# EVALUATION OF INELASTIC ANALYSIS METHODS*

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## SUMMARY

In the United States, detailed inelastic analyses may be required in the design of high-temperature reactor system components to demonstrate conformance with design codes and standards. Computer programs that employ complex constitutive theories and analysis procedures are used to perform these design calculations, and questions of validation and acceptance of the analysis results are thus introduced. We may ask ourselves, "How valid are the answers?". The purpose of this presentation is to explore this question relative to interim inelastic analysis guidelines that have been recommended for use in the design of liquid-metal-cooled reactor system components.

Interim constitutive equations have been developed for describing the inelastic behaviors of types 304 and 316 stainless steel and 2¼ Cr-1 Mo steel. Although differing in detail, the recommended constitutive equations for these three materials are basically of the same form. They rest on the fundamental assumption that an increment of total strain consists of four additive contributions: thermal, elastic, plastic, and creep. The specification of the plastic and creep strains follows from the concepts of classical kinematic hardening plasticity theory and strain-hardening creep theory. These concepts are augmented, however, with ad hoc rules to account for such things as cyclic hardening and/or softening, aging, and the effects of prior inelastic straining on subsequent plastic and creep deformations.

Uniaxial and multiaxial exploratory tests were used to identify the essential inelastic behavioral features of significance, and these features, in turn, provided the basis for choosing the interim constitutive equations. Typical exploratory test results are presented to support the basic constitutive equation framework and to illustrate how the ad hoc rules were developed, evaluated, and improved.

To assess the validity of the constitutive equations and analysis methods for high-temperature design, benchmark structural tests were conducted under time-varying loadings at elevated temperatures, and the results were compared to inelastic analysis predictions. Representative test results for beam and plate structures and for a pipe subjected to repeated thermal shocks are presented and compared to predictions. Although these comparisons are generally favorable, they did identify some specific analysis shortcomings. These are discussed, and procedures for improving the predictions are presented. Finally, the effects that normal material properties variations can have on analysis predictions are discussed and shown to be significant.

In summary, each of the comparisons indicates that the basic trends and features of inelastic response can be predicted, at least for the relatively simple structures examined to date, and the absolute agreement between theory and experiment is generally good. However, there are discrepancies, and normal material variation can produce additional discrepancies. So we again ask ourselves, "Are inelastic analysis predictions valid, and what is an acceptable correlation between analytical and experimental data?".

Actually, we should ask whether or not the combination of the analysis predictions and the associated design criteria lead to an acceptable level of structural integrity. It is believed that in this context our current analysis predictions are generally valid, and a specific example is presented to illustrate this conclusion.

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## 1. INTRODUCTION

Because of the combination of relatively high operating temperatures, large temperature differentials, and good heat transfer characteristics of the sodium coolant, inelastic behavior is often a significant consideration in the design of liquid-metal-cooled fast breeder reactor (LMFBR) components. Both elastic-plastic and time-dependent creep response may be exhibited, and the ASME Code Case 1592 (Class 1 Components in Elevated Temperature Service) [1] considers these aspects of behavior in criteria for determining the acceptability of stresses and strains. However, detailed inelastic analyses may be required to demonstrate conformance with these criteria, and the code case does not specify the techniques and methods to be used in performing the analyses.

Therefore, the Oak Ridge National Laboratory (ORNL) was given the responsibility, as a part of the U.S. LMFBR program, of providing component designers with a recommended set of inelastic analysis guidelines to be used throughout a reactor project. The guidelines consist, primarily, of mathematical descriptions (constitutive equations) of the multiaxial stress-strain response of materials to time-varying load and temperature histories. In general, these descriptions were chosen on the basis of their ability to represent characteristics of inelastic behavior observed in various uniaxial and biaxial exploratory materials tests and to be compatible with current computational techniques.

Currently, interim constitutive equations have been recommended for describing the inelastic behavior of types 304 and 316 stainless steel and 2 1/4 Cr—1 Mo steel, which are the principal pressure boundary materials in U.S. LMFBR designs. The recommended relations have been incorporated into inelastic analysis computer programs, and they have been used in design analyses. However, the use of such computer programs, which employ relatively complex constitutive theories and analysis procedures, raises a question regarding the validity of the analysis results, and that question is the topic of this compact.

