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SEVERAL PROBLEMS OF CULMULATIVE EFFECTIVE MASS FRACTION IN ANTISEISMIC ANALYSIS

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ABSTRACT

Cumulative Effective Mass Fraction(CEMF) is one of important items which sign the accuracy in antiseismic analysis. Based on the primary theories of CEMF, the paper show the influence of CEMF on the accuracy in antiseismic analysis. Moreover, some advices and ways are given to solve common problems in antiseismic analysis, such as how to increase CEMF, how to avoid the mass's lossing because of the torsional frequency's being close to the frequency correspongding to the peak of seismic response spectrum, how to avoid the mass's lossing because of the constraints, and so on..

Keywords: CEMF Frequency Antiseismic Analysis

1. FOREWORD

Cumulative effective mass is one of the important indexes in antiseismic calculation. In order to guaranttee the calculation accuracy of response spectrum, ASCE－486 has stipulated several pieces of criterion that can be used, one of which is: the total mass of response calculation is 90% of the total systematic mass at least. And surplus mass should be multiplied by the peak value of blocking frequency spectrum, the result of which should be added to structure as rigidity body strength. But in actual calculation the value is sometimes less than 90% regardless of surplus mass and the calculation accuracy can satisfy the requirement. Yet sometimes the calculation accuracy is not enough when the value exceeds 90%. This thesis gives illustration to relevant problems and tries to put forward some simple ways to solve them proceeding from basic principles.

2. BASIC PRINCIPLES

In order to state some questions in calculation, this section provides some brief explanation to basic theories.

2.1 VIBRATION EQUATION

A linear system under the influence of seismic becomes an equation group of linear algebra after being dealt with finite element method:

$$[m]\{\ddot{X}\}+[C]\{\dot{V}\}+[K]\{V\}=\{0\} \qquad \text{(Eq.1)}$$

X is the absolute displacement of each systematic point and V is the comparative displacement of seismic input point.

$$\{X\} = \{V\} + \{X_0\} \qquad \text{(Eq.2)}$$

$\{X_0\}$ is the absolute displacement of seismic input point.

Put (Eq.2)into (Eq.1)and get

$$[m]\{\ddot{V}\} + [C]\dot{V} + [K]\{V\} = [m]\{\ddot{X}_0\} \qquad \text{(Eq.3)}$$

This thesis only discusses the three-dimensional entity element models in Cartesian coordinate (0, x1, x2, x3). Suppose there is a piece of joint of n and there are three translation degrees of freedom. The vector in (Eq.3) is 3n step and the order of every element b_i for each vector in $\{B\}$ is i = 3j-2, 3j-1, 3j, and j $= 1,2 \cdots n$ corresponds to serial number of joint. $[m]$, $[C]$, $[K]$ matrix is 3n×3n matrix. For rebound seismic movement, the signs '+' and '-' don't influence the result. Here the sign '+' is adopted and so is it in the following text.

In (Eq.1), (Eq.3), $[C]$, $[K]$ are the damping matrix and rigidity matrix of vibration system respectively. $\{m\}$ is the total mass matrix of vibration system.

Here the single seismic encouragement in x1 direction is used to explain the concept of CEMF for convenience.

At this time, the formula can be written:

$$\{X\} = \{V\} + \{D\}X_0 \qquad \text{(Eq.4)}$$

$\{D\} = \{d_i\}$ is influence coefficient.

