Tearing Instability Analysis of a Completely Cracked Nozzle

J.-P. Hutin
Electricité de France, Service de la Production Thermique,
3 rue Messine, F-75008 Paris, France

C. Faidy
Electricité de France, SEPTEN,
Tour EDF-GDF, F-92080 Paris la Défense Cedex 8, France

Abstract

Because of the difficulties associated with in-service repair of nuclear reactor components, plant operators who have to deal with real crack problems cannot rely only on the generally pessimistic approach based on linear elastic fracture mechanics so that more realistic approaches are to be used. Elastic plastic theory based on J - integral and Tearing Modulus concepts is such an approach. The purpose of the present paper is to show how this theory can be applied to a practical, although very hypothetical, case : a completely cracked nozzle.

The main features of the theory and the stability criteria are presented. Justifications are given along with some limitations of the method. Algebraic expressions of the different parameters needed in the analysis are derived. Different elastic plastic models can be used depending on the relative importance of plasticity in the problem. A shape factor is introduced to take in account the effect of geometry. The paragraph ends with two expressions for the applied J and the applied T to be used in the cracked nozzle application.

Then the theory is applied to a PWR nozzle which would be completely cracked. All significant parameters are computed for the design conditions and compared with material characteristics. The conclusion is that the crack would not grow unstable by ductile tearing in the vessel wall or in the pipe wall.

At the end, some insights are given about difficulties encountered when applying this theory, what should be done to check its validity and to determine more precisely its limitations.

1 - Introduction

The safety assessment of nuclear reactor design requires an analysis to demonstrate that there is no risk of brittle fracture in any components. This demonstration is usually done by using the now well-known concepts of linear elastic fracture mechanics. Because of the material used in most cases and because of the operating conditions of a nuclear plant, this linear elastic theory is generally pessimistic. But using it can be considered as a wise practice on a safety point of view, at least at the design or manufacturing stage. However, the problem is not so easy to solve when the plant operator, after performing in-service inspection, has to deal with a real crack problem. Indeed, if it is possible, in shop, to repair a crack and control the work done in the best conditions, it is obviously not the case on site, in a reactor which has been running for several years. The repair work itself is generally difficult to perform and to control ; it is often impossible to achieve proper heat treatment of the repaired zone ; and all difficulties are enhanced by the risks associated with irradiation. So that, as far as safety is concerned, and apart from economic aspects, repairing a defect can be more harmful than the defect itself. For these reasons, when he has to take a repair or no-repair decision, the plant operator should not rely only on the pessimistic results of a linear elastic fracture mechanics analysis but should also considered the results of a more realistic approach. Then, it is easy to understand why power generating services operating nuclear reactors are very interested in the developments of elastic plastic fracture mechanics.

At this point, many elastic plastic theories have been proposed, with different parameters and criteria. Which one to choose ? The first thing to do would be to check their validity and to determine the limitation of each theory by running tests. However, it is not an easy thing to do because small scale tests may be not representative and real scale tests are difficult if not impossible to perform. Moreover, these approaches are generally based on different theoretical representations of the same physical phenomenon ; so that their validity level should be about the same. An other parameter which may be considered is the difficulty associated with the practical application of these theories. It is the purpose of this paper to show how the elastic plastic approach based on J integral and Tearing Modulus can be applied in some cases and which difficulties are encountered.

2 - J as a relevant parameter

Linear elastic fracture mechanics gives reasonnable solutions for the small scale yielding case where the stress intensity factor correctly characterizes the crack tip fields. Then Elastic Plastic Fracture Mechanics can be treated as a natural extension of small scale yielding mechanics to the case of large scale yielding. For power-law hardening material, the Hutchinson Rice Rosengren crack tip field is characterized by the J integral which plays the same role as K does in linear elasticity [1, 2] . However, one must remember the conditions under which the basic J theory is valid : both the path-independance and the energy interpretation of J are assured only for non-linear elastic materials, in the absence of residual and thermal stresses. These results may be transferred to elastic plastic materials for cases of monotonically increasing loads proportionnal to one loading parameter as shown by Hutchinson and Goldman [3]. Thus J may be expected to characterize the crack tip field and

used as a relevant parameter in elastic plastic materials as long as no unloading occurs i.e. for stable cracks [4]. This means that a criterion $J_1 = J_{1c}$ would be relevant but restricted to crack growth initiation. It is possible to relax this restriction so that a J updated for crack elongation may be used to treat also stable crack growth. This was proposed by Paris and al in their Tearing Modulus concept [5]. As it has been already said, it is not our purpose to discuss the validity of the theory itself but only to expose this theory and to show how it can be applied.

