SHEAR STRESS DISTRIBUTION DUE TO SHEAR AND RAYLEIGH WAVE PROPAGATION AT DEEP SOIL SITES

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SUMMARY

Earthquake strong motion at ground surface may be described as the combined result of body (compression and shear) waves and surface waves. In the course of earthquake engineering site response analyses, it is usually assumed that surface motions are a result of vertically propagating body waves. In studies of liquefaction potential, it is assumed that soil shear strains are a result of vertically propagating shear waves. However, shear strains are also caused by surface Rayleigh waves and solutions to strains from each source are derived and compared in the paper, and the results for shear strain in terms of surface acceleration and wave length are presented in non-dimensional form.

The results of this investigation show that the distribution of soil shear strain as a function of depth below the ground surface is different depending upon whether shear waves or Rayleigh waves are assumed to be the source of horizontal motions at the ground surface. It is shown that the shear strains derived from Rayleigh waves are greater near the surface but decrease faster with depth than the shear strains derived from vertically propagating shear waves. It is noted that the more rapid attenuation of shear strains with depth as predicted by Rayleigh wave theory is more consistent with observed cases of soil liquefaction which are usually limited to shallow depths, generally not in excess of 10 to 20 meters. On the contrary, vertical shear propagation theory predicts liquefaction occurring at loose sand sites at much greater depths. The differences are significant enough to influence the decision regarding liquefaction potential of a soil site.

1. Introduction

In simplified procedures for performing analysis of liquefaction, it is necessary to compute the maximum shear stress induced in the soil at various depths below the ground surface. Seed and Idriss,[1] developed a generalized curve to estimate shear stress distribution with depth, based on many one-dimensional analyses of vertically propagating shear waves. They found that it was possible to account for soil elasticity by defining the shear stress in an elastic soil as a fraction of the shear stress that would occur if the entire column of soil above the point of interest moved as a rigid body with acceleration equal to the ground surface acceleration. For the rigid body model, i.e. for the case when ground motions at all points from the ground surface to the depth of interest move together, the shear force at the bottom of the column can be calculated as the mass of the soil column times acceleration. As a result of soil flexibility, the real soil stress will be less than this quantity.

In reality, the ground surface motion may not necessarily be due to the vertical propagation of the shear waves alone. There is evidence to suggest[2,3] that in some cases the ground motion may be caused predominantly by horizontally propagating surface waves. The surface motion for most cases, however, is likely to be a combination of surface waves and vertically or non-vertically propagating (shear and compression) waves. Although Seed and Idriss[1] assume that the shear stresses are a result of vertically propagating shear waves alone, it is of interest to compare the shear stresses caused by vertically propagating shear waves to those caused by horizontally propagating surface waves. Hence, in this paper, Rayleigh wave as well as shear wave propagation are considered, in an undamped, uniform, elastic half-space, and the shear stresses deduced from the two models are compared. The results indicate that there is a significant difference in the shear stress distribution for Rayleigh waves and vertically propagating shear waves for equal ground surface motions.

Soil liquefaction is an extremely complex phenomenon and it is recognized that there are many factors, such as previous earthquake history, geologic time effects, soil particle cementation, as well as other factors, that significantly influence the susceptibility of soil to liquefaction.[4] The fact that the shear stress distribution on the basis of the Rayleigh wave model is significantly different to that of a shear wave model in no way suggests that current procedures used for liquefaction evaluation are unconservative. The results presented herein are intended only to serve as an illustration of alternate models of soil stress analysis. However, they indicate that, since the methods of analysis of liquefaction are based on empirical correlations between observed and computed conditions, if a Rayleigh wave model instead of the existing shear wave model is utilized, the empirical correlation factors to provide agreement with observed and predicted conditions must be revised. For instance, the fact that soil liquefaction is normally confined to the uppermost soil layers is shown to a better degree by the Rayleigh wave model than by the vertically propagating shear wave model.

Section 2 provides basic governing equations of motion for both the shear wave and Rayleigh wave models, whereas Section 3 summarizes the final results for the stress reduction factor. Section 4 discusses the implications of the results on the maximum shear stresses and on the liquefaction analyses.