If there is only encouragement in x1 direction, then:

$$\{D\}^T = (1,0,0,1,0,0\cdots) \qquad \text{(Eq.5)}$$

Introduce (Eq.4) to equation (Eq.3), the right side of the equation becomes:

$$[m]\{D\}\ddot{x}_0 \qquad \text{(Eq.6)}$$

It's obvious that $\{m_D\} = [m]\{D\}$ is just the mass related to encouragement in x1 direction and the mass in other direction is zero, i.e.:

$$\{m_D\}^T = (m_{11}, 0, 0, m_{21}, 0, 0\cdots) \qquad \text{(Eq.7)}$$

the total systematic mass is:

$$M = \{D\}^T [m]\{D\} = \sum_{i=1}^{n} m_{i1} \qquad \text{(Eq.8)}$$

2. 2 SOLVE COUPLING EQUATION

Usually the freedom degree of equation (Eq.3) is couple and the equation is described as mode of vibration coordinate for solving coupling equation. 3n pieces of separate equation can be got in mode of vibration coordinate system:

$$\{V\} = \sum_{i=1}^{3n} \{\phi_i\}\eta_i \, or \{V\} = [\phi]\{\eta\} \qquad \text{(Eq.9)}$$

$[\phi] = [\{\phi_1\},\{\phi_2\},\cdots\{\phi_{3n}\}]$, $\{\phi_i\}$ is mode of vibration I, corresponding to round frequency ω_i. As a rule, $\{\phi_i\}$ the mark i increases with ω_i. η_i is the projection of displacement $\{V\}$ in mode of vibration $\{\phi_i\}$.

put (Eq.9)into equation (Eq.3), and use $[\phi]^T$ to multiply both sides of the equation, then:

$$[\phi]^T [m][\phi]\{\ddot{\eta}\} + [\phi]^T [C][\phi]\{\dot{\eta}\} + [\phi]^T [K][\phi]\{\eta\} = [\phi]^T [m]\{D\}\ddot{x}_0 \qquad \text{(Eq.10)}$$

mode of vibration $\{\phi_i\}$ is the characteristic vector of equation (Eq.10). There is formula:

$$([K] - \omega_i^2 [m])\{\phi_i\} = 0 \qquad \text{(Eq.11)}$$

$\{\phi_i\}$ positively intersect $[K]$ and $[m]$, i.e.:

$$\{\phi_i\}^T [m]\{\phi_j\} = \begin{cases} m_i^{'} & i = j \\ 0 & i \neq j \end{cases} \qquad \text{(Eq.12)}$$

$$\{\phi_i\}^T[K]\{\phi_j\} = \begin{cases} k_i^{'} & i = j \\ 0 & i \neq j \end{cases} \qquad \text{(Eq.13)}$$

usually suppose $\{\phi_i\}$ positively intersect damping matrix, i.e.:

$$\{\phi_i\}^T[C]\{\phi_j\} = \begin{cases} c_i^{'} & i = j \\ 0 & i \neq j \end{cases} \qquad \text{(Eq.14)}$$

In addition, define

$$[\phi]^T[m]\{D\} = \{L\} \qquad \text{(Eq.15)}$$

put (Eq.12) ~ (Eq.15)into equation(Eq.10) and get coupling equation

$$m_i^{'}\ddot{\eta}_i + c_i^{'}\dot{\eta} + k_i^{'}\eta_i = l_i\ddot{x}_0 \qquad \text{i=1,2,…3n} \qquad \text{(Eq.16)}$$

the characteristic result can be got from equation (Eq.16)

$$\eta_i = \frac{l_i}{m_i^{'}} s_i \qquad \text{(Eq.17)}$$

s_i is expressed by duhamel integral

$$s_i = \frac{1}{\omega_i}\int_0^t \ddot{x}_0 e^{-\xi\omega(t-\tau)}\sin(\omega t - \tau)d\tau = s(\omega_i, \xi, t)$$

ξ is the damping ration. If $s_i = \max s(\omega_i, \xi, t)\Big|_0^t$, then s_i is the displacement of response spectrum. If expressed with spectrum acceleration $s_{ai} = \omega_i^2 s_i$ and put (Eq.17)into (Eq.9), then

$$\{V\} = [\phi]\{\eta\} = [\phi]\left(\frac{l_i}{m_i^{'}}\frac{s_{ai}}{\omega_i^2}\right) \qquad \text{(Eq.18)}$$

