

SOME CONCEPTS OF NONLINEAR THERMO-VISCOELASTICITY

Z. BYCHAWSKI, A. FOX,

Technical University of Krakow, Krakow, Poland

ABSTRACT

The subject of the present paper is the formulation of a constitutive equation for structural nonlinear viscoelastic material in which temperature appears as physical variable. Such conditions we find in materials which find application in reactor technology and are exposed to mechanical and thermal loading.

The basic principles of derivation such a constitutive equation are founded on the concepts given by Bychawski and Fox [1], [2], [3]. In addition to the complex nonlinear viscoelastic behavior the influence of a temperature field depending on time is introduced.

It is found that in order to satisfy the condition of time compatibility some restrictions should be put on the separated physical members of differential constitutive form. These are crossing relations which if satisfied enable us to consider the last form to be a total differential. Thus, a complex constitutive equation is found together with additional constitutive relations.

1. GENERAL CONCEPTS

In considering thermal effects which influence the behavior of nonlinear viscoelastic bodies we shall point out applicability of the generalized superposition principle formulated by Bychawski and Fox [2]. Thus, the expression for total strain tensor we write in the form

$$e_{ij}(t) = e_{ij}(t_0) + \int_{-\infty}^t \Theta(\tau - t_0) de_{ij}(\tau) = \int_{-\infty}^t d[\Theta(\tau - t_0) e_{ij}(\tau)] \quad (1)$$

where Θ is the Heaviside distribution defined as

$$\Theta(t - t_0) = \begin{cases} 1 & , \quad t > t_0 \\ 0 & , \quad t \leq t_0 \end{cases} \quad (2)$$

As it can be seen from eq. (1), both the forms of strain tensor are equivalent.

It should be mentioned that the integral form of eq. (1) describing the

mode of superposition in the finite time interval $\tau \in (t_0, t]$ is a consequence of a more essential differential form of superposing strains in every arbitrary small time interval . The latter may be written

$$\theta(\tau - t_0) de_{ij} = \theta(\tau - t_0) \dot{\Phi}(\tau) d\tau, \quad \tau \in (-\infty, t], \quad (3)$$

what from the mathematical point of view is equivalent to eq.(1) in accordance with the assumed class of integrability . It is clear , that this class must be related to the taken choice of measure . On the other hand , such a form as eq.(3) expressing time superposing implies that even very complex behavior of a real viscoelastic body can be characterized by a unique compact time function $\dot{\Phi}$. Thus , constitutive equation of a nonlinear thermo-viscoelastic material based on the principle of eq.(3) should also have a complex form expressed through a compact function of time . The last conclusion seems to be contrary to the usually assumed differential forms of superposing of qualitatively different physical effects in additive manner . This is done without any special care about the relations which may exist between separated quantities . In accordance with our opinion the last procedure of separating with respect to "physical superposition members" may be justified for linear materials only . In the case of nonlinear behavior one cannot neglect the existing connections of physical effects in time which one usually separates because of expediency of considerations . The assumption of time complexity of constitutive equations does not exclude the possibility of such approach even in nonlinear cases . However , it is then necessary to find out some restrictions to be put on the separated physical members in order to satisfy the condition of time compatibility of constitutive equation of a nonlinear body .

In the linear case the separated physical members corresponding to the assumed separated physical variables one can consider with justification as independent from each other in time . This must be understood in certain specific sense of "additive time compatibility" . As we shall see later , some mathematical relations, which should be satisfied with respect to the separated physical variables in nonlinear case and connected with time links between those variables, are for a linear body identically satisfied . Therefore, justification of this fact is founded on mathematical basis as result of understanding the assumed notion of time compatibility and will be pointed out later .

It should be noticed that the customary procedure and reasoning widely applied which account new additional and qualitatively different terms in the same differential form of superposed constitutive equation is regarded here critically . Since it is not possible to introduce into consideration more independent variables in an arbitrary manner , if constitutive law is considered as complex , the complexity of law in nonlinear case is expressed through the fact that independent causes , even those separated mentally , give a "mixed" effect . Therefore , it is not possible to evaluate particular shares of effects done by acting causes . Here , we have in mind total

strain and total stress states , for example , measured locally at certain point of the body .