3 - The tearing modulus as a parameter of the crack growth stability

The fundamuntal view point is that, if certain conditions are met, the severity of the situation at the tip of a pre-existing crack is correctly described by one parameter that characterizes the stress/strain field at the crack tip so that initiation and propagation of the crack are controlled by this parameter. The J- integral is supposed to be such a parameter. The criteria proposed by Paris for initiation and stability of the propagation are based on two hypothesis :

1) the resistance of a material to ductile extension of a crack under increasing load is correctly represented by a $J - \Delta a$ (or J_R) curve.

2) an extension of the crack, under constant load, will change the J- value.

A typical $J - \Delta a$ curve is shown in fig. 1. The initial length of the crack is a_o. During the first stage (from O to A), increasing the applied load (and J as a consequence) results in an apparent extension $a_A - a_o$ due to a blunting of the crack tip. Then J reaches some critical value J_{1c} and the true propagation begins. To a given value of J - let's say J_B - the amount of crack extension is given by the $J - \Delta a$ curve : $a_B - a_o$. Let's imagine that, under constant load, the crack length a_B is increased to $a_B + \Delta a_B$. This will result in a change in J which can be written as follows :

$$J\ (a_B + \Delta a_B) - J\ (a_B) = \left\{ \frac{dJ}{da} \right\}_{\text{constant load}} \cdot \Delta a_B$$

$\left\{\frac{dJ}{da}\right\}$ depends on the load, geometry and material. This quantity can be computed and is supposed to characterize the situation. It will be labelled as $\left\{\frac{dJ}{da}\right\}_{appl}$. The slope of the curve which corresponds to the increasing load case is supposed to be a constant characterizing the material resistance and will be labelled as $\left\{\frac{dJ}{da}\right\}_{mat}$.

The criteria proposed by Paris is the following : the crack propagation is stable if the calculated quantity $\left\{\frac{dJ}{da}\right\}_{appl}$ under constant load is smaller than the slope $\left\{\frac{dJ}{da}\right\}_{mat}$ under increasing load measured on the $J - \Delta a$ curve. Crack growth is unstable in the opposite case.

A somewhat more physical way to describe this criteria is to compare the J integral to a crack driving force (or to an energy release rate) and the $J - \Delta a$ curve to an R- curve. Unstable fracture will occur when the rate of change of the crack driving force (or rate of change of energy release rate) is greater than the rate of growth of material resistance to cracking or tearing [6].

The comparison between $\left\{\frac{dJ}{da}\right\}_{mat}$ and $\left\{\frac{dJ}{da}\right\}_{appl}$ is to be done on a dimensionless quantity called the Tearing Modulus and defined as :

$$T = \frac{E}{\sigma_o^2} \left\{ \frac{dJ}{da} \right\} \qquad (1)$$

E : Young's modulus $\quad \sigma_o$: flow stress of the material

So that Paris criteria can be summarized as follows :

$J < J_{1c}$: blunting of the crack - No real propagation

$J > J_{1c}$: growth of the crack $\begin{cases} \text{stable if } T_{appl} < T_{mat} \\ \text{unstable if } T_{appl} > T_{mat} \end{cases}$

4 - Limitations and practical application of the theory

The main point is that for this theory to be valid the crack propagation must be actually J- controlled. For that, Hutchinson showed that the following relation had to be verified [7]

$$\omega = \frac{b}{J} \left\{ \frac{dJ}{da} \right\} \gg 1 \qquad (2)$$

b : ligament size

In the traction case, it has been shown by Shih [7] that ω had to be greater than 80. Other limitations and conditions have been pointed out by Shih, Landes, Turner and others [7] . As far as material properties are concerned, the weak point may be the assumption of a linear $J - \Delta a$ curve. Indeed, there is evidence that, in some cases, J tends towards a plateau so that the theory has to be restricted to small amounts of crack extension [8]. Moreover, the assumption that the $J - \Delta a$ curves obtained from test specimens can be applied to real structures should be verified.