2. Basic Equations of Motion

2.1 Shear Wave Model

The general vertical shear wave propagtion problem can be studied using the procedure by Schabel et al.,[5] and the corresponding computer code, SHAKE. However, to illustrate the differences in shear wave and Rayleigh wave models, an idealized model consisting of an uniform, undamped, elastic half-space, subjected to vertically propagating harmonic shear waves, is considered in this paper. For this idealized model,[5] the horizontal and vertical particle displacements u(x,z,t) and w(x,z,t), respectively, at horizontal coordinate x, depth z below the surface, and time t, are given as:

$$u(x,z,t) = e^{i\omega t}(E\,e^{iNz} + F\,e^{-iNz}) \tag{1}$$

$$w(x,z,t) = o \tag{2}$$

where:

e = exponential

$i = \sqrt{-1}$

ω = circular frequency of the motion

E,F = constants of integration

$N = \frac{\omega}{V_s} = \frac{2\pi}{L_s}$

V_s = shear wave velocity

L_s = shear wave length

Integration constants E and F are to be determined from the boundary conditions: (1) shear stress at the surface is zero, and (2) peak acceleration at the surface is $a_o g$, where a_o is the peak ground acceleration, as a fraction of gravitational acceleration, g.

2.2 Rayleigh Wave Model

Suitable procedures and computer codes to study a general Rayleigh wave propagation model have not been developed. Hence, the idealized model considered herein consists of harmonic Rayleigh waves horizontally propagating along an axis x, through an undamped, uniform, elastic half-space. From Richart, Hall and Woods,[6] Section 3.3, the expressions for horizontal and vertical particle displacements, u and w respectively, for the Rayleigh wave model are obtained as:

$$u(x,z,t) = A_1 N' i U(z)\ \exp[i(\omega t - N'x)] \tag{3}$$

$$w(x,z,t) = A_1 N' W(z)\ \exp[i(\omega t - N'x)] \tag{4}$$

where:

A_1 = integration constant

$N' = \frac{\omega}{V_r} = \frac{2\pi}{L_r}$

$$U(z) = -\exp\left[-\frac{q}{N'}\,(zN')\right] + \frac{2\frac{q}{N'}\,\frac{s}{N'}}{\left[\frac{s^2}{N'^2} + 1\right]} \cdot \exp\left[-\frac{s}{N'}\,(zN')\right] \tag{5}$$

$$W(z) = \frac{2\frac{q}{N'}}{\left[\frac{s^2}{N'^2} + 1\right]} \cdot \exp\left[-\frac{s}{N'}(zN')\right] - \frac{q}{N'} \cdot \exp\left[-\frac{q}{N'}(zN')\right] \tag{6}$$

$$\frac{q^2}{N'^2} = 1 - \alpha^2 K^2 \tag{7}$$

$$\frac{s^2}{N'^2} = 1 - K^2 \tag{8}$$

$$\alpha^2 = \frac{V_s^2}{V_p^2} = \frac{1-2\upsilon}{2-2\upsilon} \tag{9}$$

$$K^2 = \frac{V_r^2}{V_s^2} = \frac{\omega^2}{V_s^2 N'^2} \tag{10}$$

where:

υ = Poisson's ratio

V_s, V_r and V_p = shear, Rayleigh and compression wave velocities

and, K^2 = real valued solution between 0 and 1 from the equation:

$$K^6 - 8K^4 + (24 - 16\alpha^2)K^2 + 16(\alpha^2 - 1) = 0 \tag{11}$$

The above formulation already satisfies the boundary condition that the shear stress at the ground surface should be zero. The constant A_1 can be determined from the boundary condition that the peak ground surface acceleration should be the specified value $a_o g$.

3. Solution for Stress Reduction Factor

Seed and Idriss[1] introduced the concept of a stress reduction factor to express the shear stress in elastic soil in terms of what would be expected if the soil was rigid and accelerating at peak surface acceleration at all points. If the soil column above a soil element at depth z behaved as a rigid body and the maximum ground surface acceleration were $a_o g$, the maximum shear stress, τ_{max}, on the soil element would be:

$$(\tau_{max})_r = \gamma z \, a_o \tag{12}$$

where γ is the unit weight of the soil and subscript r refers to rigid body motion.