Finally , we should keep in mind that from the said above does not follow that the mathematical form of constitutive equation cannot be additive . On the contrary , such form is fully admissible , however, in such cases is impossible to point out distinctly which from additive terms of this form corresponds to the determined type of reaction related with specific physical variable . Such a reciprocal correspondence takes place in a linear body only , because only then is possible to separate shares of specific variables . We shall point out this problem on a simple example of a linear hereditary viscoelastic body described by the constitutive equation

$$e_{ij}(t) = \frac{s_{ij}(t)}{2G} + \int_{t_0}^t s_{ij}(\tau) K(t,\tau) d\tau , \quad (4)$$

valid for $\tau > t_0$, where s_{ij} denotes stress deviator and K is kernel of integral equation .

To the equation (4) corresponds a differential superpositional form

$$\alpha de_{ij}(\tau) = \alpha \left[\frac{1}{2G} ds_{ij}(\tau) + s_{ij}(\tau) K(t,\tau) d\tau \right] , \quad \alpha = \theta(t-\tau) \theta(\tau-t_0) , \quad (5)$$

where α defines superposition interval and θ is the Heaviside distribution . The last form follows from superposing performed in the time interval $[\tau, \tau+d\tau]$ for every instant $\tau \in (-\infty, t_0]$ (see Bychawski and Fox [1]) .

It is seen that eq.(5) is of the type of eq.(3) , i.e. , is a total differential with respect to the variable τ . On the other hand , considering e_{ij} as a function of physical variables s_{ij} and τ , i.e. ,

$$e_{ij} = e_{ij}[\tau, s_{ij}(\tau)] , \quad (6)$$

and

$$\alpha de_{ij} = \alpha \left[\partial_{\tau} e_{ij} d\tau + \partial_{s_{mn}} e_{ij} ds_{mn} \right] , \quad (7)$$

where the symbols ∂ denote partial differentiation with respect to variable indicated by the index , we find comparing eq.(5) and eq.(7) that

$$\alpha \partial_{\tau} e_{ij} = \alpha s_{ij} K(t,\tau) , \quad (8)$$

$$\alpha \partial_{s_{mn}} e_{ij} = \alpha \frac{1}{2G} \delta_{im} \delta_{jn} , \quad (9)$$

where δ_{ij} is the Kronecker tensor .

Having in mind that eq.(7) should also be a total differential , we can put the following conditions to be satisfied

$$\alpha \partial_{s_{mn}\tau}^2 e_{ij} = \alpha \partial_{\tau s_{mn}}^2 e_{ij} , \quad (10)$$

which are considered as distributional crossing relations . The introduced time interval factor α appearing in the above equations is a result of necessity of determining final observation time point simultaneously with dif-

ferential superposition (see Bychawski and Fox [2]). This is an important factor as regards proper interpretation of superposition in time. As it will be seen, significance and manner of calculation of eq.(10) is somewhat different from the known classical relations. The difference consists in performing differentiation of eq.(9) with respect to time which contains distributional factor α . Carrying out differentiation we have

$$\alpha \partial_{\tau s_{mn}}^2 e_{ij} = \partial_{\tau} (\alpha \partial_{s_{mn}} e_{ij}) - \partial_{\tau} \alpha \partial_{s_{mn}} e_{ij} \quad , \quad (11)$$

where

$$\partial_{\tau} \alpha = -\delta(t-\tau) \theta(\tau-t_0) + \theta(t-\tau) \delta(\tau-t_0) = \beta \quad . \quad (12)$$

It is seen from eq. (11) that in order to evaluate left hand side, it is necessary to know in advance the form of eq.(4). However, this is not in contradiction with the statement of our problem since the integral form of eq.(4) can always be found on the basis of eq.(3). This follows from the introduced notion of complexity.

