ABSTRACT

AVINERI, TAMAR ANN. Effectiveness of a Mathematics Education Massive Open Online Course as a Professional Development Opportunity for Educators. (Under the direction of Dr. Paola Sztajn and Dr. Glenn Kleiman.)

With the recent emergence of online professional development (PD), such opportunities are increasingly available to educators. It is important, then, to assess whether and how such options can be valuable and effective for them. The goal of this study was to assess whether and how a Massive Open Online Course for Educators (MOOC-Ed) can serve as an effective PD option for educators by focusing on how and to what extent a MOOC-Ed on *Fraction Foundations* changed participants' knowledge, beliefs, and practice.

In order to address the goals of the study, a convergent, parallel mixed methods design was used in which both quantitative and qualitative data were analyzed to inform a case study on the MOOC-Ed. Data were collected through a number of instruments administered to all participants in the MOOC-Ed. Pre- and post-surveys addressed participants' attitudes and beliefs about the effectiveness of different components of the PD program, and a pre- and post-assessment addressed participants' mathematical knowledge for teaching (MKT). The course on *Fraction Foundations* was administered twice, once in the Fall and once in the Spring. Fifty-four participants submitted all surveys and assessments in the first course, and 159 participants submitted them in the second. Respondents primarily included teachers (especially of elementary school students), teacher leaders, administrators, and curriculum and instruction specialists. Additional qualitative data were collected through personal interviews and classroom observations of four elementary teachers to contextualize the interconnections between changes in participants' knowledge, beliefs, and practice. The *Interconnected Model of Professional Growth* (Clarke & Hollingsworth, 2002) was used as a

framework to analyze results from these qualitative data and develop narratives to describe the teachers' movement through their "growth cycles".

Results from this study showed that a MOOC-Ed can serve as an effective form of professional development in changing participants' knowledge and/or beliefs and having an impact on their practice in a number of ways. Design principles implemented in the development of the MOOC-Ed, which included 1) multiple voices; 2) self-directed learning; 3) peer-supported learning; and 4) job-connected learning, contributed to participants' perceptions of specific changes in their knowledge and practice. Results also showed that there is a dynamic interplay between participants' perceptions of changes in their knowledge and/or skills as a result of their participation and the impact the PD has on their practice.

Effectiveness of a Mathematics Education Massive Open Online Course as a Professional Development Opportunity for Educators

by

Tamar Ann Avineri

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APPROVED BY:

Dr. Paola Sztajn Committee Co-Chair	Dr. Glenn M. Kleiman Committee Co-Chair
Dr. Hollylynne S. Lee Committee Member	Dr. Ernest L. Stitzinger Committee Member

DEDICATION

To my mom and dad, who always lovingly supported both my personal and professional development.

BIOGRAPHY

Tamar Ann Avineri was born on October 31, 1976, in Beer Sheva, Israel. She moved to the United States with her family in August of 1981 and grew up a few miles outside of Los Angeles, CA. After graduating from high school in 1994 in Rancho Palos Verdes, CA, Tamar attended the University of California, San Diego, and graduated Cum Laude with a bachelor's degree in Applied Mathematics in 1998. Tamar continued her education at the University of California, Los Angeles, earning a master's degree in Mathematics in 2000.

Tamar's first teaching experience was as a teaching assistant in her undergraduate program, and she continued that work through her graduate program. Following her master's program, Tamar taught as an adjunct instructor at Pasadena City College and Glendale Community College, ultimately becoming a full-time faculty member at Pasadena City College in 2001. Tamar moved to North Carolina in December 2003 and began teaching as an adjunct instructor at Alamance Community College and Elon University in January 2004. In August 2004, Tamar began teaching mathematics full time at the North Carolina School of Science and Mathematics (NCSSM) in Durham, NC, and continues to teach there. She earned National Board Certification in Adolescent and Young Adult Mathematics in 2008.

With great interest and motivation to further her academic studies in mathematics education, Tamar began the Ph.D. program in Mathematics Education at North Carolina State University in Raleigh, NC, in 2010. She was enrolled on a part-time basis while working full-time at NCSSM until 2012, when she transitioned to pursuing the degree full-time and teaching at NCSSM on a part-time basis. During her full-time status at NC State University, Tamar served as a Graduate Research Assistant at the Friday Institute for

Educational Innovation and was part of the team who developed the MOOC-Ed in this study. Tamar returned to teaching full-time at NCSSM in 2014, as she worked on her dissertation, and is incredibly grateful for having had the opportunity to pursue her teaching career and graduate degree simultaneously.

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My deepest love and thanks to my family: To Netta, who has supported me with love, encouragement, and invaluable guidance and feedback throughout my program. I am the lucky one. To Oren and Eitan, for their unconditional love and for serving as examples of strength and goodness throughout our childhood to today. To Orly and Yoram, for their loving words and support. To Grandma, whose strength and endurance inspire me; and to other extended family, for their genuine love and support.

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CHAPTER ONE

Learning is a cyclical process, as we develop knowledge and skills by building upon what we know, drawing lessons from our experience, and applying what we know to situations to build new understandings. Much of what we learn is facilitated by others we engage with; in a formal education context, educators play an essential role in facilitating that learning for students. In recognition of this, just as much focus and effort toward ensuring the strength and success of education for students must be put toward developing the knowledge and skills of those who educate them. Just as there are approaches to providing high-quality and successful educational systems for students, there are approaches to professional development (PD) for educators that can have an impact on those engaged in it.

Educators are students of their own practice, and it is critical for them to have opportunities to learn and grow through professional development. Researchers have offered a number of definitions of PD, and those referenced here are adopted for purposes of this study. Guskey (2002) defined teacher PD as "systematic efforts to bring about change in the classroom practices of teachers, in their attitudes and beliefs, and in the learning outcomes of students" (p. 381). The American Federation of Teachers (AFT, 2008) added to this definition, noting that PD is a "continuous process of individual and collective examination" (p. 3). Sustained and rigorous change is a challenging goal for any professional development program to achieve. Such efforts must be great in depth and rich in complexity in order to ensure high-quality, effective PD experiences. It is important to assess whether and how any PD initiative can serve its participants in effective ways. Effectiveness of PD can be defined in a number of ways as well (e.g., extent to which participants achieved their personal goals);

in this study, it is assessed by whether and how the PD changed participants' knowledge, beliefs and/or practice.

In this chapter, I present the statement of the problem, purpose of the study and research questions, a brief description of the PD program investigated in this study, an overview of the methods used, and a description of the organization of the dissertation.

Statement of the Problem

In their study on the effects of professional development, Heck, Banilower, Weiss and Rosenberg (2008) stated the following:

> The mathematics education community has, over the last 20 years, called for a reconceptualization of the types of knowledge, skills, and dispositions that teachers need for effective practice. To make progress toward the new vision of quality mathematics education described in the NCTM's *Standards* documents, teachers need to not only upgrade their knowledge of mathematics but also develop greater understanding of how students think about and learn mathematics and broaden the toolkit of pedagogical strategies from which they can draw (NCTM, 1989, 1991, 2000, as cited in Heck et al., 2008, p. 113).

However, reports from the 2012 National Survey of Science and Mathematics Education indicate that while more than 85% of teachers at all grade levels have participated in mathematics or mathematics education-related PD in the three years prior to the survey, only 11% of elementary teachers had received more than 35 hours of PD in that time (Banilower, Smith, Weiss, Malzahn, Campbell, & Weis, 2013). Heck et al. (2008) acknowledged the need for teachers to improve their content and pedagogical knowledge and skills in light of enhanced national standards for education in mathematics, arguing that the "national need for PD to increase teacher knowledge and skills is especially heightened by the changing vision for quality mathematics education" (p. 118).

Professional development programs for teachers come in a number of forms, such as summer workshops, conferences, inservice activities for training, continuing education courses and day-to-day exchanges of ideas and reflection among colleagues. Over the past 30 years, there has been an increasing amount of research performed on different models of PD for teachers and the characteristics that make those models effective. Many studies have focused on how PD can affect teachers' beliefs, attitudes and knowledge, with some focusing on changes or impact on practice. Fewer studies have focused on effects of *online* PD (National Research Council (NRC), 2007). With current systematic challenges, such as budget cuts and implementation of new standards without professional support, it is necessary to explore new ways of maximizing the efficiency and relevancy of PD activities to motivate participation, promote learning and affect change in the classroom for both the teachers involved and the students they teach. This has great implications for the field of online education; PD programs that include accessible, personalized, and self-directed elements can provide increased opportunities for sustained, collaborative and meaningful work among teachers that can thereby have an effect on their knowledge, beliefs and practice. (Mushayikwa & Lubben, 2009; Russell, Carey, Kleiman, & Venable, 2009; Song & Hill, 2007; Vrasidas & Zembylas, 2004). Moreover, this highlights the need for research on the impact online PD can have on participants' knowledge, beliefs and practice (deWaard, Kukulska-Hulme, & Sharples, 2015; Kleiman, Wolf, & Frye, 2014; NRC, 2007).

Purpose of the Study and Research Questions

As technology has advanced, online PD opportunities have increased dramatically (Brown & Green, 2003). Massive Open Online Courses for Educators (MOOC-Eds), which are self-directed, peer-supported online courses open to the general public at no cost to participants, have recently been developed in an attempt to offer such opportunities on a large scale (Kleiman et al., 2014). The purpose of this study was to answer the following general question:

> To what extent and in what ways can a MOOC-Ed serve as a viable, effective PD option in affecting educators' knowledge and/or beliefs and impacting their practice?

In order to explore the potential of MOOC-Eds as an effective form of PD, the study focused on how one MOOC-Ed contributed to changes in participants' knowledge, beliefs and practice. Since evidence of these effects can come in both quantitative and qualitative forms, a mixed-methods approach was used to inform the study, which was guided by the following specific research questions.

1. To what extent and in what ways can the *Fraction Foundations* MOOC-Ed change participants' knowledge and/or beliefs?
2. To what extent and in what ways can the *Fraction Foundations* MOOC-Ed impact participants' practice?

The diversity of MOOC-Ed participants' roles in education (e.g., classroom teachers, teacher educators) motivated the design of these questions to remain broad and not specific to any one subgroup of educators. This allowed for anaysis and discussion of results that are relevant to a variety of groups.

MOOC-Eds

In an effort to meet the need for "large-scale, widely accessible, cost-effective professional development opportunities for educators", the Friday Institute for Educational Innovation at North Carolina State University explored new ideas to deliver such opportunities (Kleiman et al., 2014, p. 1). As MOOCs emerged as a platform for continued learning, the Friday Institute began to explore designing MOOCs specifically as professional learning opportunities for educators, identifying them as *MOOC-Eds*. Combining expertise in multiple areas related to education initiatives, technological advances, online and blended learning programs, and research on teaching and learning, the Friday Institute began developing MOOC-Eds and launched its first course, *Planning for the Digital Learning Transition in K-12 Schools*, in early 2013 (Kellogg, Booth, & Oliver, 2014). Each MOOC-Ed is designed based on research-supported principles of effective professional development and online learning (Kleiman & Wolf, 2015). The key design principles are:

- *Multiple voices*, so that participants learn about the perspectives of other teachers and administrators along with those of students, researchers and experts in the field. MOOC-Eds are purposefully *not* designed around one or two experts who present online lectures. They are about a rich set of perspectives presented within the context of activities and exchanges that reflect the additional design principles described below.
- *Self-directed learning*, so that participants can personalize their experience by identifying their own goals, selecting among a rich array of resources, and deciding

whether, when, and how to engage in discussions and activities to further their own learning and meet their goals.