Because the real soil column behaves as a deformable body, the actual shear stress at depth z, $(\tau_{max})_d$, as determined by a ground response analysis will be different to $(\tau_{max})_r$ and may be expressed as:

$$(\tau_{max})_d = r_d(z)\,(\tau_{max})_r \tag{13}$$

in which r_d is termed as the stress reduction factor, which can be viewed as the actual shear stress, but in non-dimensional terms, normalized by $(\tau_{max})_r$.

The concept of the stress reduction factor was originally developed for the shear wave model, for which its value is always less than 1.0. However, to facilitate comparison of stress levels from the shear and Rayleigh wave models, the concept of a reduction factor, defined by Equations (12) and (13), is also retained for the Rayleigh wave model, even though in this case its value may even be more than 1.0 near the surface, and up to as much as 3.4. Further, for the same facility of comparison, a concept of what may be termed as relative stress reduction factor is introduced here as:

$$r'_d(z) = \frac{r_d(z)}{r_d(o)} \tag{14}$$

where $r_d(o)$ is the stress reduction factor at the ground surface and $r'_d(z)$ is the relative stress reduction factor at depth z. Thus, by definition, the relative stress reduction factor equals 1.0 at the ground surface for both the shear wave or the Rayleigh wave models. In Section 4.2, it is reasoned that the relative stress reduction factor, r'_d, may be more pertinent to liquefaction than the absolute reduction factor, r_d.

In the following sub-sections, we shall evaluate the shear stress as a function of depth and thereby determine $r_d(o)$ and the variation of r'_d with depth for each of the two models.

3.1 Stress Reduction Factor for the Shear Wave Model

The shear stress for either the shear wave or the Rayleigh wave model can, in general, be obtained as:

$$\tau = G\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) \tag{15}$$

where:

G = shear modulus of the soil

$= \rho V_s^2$

and ρ = mass density of the soil.

With the above definitions of r_d and r'_d in view, and by manipulation of Equations (1), (2), (12), (13), and (15), the stress reduction factor at the surface, and the relative stress reduction factor at depth z, for the shear wave model can be obtained as:

$$r_{ds}(o) = 1.0 \tag{16}$$

and

$$r'_{ds}(z) = \frac{|\sin (Nz)|}{Nz} \tag{17}$$

The subscript 's' corresponds to the shear wave solution, and the vertical bars imply absolute value. It is noted that, as illustrated in Figure 1 and Table 1, for shear wave propagation, $r_{ds}(o)$ and $r'_{ds}(z)$ are not dependant on Poisson's ratio. Figure 1 illustrates the variation of r'_{ds} with the ratio of depth to shear wave length. To give an idea of the typical variation of r'_{ds} with depth, the right-hand scale of Figure 1 illustrates typical depths for a shear wave length of 140 meters which can result from a soil stratum with a shear wave velocity of 350 meters per second and a predominent frequency of motion of 2.5

Hertz. Since r_{ds} (o) = 1.0, it is obvious that the absolute and relative stress reduction factors r_{ds} and r'_{ds} are the same for the shear wave model.

3.2 Rayleigh Wave Model

For the Rayleigh wave model, considerable but straightforward mathematical manipulation of Equations (3) through (15) yields:

$$r_{dr}(o) = \left|\left(\frac{q}{N'} - \frac{s}{N'}\right) \cdot \frac{2}{K^2} \cdot \frac{q}{N'} \;/\; \left[\left(\frac{2q}{N'} \cdot \frac{s}{N'} \;/\; \left(1 + \frac{s^2}{N'^2}\right)\right) - 1\right]\right| \qquad (18)$$

and $$r'_{dr}(z) = \left|\left[\exp\,(-qz) - \exp\,(-sz)\right] / (qz - sz)\right| \qquad (19)$$

The subscript r corresponds to the Rayleigh wave solution. As illustrated in Table 1, r_{dr}(o) is a function of Poisson's ratio, while r'_{dr}(z) is only slightly influenced by Poisson's ratio, as shown in Figure 1.