Returning to our example we write in accordance with the described procedure

$$\alpha \partial_{s_{mn}\tau}^2 e_{ij} = \alpha \delta_{im} \delta_{jn} K(t,\tau) \quad , \quad (13)$$

$$\alpha \partial_{\tau s_{mn}}^2 e_{ij} = -\beta \delta_{im} \delta_{jn} \int_{t_0}^{\tau} K(\tau,\xi) d\xi \quad , \quad (14)$$

or, by satisfying eq. (10) we find

$$\begin{aligned} \alpha \delta_{im} \delta_{jn} K(t,\tau) &= \delta(t-\tau) \theta(\tau-t_0) \delta_{im} \delta_{jn} \int_{t_0}^{\tau} K(\tau,\xi) d\xi \quad (15) \\ &\quad - \theta(t-\tau) \delta(\tau-t_0) \int_{t_0}^{\tau} K(\tau,\xi) d\xi \quad . \end{aligned}$$

By integrating the above condition with respect to τ taken from the unlimited time interval containing observation interval $[t_0, t]$ we obtain the following identity

$$\delta_{im} \delta_{jn} \int_{t_0}^t K(t,\tau) d\tau = \delta_{im} \delta_{jn} \int_{t_0}^t K(t,\xi) d\xi - \theta(t-t_0) \int_{t_0}^{t_0} K(t_0,\xi) d\xi \quad . \quad (16)$$

Thus, as it can be seen from the last result in the case of a linear body the crossing relations reduce to identity. This way, it is clear that eq.(4) represents a unique form of constitutive equation for a linear body the additive shape of which is admissible. It corresponds to "unmixed" effects.

As it will be shown below, such identity of crossing relations does not take place in general, even in the case of additive forms assumed for nonli-

near bodies . As we shall point out later, for the last class of bodies the conditions following from crossing relations should be considered together with the integrated form of eq.(3) as a set of constitutive equations . Such a set should appear always , if an additive form of constitutive equation is assumed . It secures then a complex feature of such a law , i.e. , the physical "mixing" of effects . In the case of an originally differential complex constitutive equation of the form (3) the problem of crossing relations does not appear .

2. NONLINEAR THERMOVISCOELASTIC BODY

In connection with the said above we assume a differential form for a nonlinear viscoelastic material (see Bychawski and Fox [2]) in which we include thermal term

$$\alpha de_{ij} = \alpha [d(s_{ij}\phi) - s_{ij}dH + \chi_{ij}dT] \quad . \quad (17)$$

Here , T denotes time-dependent temperature , s_{ij} stress deviator ,

$$s_{mn} = s_{mn}[\tau, T(\tau)] , \quad (18)$$

are components of this deviator at certain spatial point of the body considered ,

$$\phi = \phi\{\tau, s_{mn}[\tau, T(\tau)] , T(\tau)\} , \quad (19a)$$

is a function characterizing instantaneous effects , and

$$H = H\{t, \tau , s_{mn}[\tau, T(\tau)], T(\tau)\} , \quad (19b)$$

is generalized creep function (see Bychawski and Fox [2]), describing time effects . The last term introduced in eq.(17) denotes tensor of "thermal dilatation

$$\chi_{ij} = \chi_{ij}\{\tau, s_{mn}[\tau, T(\tau)], T(\tau)\} \quad . \quad (20)$$

All the above functions should be considered in the interval $\tau \in (t_0, t]$ as distribution functions . Therefore , it is clear that eq.(17) is in a close correspondence with the form of eq.(3) , as is seen from eq.(18) - eq.(20), and can be reduced to that form easily .

We treat in our further considerations τ, \bar{s}_{mn}, T as physical variables . Here , τ is current time point taken from observation interval , \bar{s}_{mn} stress share as a result of external loading exclusively , and , therefore , temperature independent , and T is time programmed temperature . Thus , s_{mn} should be considered as stress state components at certain point of the body which result from external loading and thermal effects included . These are superposed according to the nature of the body .