- *Peer-supported learning*, through participants engaging in online discussions, reviewing each other's projects, rating posted ideas, recommending resources, crowdsourcing lessons learned, and participating in twitter chats and other exchanges appropriate to the individual course.
- *Job-connected learning*, through the use of case studies, classroom and school related projects; developing action plans; and other activities that center participants' work on critical problems of practice and data-informed decision-making in their own classrooms, schools or districts.

(Kleiman & Wolf, 2015, pp. 5-6)

Overview of Methods

Multiple measures and sources of data were used to help answer the research questions. Quantitative data included a registration form (pre-survey), post-course survey, and a pre-post assessment adapted from the Mathematical Knowledge for Teaching (MKT) assessment developed at the University of Michigan (Hill, Blunk, Charalambous, Lewis, Phelps, Sleep, & Ball, 2008; Learning Mathematics for Teaching (LMT) Project, n.d.). The pre-survey consisted of self-reported demographic data – including information on professional roles and work settings, years of experience, gender, level of education, and personal learning goals and was administered when participants first registered for the course. The post-course survey consisted of approximately 30 Likert-scale items and four open-ended items designed to solicit participants' perceptions of the impact and effectiveness of the MOOC-Eds. The survey was made available to participants in the final weeks of the course. Items included in the MKT pre-post assessments were selected based on their alignment with the three major topics covered in the MOOC-Ed: fractions in fair-sharing contexts; fractions as measures; and understanding operations with fractions. The pre-MKT assessment was offered in the first two weeks of the course, and the post-MKT assessment was offered beginning in the final unit of the course.

Basic descriptive statistics, including data such as primary area of responsibility and number of years as an educator, were used to describe the sample of participants in the MOOC-Ed. Spearman's rank correlation coefficient, r_s, was used to measure associations between different elements of perceived effectiveness. For example, I explored the strength of association between perceptions of enhanced knowledge and skills in certain areas of practice and self-reported intentions to change practice by calculating rank-order correlations between individual responses to the post-survey questions that aligned with those elements.

The pre-post assessment on *Mathematical Knowledge for Teaching* (MKT) provided data as an additional quantitative measure of the extent to which participants experienced change as a result of their participation in the MOOC-Ed. Basic descriptive statistics, paired two-sample *t*-tests on mean scores, graphical representations of change scores, and effect size estimates (Cohen's *d*) were used to analyze results from the pre- and post-assessments (Cohen, 1992; Sullivan & Feinn, 2012).

Qualitative data included responses to open-ended survey questions that helped inform answers to the research questions in this study, semi-structured interviews with four elementary mathematics teachers, and observations of their classrooms. Personal interviews and classroom observations with three of the teachers were conducted once before the course launched and twice following the conclusion of the course. The fourth teacher was interviewed and observed once during the MOOC-Ed and a final time after the course had officially closed.

The open-ended questions on the post-survey were analyzed to help contextualize the quantitative results. Responses were analyzed using grounded theory coding methods suggested by Charmaz (2006, as cited in deWaard et al., 2015, pp. 237-238). Analysis of data gathered through interviews and observations included multiple stages that will be described in Chapter Three. Statements from interviews and observations were used to illustrate each participant's profile in the study, and the *Interconnected Model of Professional Growth*[1] served as a framework for analysis (Clarke & Hollingsworth, 2002).

Organization of Dissertation

This dissertation is organized into five chapters. Chapter One provides a statement of the problem this study focused on, the purpose and research questions that guided this study, a brief introduction to the overall PD initiative investigated in this study, and an overview of the methods used. Chapter Two provides a discussion of the theoretical framework used in the analysis of the study and a review of the research literature that informed and motivated it. The review focuses on three main topics relevant to this study: Mathematics Education for Elementary Teachers; Principles of Effective Professional Development; and Online Professional Development. Chapter Three provides a detailed description of the methodology used to conduct the study, rationale for the choice of procedures, description of sources of

[1]See Chapter Two for more on this model.

data, and methods of analysis used. Chapter Four presents the findings from the study organized to address each of the research questions. Findings include results from post-surveys administered at the conclusion of the MOOC-Ed, the pre- and post-MKT assessment, and data from personal interviews and classroom observations of a small sample of participants. Chapter Five discusses the findings and conclusions aligned to the research questions, as well as the significance, implications, limitations and future research related to the study.

CHAPTER TWO

Theoretical Perspective

This study was conducted under the theoretical perspective that changes in individuals' knowledge, beliefs and professional practice can be realized through multiple and diverse pathways. Educators base their learning in the needs of those they serve and approach learning opportunities with many of their own goals (Daley, 2001). As diverse as educators' goals are for professional development, so are the paths through which they change. Boling and Martin (2005) added to this perspective:

> Teacher change is defined as a highly personal process accomplished by the individual through experiences, emotions, cognitions, and behaviors over a period of time which transform a teacher's values and beliefs (Treacy et al., 2002; Prochaska, DiClementa, & Norcross, 1997). This process of change assumes the teacher recognizes a need, makes plans to improve, engages in the improvement, and allows time to evaluate the effectiveness of the new practices. Authentic change occurs when a teacher's belief system and core values concerning teaching and learning are modified. In order for authentic change to occur, teachers need the opportunity to learn new information, time to experiment with the new concept, and the opportunity to evaluate its effectiveness (Levy & Murnane, 2004; Richardson, 2003). (p. 4)

Clarke and Hollingsworth (2002) represented this in their *Interconnected Model of Professional Growth* (see Figure 1), which they developed based on results and analysis of three Australian studies on professional development[2]:

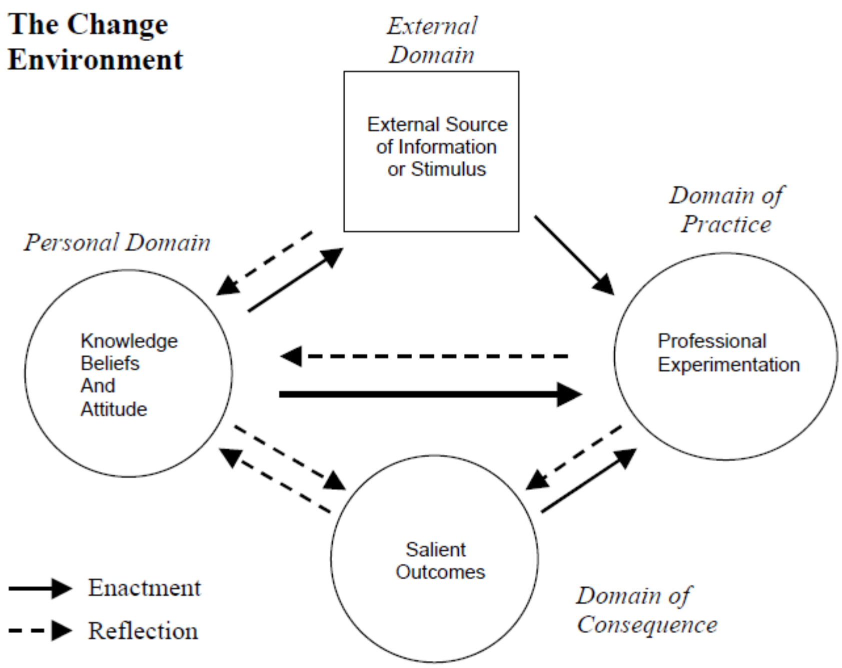


Figure 1. The Interconnected Model of Professional Growth (Clarke & Hollingsworth, 2002, p. 951)

The model divides the teacher's "change environment" into four domains: the *personal domain* (knowledge, beliefs and attitudes), the *domain of practice* (implementation of new approaches/activities), *domain of consequence* (outcomes and conclusions), and the *external domain* (resources, professional development) (Clarke & Hollingsworth, 2002, p. 950). The model illustrates that movement between the respective domains occurs as a result of *enactment* and *reflection* (indicated by the arrows in the figure). For example, consider the

[2] The ARTISM study (Clarke, Carlin, & Peter, 1992); the EMIC study (Hollingsworth, 1999); the negotiation of meaning project (Clarke, 1998, 2001)

solid arrow originating from the *personal domain* pointing toward the *external domain,* and the dotted arrow in the opposite direction. This can represent a teacher “beginning” their process with a certain belief that an instructional approach (e.g., project-based learning) could have a positive effect on their students’ learning. In “enacting” that belief, the teacher would subsequently work to improve or enhance their skills in using that approach by participating in professional development. Upon “reflection” during and after the PD experience, the teacher then adjusts their belief based on that experience. The teacher will have then experienced “change” as part of the process depicted in the model. As the model illustrates, this is just one of multiple pathways through the “change environment” that educators can follow. Recognizing the non-linear process of professional growth, this *Interconnected Model of Professional Growth* serves as a lens for this study in interpreting and analyzing the pathways that participants took as they engaged in the MOOC-Ed and as a guiding framework through which conclusions can be made from the results of the study.

Review of Literature

This study integrates several areas of research. Examining the literature in each of those areas reveals connections that help to support the rationale for the study. This review is presented in three sections: Mathematics Education for Elementary Teachers; Principles of Effective Professional Development; and Online Professional Development. In the first section, literature on the mathematics education of elementary teachers is discussed, as the target audience for the MOOC-Ed in this study, *Fraction Foundations,* was K-8 educators. This literature supports the premise that this study is well-situated to help address the need for ongoing, deeper learning of mathematics, and specifically that of fractions, for elementary educators. The first section also explains methodological decisions made in this study. The second section examines literature on principles of effective PD models so as to help frame characteristics of PD that this study examines. The final section is a review of literature on online PD and its potential to provide opportunities for effective professional development. These sections come together to inform the research that was conducted in this study.

Mathematics Education for Elementary Teachers

In his seminal piece, *Those Who Understand: Knowledge Growth in Teaching*, Shulman (1986) offered a framework to describe the different types of knowledge teachers need in order to be effective practitioners. Shulman proposed three categories of content knowledge: 1) subject content knowledge, 2) pedagogical content knowledge, and 3) curricular knowledge. In the context of mathematics, the first category is characterized by knowledge of both facts of the discipline (i.e. “truths in the domain”) and the reasons those facts are true (Shulman, 1986, p. 9). Teachers must “be able to explain why a particular proposition is deemed warranted, why it is worth knowing, and how it relates to other propositions, both within the discipline and without, both in theory and in practice” (Shulman, 1986, p. 9). This heightens the level of rigor at which teachers need to know the content they teach.

Shulman described *pedagogical content knowledge* as that which is needed to best represent the subject matter to students. In the context of mathematics, this knowledge would include forms of representation (e.g., graphical, symbolic), problem examples, and explorations of mathematical ideas through classroom tasks. Shulman also included the knowledge of common student misconceptions and challenges when learning certain concepts in this category.

The third category of content knowledge, *curricular knowledge*, includes knowledge of the ideas and concepts students learn in previous and future years both within and across disciplines (Shulman, 1986). With this knowledge, teachers can design lessons and tasks that support that learning, drawing from students' previous knowledge and building toward concepts they know their students will apply in future years.