2. 3 CUMULATIVE EFFECTIVE MASS

In equation (Eq.1), the mass force that every particle receives

$$\{f\} = [m]\{\ddot{X}\} \qquad \text{(Eq.19)}$$

regardless of contribution to force by damping, from equation (Eq.1):

$$\{f\} = [K]\{V\}$$

put (Eq.9) in, then

$$\{f\} = [K][\phi]\{\eta\} \qquad \text{(Eq.20)}$$

the rsultant of forces of every joint in x1 direction:

$$F_1 = \{D\}^T\{f\} = \{D\}^T[K][\phi]\{\eta\} \qquad \text{(Eq.21)}$$

use (Eq.11) and put (Eq.18)into (Eq.21), then

$$F_1 = \{D\}^T[m][\phi]\left\{\frac{l_i}{m_i^{'}} s_{ai}\right\} \qquad \text{(Eq.22)}$$

introduce (Eq.15)

$$F_1 = \{L_i\}^T\left\{\frac{l_i}{m_i^{'}} s_{ai}\right\} = \sum_{i=1}^{3n}\frac{l_i^2}{m_i^{'}} s_{ai} \qquad \text{(Eq.23)}$$

if $s_{ai} = 1, i = 1,2\cdots 3n$, in number value:

$$F_1 = \sum_{i=1}^{3n}\frac{l_i^2}{m_i^{'}} \qquad \text{(Eq.24)}$$

obviously $\sum_{i=1}^{3n} \frac{l_i^2}{m_i^{'}}$ is the resultant of forces encouraged in x1 direction. It can be proved that it is the systematic mass in x1 direction. The brief illustration is as follows:

like (Eq.9)vector $\{D\}$ can also be expressed with characteristic vector

$$\{D\} = [\phi]\{\alpha\} = \sum_{i=1}^{3n} \{\phi_i\}\alpha_i \quad \text{(Eq.25)}$$

put (Eq.15)in

$$\{L\} = [\phi]^T [m]\{D\} = [\phi]^T [m][\phi]\{\alpha\}$$

component $l_i = \{\phi_i\}^T [m] \left\{ \sum_{i=1}^{3n} \{\phi_i\}\alpha_i \right\}$

from (Eq.12) $l_i = \{\phi_i\}^T [m]\{\phi_i\}\alpha_i = m_i^{'}\alpha_i$ to

$$\alpha_i = \frac{l_i}{m_i^{'}} \quad \text{(Eq.26)}$$

put (Eq.25)in

$$\{D\} = [\phi]\left\{\frac{l_i}{m_i^{'}}\right\}$$

thus a further result can be got

$$\{D\}^T [m]\{D\} = \left\{\frac{l_i}{m_i^{'}}\right\}^T [\phi]^T [m][\phi]\left\{\frac{l_i}{m_i^{'}}\right\} \quad \text{(Eq.27)}$$

use the positive intersection of $\{\phi_i\}$ to $[m]$

$$\{D\}^T [m]\{D\} = \left\{\frac{l_i}{m_i^{'}}\right\}^T [\phi]^T [m][\phi]\left\{\frac{l_i}{m_i^{'}}\right\} = \sum_{i=1}^{3n} \frac{l_i^2}{m_i^{'}} \quad \text{(Eq.28)}$$

compared with (Eq.8), it's obvious that the right side of (Eq.28)is the total systematic mass in x1direction. Thus $F_1 = \sum_{i=1}^{3n} \frac{l_i^2}{m_i^{'}}$ in number value it's the total systematic mass in x1 direction. For $\frac{l_i^2}{m_i^{'}}$ corresponds to $\{\phi_i\}$, we can refer to $\frac{l_i^2}{m_i^{'}}$ as mode of vibration mass. It proves that under circumstance of all the 3n modes of vibration participating in responding,the total corresponding mass in x1 direction $\sum_{i=1}^{3n} \frac{l_i^2}{m_i^{'}}$ is the cumulative effective mass and equal total systematic mass in x1 direction.