On the basis of the assumptions made about physical variables , eq.(17)

can be formed in such a way as to be expressed explicitly through differentials of physical variables . Doing so , we obtain

$$\alpha de_{ij} = \alpha \left[\partial_{\tau} e_{ij} d\tau + \partial_{\bar{s}_{mn}} e_{ij} d\bar{s}_{mn} + \partial_T e_{ij} dT \right] , \quad (21)$$

where

$$\alpha \partial_{\tau} e_{ij} = \alpha s_{ij} \partial_{\tau} (\phi - H) , \quad (22)$$

$$\alpha \partial_{\bar{s}_{mn}} e_{ij} = \alpha \left[\phi \delta_{im} \delta_{jn} + s_{ij} \partial_T (\phi - H) \right] , \quad (23)$$

$$\alpha \partial_T e_{ij} = \alpha \left[\phi \partial_T s_{ij} + s_{ij} \partial_T (\phi - H) + s_{ij} \partial_T s_{kl} \partial_{s_{kl}} (\phi - H) + \chi_{ij} \right] , \quad (24)$$

which are partial distributional derivatives in the interval considered .

In such a way , as is seen from eq.(21) , although we admit constitutive integral form to be additive , we secure complexity by imposing of crossing relations . The last follow from eq.(22) - eq.(24) .

We notice that eq.(23) has a different , compared with eq.(21) and eq.(23) , character . It is a tensorial equality . It should be also noticed that all the mentioned distributional equalities have a local time significance , but contrary to classical crossing relations , the distributional crossing relations following from those equalities do not show a local significance . It must be related to the fact that they contain integrals taken over observation interval up to the time point considered . This follows from necessity to secure complexity in the whole time interval of process duration and due hereditary properties of the body assumed . It also means that in order to write down a constitutive relation for such material at a given instant , is necessary to possess information about the behavior of the body up to this instant (non-local information , i.e. , integral) . Evidently , it happens in our case of distributional crossing relations . Therefore , we fully justify the presentation of crossing relations in integrated form which we shall show and discuss in the following .

3. DISTRIBUTIONAL CROSSING RELATIONS FOR THERMOVISCOELASTIC BODY

In calculating crossing relations following from eq. (22)- eq.(24) according to the principle of eq.(4) , we meet the necessity to possess the integral form of constitutive equation corresponding to eq.(17) integrated up to the current time point and also its distributional derivatives with respect to \bar{s}_{mn} and T .

Thus , performing integration of eq.(17) we find

$$e_{ij}(\tau) = \phi s_{ij} - \int_{t_0}^{\tau} s_{ij} d_{\xi} H d\xi + \int_{t_0}^{\tau} \chi_{ij} \partial_{\xi} T d\xi . \quad (25)$$

The above equation furnishes the information about the history of the body up to instant τ .

On the basis of eq.(25) we obtain the derivative with respect to \bar{s}_{mn} and T , respectively

$$\begin{aligned} \frac{\partial}{\partial \bar{s}_{mn}} e_{ij}(\tau) = & \delta_{im} \delta_{jn} \phi + s_{ij} \partial_{\bar{s}_{mn}} \phi + \delta_{im} \delta_{jn} H \Big|_{\xi=t_0} - \int_{t_0}^{\tau} s_{ij} d_{\xi} (\partial_{\bar{s}_{mn}} H) d\xi + \\ & + \int_{t_0}^{\tau} \partial_{\bar{s}_{mn}} \chi_{ij} \partial_{\xi} T d\xi, \end{aligned} \quad (26)$$

$$\begin{aligned} \partial_T e_{ij}(\tau) = & \partial_T s_{ij} \phi + s_{ij} \partial_T \phi - \int_{t_0}^{\tau} \partial_T s_{ij} d_{\xi} H d\xi - \int_{t_0}^{\tau} s_{ij} d_{\xi} (\partial_T H) d\xi + \\ & + \int_{t_0}^{\tau} \partial_{\xi} T \partial_T \chi_{ij} d\xi. \end{aligned} \quad (27)$$

We notice that eq.(26) gives us information at instant τ about the behavior of the body related to history of stress state realized by external loading in the interval $[t_0, \tau]$. On the other hand, eq.(27) furnishes a similar information related with history of temperature changes.

