the above equation. The rotation γ between the ends of the assembly can be shown to be [11],

$$\gamma = \frac{1}{R}\sum_{j=1} [F_j(\frac{\alpha}{2} - (\frac{\alpha}{j\pi})\sin(\frac{j\pi}{2}) + D_j(\frac{j\pi}{\alpha})\sin(\frac{j\pi}{2}) + G_j 2(j+1)(\ell/R)^j] \tag{5}$$

The displacement series coefficients are obtained by minimising the TPE, eq.(4), with respect to all of the unknown coefficients. This was performed using a specially developed numerical integration and minimisation FORTRAN computer program which took about five minutes to run for any set of bend parameters. It is important to include sufficient numbers of terms in each series to ensure that the solution has converged. The details cannot be discussed here but all the results presented can be considered for practical purposes to be converged.

3. Flexibility Factors

Bend flexibility factors are defined from the end rotation, as

$$K = (\gamma - 2\gamma^T)/\gamma_o$$

where $\gamma^T = (1 - \nu^2)\frac{M\ell}{EI}$, $\quad \gamma_o = \frac{MR\alpha}{EI}$

and γ_o is the end rotation of a straight pipe of the same length as the bend ($R\alpha$) under the same load and γ^T is the rotation between the ends of one of the tangent pipes loaded on its own, as would be predicted by a lower bound energy analysis of the type used here. The above equation causes any additional flexibility of the assembly, created by the bend or the tangents to be included in the bend flexibility factor. Thus additional factors will only be required for the bends in the analysis of piping systems.

Typical variations of flexibility factors with tangent pipe length are shown in Fig.2. It can be seen that the flexibility of a bend falls appreciably when it is constrained by short length tangents or flanges. The flexibility tends toward a constant value for

$\ell/r > 2\pi$. The flexibility factors for all types of constraints are found to vary with pipe factor, $\lambda = \frac{Rt}{r^2}$, bend angle, α, tangent pipe length ratio, R/r, and to a lesser extent with Poisson's ratio, ν. Herein, $\nu = 0.3$.

3.1 Bends terminated by Rigid Flanges

In Fig.3 the present results are compared with previous theoretical work on flanged bends ($\ell/r = 0$). The results of Thailer-Cheng [2] and Findlay and Spence [3] show little or no variation with radius ratio. Both theories erroneously ignore the contribution of the u_D displacement to the strains. The results of Axelrad [45], which are only a first approximation, show reasonable correlation with the present work, particularly for larger pipe factors. The best comparison is obtained with the flexibility factors obtained by Whatham [6, 7], which were derived from a method involving the solution of the governing differential equations. The small difference is probably due to different solution techniques. Fig.4 gives the flexibility factors for 90^0 bends for several radius ratios.

3.2 Bends with Long Tangent Pipes

Fig.5 gives a typical comparison of the present results with published theoretical and finite element results for bends with long tangent pipes. It can be seen that the results from all sources are reasonably close. The theoretical results of Whatham [7] and the finite element results of Ohtsubo and Watanabe [8] and Rodabaugh et al [9] are slightly higher than the present results or the finite element results of Natarajan and Blomfield[10]. Comparison is also given in Fig.5 with the present ASME code. The tangent pipe results are less influenced by R/r than those for flanges. Fig.6 summarises some of the present work by giving formulae which approximate the flexibility factors for bends with connected long tangents, with an accuracy of better than 5% for flexibilities greater than about two.

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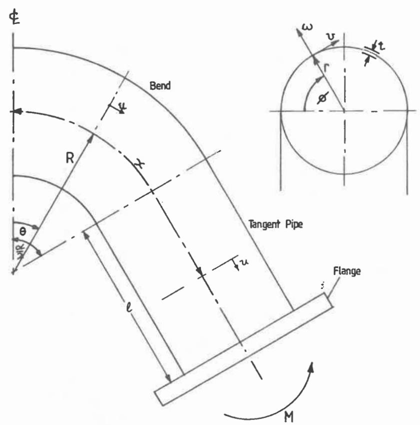