Questions of validation and acceptance of inelastic design analysis results involve the concepts of verification and qualification. *Verification* is a demonstration that a computer program does what it is supposed to do, regardless of whether or not the model programmed is a valid representation of any particular realistic system. *Qualification* is concerned with the use of a computer program to solve real problems. That is, does the combination of mathematical model, constitutive equations, material properties, geometric discretization, representation of the mechanical and thermal loading histories, and boundary conditions, all consistent with the program limitations, give an acceptable solution to the physical problem?

Verification and qualification can best be carried out by performing benchmark calculations; that is, by solving a number of carefully chosen problems with known solutions. Verification can often be accomplished by comparison with relatively simple, but carefully chosen, analytical solutions, and exact correlations can be expected. Qualification, on the other hand, must be by comparison with experimental data, and, in the inelastic case, exact correlations cannot be expected.

In the evaluation of inelastic analysis methods, we are concerned with the question of qualification, and this, in turn, requires that we have inelastic structural test data available for benchmark calculations. As a result of this recognized need, a major portion of the ORNL program to develop high-temperature structural design methods has been devoted to the generation of high-temperature, inelastic structural test data on a variety of structural geometries ranging from the simple to the complex. A number of representative inelastic structural test results from this series of tests has been published by the ASME [2], and data from the tests have been used to evaluate the interim constitutive equations and to verify and qualify inelastic analysis computer programs.

A sizable body of test data thus exists for use in qualifying an analysis method and computer program for a specific job. Unfortunately, however, the correlation of analytical predictions with experimental results for inelastic problems is not straightforward. Inelastic material behavior is a highly complex phenomenon consisting of processes and interactions that defy complete understanding and exact description. There can be considerable variations in behavior from specimen to specimen. Current constitutive equations are only first approximations. Finally, accurate high-temperature structural test data are difficult to obtain. What, then, is an *acceptable* correlation of analytical and experimental data that will *qualify* an inelastic analysis method and computer program? Hopefully, this compact will help answer both this question and the more basic one, "Are inelastic analysis predictions valid?"

The currently recommended constitutive equations are briefly outlined, together with their bases, in the following section. Section 3 then addresses the evaluation and improvement of the analysis methods through benchmark calculations. Section 4 examines the question of material variability and its effect on the validity of analysis predictions. The final section contains conclusions.

## 2. CURRENTLY RECOMMENDED CONSTITUTIVE EQUATIONS FOR INELASTIC DESIGN ANALYSIS

The currently recommended constitutive equations have been described by Robinson et al. [3], and much of the uniaxial exploratory test information on which they are based has been reviewed by Swindeman and Klueh [4]. Although differing in detail, the recommended constitutive equations for types 304 and 316 stainless steel and 2 1/4 Cr–1 Mo steel are basically of the same form. The equations are limited in application to infinitesimal deformations, and they rest on the fundamental assumption that the total strain rate, or strain increment, consists of four additive contributions — thermal, elastic, plastic, and creep. Although each contribution is treated as being essentially independent, provision is made in the recommended procedures for limited interaction between the plasticity and creep contributions.

The specification of the plastic strain increment follows from the classical linear kinematic hardening model. The von Mises yield criteria and the associated flow law of von Mises are used. The flow law is of the form:

$$d\varepsilon_{ij}^{P} \sim \frac{\partial f}{\partial \sigma'_{ij}} , \tag{1}$$

where $d\varepsilon_{ij}^{P}$ is the plastic strain increment and $\sigma'_{ij}$ represents the components of the stress deviator. The function f which describes both the initial yield surface and the subsequent loading surfaces is given by

$$f = \frac{1}{2}\left(\sigma'_{ij} - C\varepsilon_{ij}^{P}\right)\left(\sigma'_{ij} - C\varepsilon_{ij}^{P}\right) = \kappa(T) , \tag{2}$$

where C is the hardening coefficient, which is taken to be constant, and $\kappa$ relates to the size of the surface. With C constant, plastic hardening is linear, and when coupled with linear-elastic response, constitutes a bilinear idealization of elastic-plastic behavior.*

With κ taken to be independent of deformation history, the yield function [Eq. (2)] represents *true* kinematic hardening. In reality, however, prior plastic and creep strains change the size of the yield surface as defined by the bilinear representation; that is, some isotropic hardening occurs.** To account for this, ad hoc rules are provided which allow for changes in the value of κ as plastic and creep strains are accumulated.