Members of both the education research and practitioner communities have advocated for teacher education programs and professional development focused on the specific types of mathematical knowledge for teaching that Shulman proposed and argue that providing such opportunities in a mathematics context can improve teacher quality (Ball, Lubienski, & Mewborn, 2001; Ball, Thames, & Phelps, 2008; Conference Board of the Mathematical Sciences, 2001, 2012; Hill, Blunk, Charalambous, Lewis, Phelps, Sleep, & Ball, 2008). In 2001, the American Mathematical Society (AMS), along with the Mathematical Association of America (MAA), published the first edition of the Conference Board of the Mathematical Sciences' (CBMS) special issue on *The Mathematical Education of Teachers* (MET I) as a resource for mathematics educators who play a role in the mathematical education of pre-service teachers (CBMS, 2001). One of the overarching themes of the book is the recognition that mathematical knowledge for teaching (MKT) is different from mathematical knowledge needed for working in other fields and that mathematics educators (e.g., university faculty, department heads) need to pay special attention to how they help develop and strengthen pre-service teachers' MKT (CBMS, 2001, Preface). In 2012, after the

Common Core State Standards for Mathematics (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010) were adopted, and focus in the mathematics community increased on what teachers need to know for teaching mathematics and how they should strengthen that knowledge (e.g., Math Science Partnerships funded by the National Science Foundation and the US Department of Education), the AMS and MAA published a second volume of the CBMS special issue to address those changes, titled *The Mathematical Education of Teachers II* (CBMS, 2012, p. xi). This report broadened the scope of its recommendations to include both pre-service *and* in-service teachers, thereby providing a resource for mathematics educators in designing and providing professional development for practicing teachers. Both volumes of the book were written with the claims that:

- Proficiency with school mathematics is necessary but not sufficient mathematical knowledge for a teacher.
- The mathematical knowledge needed for teaching differs from that of other professions.
- Mathematical knowledge for teaching can and should grow throughout a teacher's career. (CBMS, 2012, p. xii)

Ball et al. (2008) explored the second claim to examine the special nature of mathematical knowledge for teaching. Specifically, Ball and colleagues sought to answer this question:

> What fundamental activities are demanded by the broad aims of developing a classroom in which mathematics is treated with integrity, students' ideas are taken

> seriously, and mathematical work is a collective as well as an individual endeavor? (Ball et al., 2008, p. 395)

In their work, Ball et al. (2008) took a longitudinal, mixed-methods approach to investigate the practice of teaching mathematics through observations of teaching and specific, quantitative measures of MKT (cf., Hill et al., 2008). The team gathered qualitative data from video and audio recordings of classroom lessons, transcripts, student work samples, homework assignments, assessments, teachers' lesson plans, and teachers' reflections (Ball et al., 2008). Quantitative data came in the form of results from instruments designed and validated by members of the research team to assess MKT. In their analysis, Ball et al. (2008) considered a broad set of actions that they believed define the teaching profession (e.g., planning, evaluating students' work, interactive work of teaching lessons) and emphasized that each of those actions require strong content knowledge, "skills of mathematical reasoning, fluency with examples and terms, and thoughtfulness about the nature of mathematical proficiency" (p. 395). Effective teaching involves understanding, for example, how an algorithm works, why it works that way, how to engage students in tasks that highlight why it works, and "being able to size up the source of a mathematical error" when assessing students' thinking (Ball et al., 2008, p. 397). Through their work, Ball et al. (2008) concluded that "mathematical ability does not fully account for the knowledge and skills entailed in teaching mathematics" (Ball et al., 2008, p. 396), which supports the argument that mathematical knowledge for teaching must be addressed explicitly in teacher training and ongoing professional development.

The CBMS (2012) argued that student success is greatly dependent on strong educators in their field, and "widespread expertise is aided by high standards for entry into the profession and continual improvement of mathematical knowledge and teaching skills" (p. 2). To this point, in 2008, researchers at the National Council on Teacher Quality (NCTQ) published their report *No Common Denominator: The Preparation of Elementary Teachers in Mathematics by America's Education Schools*, strongly recommending that elementary teachers engage in extended and rigorous study of the mathematics they will ultimately teach (Greenberg & Walsh, 2008b). The report, based on a study of 77 education schools spanning the United States, focused on the mathematics programs required for prospective elementary school teachers to graduate by considering three factors: 1) relevance of coursework to the job of elementary school teachers; 2) breadth of coverage of mathematics topics; and 3) depth of attention to the essential topics (Greenberg & Walsh, 2008a). Several findings specifically addressed the level of mathematical depth of those programs; they are as follows (Greenberg & Walsh, 2008b):

- "Few education schools cover the mathematics content that elementary teachers need." (p. 23)
- "While most state education agencies issue guidelines for the mathematics preparation of elementary teachers, states do not appear to know what is needed." (p. 34)
- "The elementary mathematics in mathematics methods coursework is too often relegated to the sidelines. In particular, any practice teaching that may occur fails to emphasize the need to capably convey mathematics content to children." (p.

43)

- “Too often, the person assigned to teach mathematics to elementary teacher candidates is not professionally equipped to do so.” (p. 46)
- “Elementary mathematics courses are neither demanding in their content nor their expectations of students.” (p. 47)

With these findings, the NCTQ report highlighted the need for elementary teachers to deepen both their own mathematical knowledge and content knowledge *for teaching*. This was supported by results from the 2012 National Survey of Science and Mathematics Education, as cited earlier (Banilower et al., 2013).

Researchers have more specifically identified the need for both pre-service and in-service elementary teachers to deepen their understanding of fractions and strengthen the knowledge they need for teaching fraction concepts to their students (e.g., Ball, 1990; Ma, 1999; Newton, 2008; Post, Harel, Behr, & Lesh, 1991; Tirosh, Fischbein, Graeber, & Wilson, 1998). Indeed, a number of empirical studies have provided evidence that many prospective and practicing elementary teachers do not have a strong command of fraction concepts or procedures. In their study of 147 prospective elementary teachers, Tirosh et al. (1998) found that the pre-service teachers’ knowledge of fractions and student learning of fraction concepts was limited. They concluded the following:

> Most of [the prospective teachers] operated almost totally with rigid algorithms and procedural knowledge about rational numbers concepts, were generally unable to justify the successive steps of the algorithms, held primitive models of the operations, could not produce adequate representations of rational number concepts or operations

> with rational numbers…[and] were generally unable to list students' common ways of thinking about rational numbers. (Tirosh et al., 1998, p. 11-12)

Tirosh et al. (1998) claimed that these deficits stemmed from the prospective teachers' own "fragmented" knowledge that they attained in elementary school and were not adequately addressed in their teacher education programs. Evidence from their study supported this claim, as a number of the challenges their prospective teachers had in solving rational number problems mirrored those often demonstrated by students (e.g., multiplying and dividing fractions; adding fractions with unlike denominators) (Tirosh et al., 1998).

Post et al. (1991) provided further evidence of this claim in their study on elementary teachers' knowledge of fraction concepts. As part of their study, Post et al. (1991) assessed 218 elementary teachers (grades 4, 5, and 6) on their command of rational number concepts. In developing a "knowledge profile" of their participants, the researchers designed an assessment containing three main parts: 1) short-answer content items on various rational number concepts, such as part-whole, ratio, and computation; 2) items requiring explanations designed for students; and 3) a two-hour interview on rational number concepts and students' understanding of rational numbers. Results showed that a significant percentage of the teachers in the study answered fewer than 50% of the items correctly in each of the topic categories in Part 1 of the assessment, and approximately 30% of the teachers earned an overall score of less than 50% (Post et al., 1991). Results from Part 2 of the assessment showed that the teachers had difficulty providing sound explanations for their solutions, even when their solutions were correct. Results from Parts 2 and 3 led the researchers to conclude that teachers share a number of the "same misunderstandings and 'naïve conceptualizations'" that students have, such as *multiplication makes bigger and division makes smaller* (Post et al., 1991, p. 181). This echoed findings from previous research (e.g., Ball, 1990) and supported future research on prospective and practicing elementary teachers' mathematical knowledge (e.g., Klein & Tirosh, 1997).

Without proper and frequent opportunities to engage in mathematics or mathematics education-specific PD, deepening of both mathematical knowledge and that knowledge for teaching would prove challenging. Since 2007, in an effort to address current challenges in education, the National Center for Education Evaluation and Regional Assistance (NCEE) of the Institute of Education Sciences (IES) at the U.S. Department of Education has published guides to provide practical recommendations for addressing those challenges. With significant evidence that teachers "lack a deep conceptual understanding of fractions" (Siegler, Carpenter, Fennell, Geary, Lewis, Okamoto, Thompson, & Wray, 2010, p. 42), citing a number of the studies discussed in this review, a panel of researchers and practitioners in the field was brought together to develop the *Developing Effective Fractions Instruction for Kindergarten Through 8th Grade Practice Guide* (Siegler et al., 2010). The authors used empirical evidence gathered from a comprehensive collection of studies that evaluated teaching and learning fractions to inform the development of the guide and its five main recommendations. The first four recommendations focus on student learning of and approaches to teaching fraction concepts. The fifth recommendation focuses on the importance of PD programs specifically designed to "improve teachers' understanding of fractions and of how to teach them" (Siegler et al, 2010, p. 42), a call made by many others (e.g., Li & Kulm, 2008; Tirosh et al., 1998). The guide outlines three key ways in which the recommendation should be implemented. First, PD programs should "build teachers' depth of understanding of fractions and computational procedures involving fractions" by including activities that require them to solve problems, explore the meaning and justification for algorithms, and discussing challenges associated with solving those problems (Siegler et al., 2010, p. 43). Second, PD should "prepare teachers to use varied pictorial and concrete representations of fractions and fraction operations" by including activities in which they develop tasks for their students that integrate these representations (Siegler et al., 2010, p. 44). Third, PD programs should "develop teachers' ability to assess students' understandings and misunderstandings of fractions" by incorporating research on teaching and learning fractions as well as activities designed around critical analysis of student thinking (e.g., discussion of students' written work and/or video segments) (Siegler et al., 2010, p. 44).

The CBMS (2012) also provided recommendations for PD specifically designed to help meet the need for greater depth of mathematical knowledge for elementary teachers. In addition to reforms of elementary teacher preparation programs, the CBMS (2012) recommended that PD for classroom teachers be both content-focused and "directly relevant to the work of teaching mathematics" (p. 32). They provided ideas for social PD activities that bring mathematics specialists and other content experts together with practitioners to help strengthen teachers' MKT, such as solving problems and deeply exploring the mathematics in a professional learning community (i.e. fellow practitioners), analyze authentic student work (e.g., from participating teachers' classrooms), and participate in collaborative task design with colleagues (CBMS, 2012). Through these activities, engaging in mathematical practices, such as "reasoning abstractly", "constructing viable arguments" and using multiple representations and strategies to solve problems and communicate thinking and process, should be integrated throughout (CBMS, 2012, p. 33). Doing so would strengthen both teachers' knowledge of the mathematics and their skills in teaching the content to their students.

As described here, literature suggests that elementary teachers must be afforded opportunities, both in their preparation and their ongoing professional growth, to engage in learning that strengthens both their mathematical content knowledge and that knowledge needed for teaching, specifically of fraction concepts and procedures. This type of education is complex and multifaceted. The knowledge that teachers need to teach mathematics involves skills in many dimensions, and research suggests that overall, elementary teachers in particular are unfortunately not being prepared or supported in strengthening that knowledge (e.g., Banilower et al., 2013; Heck et al., 2008). This has great implications for MOOC-Eds for elementary educators, as developers of the PD experience can design programs that support educators in these multiple dimensions.

Principles of Effective PD

In his statement on the "Knowledge Gap in Professional Development", Elmore (2002) argued that in order for PD to be effective, it must be designed to address issues that are relevant and specific to teachers' practice and students' learning. He wrote that PD:

> must be tailored to address the difficulties encountered by real students in real classrooms as well as broader systemic objectives. Similarly, effective professional development is connected to questions of content and pedagogy that educators are asking—or should be asking—about the consequences of their instructional practices

> on real students as well as in general questions about effective teaching practice. (Elmore, 2002, p. 7)

He added that “professional development brings the general and the externally validated in contact with the specific and the contextual” (p. 7), making the point that PD can provide opportunities to take what is known (e.g., low test scores in algebra; declining student motivation) and take action to help understand and address what is known (e.g., PD on teaching specific algebra concepts; PD on engaging students in the classroom).