Different methods for measuring T_{mat} and computing T_{appl} have been exposed and described in ref. [7] .

5 - Derivation of some parameters used in the stability analysis

The stability analysis relies mainly on the calculation of two parameters, J_{appl} and T_{appl} , and on their comparison with two material characteristics J_{1c} and T_{mat} .

5.1 - Infinite solid case

In linear elastic fracture mechanics, the J- integral can be related to the stress intensity factor K through :

$$J = \frac{K^2}{E} \qquad (3)$$

E : Young's Modulus if plain stress conditions are met.

In an infinite body :

$$J = \frac{\pi \sigma^2 a}{E}$$

Introducing some plasticity can be done, in a first step, through the classical Irwin's correction :

$$K = \sigma \sqrt{\pi a_E}$$

where $a_E = a + ry$ and $ry = \frac{1}{\beta \pi} \left\{ \frac{K}{\sigma_0} \right\}^2$ ($\beta = 2$ for plane stress and $\beta = 6$ for plane strain.)

so that a_E can be written as :

$$a_E = a \left\{ 1 + \frac{1}{\beta} \left\{ \frac{\sigma}{\sigma_0} \right\}^2 \right\} \simeq \frac{a}{1 - \frac{1}{\beta} \left\{ \frac{\sigma}{\sigma_0} \right\}^2} \qquad (4)$$

if $\frac{1}{\beta} \left\{ \frac{\sigma}{\sigma_0} \right\}^2$ is small enough

This yields the following expression for J :

$$J = \frac{\sigma_0^2 a}{E} \cdot \left\{ \frac{\pi \left(\frac{\sigma}{\sigma_0} \right)^2}{1 - \frac{1}{\beta} \left\{ \frac{\sigma}{\sigma_0} \right\}^2} \right\} \qquad (5)$$

Under constant load :

$$T = \frac{E}{\sigma_0^2} \left\{ \frac{dJ}{da} \right\} \sigma = \text{constant} = \frac{\pi \left\{ \frac{\sigma}{\sigma_0} \right\}^2}{1 - \frac{1}{\beta} \left\{ \frac{\sigma}{\sigma_0} \right\}^2} \qquad (6)$$

$$\text{and} \quad \frac{J}{T} = \frac{\sigma_0^2 a}{E} \qquad (7)$$

The quantity $\pi \left\{ \frac{\sigma}{\sigma_0} \right\}^2 . \left\{ 1 - \frac{1}{\beta} \left\{ \frac{\sigma}{\sigma_0} \right\}^2 \right\}^{-1}$ will be called "stress factor". As the amount of plasticity becomes larger and larger, Irwin's correction is not sufficient any more. Other elastic plastic models have to be used to derive new expressions of the stress factor (Strip yield model, Ramberg-Osgood model, Power Hardening model...). This has been done by P. Paris [9] and leads to several formulas for the stress factor, each of them having its own validity range. Putting together the different corresponding curves yields a unique curve giving the stress factor as a function of $\frac{\sigma}{\sigma_0}$ (figure 2).

It must be noticed that in all cases, $\frac{J}{T}$ is found equal to $\frac{\sigma_0^2 a}{E}$.

5.2 - Shell corrections

To take in account the shell effect, a shape factor Y can be introduced in the K expression.

$$K = Y \sigma \sqrt{\pi a}$$

For cylinders, this shape factor is a function of the parameter $\lambda = \frac{a}{\sqrt{Rt}}$ where a is half the crack length, R the radius of the cylinder and t its thickness.