The effects that prior inelastic strains can have on hardening are depicted in Figs. 1 and 2 for type 304 stainless steel. The results of several uniaxial cyclic stress-strain

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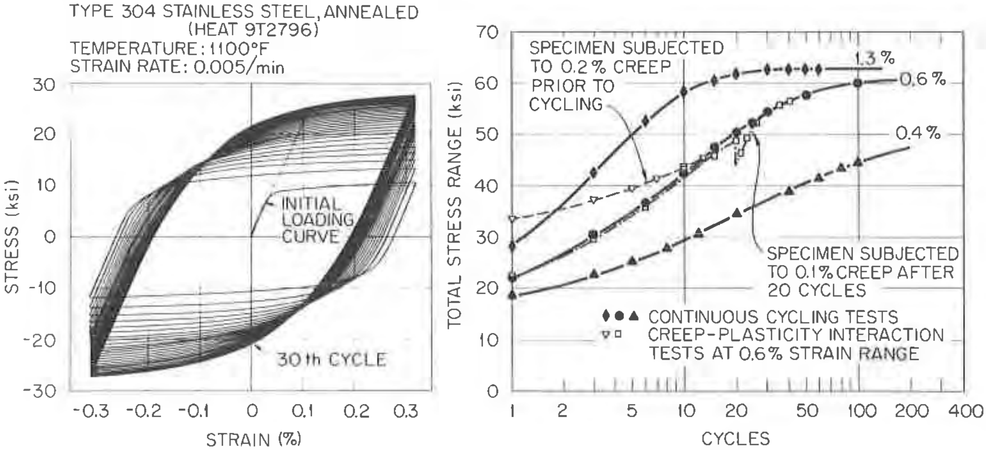
*The constant C is determined from the respective moduli of the segments of a bilinearized uniaxial stress-strain curve, and κ is related to the yield stress of the idealized curve.

**In the case of 2 1/4 Cr—1 Mo steel, isotropic softening can also occur.



(*a*) TYPICAL CYCLIC STRESS-STRAIN CURVE (0.6% TOTAL STRAIN RANGE)

(*b*) TOTAL STRESS RANGE VERSUS NUMBER OF CYCLES

Fig. 1. Examples of cyclic stress-strain behavior of type 304 stainless steel at 593°C (1100°F). [1 ksi = 6.895 MPa]

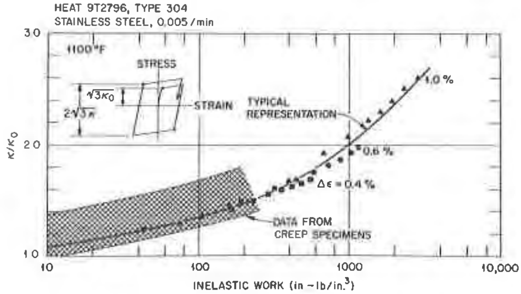


Fig. 2. Yield parameter ratio, $\kappa/\kappa_0$, vs accumulated inelastic work for annealed type 304 stainless steel at 593°C (1100°F). Data are from cyclic stress-strain tests of virgin specimens and stress-strain tests of precrept specimens. [in.-lb/in.$^3$ × 6895 = J/m$^3$]

tests are shown in Fig. 1, illustrating that both prior plastic strains and prior creep strains can cause hardening. Figure 2 illustrates that the change in $\kappa$ can be correlated with prior inelastic work, and the use of such correlations is recommended in the interim constitutive equation guidelines.

The recommended procedure for calculating the creep strain increments is based on the classical creep strain-hardening theory. For the situation of stress reversals in creep, the model is modified according to auxiliary hardening rules suggested by Corum et al. [5]. Multiaxial creep behavior is represented by the strain-hardening equation

$$d\varepsilon_{ij}^{C} = \frac{3}{2} \frac{\dot{\bar{\varepsilon}}^{C}(\bar{\sigma}, \varepsilon^{C})}{\bar{\sigma}} \sigma'_{ij} \, dt \ , \tag{3}$$

where $d\varepsilon_{ij}^{C}$ is the creep strain increment, $\dot{\bar{\varepsilon}}^{C}$ is the effective creep strain rate obtained from the uniaxial creep relation, $\bar{\sigma}$ is the effective stress, $\varepsilon^{C}$ is a modified effective accumulated creep strain, and dt is the time increment.