While connection between the “externally validated” and the “specific” and “contextual” with regard to educators’ practice is necessary for effective PD, it is not sufficient. Researchers have developed theories about and tested characteristics of PD programs that make them effective; defining and measuring the effectiveness of PD is however challenging and complex (Desimone, Porter, Garet, Yoon, & Birman, 2002; Herrington, Herrington, Hoban, & Reid, 2009; Ingvarson, Meiers, & Beavis, 2005). Sowder (2007) noted that effectiveness of PD programs for mathematics teachers is primarily measured by “teacher changes in knowledge of mathematics, beliefs about mathematics, and instructional practice, as well as increased student learning” (p. 169). Though she made the argument in a mathematics PD context, this view can be generalized to all instructional disciplines.

Guskey (2002) offered a linear *Model of Teacher Change* that begins with the PD experience as a *treatment*; teachers who receive effective PD thereby make changes in their classroom practice; once those changes have been implemented, teachers observe a change in student learning outcomes, which then affect changes in teachers’ beliefs and attitudes

(Guskey, 2002). Clarke and Hollingsworth (2002) similarly argued that effectiveness is measured by professional growth; however, their *Interconnected Model of Professional Growth* represents the non-linear process of professional growth and recognizes the "idiosyncratic and individual nature of teacher professional growth" (Clarke & Hollingsworth, 2002, p. 965). Other researchers, particularly in the field of online learning, promote the use of Social Network Analysis (SNA), a method of "mapping and measuring relationships and flows between connected information/knowledge entities" (e.g., people, web resources), to assess whether and how the online experience supports users' needs (Krebs, n.d.). While this type of analysis tends to focus more on groups of participants than on individuals themselves, in the context of professional learning, SNA can be used to study participants' interactions in digital networks (e.g., discussion threads) to identify the strength of collaboration and support as a measure of effectiveness of the learning opportunity (Fournier, Kop, & Sitlia, 2011; Kellogg, Booth, & Oliver, 2014).

While measures of effectiveness vary based on definitions of goals and "success" of PD programs, a number of key principles have emerged that describe effective programs. Elmore (2002) provided a "consensus view" of these principles (Figure 2), summarized through an extensive review of literature and standards adopted by the National Staff Development Council in 1995 (Sparks, 1995; Sparks & Hirsch, 1997, as cited in Elmore, 2002).

Professional Development: The Consensus View

- Focuses on a well-articulated mission or purpose anchored in student learning of core disciplines and skills
- Derives from analysis of student learning of specific content in a specific setting
- Focuses on specific issues of curriculum and pedagogy
    - Derived from research and exemplary practice
    - Connected with specific issues of instruction and student learning of academic disciplines and skills in the context of actual classrooms
- Embodies a clearly articulated theory or model of adult learning
- Develops, reinforces, and sustains group work
    - Collaborative practice within schools
    - Networks across schools
- Involves active participation of school leaders and staff
- Sustains focus over time—continuous improvement
- Models of effective practice
    - Delivered in schools and classrooms
    - Practice is consistent with message
- Uses assessment and evaluation
    - Active monitoring of student learning
    - Feedback on teacher learning and practice

Figure 2. Professional Development: The Consensus View (Elmore, 2002, p. 7)

Elmore (2002) did note that while theories proposed in the literature are “sensible” and can contribute to the design of effective PD for teachers, “the research is rarely grounded in hard empirical evidence about its effects on practice or on student learning” (p. 6). He admitted that the characteristics described in Figure 2 “embody some very heroic assumptions about the organizational context in which the activity occurs" (Elmore, 2002, p. 14), highlighting the point that the effectiveness of PD for educators depends on both the PD (design and implementation) and the system within which it is being provided (e.g., administrative support, infrastructure).

Some research both before and since Elmore's discussion has provided empirical evidence of what makes PD effective and will be presented in this review. Principles that have emerged in both theoretical and empirical research discussed here include: 1) a focus on content; 2) engagement in active learning (e.g., analyzing student work, implementing new practices); emphasis on student thinking; and, participants' ongoing learning and support.

Focus on content

It has been argued that in order to be effective in their practice, teachers must have a fundamentally strong command of the material they teach (CBMS, 2001, 2012). Indeed, Borko (2004) argued that in order "to foster students' conceptual understanding, teachers must have rich and flexible knowledge of the subjects they teach" (p. 5). Their command of and confidence in teaching the subject "affects both what they teach and how they teach it" (National Council of Teachers of Mathematics, NCTM, 1991, Standard 2-Knowing Mathematics and School Mathematics, para. 1). PD programs that focus on subject-specific content can help teachers enhance their understanding of their subject, thereby helping teachers be better equipped to teach the subject at a deep level (Borko, 2004; Monk, 1994; Sowder, Philipp, Armstrong, & Schappelle, 1998). With this enhanced understanding, teachers can better design and implement effective tasks and assessments, engage students in rich, conceptual discussions and anticipate students' questions and misconceptions. During classroom discussions, for example, teachers would have tools to ask higher-level thinking questions (the *whys* and *what ifs*) and confidently field responses with even greater depth, thereby fostering student learning.

As part of their work toward identifying "What Makes Professional Development Effective", Garet, Porter, Desimone, Birman, and Yoon (2001, Title) highlighted a "focus on content" as a "core feature" of PD that contributes to its effectiveness (p. 923). In their review of literature on the effectiveness of PD programs, they explained that the "content" focused on in PD programs can refer to different things. Some studies focused on "content knowledge" as knowledge specific to subject matter content, while others considered it the teaching strategies needed to teach subject-specific content (i.e., *pedagogical content knowledge*).

Research on PD programs that focus on strengthening participants' mathematical content knowledge has provided empirical evidence that such programs are more likely to enhance teachers' knowledge and skills (e.g., Garet et al., 2001) and are "contributing to major changes in teachers' instructional practices that have resulted in significant gains in students' achievement" (NRC, 2001, p. 391). This is consistent with findings from other empirical research on effectiveness of PD that focuses on enhancing subject matter knowledge (e.g., Ingvarson et al., 2005).

Research also suggests that PD programs that focus on both subject-specific content and pedagogical content knowledge have significant effects on teaching practices and student outcomes. Indeed, Corcoran (1995) suggested that such PD programs are critical in affecting changes in teacher practice. In their analysis of data collected from the Teacher Activity

Survey (TAS) as part of a national evaluation of the federally-funded Eisenhower Professional Development Program for teachers, Garet et al. (2001) found that PD programs that included a focus on content as a "core feature" had a substantial positive effect on participants' knowledge and skills (Garet et al., 2001, p. 933). Cohen and Hill (2000) found that programs that focused on content knowledge (subject matter and pedagogical content knowledge) had statistically greater effects on student outcomes than did programs that had a broader focus, a finding consistent with Kennedy's (1998) review. In their 2009 status report on teacher professional development, the National Staff Development Council (NSDC) found that a plurality of teachers surveyed (23%) ranked increased content knowledge as their highest priority in their professional growth (Darling-Hammond, Wei, Andree, Richardson, & Orphanos, 2009). With interest in enhanced content knowledge as a goal for teachers, this suggests that PD programs that facilitate that enhancement are effective.

Focus on active learning

Learning is enhanced when it is done purposefully and actively. Teachers should have opportunities to engage in active learning (e.g., solve problems, address questions, complete activities in the role of students) that promote their growth and help them "make sense of what they learn in meaningful ways" (Darling-Hammond et al., 2009). As part of their *Standards for Professional Learning,* Learning Forward (2011) argued that active learning provides participants with motivation to change through personally meaningful experiences:

> Active engagement respects adults as professionals and gives them significant voice and choice in shaping their own learning. Through active engagement, educators

> construct personal meaning of their learning, are more committed to its success, and identify authentic applications for their learning. Active learning processes promote deep understanding of new learning and increase motivation to implement it. (Learning Designs, para. 8)

This parallels the experiences teachers want to have for their students. Schorr, Warner, Gearhart, and Samuels (2007) made that connection with the following argument:

> Telling teachers, researchers, and teacher educators about better ways to teach or about their students' mathematical thinking, or even "modeling" the desired behaviors or activities for them is no more effective than "telling" students about an idea or "modeling" a solution for them and then expecting them to "understand" the concepts involved and be able to apply them to new or unique situations. (p. 445)

In order for learning, reflection, analysis, and growth to occur, teachers need to engage actively in their PD experiences. Lave and Wenger (1991) touted PD opportunities that are delivered in *situated* environments in which teachers collaboratively engage in activities just as their students would, as they provide laboratories for learning to grow. Indeed, they argued that "activities, tasks, functions, and understandings do not exist in isolation; they are part of broader systems of relations in which they have meaning" (Lave & Wenger, 1991, p. 53). Clarke (1994) included this as one of his ten key features of effective PD and contended that those settings provide teachers with rich opportunities to gain a "clear vision of the proposed changes" promoted by the PD program (Clarke, 1994, p. 38).

Herrington and Oliver (2000) developed a framework describing specific characteristics of situated learning environments in the context of professional development that aligns with much of Lave and Wenger's philosophy. The authors contended that these environments must do the following in order to be effective:

1. Provide *authentic contexts* that reflect the way the knowledge will be used in real life
2. Provide *authentic activities*
3. Provide access to *expert performances* and the modeling of processes
4. Provide *multiple roles and perspectives*
5. Support *collaborative construction of knowledge*
6. Promote *reflection* to enable abstractions to be formed
7. Promote *articulation* to enable tacit knowledge to be made explicit
8. Provide *coaching* and *scaffolding* by the teacher at critical times
9. Provide for *authentic assessment* of learning within the tasks. (p. 4)

Garet et al. (2001) found that PD programs that include active learning can have a positive impact on teachers' practice, a finding supported by many others (e.g., Learning Forward, 2011; Lieberman, 1996; Loucks-Horsley, Hewson, Love, & Stiles, 1998). This result was also highlighted by Ingvarson et al. (2005) in their review of evaluations of the quality and impact of PD programs performed by the Australian Council for Educational Research (ACER). Ingvarson et al. (2005) identified three primary measures of active learning in their review: (a) the extent to which the PD program provides opportunities for teachers' reflection on their practice, (b) the level at which teachers actively engage in identifying their own needs and goals for the PD program, and (c) the extent to which the PD program offers opportunities to implement new practices. Their study suggests that PD programs that include each of these *opportunities to learn* are effective in changing teachers' practice and positively affecting student achievement (Ingvarson et al., 2005). Ingvarson et al.'s three categories are discussed here to provide additional insight into their definition of active learning and its role in supporting effective PD.