Fig.1 Geometry of Bend Tangent Assemblies

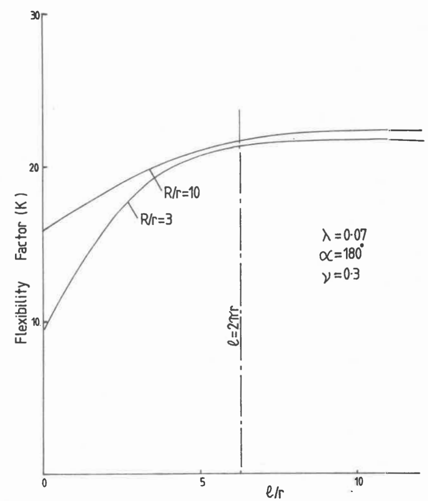


Fig.2 Influence of Tangent Pipe Length

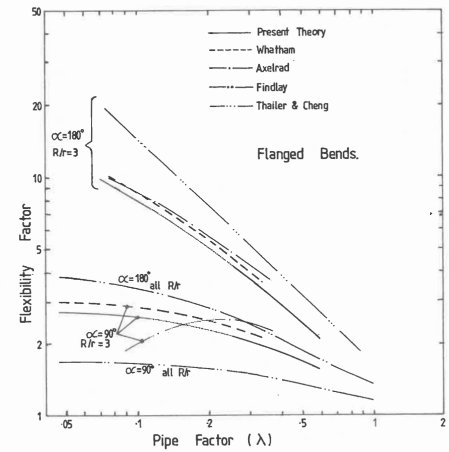


Fig.3 Comparison of Flexibility factors for Flanged Bends

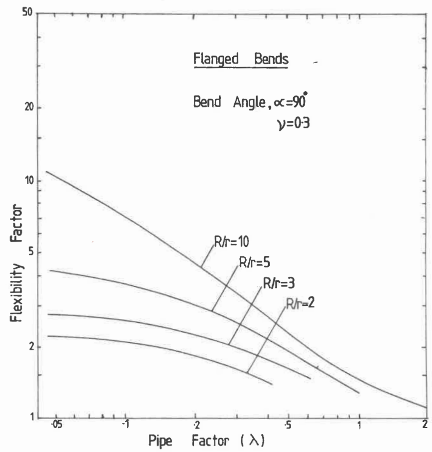


Fig.4 Theoretical Flexibility Factors

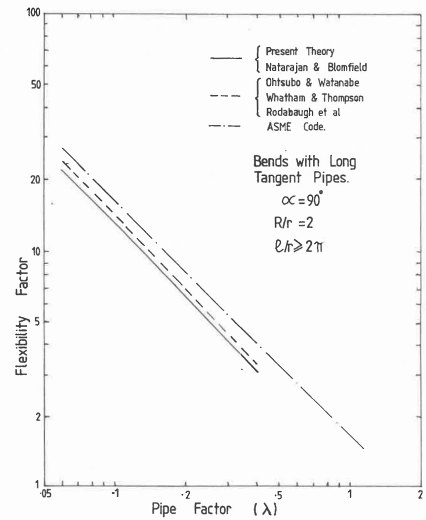
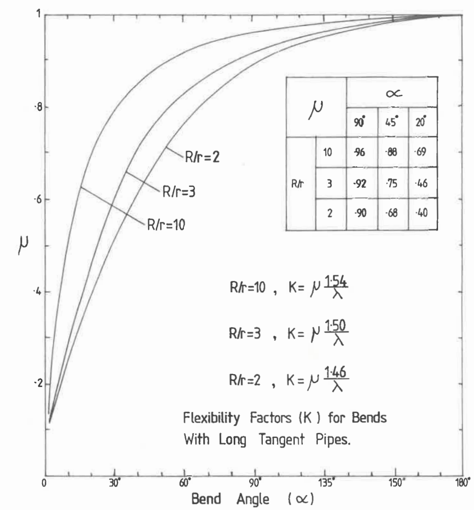


Fig.5 Bends with Tangent Pipes



μ		α 90°	45°	20°
R/r	10	·96	·88	·69
	3	·92	·75	·46
	2	·90	·68	·40

Fig.6 Approximate Formulae for Tangent Bends