The suitability of the modified strain-hardening theory is illustrated, at least for the uniaxial loading case, in Figs. 3 and 4 for 2 1/4 Cr—1 Mo steel. As shown in Fig. 3, the theory provides reasonable predictions of the behavior of a uniaxial specimen subjected to step changes in both load and temperature. Figure 4 shows that the effects of stress reversals are also predicted reasonably well when the modified hardening rules are used.*

Just as prior creep was shown to have an effect on subsequent plasticity, prior plasticity can have an effect on subsequent creep. Data for type 304 stainless steel do not show this effect to be large. However, there is a significant effect in the case of 2 1/4 Cr—1 Mo steel. This is discussed in the next section along with modifications to the constitutive procedures to account for the effect.

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*The auxiliary rules are based on the premise that creep strain hardening accumulated in one direction does not exhibit itself as hardening when the stresses are reversed.

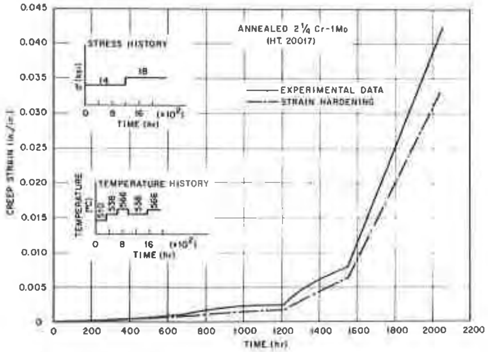


Fig. 3. Comparison of experimental creep data and strain-hardening prediction for stepped stress/temperature test on 2 1/4 Cr—1 Mo steel. [1 ksi = 6.895 MPa; °C = (°F — 32) × 0.5556]

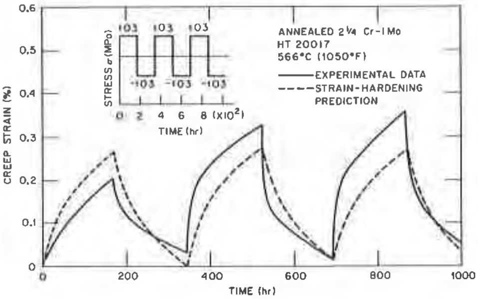


Fig. 4. Comparison of experimental creep data and modified strain-hardening prediction for reversed creep test on 2 1/4 Cr—1 Mo steel. [1 MPa = 145 psi]

## 3. EVALUATION AND IMPROVEMENT OF ANALYSIS METHODS THROUGH BENCHMARK CALCULATIONS

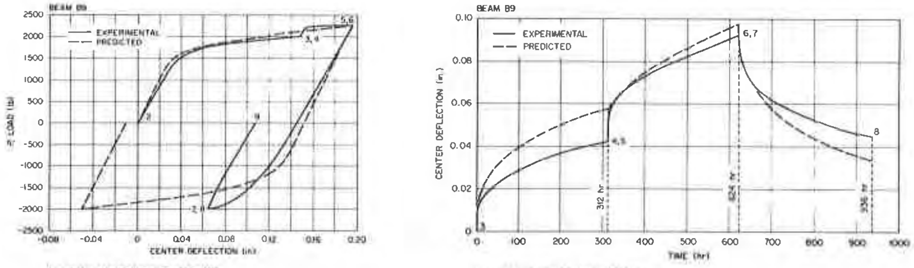
Inelastic analysis predictions based on the procedures outlined in the previous section have been compared with experimental benchmark problem data from tests of structures ranging from simple beams, circular plates, and nozzle-to-spherical shell attachments to lengths of pipes tested in sodium and subjected to repeated thermal downshocks. In most cases, the comparisons have been good; in a few, however, the agreement was relatively poor and led, in fact, to improvements in the constitutive equations.

In this compact, just four representative benchmark problem comparisons are presented, but these are typical of the broader range of comparisons that have been made, and they will, hopefully, provide the reader with a feeling for the agreement that can be expected. Two of the four problems deal with the simple case of a beam, and two deal with thermal ratchetting of pipes under sodium thermal downshock conditions. In each case, the analysis predictions were based on the recommended constitutive equations, and they used, as input, cyclic stress-strain data and creep data for the particular heat of material of the specimen. The question of heat-to-heat material property variations is addressed in Section 4.