The crossing relations we look for are put in the following order

$$I. \alpha \partial_{\tau} \frac{\partial}{\partial \bar{s}_{mn}} e_{ij} = \alpha \frac{\partial}{\partial \bar{s}_{mn}} \partial_{\tau} e_{ij}, \quad (\text{eq. 22 and eq. 23}), \quad (28)$$

$$II. \alpha \partial_{\tau} \partial_T e_{ij} = \alpha \partial_T \partial_{\tau} e_{ij}, \quad (\text{eq. 23 and eq. 24}), \quad (29)$$

$$III. \alpha \frac{\partial}{\partial \bar{s}_{mn}} \partial_T e_{ij} = \alpha \partial_T \frac{\partial}{\partial \bar{s}_{mn}} e_{ij}, \quad (\text{eq. 24 and eq. 25}). \quad (30)$$

Performing the indicated in eqs.(28),(29) and (30), operations and taking into account eqs.(26) and (27) we find, successively,

$$\begin{aligned} I. \alpha \delta_{im} \delta_{jn} \partial_{\tau} H = & \beta [s_{ij} \partial_{\bar{s}_{mn}} H + \delta_{im} \delta_{jn} H \Big|_{\xi=t_0} - \int_{t_0}^{\tau} s_{ij} d_{\xi} (\partial_{\bar{s}_{mn}} H) d\xi + \\ & + \int_{t_0}^{\tau} \partial_{\bar{s}_{mn}} \chi_{ij} \partial_{\xi} T d\xi], \end{aligned} \quad (31)$$

$$II. -\alpha \partial_T s_{ij} \partial_{\tau} H = \alpha [s_{ij} \partial_T s_{kl} \partial_{\tau} \partial_{\bar{s}_{kl}} (\phi - H) + \partial_{\tau} \chi_{ij}] + \beta [-s_{ij} \partial_T H + \quad (32)$$

$$+ s_{ij} \partial_T s_{kl} \partial_{s_{kl}} (\phi - H) + \chi_{ij} + \int_{t_0}^{\tau} \partial_T s_{ij} d_{\xi} H d\xi + \int_{t_0}^{\tau} s_{ij} d_{\xi} (\partial_T H) d\xi - \int_{t_0}^{\tau} \partial_{\xi} T \partial_T \chi_{ij} d\xi,$$

$$\text{III. } \alpha \partial_T s_{kl} \left[\int_{im} \int_{jn} \partial_{s_{kl}} H - \int_{ik} \int_{jl} \partial_{s_{mn}} H \right] + \int_{im} \int_{jn} \partial_T H = \quad (33)$$

$$= \int_{im} \int_{jn} \partial_T s_{kl} \partial_{s_{kl}} \phi + s_{ij} \partial_T s_{kl} \partial_{s_{mn}} (\phi - H) + \partial_{s_{mn}} \chi_{ij} .$$

The above distributional conditions relate all physical characteristics and , according to our opinion , they should constitute an additional set of constitutive relations . Thus , the complexity principle concerned with the additive form of constitutive law given by eq.(25) requires that not only the latter but a complete set , i.e. , eqs.(25), (31), (32) and (33) must be regarded as constitutive system of equations in the sense of this principle .

The equations (31), (32), (33) contain the Dirac distribution and , therefore , it is convenient to interpret the results on the integrated form of these relations . This should be done in the interval including observation interval $(t_0, t]$. In performing integration we make use of an additional condition put on the generalized creep function (see Bychawski and Fox [1]) given by eq.(19b)

$$H \Big|_{\tau=t} = 0 . \quad (34)$$

Carrying out integration in the said interval of eqs.(31), (32) and (33), after some transformations we obtain

$$\int_{im} \int_{jn} \left[H \Big|_{\tau=t_0} + \int_{t_0}^{\tau} \partial_{\xi} H d\xi \right] = - \int_{t_0}^{\tau} d_{\xi} s_{ij} \partial_{s_{mn}} H d\xi - \int_{t_0}^{\tau} \partial_{\xi} T \partial_{s_{mn}} \chi_{ij} d\xi , \quad (35)$$

$$\int_{t_0}^{\tau} \partial_T s_{ij} \partial_{\xi} H d\xi = \int_{t_0}^{\tau} \partial_T s_{ij} d_{\xi} H d\xi + \int_{t_0}^{\tau} s_{ij} \partial_T s_{kl} d_{\xi} s_{mn} \partial_{s_{kl}} \partial_{s_{mn}} H d\xi + \quad (36)$$

$$+ \int_{t_0}^{\tau} \partial_{s_{kl}} (\phi - H) d_{\xi} (s_{ij} \partial_T s_{kl}) + \int_{t_0}^{\tau} s_{ij} \partial_T s_{kl} \partial_{\xi} T \partial_{s_{kl}} \partial_T H d\xi + \int_{t_0}^{\tau} d_{\xi} s_{mn} \partial_{s_{mn}} \chi_{ij} d\xi .$$