In L.E.F.M. and after plastic zone correction, the J expression becomes :

$$J = \frac{\sigma_o^2\, a}{E} \text{ (stress factor)} \cdot Y^2 \qquad (8)$$

Going through some algebra and calling Y' the derivative of the shape factor Y with respect to λ, the tearing modulus can be written as :

$$T = \left(\text{stress factor} \right) \cdot (Y^2 + 2\,\lambda\, YY') \qquad (9)$$

and $\frac{J}{T}$ becomes :

$$\frac{J}{T} = \frac{\sigma_o^2\, a}{E} \cdot \frac{1}{1 + 2\lambda \frac{Y'}{Y}} \qquad (10)$$

Different shell correction factors have been evaluated by P. Paris. Those for longitudinal cracks in a cylinder are plotted against λ in figure 3 and 4.

6 - Stability analysis of a completely cracked nozzle

6.1 - Modelisation

The starting point of the analysis is a typical PWR reactor outlet nozzle which would be completely cracked as if it was split in two parts. The question is to determine wether there is a possibility of unstable propagation by ductile tearing in the vessel wall or in the primary pipe wall under design pressure. For this analysis, the cracked nozzle is modeled as a through-crack in the vessel, then in the pipe, the size of this crack being the largest dimension of the nozzle (fig.5) . Flow stresses are taken equal to 500 MPa for nozzle and vessel steel and 300 MPa for the pipe steel at design temperature. Young's modulus is 2×10^5 MPa and the design pressure is 17.2 MPa. Materials are indicated on fig.5.

The crack is supposed to be a longitudinal crack. It can be shown that it is the worst case.

6.2. - Propagation in the vessel wall

The crack length is taken equal to the diameter of the nozzle-vessel welded joint.

$$a = 650 \text{ mm}$$

At the design pressure : $\sigma_{hoop} \simeq 150$ MPa

and $\frac{\sigma}{\sigma_o} \simeq 0.3$

This ratio value justifies that the stress factor expression coming from the plastic zone correction model can be used (see fig.2) . In this stress factor formulation, β is taken equal to 2 as the worst case value (plane stress condition). Using the previously mentionned figures and formulas yields $J_{appl} = 517$ kJ/m^2 and $T_{appl} = 1.31$.

In the published litterature, the J_{1C} value for this kind of steel is found around

250 kJ/m^2. In an important but yet unpublished study of cracks in nozzles, a conservative value of 120 kJ/m^2 has been recently used. So that some propagation of the crack by ductile tearing is likely to occur in the vessel wall. Published J - Δa curves show that this propagation should be less than 10 mm. For most steels used in the nuclear industry, the material tearing modulus T_{mat} is likely to be somewhere between 100 and 300 and almost certainly above 20. In the previously mentionned study, T_{mat} has been conservatively taken equal to 80 for crack growth up to 1.5 mm and equal to 26 for crack growth up to 7 mm. According to the tearing modulus criteria, there should be no tearing instability of the crack in the vessel wall. The validity condition $\omega \gg 1$ (see par. 4) is met since ω is equal to about 50 times the ligament.

One point may be worth to be noticed. If we take $\sigma_o = 1/2 \ (\sigma_y + \sigma_u)$ with ASME code values for σ_y and σ_u we get $\bar{\sigma}_o \simeq 420$ MPa at design temperature. If we go through all the procedure again, we find $J_{appl} = 524$ kJ/m^2, $T_{appl} = 1.9$ and the lower bound for T_{mat} goes from 26 to 37 so that the predicted behaviour and the margin of safety are not significantly changed.

6.3 - Propagation in the primary pipe

Half the crack length is taken equal to the nozzle length (900 mm). Hoop stress is 86.7 MPa and $\frac{\bar{\sigma}}{\sigma_o}$ is 0.289. Once again, the plastic zone correction model is used and the stress factor is 0.274. Going through all the calculation yields $J_{appl} = 282.9$ kJ/m^2 and $T_{appl} \simeq 20$. Although there is a large discrepancy in the publihed data for J_{1c} and T_{mat} of this material, it can be said that 80 kJ/m^2 for J_{1c} and 150 for T_{mat} are likely to be quite conservative values. So that, according to the tearing modulus criteria, there should be no unstable propagation by ductile tearing in the pipe wall. Once again, it can be noticed that changing $\bar{\sigma}_o$ (between 200 and 400 MPa) leads to about the same predicted behaviour.