4. Discussion of the Above Results - Comparison of the Two Models

4.1 Stress Reduction Factors and Maximum Shear Stresses

From Equation (13) it is apparent that the maximum shear stresses at a given depth for both the shear wave and Rayleigh wave models are directly proportional to the stress reduction factors. Additionally, it is noted from Table 1 that near the ground surface the reduction factor for the Rayleigh wave model is much higher than that for the shear wave model and, therefore, correspondingly, shear stresses for the Rayleigh wave model will be about two to three times as large as those for the shear wave model.

Higher stress in the Rayleigh wave model can be explained from Equations (2), (4) and (15). Equation (15) indicates that horizontally propagating waves with vertical displacement would also add to shear stresses. However, the shear wave model (Equation (2)) has no vertical displacement, whereas in the Rayleigh wave model the vertical displacements necessarily accompany the horizontal displacements (Equations (3) and (4)). Thus, for the same horizontal ground surface motion in both models, the presence of the vertical displacements in the Rayleigh wave model yields much higher shear stresses near the ground surface than does the shear wave model.

The absolute stress reduction faction at any depth 'z' can be obtained by multiplying the near surface stress reduction factors from Table 1 with the relative stress reduction factors defined in Figure 1. The difference between the stress reduction factors for the two models narrows with depth, as the relative shear stress reduction factor for the Rayleigh wave model drops more rapidly than that for the shear wave model.

4.2 Relative Stress Reduction Factor and Liquefaction Potential

It can be reasoned that the liquefaction potential near ground surface should be the same for both models. In other words, the relative stress reduction factor, r'_d, and not the stress reduction factor, r_d, should determine the liquefaction potential.

As Seed and Idriss[1] indicate, the actual field stresses during liquefaction are not measured and since it is not possible to simulate field behavior exactly in the laboratory, empirical coefficients related to the observed and estimated behavior have been developed. The value of these factors must, therefore, necessarily depend upon the model used to estimate the field behavior, but always in such a manner that the combined effect of the model used to estimate the field behavior and the corresponding empirical coefficient value remains the same for all models. In other words, although the actual shear stresses predicted from the Rayleigh model are higher, the corresponding empirical coefficients must necessarily be such that the combined result of the shear stress and the corresponding empirical coefficient value, should remain the same as those derived on the basis of the shear wave model. Since most of the case histories studied have been for soil layers close to the ground surface,[7] this similarity of liquefaction potential for the shear and Rayleigh wave models should be valid near the ground surface and, therefore variations in liquefaction potential with depth should be proportional to the variation in r_d' with depth for both models.

With this hypothesis in view, it can be observed from Figure 1 that the risk of failure for the two wave propagation models is the same near the surface, but decreases much more rapidly with depth for the Rayleigh wave model than for the shear wave model. Further, since the natural occurrence of liquefaction is generally limited to the top ten to twenty meters of ground surface,[7] whereas the shear wave model will often predict liquefaction at greater depths, it would seem that the Rayleigh wave model results furnish a better explanation of the field behavior than the results of the shear wave model.

5.0 Discussion and Conclusion

Although the results presented above are not conclusive, insomuch as they have been obtained for an idealized, homogeneous, undamped half-space, they do indicate that there is a significant difference in the shear stress distribution for Rayleigh waves and vertically propagating shear waves for equal ground surface motions. This difference does not lead to the suggestion that current procedures used for liquefaction evaluation, based on the shear wave model, are unconservative. However, since the methods of analysis of liquefaction are based on empirical correlations between observed and computed conditions, if a Rayleigh wave model instead of the existing shear wave model is utilized, the empirical correlation factors to provide agreement with observed and predicted conditions must be revised. Specifically, it can be reasoned that the risk of liquefaction near surface is the same for both models, but it decreases much more rapidly with depth for the Rayleigh wave model than it does for the shear wave model, even though the computed stresses near the surface are higher due to the Rayleigh wave model than they are due to the shear wave model. The liquefaction failures actually observed have been limited to the top ten to twenty meters of ground surface,[7] whereas the shear wave propagation model may often predict liquefaction for much greater depths. Thus, it would seem that the Rayleigh wave model provides a better explanation of natural behavior. Even though this conclusion is tentative, these observations do at least raise the need for greater attention to Rayleigh and, perhaps, even Love wave models.