The remaining relation given by eq.(33) can be discussed in differential form . This is due to the fact that it does not include any time derivative. However , it should be mentioned that the last relation is considered in observation time interval , since it contains time factor .

We notice that integrated crossing relations of eqs.(35) and (36) and

differential form of eq.(33) as well , are tensorial relations of the orders four , two and four , respectively . From this fact it follows the necessity of inspecting and discussing them from the point of view of tensor calculus.

4. SOME FINAL REMARKS AND CONCLUSIONS

As we can observe by inspecting eqs.(33), (35) and (36) their form is rather complicated . It is obvious that is a result of general assumptions made in formulation of the fundamental differential form given by eq.(17) . On the other hand , the said relations are potentially a strong source of information as regards mutual connections existing between physical functions characterizing materials . Usually , these are considered in our mental inspection as independent quantities . The number of connections is conditioned by the mathematical form assumed of eq.(17) and increases as new terms appear in additive manner . According to our opinion this problem has not yet been revealed and discussed in the literature .

We lay stress on the above remarks and conclusions , because the given additional relations should assume , according to our opinion , the rank of complementary constitutive conditions if ,for conveniency of inspection of material properties,an additive splitting of qualitatively different effects is done in fundamental constitutive equation . It would be rather indicated, if possible , to avoid such formulation . More profound conclusions following from the theory presented here are actually the subject of our further and careful investigation . Therefore , we restrict ourselves only to some suggestions which may be are of direct interest .

In particular , it is possible to evaluate from eqs.(33), (35) and (36) the dependence of dilatation tensor χ_{ij} on the remaining characteristics , i.e. , in our case ϕ and H and also physical variables . We shall clarify this point below . On the other hand , we can eliminate from eqs.(35) and (36) dilatation tensor χ_{ij} by means of eq.(33). Thus , we are able to obtain independent set of integral relations with respect to ϕ and H . The solution of this set will then give us the form of dependence of those functions on physical variables which is admissible from the point of view of performed splitting into additive parts in eq.(17) (as we shall show on an example,the said set will be reduced to one equation only on temperature level $T = \text{const.}$). It indicates that ϕ and H are not so arbitrary, as it may be generally guessed or is even sometimes the object of common conviction .

As regards the problem of thermal effects in nonlinear viscoelastic bodies , we introduce temperature into characteristics ϕ and H and , separately , we include into differential form of eq.(17) an additive thermal effect as deformation contribution caused by increase of temperature . Here , we take into account thermal "input" state of the material . It simply means that we consider actually materials starting to deform on different temperature levels as physically different bodies . Therefore , the characteristics ϕ and H must differ from each other on different isothermal levels, respectively. In differential form of constitutive equation we separate thermal term

because of the reasons which follow . Temperature as a measure of internal energy changes at certain point of the body is the cause of thermal deformation and thermal stresses independent of programmed external loadings . If there are external loadings and the influence of temperature changes is taken into account , a mixed stress state is realized . The latter is considered as basic in our investigations as expressed by eq.(18) . It is clear that in no case of simultaneous changes of external loadings and temperature is possible to separate stress state into thermal stress state and that dependent on loadings in an additive form . However , it is possible to realize a pure thermal stress state and pure mechanical stress state , respectively . The latter takes place at a fixed temperature level only . If it happens , then for conveniency of considerations we can assume a family of generalized creep functions in the following form

$$H\{t, \tau, s_{ij}[\tau, T(\tau)], T(\tau)\} = H[t, \tau, s_{ij}(\tau)] \Gamma[T(\tau)] . \quad (37)$$

The form of eq.(37) is related to the remark given above about variability of H at different isothermal levels . Indeed , for fixed $T_1 = T_2$, on the basis of eq.(37) , we obtain two different generalized creep functions at temperature levels T_1 and T_2 , respectively . Simultaneously , the form assumed of eq. 37 satisfies the condition (see eq.(34)) ,

$$H[t, \tau, s_{ij}(\tau, T), T] \Big|_{\tau=t} = 0 , \quad (38)$$

where T is an arbitrary fixed temperature or , in general , for the case of variable temperature $T = T(\tau)$ the condition is satisfied for its value at instant $\tau = t$.