Opportunities for reflection on practice. As it is defined, reflection refers to a "casting back" or image of what is being reflected on ("Reflect", def. 1). Reflecting on practice provides those who engage in it with an image of their work as they planned and implemented that practice, which gives them an opportunity to take a critical view of that work. These opportunities can be used, for example, to refine practices by considering alternate approaches to teaching certain topics, asking better-defined questions, and analyzing interactions with students during classroom discussions. They can also serve as time to identify effective strategies that can then be shared with other practitioners. Teachers in the United States, however, are not afforded many opportunities to actively plan, reflect and receive feedback on their practice (Clarke, 1994). One of the key principles from research on effective PD explains that such activities allow teachers to "report successes and failures to the group, to share 'wisdom of practice,' and to discuss problems and solutions regarding individual students and new teaching approaches" (Clarke, 1994, p. 38). Others have included these opportunities among components critical to providing effective PD, recognizing that reflection on practice is a fundamental tenet of adult learning (Darling-Hammond et al., 2009; Elmore, 2002; Lambert, 2003; Learning Forward, 2011). The exercise of exchanging ideas, experiences and challenges provides teachers with opportunities to learn effective (and ineffective) approaches to teaching, validate some of their practices, and build a sense of community populated by others who share similar struggles and successes. Through reflection, teachers “gain an awareness of their tacit assumptions, beliefs and views, and how these relate to their practice… [They] develop coherent rationales for their views, assumptions, and actions, and become aware of viable alternatives” (Thompson, 1992, p. 139). Teachers who are supported by colleagues who share common experiences and a sense of camaraderie are more likely to incorporate what they learn into their instructional practices (Simoneau, 2007). This may stem from a feeling of decreased vulnerability in testing new approaches in the classroom, as colleagues will engage in analysis and reflection about those approaches together, with a focus on improvement as opposed to criticism. Indeed, as Simoneau (2007) contended:

> Learning, defined as the process of constructing personal understanding through interactions with others while collectively engaging in challenges that are novel and transferable to other situations and settings, is transformational. (Abstract)

Actively engaging in identifying personal needs and goals. Effective PD programs recognize that “change is a process, not an event; [the] individual must be the primary focus of interventions; and change is a highly personal process” (Clarke, 1994, p. 39). Professionals base their learning in the needs of those they serve and approach learning opportunities with many of their own goals (Daley, 2001). Research suggests that PD programs that engage participants in identifying their own goals have a positive impact on teachers’ practice (Clarke, 1994; Ginsburg, Gray, & Levin, 2004; Hawley & Valli, 1999). Ginsburg et al. (2004) referred to this as PD that is *learner-centered,* or “addresses the learner’s perspective with respect to their pedagogical and content needs as well as their extremely limited time for professional training" (p. 2). This exercise provides participants with an opportunity to reflect on their needs and focus their attention and learning on components of the PD program that are most relevant to them (Sparks & Loucks-Horsley, 1989; Ginsburg et al., 2004). With this, participants can make direct connections between what they learn in the PD program and their practice, better ensuring positive change.

In his *Guide for Learners and Teachers*, Knowles (1975) defined *self-directed learning* (SDL) as:

> a process in which individuals take the initiative, with or without the help of others, to diagnose their learning needs, formulate learning goals, identify resources for learning, select and implement learning strategies, and evaluate learning outcomes." (p. 18)

deWaard et al. (2015), in their study of the existence of SDL on the part of participants in MOOCs, intimated that SDL is "personal" and critical to learner success, as it allows participants to determine whether and how they achieved what they sought in their engagement in the learning environment. This, thereby, increases the potential for self-directed PD to be effective.

Opportunities to implement new practices. A significant measure of effectiveness of a PD experience for teachers is impact on student learning outcomes (Ingvarson et al., 2005). In order to observe changes in those outcomes, teachers need opportunities to implement new practices, thereby giving them evidence of whether those practices have an impact. Webster-Wright (2009) argued that "professionals learn through practice experience, and that learning is contextually mediated" (p. 720). For example, a new approach to teaching a certain topic may sound effective in theory, but a teacher cannot anticipate the refinements they would need to make to the approach until they have presented it in the classroom. Through implementation, the teacher learns what works well for their students and how to adjust the practice to best serve their students. NCTM (1991) also recognized the need for teachers to have such opportunities in their *Standards for Professional Development of Teachers of Mathematics*; they viewed one of teachers' critical activities in promoting their own professional growth as "experiment[ing] thoughtfully with alternative approaches and strategies in the classroom" (Standard 6-The Teachers' Role in Professional Development, bullet 1). PD programs that encourage teachers to test new practices in the classroom have been shown to be effective in changing teacher practice and affecting student learning outcomes (Guskey, 1986; Ingvarson et al., 2005). These experiences can provide rich opportunities to reflect on what happened in the classroom as a result of the implementation, thereby engaging participants in the cycle of active learning.

Emphasis on student thinking

PD programs that facilitate understanding of student learning are effective in promoting professional growth and affecting student outcomes (Darling-Hammond et al., 2009; Kennedy, 1998). With understanding and analysis of student thinking, teachers can better anticipate and address their students' needs. With a lens into how students learn specific content, teachers can better design their lessons (choosing meaningful tasks), address misconceptions, and better understand what prior knowledge their students have before entering their classroom. Sztajn, Confrey, Wilson, and Edgington (2012) posited that increased focus on teachers' understanding and incorporation of students' *learning* *trajectories* (LTs), or *learning progressions*, into their practice demands an increased focus on facilitating that link from knowledge and understanding to practice. PD programs that are designed to help facilitate that link can be effective in impacting teachers' practice and affecting student outcomes. In his 3-month study on teachers' use of LTs in their instruction, Wilson (2009) found that the LTs provided teachers with valuable knowledge and skills in selecting meaningful, effective tasks for their students, facilitating classroom discussion, and analyzing student work. Mojica (2010) found similar results in her study of teachers and the effects of LT-based PD on their knowledge and instructional practice. She found that LTs enhanced teachers' mathematical content knowledge and helped equip them with skills to better understand their students' thinking (Mojica, 2010). This was further evident in work done by Clements, Sarama, Spitler, Lange and Wolfe (2011) in their examination of the effectiveness of research-based teaching focused on LTs. Clements et al. (2011) found that, with PD support for teachers, instruction based on LTs had a positive effect on student achievement as measured by growth in their mathematical knowledge.

In their work on PD focused on *Cognitively Guided Instruction* (CGI), which is designed to build teachers' understanding of how students' mathematical thinking develops, Carpenter, Fennema, Peterson, Chiang and Loef (1989) found that teachers' understanding of student thinking served to positively affect their practice and their students' outcomes. For example, after participating in an intensive CGI workshop, teachers were observed to pose rich questions and listen more carefully to their students than those who had not participated in the workshop (Carpenter et al., 1989). With these practices, teachers can better assess their students' knowledge and make appropriate and informed subsequent decisions on how to proceed (Carpenter et al., 1989). Participants of the workshop were also observed to encourage students to use multiple strategies when solving problems, which promoted students' mathematical flexibility and self-confidence (Carpenter et al., 1989). Carpenter et al.'s (1989) work provides additional evidence that PD programs that include a focus on understanding students' thinking are effective in promoting changes in teachers' beliefs and practice as well as their students' self-efficacy and achievement.

Sustained, ongoing learning and support

PD programs that offer ongoing support and guidance, with a particular focus on interaction with and collaboration of participants, have a positive impact on teachers' practice (Abdal-Haqq, 1996; Ball & Cohen, 1999; Hawley & Valli, 1999; Ingvarson et al., 2005; Wilson & Berne, 1999). Changes in attitudes, beliefs and practice take significant time and effort to evolve (Clarke, 1994). Without sustained, ongoing work to facilitate the changes, it is more difficult to experience growth. Indeed, as part of their *Standards for Professional Learning*, Learning Forward (2011) noted the following:

> Any single professional learning activity is more likely to be effective in improving educator performance and student learning if it builds on earlier professional learning and is followed up with later, more advanced work to become a part of a coherent set of opportunities for ongoing professional learning. Coherence also ensures that professional learning is a part of a seamless process that begins in the preparation program and continues throughout an educator's career. (Outcomes, para. 7)

Daley (2001) also expressed the importance of sustained, ongoing learning by noting that "professionals make meaning by moving back and forth between continued professional education and their professional practice" (p. 39). These learning opportunities could include teachers' analysis of their own teaching through videos as they view them with colleagues during planning periods or regular professional learning community (PLC) meetings. These activities can be used as "case studies" in which teachers discuss strengths and weaknesses of the episodes and develop strategies for improvement (Clements et al., 2011). Such exercises provide teachers with ongoing and frequent reflection within a teacher's own context that is critical to professional learning (Borko, Stecher, Alonzo, Moncure, & McClam, 2005**;** Daley, 2001; Webster-Wright, 2009).

In their study of PD experiences for teachers of 'gifted' students, Wycoff, Nash, Juntune, and Mackay (2003) found that "few teachers made a meaningful transition between what was learned in their in-service and what was implemented in the classroom" (p. 39), suggesting that in order to make a meaningful transition, teachers need opportunities to connect what they learned in their PD activities to their practice. These opportunities are frequently facilitated by ongoing support from others (e.g., colleagues, PD facilitators, teacher leaders) to revisit, reflect on and assess professional growth as teachers implement new practices. Ongoing support from colleagues is a crucial component of effective PD programs, as it builds motivation to make a commitment to continue to grow and learn (Banilower, Boyd, Pasley, & Weiss, 2006; Darling-Hammond et al., 2009). Researchers spanning decades have contended that a connection to a supportive learning environment or community of practice (CoP) serves as one of the most valuable components of a PD program (Clarke, 1994; Hawley & Valli, 1999; Johnson, 2001; Kop, Fournier, & Mak, 2011; Wenger, 2000). For purposes of this discussion, *communities of practice* will be defined as

"Groups of people who share a concern or a passion for something they do and learn how to do it better as they interact regularly" (Wenger, n.d., p. 1). Lave and Wenger (1991) articulated their view that learning occurs when it is socially mediated through such communities of practice:

> Learning is a process that takes place in a participation framework, not in an individual mind. This means, among other things, that it is mediated by the differences of perspective among co-participants. It is the community…who learn under this definition. (p. 15)

Bandura (1977) made a more urgent argument by claiming that "Learning would be exceedingly laborious, not to mention hazardous, if people had to rely solely on the effects of their own actions to inform them what to do" (p. 22).

Learning is enhanced by social activities that provide for open discussion and exploration among colleagues. Indeed, Lambert (2003) highlighted the benefits of an "interactive professional culture" as it fuels the "generative" process of professional growth (p. 423). This "generative" process is characterized by continued addition, adaptation, and revision of current understandings to become new understandings. This is done by "applying learners' knowledge to learn new topics and solve new and unfamiliar problems" (Franke, Carpenter, Levi & Fennema, 2001, p. 656). Lesser and Storck (2001, as cited in Smith, 2003) explained that "the social capital resident in communities of practice (CoPs) leads to *behavioral* change that results in greater knowledge sharing, which in turn positively influences…performance" (p. 5), suggesting that quality CoPs enhance the effectiveness of PD programs.

As is evident from this discussion, research, both empirical and theoretical, has identified features of PD that can play a role in its effectiveness in changing participants' knowledge, beliefs and practice. The principles implemented in the design of the MOOC-Ed in this study align with many of these features, including a focus on active learning through meaningful tasks (*job-connected learning*) and opportunities to participate in a community of practice (*peer-supported learning*). The ideas discussed in this section informed this study by highlighting features that would be further investigated in the collection and analysis of data. Other principles, such as a focus on content (pedagogical and/or mathematical) and student thinking, informed this study by helping to identify themes in qualitative responses and contributing to the collection and analysis of data through interviews and observations.

Online PD

With recent advances in technology and interest in using them to offer alternative methods of PD, the number of online PD (OPD) opportunities has increased dramatically. The NRC (2007) claimed that:

> The provision of professional development through online media has had a significant influence on the professional lives of a growing number of teachers. Growing numbers of educators contend that online teacher professional development (OTPD) has the potential to enhance and even transform teachers' effectiveness in their classrooms and over the course of their careers. (p. 2)

As a result, researchers with diverse foci have begun to study such opportunities. In their work on a research synthesis of 40 studies on OTPD, Whitehouse, Breit, McCloskey, Ketelhut, and Dede (2006) identified five "main areas of concern" of these studies: 1) design of PD; 2) effectiveness of PD; 3) technology to support PD; 4) online communication and PD; and, 5) research methods (p. 16). They noted that, unfortunately:

> Few studies were concerned with measuring how effectively the desired educational improvements at the heart of these interventions or programs were realized…Rather, the studies tend to look more closely at…effects such as discourse patterns, contextual influences, and the formation of practice communities with conditions that support interaction, collaboration, participation, and teacher self-efficacy within these communities. (Whitehouse et al., 2006, p. 20)

This suggests that this case study is well-situated to contribute to the discussion of OPD that can have measurable impact on its participants.