7 - Discussion and conclusion

The application of the theory to the very simple case of through cracks under design conditions seems quite straightforward. Difficulties come out when material characteristics have to be introduced. There is generally an important scattering of experimental points when plotting a J-Δa curve so that not only the value but even the existence of a constant slope in some Δa range does not appear very clearly (standard test techniques are yet to be defined). Then it should be shown that the J-Δa curves obtained from testing specimens are applicable to the real structure. However, the lack of definitive tests does not make the approach unusable as pointed out by Milne if a proper sensitivity analysis can be performed showing that conclusions are relatively unaffected by change in poorly defined data [10].

Other difficulties will arise when applying the theory to other operating conditions. For example, under faulted conditions, the temperature and stress distribution can be severe enough for cleavage fracture to occur either during the development of slow growth or after the onset of instability. Because of the intricate influence of residual stresses, work-hardening, change in toughness, strain rate etc... it is not yet feasible to predict such a change in fracture mode [6,11] .

An other point of concern is the extension of the theory itself in order to be able to take in account other parameters like residual stresses and to handle more general problems (three-dimensionnal, mixed modes etc...).

8 - Acknowledgments

The authors wish to thank Pr. P. Paris for suggesting the idea of the present application and for providing much of the information on which it is based. The first work on this analysis method was done by Pr. P. Paris for Mr. Giles Argy of Technigaz (Maurepas) and later for Dr. R. Johnson of USNRC. The authors wish also to thank Electricité de France staff members, and particularly Mr. Noël, for their assistance and support.

REFERENCES

1 HUTCHINSON, J. Mech. Phys. Solids, 16, pp. 13 - 31 (1968)
2 RICE, ROSENGREN, J. Mech. Phys. Solids, 16, pp. 1 - 12 (1968)
3 GOLDMAN, HUTCHINSON, Int. J. Solids Structures, 11, pp. 575 - 591 (1975)
4 LABBENS, "Introduction à la mécanique de la rupture", Ed. Pluralis (1980)
5 PARIS et al., Washington University, US - NUREG - 0311 (1977)
6 TURNER, "Slow stable crack growth and unstable fracture", Ispra (1979)
7 CSNI Specialist Meeting on Plastic Tearing Instability, St Louis (1979)
8 MILNE, CHELL, 4th Int'l Conference Pressure Vessel Technology, London (1980)
9 PARIS, Private communication
10 MILNE, HARRISON, DOWLING, 4th Int'l Conference Pressure Vessel Technology, London (1980)
11 TURNER, "A J- based design curve", 2d Adv. Seminar on Fract Mech., Ispra (1979)