The first comparison is shown in Fig. 5 and is for a simply-supported type 304 stainless steel beam which was subjected to a time-varying center load history at 593°C (1100°F). The beam was 50.8 mm (2 in.) high, 25.4 mm (1 in.) wide, and 0.61 m (24 in.) between end supports. A center load which produced plastic deformation of the beam was applied from points 2 to 3 and was then held constant from points 3 to 4; the load was then increased (points 4 to 5) and was again held constant from points 5 to 6; and, finally, the load was reversed from points 6 to 7 and held constant from points 7 to 8.

The large discrepancy in Fig. 5*a* is caused by the fact that the constitutive equations used did not adequately account for the hardening effect of prior creep on subsequent plastic hardening. Comparisons of analysis predictions with other test results identified this same shortcoming, and consequently the recommendations were modified to explicitly relate the plastic hardening (κ change) to prior inelastic work, as was shown in Fig. 2. Subsequent predictions, based on the revised recommendations and the correlation in Fig. 2, are compared with the time-independent response of the beam in Fig. 6. The comparison is much more reasonable than in Fig. 5*a*.

The second comparison is based on the test of a specially modified beam of 2 1/4 Cr—1 Mo steel which was subjected to pure bending near the center. The center portion of the beam is



(a) TIME-INDEPENDENT RESPONSE

(b) TIME-DEPENDENT RESPONSE

Fig. 5. Comparison of measured and predicted response of a beam subjected to a time-varying loading history consisting of 8896 N (2000 lb) for 312 hr, 10,008 N (2250 lb) for 312 hr, and —8896 N (—2000 lb) for 312 hr. [1 lb = 4.448 N; 1 in. = 2.54 cm]

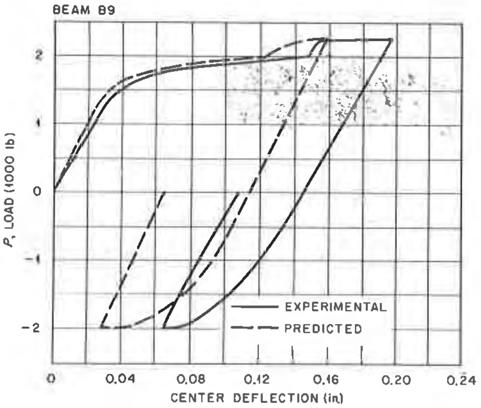


Fig. 6. Comparison of measured time-independent response of beam B9 with revised predictions based on $\kappa/\kappa_0$ as a function of inelastic work (see Fig. 2).

shown in Fig. 7; it consisted of two thin flanges, under near-uniaxial loading — one in tension, the other in compression. The beam was loaded so that a strain cycle, typical of that which might be encountered in design, was imposed, and the results, from one of the two flanges, are shown in Fig. 8. The specimen was first strained from the origin to point *a* (Fig. 8*a*) and back to point *b*, corresponding to the effects of a thermal shock. The strain at point *b* was held constant for 240 hours and the *residual* stress allowed to relax. The specimen was subsequently subjected to four additional cycles (*c*—*a*—*b*—*c*) in which the strain limits at *a* and *b* were maintained.

The measured relaxation behavior for the first and fifth of these five cycles is shown in Fig. 8*b*. The amount of observed relaxation diminished very little from cycle to cycle. Yet, because of the continual accumulation of creep strains during the relaxation periods, the usual creep strain-hardening theory would predict considerably less relaxation from one cycle to the next. Less relaxation means higher stresses and consequently more predicted creep-rupture damage. This, then, was a significant deficiency in the initial constitutive equations recommended for 2 1/4 Cr—1 Mo steel.

It was hypothesized that the occurrence of reversed plastic strains (from *c* to *a* in Fig. 6*a*) negated the creep strain hardening accumulated during the prior relaxation period.