In this section, I review literature on the benefits and challenges of OPD, principles of effective OPD that have considerations unique to online environments, and MOOCs as online resources for professional learning.

Affordances and constraints. Some researchers argue that online PD offers a number of valuable benefits for its participants. One is that OPD programs provide increased opportunities for sustained, collaborative and meaningful work among teachers, as they connect practitioners who may not otherwise engage with each other (Boling & Martin, 2005; NRC, 2007; Thomas, 2009). Boling & Martin (2005) touted OPD programs as providing "supportive learning environments" (p. 2), and others have highlighted the value of the opportunities afforded by online PD as being available "anytime, anywhere", encouraging participation by those who may not otherwise have access to programs (NRC, 2007; Song & Hill, 2007; Thomas, 2009; Vrasidas & Zembylas, 2004). In her mixed methods study on identifying effective models of online PD for teachers, McNamara (2010) noted growing interest among teachers in participating in OPD activities because such activities are viewed as saving time and money and providing personally relevant instruction. Moreover, participants in online PD have praised the model as fostering an environment that allows for continuous training and support--two significant measures of effective PD in general--that traditional face-to-face PD does not typically afford, as teachers can engage in PD activities while they are actively working (McNamara, 2010; NRC, 2007; Thomas, 2009). This is supported by Dede, Ketelhut, Whitehouse, Breit, and McCloskey (2009) in their explanation that online PD can provide "real-time, ongoing, work-embedded support" (p. 10).

As an added benefit, one seldom mentioned in the literature, is that participants in online PD have opportunities to improve their skills in using technology (e.g., engaging with the online PD platform to participate in discussions and exchange resources electronically) (NRC, 2007). Indeed, in her study on OTPD, McNamara (2010) found that participants became more motivated to integrate technology into their teaching, thereby offering their students a richer learning experience. The technology they incorporated into their classroom instruction (e.g., SMART board, wikis) was not necessarily the same as that used in the PD experience, but teachers seemed more confident in using technology for teaching after increased familiarity with it through the online experience (McNamara, 2010).

Research has also identified a number of constraints of online learning environments. The NRC (2007) highlighted that, due to the nature of the medium, participants of OPD would need regular and reliable access to the technologies needed to engage in it. For some, this is not always possible. Further, the NRC (2007) cautioned that engaging in online learning can present opportunities for participants to become distracted from what they are learning, perhaps by accessing unrelated online resources:

> Content and use of online technologies…can isolate some users just as easily as they can build community among others. Today, for some [people], online technologies more often act as a distraction from learning than as a tool for learning. (p. 20)

The NRC (2007) also highlighted challenges associated with transferring what is learned from online resources into practice. Indeed, they quoted Chris Dede as acknowledging the following:

> It is hard to watch somebody else's practice richly captured and then transfer it into your own strengths, your own material, your own students, and your own context…It is a very demanding, labor-intensive, and expensive form of pedagogy. (as cited in NRC, 2007, p. 21)

Song and Hill (2007) emphasized that communication in online learning environments can present a challenge unique to the medium, as the "lack of facial expressions and body language in written communication may lead to misinterpretation" (p. 34), thereby potentially limiting productive discussion among participants as part of a learning community. They cautioned, too, that participants may not necessarily be motivated to engage in discussions with rich and meaningful thoughts, as the amount of time it takes to communicate in writing is often greater than that needed in doing so orally. Song and Hill (2007) also pointed to the timing of responses to discussion posts as another challenge to

OPD, as delayed responses from facilitators and peers can affect participants' motivation to continue their engagement.

Principles of effective OPD. Recent research on OPD has identified design principles that help inform the development and implementation of future OPD opportunities that can have an impact on participants. Many align closely with those of effective PD in general, such as focus on content, active learning, and facilitation of communities of practice. Some have unique considerations in online environments. Those discussed here are: 1) personalization and choice; 2) opportunities for discussion and interaction; and 3) the facilitators' role in the online environment.

Personalization and choice. In their mixed methods study on *Motivation and Learning in an Online, Unmoderated Mathematics Workshop for Teachers*, Renninger, Cai, Lewis, Adams, and Ernst (2011) argued that OPD can be effective in changing teacher beliefs and instructional practices if it is designed to address the varied needs and abilities of the participants, a key point made by others (e.g., Guskey, 2000; Yang & Liu, 2004). Indeed, Renninger et al.'s findings suggested that:

> the potential of [OPD] lies in its designers' abilities to support participant stake by providing for multiple ways into thinking and working with disciplinary content—design that both accommodates and supports those with differing strengths and needs. (2011, p. 229)

This can be interpreted as indicating that one of the objectives of OPD programs should be to provide enough choice, or *personalization*, among the included activities (e.g., varied tasks, opportunities for reflection on different practices) to give participants options to engage with components that are most useful to them and their practice (Ginsburg et al., 2004). Vrasidas and Zembylas (2004) supported this by suggesting that designers be especially mindful of the activities they include in their program, as meaningful, accessible and relevant tasks encourage participants to then apply their knowledge to the classroom. Indeed, McNamara (2010) showed that participants valued the opportunity OPD affords to engage in activities that address pedagogical and content issues specifically relevant to them and at their own pace. The NRC (2007) went further and cited participants in a PD workshop they studied as saying that an additional way to provide choice and personalization is to "involve [participants] in the development of materials, so that online tools reflect what [they] want and need" (p. 20). This discussion supports the principle that OPD be designed with special consideration for providing choice and opportunities for personalization of the experience to its participants.

Opportunities for discussion and interaction. One principle that parallels that described in the context of general PD is the development and facilitation of an online community of practice (CoP). Much of the literature on online CoP's suggests that they serve as a benefit of online PD that is not always afforded in traditional face-to-face PD (e.g., Breslow, Pritchard, DeBoer, Stump, Ho, & Seaton, 2013; Mackey & Evans, 2011; Sheu, Lee, Bonk, & Kou, 2013). There has been increased interest in recent years in designing online learning programs to support CoPs as the interest in online professional communities has grown. PD designers are beginning to recognize the opportunities online CoPs provide for sustained, ongoing and supportive PD (Vrasidas & Zembylas, 2004). In her multiple case study on CoPs in a blended PD model (both synchronous and asynchronous components), Wagner (2010) investigated the presence of Wenger's (2000) four characteristics of effective CoPs (*shared practice, mutual communication among members of the community, meaning making*, and *shared minimum knowledge)* and Johnson's (2001) eight characteristics of effective *online* CoPs (*varied levels of expertise, relevant tasks, movement from "novice to expert," constructivist approach, shared knowledge, trust, scaffolding, and participant facilitation)* in two blended programs for teachers (as cited in Wagner, 2010, p. 367). While the two programs Wagner (2010) analyzed contained only some evidence of each of these characteristics, and her review of literature cautioned that research on the formation of CoPs in online PD programs is limited, Wagner's results suggest that there is great potential for online PD programs to provide the infrastructure and environment that foster the development of CoPs, a finding supported by others. Indeed, citing Mackey's case study on connections between social learning theory and online CoPs, Mackey and Evans (2011) argued that online CoPs provide members with "extended access to resources and expertise beyond the immediate school environment" (p. 11), thereby offering ongoing PD and the potential for increased application of learning into the classroom. In their multiple case study on online PD for teachers, exploring the "impact and transfer of knowledge" (p. 191), Herrington et al. (2009) found that teachers succeeded in implementing new pedagogical strategies in their classrooms when they felt supported by their online CoP. Thus, online CoPs can provide an opportunity for teachers to receive ongoing support and guidance that foster professional growth.

In order to maximize the benefits that CoPs provide, designers of online PD programs must be creative in building the infrastructure necessary to support such communities, as participants have the challenge of not being physically in the same place when engaging in online activities. Simoneau (2007) argued that “online learning communities organized around collaborative inquiry and collective problem solving become co-creators of knowledge in a risk free, trusting environment” (Abstract). Programs that include asynchronous discussion forums where participants respond to carefully-designed, relevant prompts provide opportunities for participants to reflect on their practice, exchange ideas, and discuss strategies for improvement on their own schedules and with colleagues they may not interact with otherwise. Indeed, Treacy, Kleiman, and Peterson (2002) made the following argument on this point:

> Because participants and facilitators are able to take time to prepare comments and responses, online discussions can be more reflective than synchronous discussions or face-to-face workshops, and provide all participants ample opportunity to contribute to the discussion. A record of each online discussion is kept automatically, so participants and facilitators can always review previous discussions to build on them in later discussions. This also contributes to the depth and inclusiveness made possible by the learning community model. (p. 43)

It is important, then, that OPD providers construct thoughtful prompts and questions that are open-ended to a degree that motivates participants to engage in discussions that are relevant to them. All of this suggests that opportunities for interaction and discussion offer participants the opportunity to engage in learning that will be sustained and relevant, as knowledge is enhanced through the exchange of thoughts and insights among a CoP, and skills are developed with a focus on specific needs (Simoneau, 2007).

Facilitators' role. Empirical evidence suggesting that the role of a facilitator in OPD programs has a significant effect on participants and their practice is rather limited (Russell et al., 2009). Researchers in the field, however, theorize that the support and skill of the facilitator can play a significant part in programs' effectiveness (e.g., Anderson, Rourke, Garrison, & Archer, 2001; Treacy et al., 2002; Yang & Liu, 2004). A common theme in the literature suggests that such programs can be enhanced by facilitators' attending specifically to participants' needs; this may be more challenging in an online environment, as opportunities for immediate exchange of communication are not as frequent (Russell et al., 2009). Indeed, Song and Hill (2007) cautioned that "delayed response time from the instructor makes it a difficult task for online learners to effectively take advantage of the instructor as an expert human resource in their online learning" (p. 33). Researchers emphasize the importance of a facilitator who provides frequent, meaningful, and focused feedback (Wagner, 2010; Wenger, 2000; Yang & Liu, 2004). Facilitators play a key role in creating an environment that invites multiple perspectives, mediating discourse in discussion forums, and motivating continued participation (Anderson et al., 2001). Indeed, "more meaningful discourse arises only when facilitators request participants to analyze, compare, and reflect upon concrete cases" (Nemirovsky & Galvis, 2004, as cited in Russell et al., 2009, p. 446). Citing participant attrition and lack of motivation as some of the most significant challenges of OPD, Wagner (2010) found that strong facilitation and leadership helps to prevent such issues.