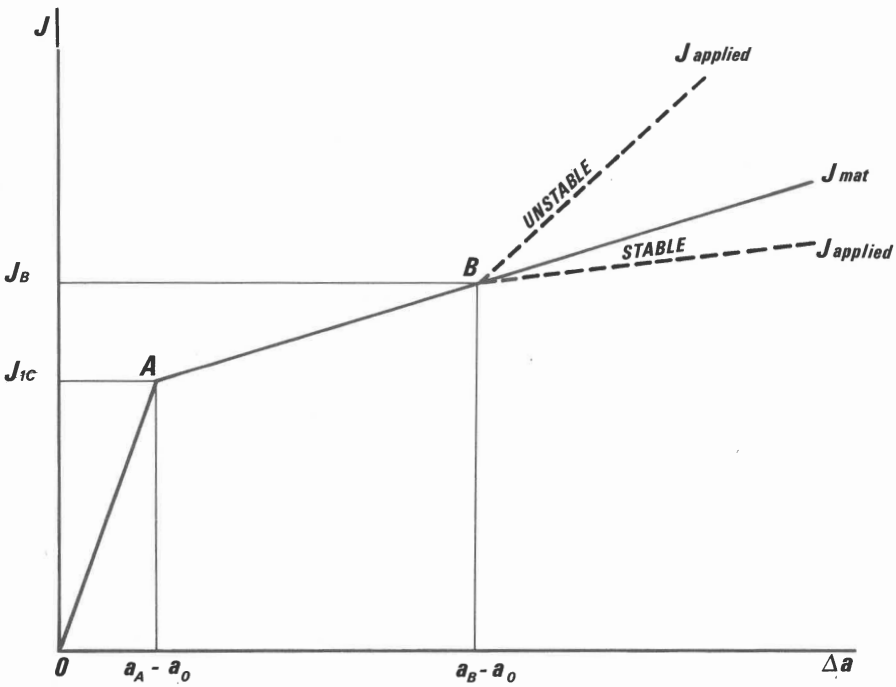

fig. 1 Typical J - Δa curve

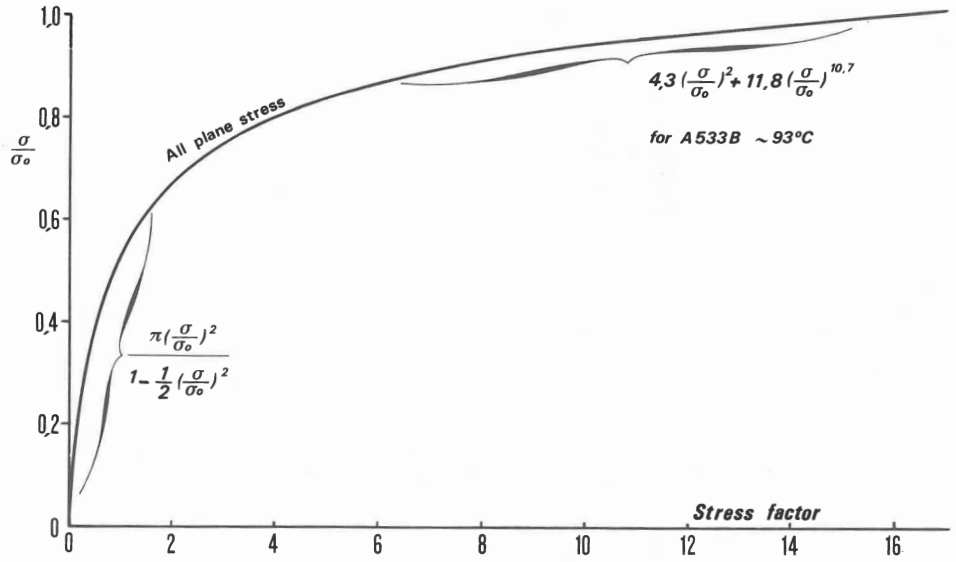

fig. 2 Stress factor for J and T calculation

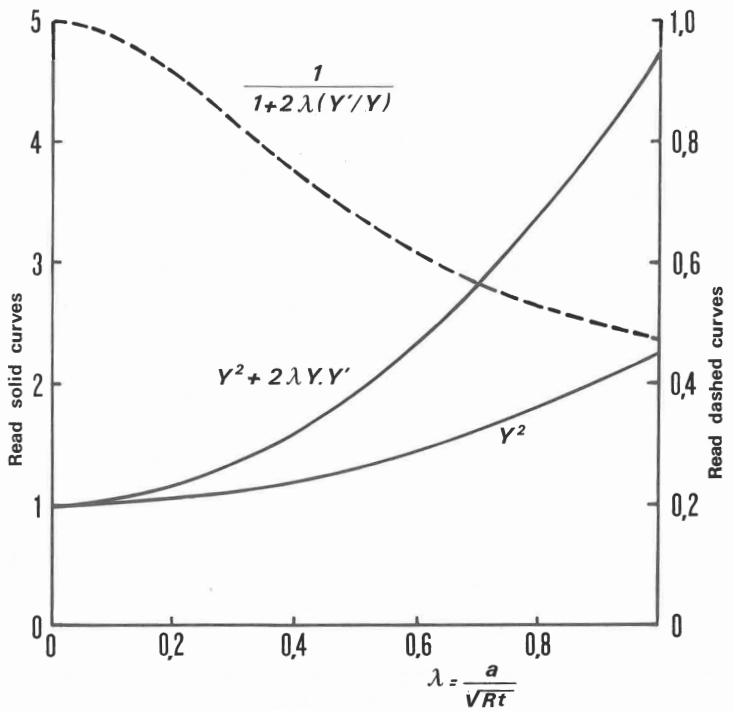

fig. 3 Shell correction for longitudinal cracks in cylinder (for $\lambda \leqslant 1$)