As it is seen from eq.(37) , the assumption made corresponds to introducing a stress deviator of the form

$$s_{ij} = \bar{s}_{ij} \gamma [T(\tau)] . \quad (39)$$

The latter indicates that stress deviators of the mixed states which appear at the given point of the body are here amplified with respect to programmed stress deviators . This is done by a temperature dependent factor γ which accounts the fact that stress states at different isothermal levels differ from each other , i.e. , the increments of mixed states are

$$ds_{ij} \Big|_{T=T_1} = ds_{ij} \Big|_{T=T_2} . \quad (40)$$

Further , we introduce the right hand sides of eqs.(37) and (39) into eq.(33) , and then we put temperature constant . Thus , taking into consideration that in such a case

$$\partial_T \gamma \Big|_{T=\text{const}} = 0 , \quad \partial_T \Gamma \Big|_{T=\text{const}} = 0 , \quad (41)$$

we find from eq.(33) a simple result , namely ,

$$\alpha \partial_{s_{mn}} \chi_{ij} = 0 , \quad (42)$$

i.e. , that for every instant $\tau \in (t_0, t]$ dilatation tensor does not depend on stress state .

Simultaneously , both the remaining relations of eqs.(35) and (36) are identically satisfied for the assumed constant temperature . Thus , if the process considered occurs in the presence of isothermal conditions , we are not able to observe the variability of dilatation tensor with the changes of stress states . However , we cannot conclude on this basis that , in general dilatation tensor is not a function of stress state . If there happens that dilatation tensor is actually independent on stress state, but may be a function of time and temperature , then eqs.(33), (35) and (36) do not include χ_{ij} . The latter set reduces to a simpler form from which one is able to take conclusions about the links existing between the remaining physical characteristics . It also furnishes the information about their dependence on stress state .

On the other hand , assume , for example , that we take into consideration the representation given by the left hand sides of eqs.(37) and (39) in the discussed relations . Then on an isothermal level T_0 we obtain from eq.(36)

$$\int_{t_0}^t d\xi s_{mn} \partial_{s_{mn}} \chi_{ij} d\xi = 0 , \quad (43)$$

or , equivalently ,

$$\chi_{ij} [t, \bar{s}_{mn}(t) \chi(T_0, T_0)] - \chi_{ij} [t_0, \bar{s}_{mn}(t_0) \chi(T_0, T_0)] = \int_{t_0}^t \partial_{\xi} \chi_{ij} d\xi . \quad (44)$$

It follows from the above relation that , if dilatation tensor χ_{ij} is not dependent explicitly on time , i.e. , the material considered is not an thermally ageing one , it does not depend on stress state , too . However , in the latter case χ_{ij} may be temperature dependent .

Further , let us remark that , in general , for a thermally ageing material the relation of eq.(35) is identically satisfied , while eq.(33) gives for $i, j = m, n$, respectively ,

$$\alpha \partial_{s_{mn}} \chi_{ij} \Big|_{T=T_0} = \alpha \delta_{im} \delta_{jn} \partial_{TH} \Big|_{T=T_0} . \quad (45)$$

The last result indicates the links between dilatation tensor components and corresponding stress deviator components .

On the other hand , in the discussed case of non-ageing material eq.(45) points out that in the presence of isothermal conditions generalized creep function H depends explicitly on temperature . Nevertheless , if such is the case , one should use in calculating deformations this form belong-

ing to the family of creep curves which corresponds to the actual isothermal level T_0 .

Finally, in general we state that additive splitting of a complex constitutive law is true only then if the physical characteristics ϕ , H and χ_{ij} are solutions of the presented distributional relations of eqs. (33), (35) and (36).

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