It should be stated that this conclusion is suitable in other directions. And it can also be proved that effective rotating inertia can be derived by runnin encouragement. It's the total corresponding systematic rotating inertia.

3. SEVERAL CONCRETE PROBLEMS

3.1 CEMF<90% WHILE THERE IS POSSIBILITY OF ENOUGH ACCURAY

Section 2 has proved that CEMF is the total systematic mass when all the modes of vibration participate in the responding. But for large-scale dynamic equation group, people usually use limited modes of vibration to calculate in order to save calculating time and reduce memory taken up, such as adopting subspace to change, lanczos and so on. Therefore, CEMF, the ration of effective mass to total systematic mass is smaller than 100%. ASCE－486 requires it>90% and stipulates at the same time: for mass that doesn't participate in calculation, namely, surplus mass should be multiplied by peak of blocking frequency spectrum and the result should be added to structure as rigidity body strength. But in calculation actually sometimes the CEMF<90% regardless of contributions by surplus mass, the stress calculation is still accurate enough. From (Eq.27) it can be inferred this results from two reasons.

(1) Mass of vibration mode decreases with the increase of vibration mode steps.

In actual calculation, most equipment, take one container supported with skirt support for example, the contribution by roof beam mode of vibration is greater than that by shell mode of vibratin, and in roof beam mode of vibration the contribution by crooked mode of vibration is greater than that by rotating mode of vibration. Though the mode of vibration mass doesn't decrease with increase of serial number, the main trend is dropping. For example, Fig. 1 is the simple roof beam with 5 nodes. At 1/4 and 3/4 points the centralized mass is m and the mass in the middle is 2m, assuming seismic encouraged in x direction.

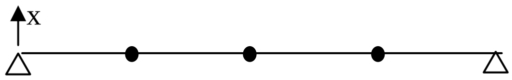

Fig.1

Corresponding mode of vibration

$$\{\phi_1\} = \begin{Bmatrix} 1 \\ 1.4277 \\ 1 \end{Bmatrix} \quad \{\phi_2\} = \begin{Bmatrix} -1 \\ 0 \\ 1 \end{Bmatrix} \quad \{\phi_3\} = \begin{Bmatrix} 1 \\ -0.7004 \\ 1 \end{Bmatrix}$$

corresponding mass of vibration mode

$$\frac{l_1^2}{m_1^{'}} = 3.878\text{m}, \quad \frac{l_2^2}{m_2^{'}} = 0, \quad \frac{l_3^2}{m_3^{'}} = 0.1220$$

total $\sum_{i=1}^{3} \frac{l_i^2}{m_i^{'}} = 4\text{m}$. by $\dfrac{\frac{l_1^2}{m_1^{'}}}{\frac{l_3^2}{m_3^{'}}} = 31$, the mass of vibration mode decreases with increase ofmass of vibration mode i. In addition, $\frac{l_2^2}{m_2^{'}} = 0$ lies that the direction of seismic encouragement positively intersect $\{\phi_2\}$

(2) Acceleration value of spectrum decreases with increase of frequency after several natural frequency.

For Chinese experimental pile, the value of spectrum acceleration s_{ai} reduces with the growth of ω_i after about 8Hz. There is a quantity grade between the peak value of spectrum acceleration and zero cycle. Under the influence of the two above-mentioned factors, the contribution by lower mode of vibration is far more greater than that by higher mode of vibration. For containers supported by skirt seat, the mass of vibration mode in crooked mode of vibration is far more greater than that in other modes of vibration. Therefore, there is enough accuracy when the mass of vibration mode is only 75%. For cases that the frequency is greater than 33 Hz in every step, the mode of vibration should be enough to guaranttee the accuracy of calculation.