The general problem of the shear wave model has been comprehensively studied using existing computer codes such as SHAKE,[5] and LUSH.[8] Conclusive evidence of the influence of Rayleigh and Love wave models will have to wait until generalized computer codes to study them, similar to SHAKE and LUSH, have been developed.

References

(1) SEED, H. B., IDRISS, I. M., "Simplified Procedure for Evaluating Soil Liquefaction Potential", Journal of the Soil Mechanics and Foundations Division," ASCE, Volume 97, No. SM3 (September, 1971).

(2) HALL, W.J., NEWMARK, N. M., MOHRAZ, B., "Comments on Earthquake Transmission from Basement Rock to Surface," Proceedings, Fifth World Conference on Earthquake Engineering, Rome (1973).

(3) DONOVAN, N.C., DEGENKOLB, H. J., "Field Measurement of Relative Ground Displacement," Proceedings, U.S. Conference on Earthquake Engineering, Ann Arbour, Michigan (June, 1975).

(4) SEED, H. B., "Evaluation of Soil Liquefaction Effects on Level Ground during Earthquakes," Proceedings, Liquefaction Problems in Geotechnical Engineering, ASCE, Philadelphia (September, 1976).

(5) SCHNABEL, P. B., LYSMER, J., SEED, H.B., "SHAKE - A Computer Program for Earthquake Response Analysis of Horizontally Layered Sites," Report No. EERC 72-12, Earthquake Engineering Research Center, University of California, Berkeley (1972).

(6) RICHART, JR., F. E., HALL, JR., J. R., WOODS, R. D. "Vibrations of Soils and Foundations," Prentice Hall, Inc. (1970).

(7) CHRISTIAN, J. T., SWIGER, W. F., "Statistics of Liquefaction and SPT Results," Journal of the Geotechnical Engineering Division, ASCE, Volume 101, No. GT11 (November, 1975).

(8) LYSMER, J., UDAKA, T., SEED, H.B., HWANG, R., "LUSH - A Computer Program for Complex Response Analysis of Soil-Structure Systems," Report No. EERC 74-4, Earthquake Engineering Research Center, University of California, Berkeley (April, 1974).

TABLE 1

$r_d(o)$ - STRESS REDUCTION FACTOR AT SURFACE

Poisson's Ratio υ	Due to Shear Wave Model	Due to Rayleigh Wave Model
0.30	1.0	2.32
0.35	1.0	2.52
0.40	1.0	2.75
0.45	1.0	3.03
0.50	1.0	3.38

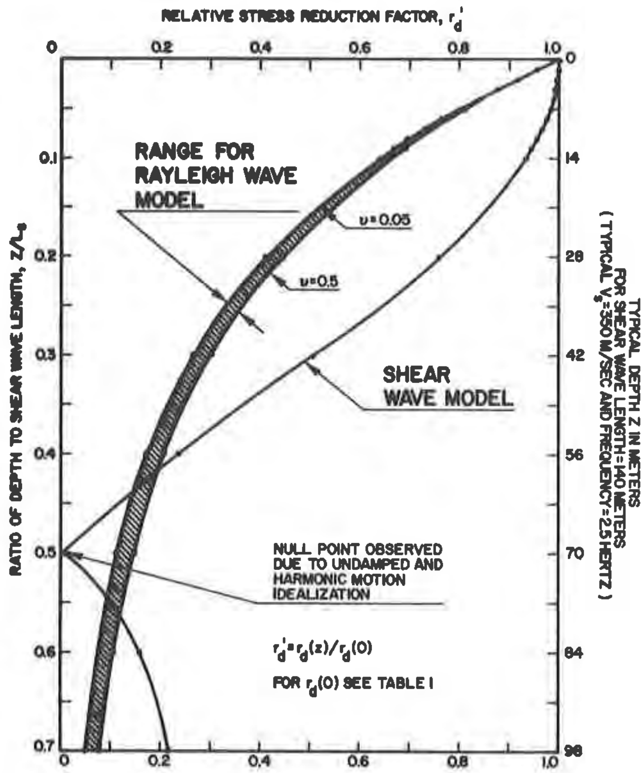

Figure 1 – Distribution of Relative Stress Reduction Factor r_d' with Depth Due to Shear and Rayleigh Wave Models