Massive Open Online Courses (MOOCs). *History.* In an effort to make education available and accessible, particularly to underserved populations, the OpenCourseWare

(OCW) project was initiated at the beginning of the 21st century by MIT president Dr. Charles Vent (Sheu, Lee, Bonk, & Kou, 2013). The project made course materials available in an open access, online environment. Portals and repositories of open courses and resources for mass consumption and self-directed learning, known as Open Educational Resources (OER), emerged thereafter (Sheu et al., 2013). One very recent addition to the field of online learning that emerged out of the work done in the OCW and OER projects is that of MOOCs. MOOCs made their first appearance in the form of university-level courses in the late 2000s, the first of which, called *Connectivism and Connective Knowledge,* was based on the theory of *connectivism* designed and delivered by Dr. George Siemens at Athabasca University and Dr. Stephen Downes of the Canadian Research Council in 2008 (Brooks, 2012; Daniel, 2012; Sheu et al., 2013). The theory recognizes that learning is done on a continuum and that this continuum is characterized by making connections between different ideas and concepts across fields (Siemens, 2005). Key principles of connectivism include that "learning and knowledge rests in diversity of opinions" and "learning is a process of connecting specialized nodes or information sources" (Siemens, 2005, p. 4). The theory served as both the content focus of Siemens' and Downes' course and as the framework under which MOOCs emerged, using the platform to connect students with different opinions at scale and connecting "nodes" of information within them. This platform was well-situated to serve as a medium through which professional development for educators could be delivered (Kleiman, Wolf, & Frye, 2014).

Over the span of roughly 5 years, with increased technological and analytic capabilities as well as increased demand for open and accessible information, MOOCs became a more significant part of the language in online education, ultimately making international news as the new form of open and accessible education (Pappano, 2012). Generally, they are described as open (i.e., accessible by anyone with internet access), online courses with a large number of participants enrolled (i.e. "*massive*"). MOOCs are designed and delivered in a variety of styles in order to serve different target populations and provide diverse experiences for learners, depending on the learning goals for participants (Clark, 2013).

As noted in Chapter One, in recognizing the potential for MOOCs to serve as valuable opportunities for continued professional learning, the Friday Institute for Educational Innovation began developing MOOCs for PD for educators, launching their first course in April 2013 and designed for technology leaders in educational settings (Kleiman et al., 2014). Along with the course, as part of the larger initiative, the Friday Institute launched their website *mooc-ed.org* as a platform for research, information about and development of what they titled "MOOC-Eds", or MOOCs for Educators (Kellogg et al., 2014). A few months later, in partnership with seven established university education programs such as University of Washington's College of Education, the Curry School of Education at UVA, Johns Hopkins University's School of Education, and Peabody College of Education and Human Development at Vanderbilt University, Coursera (2013) began launching MOOCs for Teacher Professional Development on topics such as curriculum development to blended learning models.

MOOC research. As researchers began to study MOOCs as an effective learning option, questions about how to measure "success" or "achievement" in them were explored.

These were complex questions, as participants had different measures of success, based on their own goals. Indeed, DeBoer et al. (2013) posited that success in MOOCs needs to be *reconceptualized*—away from definitive measures of "completion" or "level of engagement" and more toward participants defining success for themselves. This sentiment was echoed by Breslow, Pritchard, DeBoer, Stump, Ho, and Seaton (2013) in their research on the first MOOC offered in a joint effort between MIT and Harvard in 2012. The course on "Circuits and Electronics", was taught by members of the MIT faculty and edX scientists (Breslow et al., 2013). Two major classifications of "success" emerged from the results of their work: *Success as Achievement* (e.g., grades earned) and *Success as Persistence* (i.e. continuing through the duration of the course) (Breslow et al., 2013). Results from their examination of describing *Success as Persistence* proved to be particularly complex, as "persistence" can be defined in multiple ways in the context of MOOCs. For example, persistence may be measured by "interaction with any part of the course in a subsequent week" or "interaction with a specific course component in any subsequent week" (Breslow et al., 2013, p. 22). This speaks to the challenge of defining and measuring success in, and therefore defining and determining the *effectiveness* of, an online learning medium such as MOOCs.

Success may be defined by the level of sustained interaction and peer support that participants experience in a MOOC. MOOCs can provide for extensive connections between participants through both asynchronous (e.g., discussion boards) and synchronous means (e.g., online webinars), promoting peer support (Kellogg et al., 2014), and self-directed learning (McAuley, Stewart, Siemens, & Cormier, 2010). In their case study on the *Digital Learning Transition* MOOC-Ed designed for educators, through social network analysis, Kellogg et al. (2014) found that MOOCs can be effective in facilitating supportive connections among participants that result in growth in knowledge.

Much has been written both in the research literature and in the media about the striking level of attrition of students in MOOCs and whether that attrition can serve as a measure of MOOCs' success (or lack thereof). Daniel (2012) noted that some institutions, including MIT and Coursera (from Stanford University), have had to defend low retention rates in their MOOC offerings and the value of developing courses despite that. For example, in response to concerns about the low completion rate in the "Circuits and Electronics" course discussed earlier, Anant Agarwal, the president of edX and professor of Computer Science and Engineering at MIT, argued that "if you look at the number in absolute terms, it's as many students as might take the course in 40 years at MIT" (Hardesty, 2012). Agarwal also defended the low completion rate by noting that the "Circuits and Electronics" course required a significant number of high-level prerequisite courses in physics and mathematics and that future courses will be advertised with more clear prerequisites needed in order to succeed in them (Hardesty, 2012).

In their discussion on interpretations of attrition rates, deWaard et al. (2015) cited a few, more optimistic, perspectives. In his discussion on learning analytics in the context of MOOCs, and the large "funnel of participation" characteristic of them, Clow (2013, as cited in deWaard et al., 2015) suggested that some participants "climb out", as opposed to "drop out", of MOOCs, since that they may have met their self-determined goals. deWaard et al. (2015) described this as a phenomenon connected to self-directed learning, since, as they cite, "in an open online learning environment, the control of learning no longer rests with an educational institution but with the learners themselves" (Fournier, Kop & Durand, 2014, p. 2). deWaard et al. (2015) further suggested that "early exit [from a MOOC] can be an indication of failure to self-direct and manage personal time and resources" (p. 236).

Sheu et al. (2013) also addressed issues of attrition and highlighted a number of examples of MOOCs for which the issue was clear (e.g., MOOCs@Edinburgh, 2013). Even further, DeBoer et al. (2013) noted that "users are neither accountable to nor interested in completing the full class experience" (p. 4). They argued, however, that the data does not necessarily suggest that this signals a failure of MOOCs. Indeed, the report issued by MOOCs@Edinburgh (2013), providing key findings in an analysis of six MOOCs offered at the University of Edinburgh, noted a high satisfaction rate by students enrolled in the courses. Specifically, the report indicates that 77% of respondents felt "good" about their MOOC experience, and 98% responded that the course they took either completely met or exceeded their expectations (MOOCs@Edinburgh, 2013).

This suggests that success of MOOCs for participants can be measured in as many ways as there are participants enrolled. Each participant enrolls with their own set of goals (e.g., gather information solely on one topic presented in the course, complete the course for certification), and success may be measured by the extent to which the participants believe they achieved those goals. This also suggests that success of a MOOC for the MOOC designers can be measured by the extent to which their participants believe the MOOC was successful for them.

MOOCs for PD. As the discussion in the previous section makes clear, views differ in the value of MOOCs as professional learning opportunities. However, literature does suggest that MOOCs are unique in what they can offer with regard to scalable professional development, including accessibility and flexibility, reduced time commitment and cost for participants, and a connection to a supportive learning environment or community of practice (Boling & Martin, 2005; Breslow et al., 2013; Daniel, 2012; Sheu et al., 2013), all of which are considered benefits of OPD. Even further, MOOCs have the potential of connecting teachers with colleagues across regions, thereby "widening perspectives and fostering professional connections that would not occur otherwise" (Russell, Carey, Kleiman, & Venable, 2009, p. 71). In MOOCs, participants are often allowed to navigate their own way through the courses, identifying content that is useful and relevant to them (DeBoer et al., 2013; McAuley et al., 2010), which has also been touted as a benefit of OPD (e.g., deWaard et al., 2015). Moreover, while research on MOOCs for PD is in its infancy, findings from Kellogg et al.'s (2014) case study on the potential of MOOCs to foster peer-supported learning and knowledge construction through social networks support the notion that MOOCs have potential as effective models of PD. Indeed, through social network analysis, Kellogg et al. (2014) found that the majority of discussions in the two MOOC-Eds they studied contributed to participants' co-construction of knowledge.

Summary

In this chapter, I first reviewed literature on the mathematical education of elementary teachers to make the case that this study is well-situated to help address the need for ongoing, deeper learning of mathematics for elementary educators. I included specific discussion of literature that addressed pre- and in-service teachers' mathematical knowledge for teaching fractions, as the MOOC-Ed in this study was designed to help address the need for high-quality PD on the topic. The section also supports the inclusion of measures of effectiveness of the MOOC-Ed that are informed by both self-reported and assessment data related to changes in mathematical content and pedagogical knowledge as well as the potential impact the PD has on participants' practice.

The second section of this review examined literature on principles of effective PD, which helped to highlight features and characteristics of PD that can serve as catalysts to changes in participants' knowledge, beliefs and practice. These include features related to content-focus, engagement in active learning, emphasis on student thinking, and participants' peer support. The review helped to frame the examination of how the effectiveness of the MOOC-Ed in this study would be assessed, as it narrowed the focus on particular characteristics of PD that have been found to contribute to changes.

In the final section, I reviewed literature on online PD and its potential to provide opportunities for effective professional development. Both affordances and constraints were studied in order to identify how PD delivered online can serve as a benefit to some but not all participants. Principles of effective OPD were also examined in attempts to identify how OPD can provide unique opportunities for participants and how they can promote changes in knowledge, beliefs and practice. Literature on MOOCs was reviewed in order to first understand the history of their emergence in the field of online learning and their potential as opportunities for professional learning. Research on MOOCs was reviewed in order to identify how their "success" or "effectiveness" was being examined and to identify gaps in the literature.

Each section in this chapter helped to inform the development of survey and assessment instruments, and interview and observation protocols, as well as the methods of analysis of results to help assess the effectiveness of the MOOC-Ed in this study in changing participants' knowledge, beliefs and practice.