3.2 LARGE ROTATING INERTIA ISSUES

The equipment in Fig. 2, the top mass is great and underpart is a frame structure. There is large distance between the rigidity centre and masss centre of the equipment. Thus great torsion is produced under the influence of horizontal seismic. In addition, the torsion frequency is close to peak value of floor spectrum acceleration.

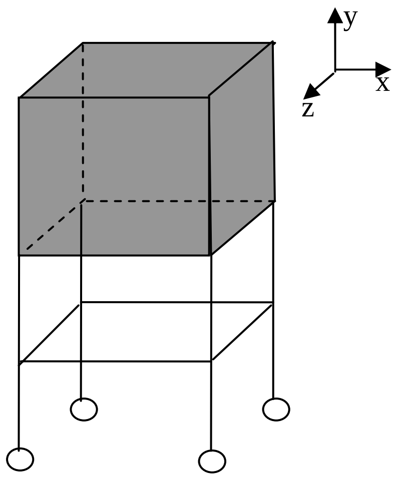

Fig. 2

The calculation proves that the result is not reliable yet though the ratio of effective mass in translation direction to total systematic mass is more than 90% for many modes of vibration. The basic reason is that only three encouragements in translation direction are considered by floor spectrum. But under the influence of actual seismic as the rigidity centre of the floor is not in accordance with mass centre, the floor will be caused to wind around the vertical axis. For equipment in Fig. 2, the encouragement of torsion should not be igored as it's as important as the three translation encouragement.

Moreover, large distance of torsion can be caused if there's only translation in floor spectrum input. Its contribution to equipment stress is great, even making the supporting frame fail to pass ASME evaluation. By the way, because of great torsion, the equipment quiet strength law is not applicable.

The content in this section doesn't state that the conclusion by ASCE-486 is not applicable. We advocate to change the design of the equipment. The resisting spraining rigidity of the equipment has been increased, such as increasing the set bolt, increasing the oblique roof beam in the frame, etc. This makes the torsion frequency rise to 25Hz and rank after several frequency of roof beam crooked mode of vibration. Thus the stress of equipment drops obviously and get through the ASME evaluation.

3.3 CONSERVATIVE METHOD TO DEAL WITH SURPLUS MASS

According to regulations of ASCE—486, the surplus mass should be multiplied by the peak value of blocking frequency. The product should be added to structure as rigidity body strength. In a situation with leeway, the following simple ways can be used to deal with the problems. The steps are as follows:

(1) The first calculation

Suppose the effective mass of antiseismic calculation is m_0 which includes contribution by the first i mode of vibration mass and the total systematic mass is M_0, CEMF $\eta = \frac{m_0}{M_0}$. If η<90%, the mass should be increased, for instance, the centralized mass and density of the system is multiplied by k.

(2) The second calculation

Suppose the total increased systematic mass $M_1 = kM_0$. If the vibration mode of participating in contribution by system M_1 is equal to that by system M_0, then the approximate effective mass of system M_1 is $m_1 = \eta M_1$. For $\frac{m_1}{M_0} \geq 90\%$, requiring $k \geq \frac{0.9}{\eta}$. For example, if $\eta = 70\%$, then $k > 1.29$ The calculated frequency at this time is different from that in the first calculation. But the spectrum value got in the first calculation should be calculated.

(3) The third calculation

Update spectrum acceleration. If calculated with original spectrum value, the result will become bigger or smaller for change of natural frequency. So updating spectrum value can make the acceleration spectrum value of every natural frequency in the third calculation equal to that in the first calculation.

This method is conservative because surplus mass corresponds to high mode of vibration but the large part of increased mass here is added to low mode of vibration.

4. CONCLUSIONS

This thesis has given explanation to correct use of ‘ The total mass of response calculation is 90% of the total systematic mass at least’ stipulated by ASCE－486 and proposed simple ways to deal with ‘surplus mass’.

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