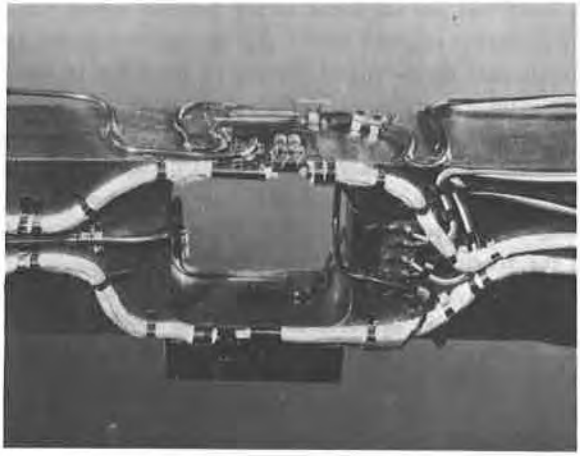
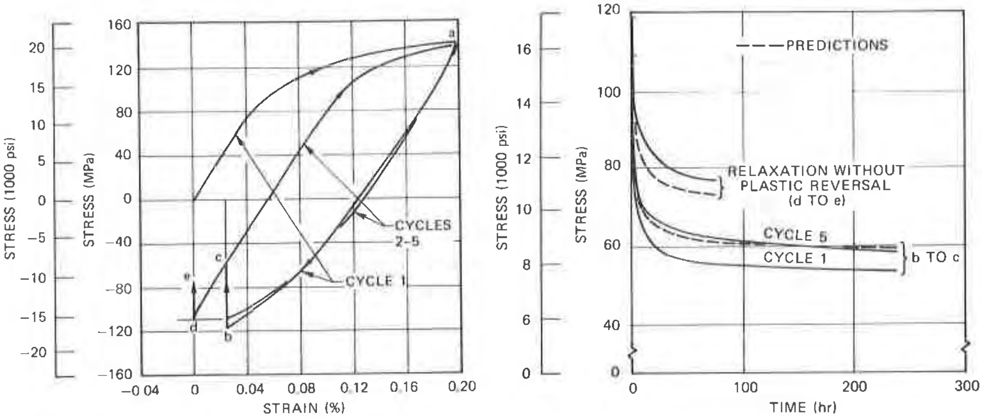


Fig. 7. Center portion of *slotted* beam tested in four-point bending. Capacitive strain gages were mounted on each flange.



(*a*) STRESS-STRAIN RESPONSE

(*b*) RELAXATION RESPONSE

Fig. 8. Measured and predicted response for 2 1/4 Cr—1 Mo steel *slotted* beam tested at 538°C (1000°F).

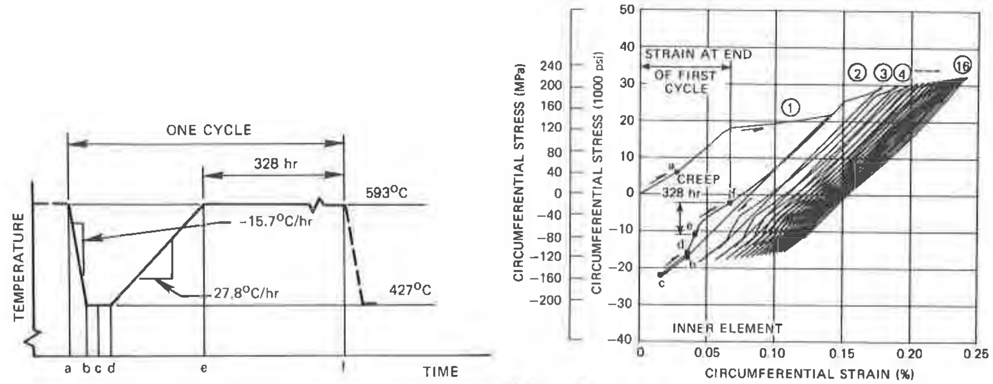
To check this premise, an additional sequence was added to the test series. At the end of the fifth relaxation period the stress level was increased from *c* to *d* (Fig. 8*a*) without going through a plastic reversal, and relaxation was then allowed to occur from *d* to *e*. The resulting relaxation response for 72 hours is shown in Fig. 8*b*. It is much less than in the previous five cycles and supports the above premise.

On the basis of these, and similar, test data, the recommended constitutive equations for 2 1/4 Cr—1 Mo steel were modified to more accurately account for the effects of prior plasticity on subsequent creep. Analytical predictions based on the improved constitutive relations are compared with the measured responses in Fig. 8*b*, and the agreement is seen to be good.

The third comparison is for the case of a 195.6 mm ID × 12.7 mm wall (7.7 × 0.5 in.) pipe of 2 1/4 Cr—1 Mo steel that was subjected to the internal sodium temperature histogram shown in Fig. 9*a*. The predicted response of the pipe is typified in Fig. 9*b* where the circumferential stress-strain path on the inside surface is shown for 16 thermal cycles. The path from the origin to point *a* corresponds to the initial pressurization; the first thermal downshock, from *a* to *b*, produces plastic yielding; the pressure is removed and reapplied from *b* to *d*; the temperature is slowly increased from 427°C (800°F) back to 593°C (1100°F) between points *d* and *e*, during which additional yielding occurs due to the decrease in yield strength with increasing temperature; and, finally, creep and relaxation of the residual stresses occur during the 328-hr hold period from points *e* to *f*.