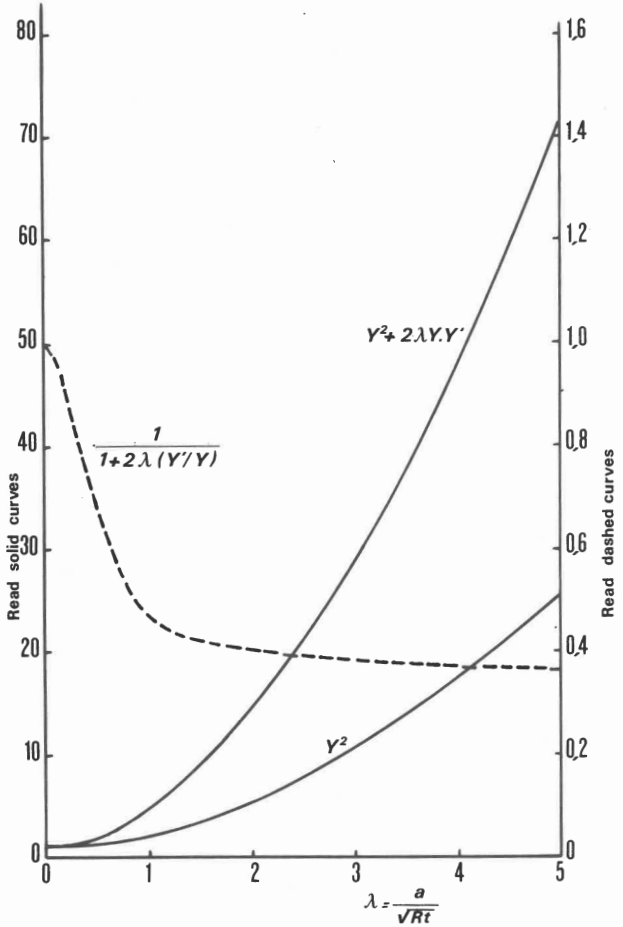

fig. 4 Shell correction for longitudinal cracks in cylinder ($1 < \lambda < 5$)

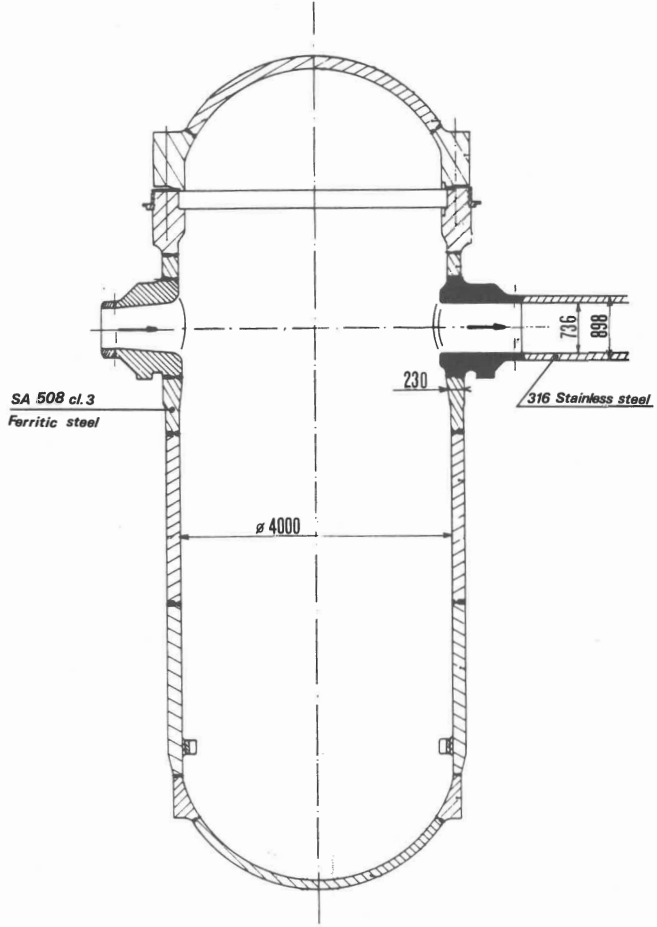

fig. 5 PWR vessel with a completely cracked nozzle