How valid are these relatively complex predictions? This question is answered, at least in part, by Fig. 10, where the measured and predicted circumferential strains on the outside surface are compared for nine thermal cycles. Considering the complex behavior involved, the comparison is considered to be good.

A similar comparison is presented in the following section for a type 304 stainless steel pipe.



(*a*) TEMPERATURE HISTOGRAM (*b*) PREDICTED RESPONSE

Fig. 9. Predicted circumferential stress-strain path at the inside surface of a 2 1/4 Cr—1 Mo steel pipe for 16 cycles of ratchetting response. The internal pressure of 4.83 MPa (700 psi) was removed and immediately reapplied at point *c*. [°C = (°F — 32) × 0.5556]

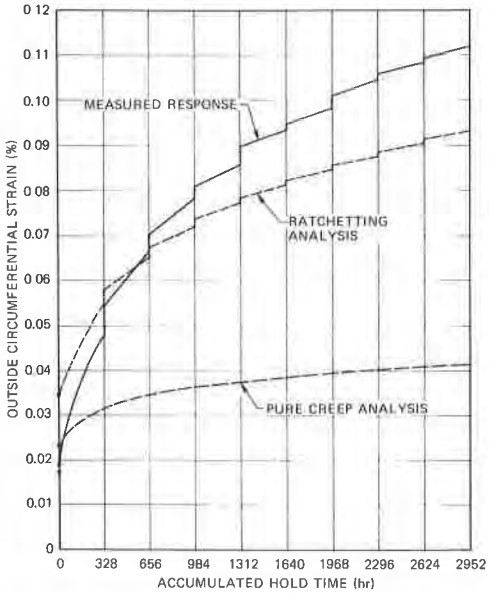


Fig. 10. Comparison of measured and predicted circumferential ratchetting strains at the outside surface of the 2 1/4 Cr—1 Mo steel pipe. The pure creep curve shows the predicted behavior had there been no thermal shocks and consequently no thermal ratchetting.

## 4. SENSITIVITY OF ANALYSIS PREDICTIONS TO MECHANICAL PROPERTY VARIATIONS

Generally, the agreement between measured and predicted results is reasonable for the range of benchmark problems examined. As typified by the examples of the previous section, the basic inelastic trends were captured, although there were pronounced quantitative discrepancies.

It should be remembered, however, that these comparisons were made under ideal conditions, in that the materials data used in the inelastic analyses were obtained from the same heat of material as the structural test specimens. The designer generally does not have this situation. Questions of specimen-to-specimen variations and heat-to-heat variations thus arise. These questions are briefly addressed in this section.

The ORNL Metals and Ceramics Division has examined some 20 different heats of type 304 stainless steel. Stress-strain and creep tests have been performed at various temperatures. As an example, stress-strain tests at 593°C (1100°F) show a 0.2% offset yield strength variation ranging from about 103 MPa to 172 MPa (15,000 to 25,000 psi), which represents a variation of about 25% about the mean.

Creep tests also show a significant variation in results. For example, specimens tested at a creep stress level of 117 MPa (17,000 psi) at 593°C exhibited creep strains, at any given time, which varied by almost a factor of 4.

What effect can such variations have on design calculations? To briefly examine this question, a small sensitivity study was conducted in which a 195.6 mm ID × 9.53 mm wall (7.7 × 0.375 in.) type 304 stainless steel pipe thermal ratchetting test specimen was analyzed, first using the actual material properties of the specimen heat, and second with the yield stress or the creep response varied. The results are shown in Fig. 11.

The inset of Fig. 11 shows the sodium temperature and pressure cycle imposed on the specimen. The data points show the specimen response for 13 cycles as measured by two diametrically opposed capacitive strain gages, and the solid curve shows the predictions based on the actual properties of the specimen heat.

The sensitivity study results show, for example, that if the yield stress at all temperatures is lowered by 20%, the predicted ratchetting strain at the end of 13 cycles is *increased* by 40% (a conservative prediction), but the predicted creep-fatigue damage factor is *decreased* by 54% (an unconservative prediction). This example illustrates that when elevated-temperature elastic-plastic-creep behavior and the attendant modes of failure are considered, the traditional concept of a conservative calculation begins to lose its obvious rationale and must be reevaluated.

## 5. CONCLUSIONS

In summary, the benchmark comparisons that have been made indicate that the basic trends and features of the inelastic response are predicted, and the absolute agreement between theory and experiment is generally good. However, there are obvious discrepancies, and, in addition, Fig. 11 illustrates the significant differences that can result from normal material variation. Thus, we might again ask the questions, "Are inelastic analysis predictions valid, and what is an acceptable correlation between analytical and experimental data that will qualify an inelastic analysis method and computer program?"

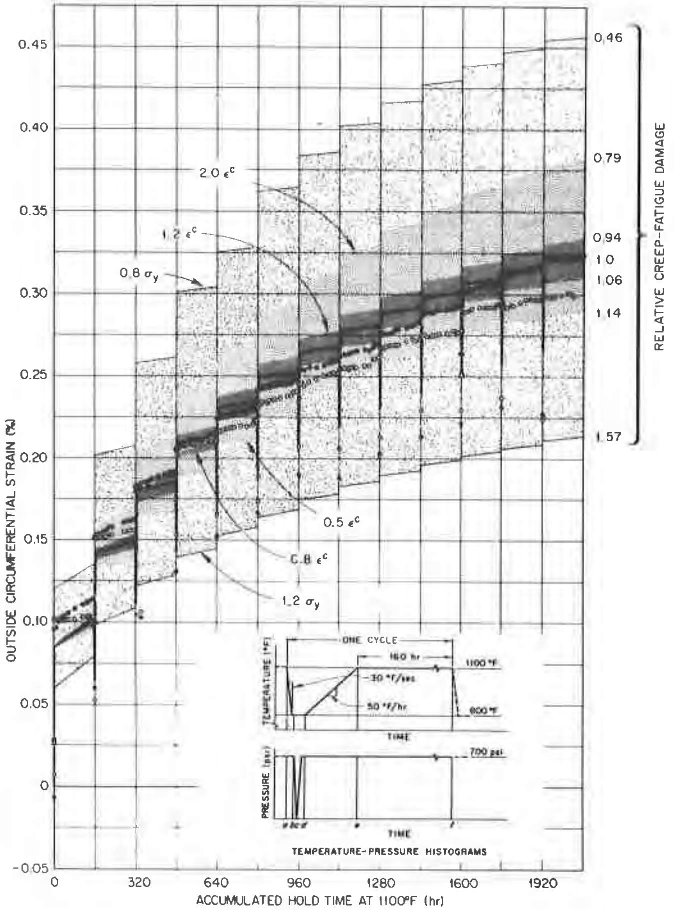


Fig. 11. Comparison of measured and predicted circumferential ratchetting strains for a type 304 stainless steel pipe. The solid prediction curve is based on the best available elastic-plastic and creep data for the specimen material. The extremes of the shaded bands represent predictions based on increasing or decreasing the yield stresses and the creep strain response as indicated. The relative creep-fatigue damage factors shown to the right were calculated according to the inelastic analysis rules of ASME Code Case 1592 [1]. This figure illustrates the range of behavior that can result from normal heat-to-heat material variations. [°F = 1.8°C + 32, psi × 6895 = Pa]

Actually, we should ask whether or not the combination of the analysis predictions and the associated design criteria lead to an acceptable level of structural integrity. It is believed that in this context the analysis predictions are generally valid, even though exact correlations between predictions and actual behavior cannot be expected. For example, the creep-fatigue damage calculations shown in Fig. 11 show that if the yield stress is 20% less than expected, the creep-fatigue damage can be underpredicted by approximately 54%. This damage is essentially all creep damage, and the ASME high-temperature code case criteria for creep rupture include a factor of safety of at least 20 on life. Thus, at least in this case, the Code contains an adequate margin of safety to take care of the variation in material behavior.

In conclusion, the adequacy of inelastic analysis methods must be examined by considering not only the analyses themselves but also the associated design criteria as was done in the paragraph above. Final judgment must be reserved for each specific design situation.

The objectives of inelastic design analyses should be to (1) predict the basic characteristics, or the essential features, of inelastic response in critical situations and/or critical regions, (2) provide a consistent basis for evaluating critical areas throughout a high-temperature reactor project, and (3) when coupled with design criteria, provide a satisfactory level of structural integrity. Standards for judging qualification and validity must be established for each particular application based upon these objectives.

## 6. ACKNOWLEDGMENTS

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