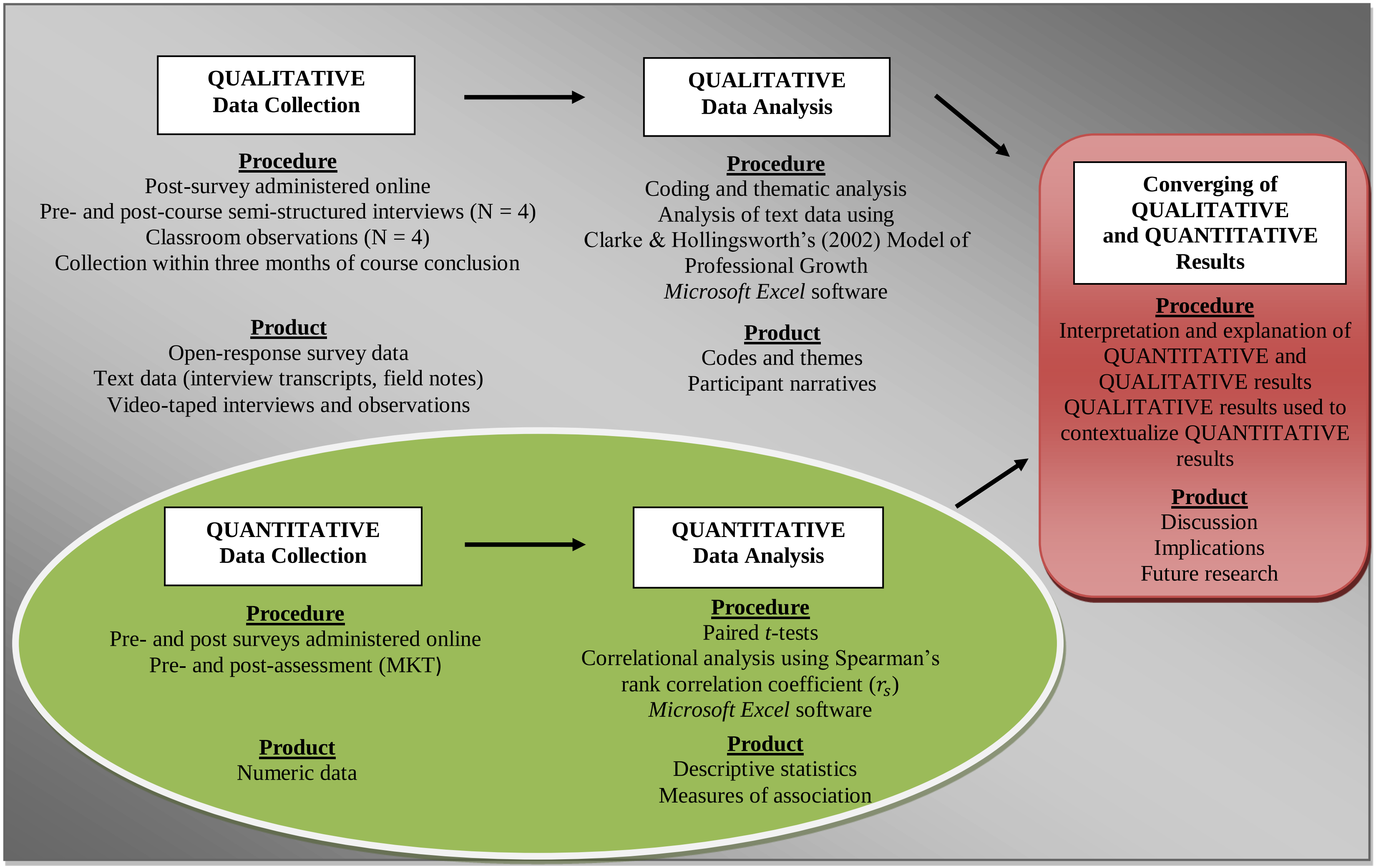
CHAPTER THREE

This study was conducted using a mixed methods approach, as its purpose was to examine both the extent to and ways in which a MOOC-Ed can serve as effective PD. A number of definitions of mixed methods research have emerged in attempts to foster a common understanding of the "third methodological movement" (Johnson, Onwuegbuzie, & Turner, 2007, p. 118). The following definition was adopted for this study:

> Mixed method inquiry is an approach to investigating the social world that ideally involves more than one methodological tradition and thus more than one way of knowing, along with more than one kind of technique for gathering, analyzing, and representing human phenomena, all for the purpose of better understanding. (Greene, 2006, as cited in Johnson et al., 2007, p. 119)

Specifically, a convergent, parallel mixed methods design (QUAL+QUAN) was used, in which quantitative and qualitative data informed a case study (Creswell & Clark, 2011; Morse, 2003). The case analyzed herein is the *Fraction Foundations: Helping Students Understand Fractions* MOOC-Ed course; the participants' experiences in this course informed the study of the case. The convergent, parallel design was appropriate for this study as the purpose was to "obtain different but complementary data on the same topic" (Morse, 1991, p. 122), primarily simultaneously, and "[synthesize] results to develop a more complete understanding of [the] phenomenon" (Creswell & Clark, 2011, p. 77) that was the PD experience. A visual model of the methodology used in this study is provided in Figure 3.

Figure 3. Visual Model for Convergent, Parallel Mixed Methods Design


QUALITATIVE Data Collection
Procedure
Post-survey administered online
Pre- and post-course semi-structured interviews (N = 4)
Classroom observations (N = 4)
Collection within three months of course conclusion
Product
Open-response survey data
Text data (interview transcripts, field notes)
Video-taped interviews and observations
QUALITATIVE Data Analysis
Procedure
Coding and thematic analysis
Analysis of text data using
Clarke & Hollingsworth's (2002) Model of
Professional Growth
Microsoft Excel software
Product
Codes and themes
Participant narratives
Converging of QUALITATIVE and QUANTITATIVE Results
Procedure
Interpretation and explanation of
QUANTITATIVE and
QUALITATIVE results
QUALITATIVE results used to
contextualize QUANTITATIVE
results
Product
Discussion
Implications
Future research
QUANTITATIVE Data Collection
Procedure
Pre- and post surveys administered online
Pre- and post-assessment (MKT)
Product
Numeric data
QUANTITATIVE Data Analysis
Procedure
Paired *t*-tests
Correlational analysis using Spearman's
rank correlation coefficient (r_s)
Microsoft Excel software
Product
Descriptive statistics
Measures of association

This MOOC-Ed is of particular interest for a number of reasons. With the focus on a high-need topic for different groups of educators, the *Fraction Foundations* MOOC-Ed provided the potential for a case with data from participants with multiple perspectives and diverse needs (e.g., classroom teachers, teacher coaches, content specialists). Further, with its focus on a topic in mathematics, the MOOC-Ed provided a case in which I as the researcher could better observe and analyze changes in participants' knowledge, beliefs and practice, as my background and expertise is in teaching mathematics. Moreover, the course was developed after lessons had been learned through development and implementation of a number of MOOC-Ed courses, thereby providing a course in which design principles had been refined and then implemented. The study would then investigate whether a MOOC-Ed developed to align with those design principles was effective. Other PD providers and developers can generalize the findings of the case study as they use what they learned here in their own design and development.

Description of Course Context

Recognizing the critical need for professional development in K-12 mathematics, the Friday Institute for Educational Innovation at North Carolina State University considered developing MOOC-Eds on topics for which those needs were most critical. Decisions on course subjects were based on research on professional development and teacher education; *fractions* emerged as one of those subjects (e.g., Ball, 1990; Post et al., 1991; Tirosh et al., 1998). As part of the MOOC-Ed initiative, with a team of K-12 mathematics teachers, education researchers, university faculty, and technology experts, the *Fraction Foundations: Helping Students Understand Fractions* course began development in Spring 2014 to be launched in Fall 2014. I served as a graduate student member of that team. The course was offered twice, with the second course designed as a revised version of the first based on feedback from participants in the first course and the research team.

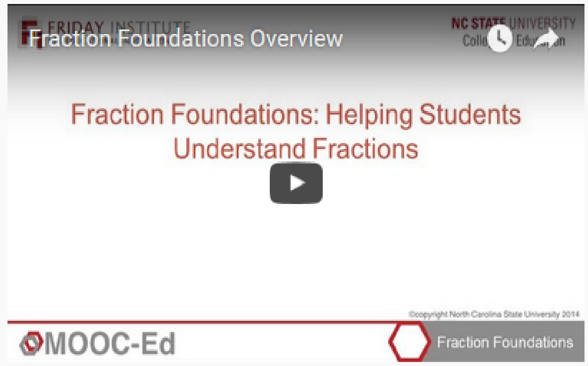
The MOOC-Ed was designed around the recommendations of the IES practice guide, *Developing Effective Fractions Instruction for Kindergarten Through 8th Grade* (Siegler et al., 2010), described in Chapter Two, which were based on research on teaching and learning fractions (e.g., Ma, 1999; Post et al., 1991; Tirosh et al., 1998). As discussed in Chapter Two, the guide emphasizes that PD programs on the topic of fractions should focus on: 1) building participants' knowledge and understanding of fractions (e.g., discuss challenges associated with solving problems; explore the meaning and justification of algorithms); 2) presenting various representations of fractions and fraction operations (e.g., provide activities for participants to design tasks that integrate multiple fraction representations, such as diagrams and fraction strips); and 3) helping to refine participants' ability to analyze student thinking about fractions (e.g., discussion of students' written work and common misconceptions) (Siegler et al., 2010). These elements are highlighted in the course description, outline and components of the *Fraction Foundations* MOOC-Ed, as described below.

The *Fraction Foundations* course description and participant goals were displayed as part of the home webpage and included the information provided in Figure 4.

Figure 4. *Fraction Foundations* Course Description and Introduction (Friday Institute for Educational Innovation, n.d.)

Fraction Foundations: Helping Students Understand Fractions

The Fraction Foundations MOOC-Ed will help you teach fractions concepts and skills more effectively through understanding students' thinking and implementing research-based approaches in your classroom. It will help you address rigorous curriculum standards for fractions, whether from the Common Core State Standards or from other up-to-date standards.



Though this MOOC-Ed focuses on teaching fractions in grades 3-5, the concepts are applicable for other K-8 teachers and will help you address the needs of students who have learning differences that impact their mastery of mathematics.

While classroom teachers are the primary audience, this MOOC-Ed is also relevant for teacher educators who provide preservice or inservice professional learning programs, along with teacher leaders, coaches and administrators who support classroom teachers.

Register

We invite participation from public, charter and private schools. While the course materials are U.S. focused, educators from around the world are welcome. There is no cost for participating in the Fraction Foundations MOOC-Ed.

We recommend, when possible, participation with local colleagues so that you can work together and engage in peer-coaching. Individuals are welcome to participate on their own.

Jump to Course Schedule

The Fraction Foundations MOOC-Ed is organized around the recommendations of the *Practice Guide on Developing Effective Fractions Instruction for Kindergarten Through 8th Grade*, from the U.S. Department of Education's Institute of Education Sciences (published in Sept 2010). The overall goals are that participants will:

- Develop a deeper understanding of the fractions content standards, and relevant practice standards, that apply in their own schools.
- Investigate common student misconceptions about fractions and why fractions are hard for children (and adults) to understand.
- Analyze students' thinking about fractions to inform instruction.
- Address students' learning differences when teaching fraction concepts and skills.
- Learn to effectively use:
    - *Fair-sharing* activities to help students understand key concepts of fractions, such as fractions representing the relationship between parts and wholes, equivalent fractions and comparing fractions. (Focus of Unit 2)
    - *Measurement and number line* activities to help children understand fractions as part of the number system and key concepts such as equivalent fractions, comparing fractions, and the relationship of fraction and integer operations. (Focus of Unit 3)
    - Activities to help students understand *why procedures for computations with fractions make sense*. (Focus of Unit 4)

The course contained five units; the first and last units were each planned for one week, and the middle three were each planned for two weeks. The course outline and essential questions for each unit are shown in Table 1.

Table 1
Fraction Foundations Course Outline and Essential Questions (Friday Institute for Educational Innovation, n.d.)

Unit Description	**Essential Questions**
Unit 1: The Foundations for Understanding Fractions, will explore why fractions are hard for children and even adults, recommendations for effective fraction instruction, curriculum standards for fractions concepts and operations, and the concept of deeper learning. Participants will discuss successes and challenges of their current approaches to teaching fractions and begin to plan their project.	1. Why are fractions so challenging to learn? 2. What misconceptions about fractions and fraction operations are common among students? 3. What informal strategies and language do students use to solve fraction problems and how can teachers build upon those to help students learn the mathematics of fractions?
Unit 2: Fair Sharing Activities, introduces several instructional strategies for meaningful fractions learning, including building on students' understanding of fractions in the context of fair-sharing and analyzing students' thinking to inform instructional decisions.	1. What are fair-sharing activities and why are they recommended to help students build a foundation for understanding fractions? 2. How can fair-sharing activities be used to address misconceptions? 3. How can fair-sharing activities be used with students with different levels of understanding and different learning strengths?

Table 1 continued

Unit 3: Measurement and Number Line Activities, focuses on interpreting fractions as numbers through partitioning and iterating (repeatedly using) fractions. Fractions as measures on a number line, as emphasized in the Common Core State Standards, and measures of area and sets, will be explored.	1. What are measurement activities and why are they important? 2. How can the number line activities be used to help students understand fractions within the number system? 3. How can measurement and number line activities be used to address students' misconceptions? 4. How can measurement and number line activities be used to help students with different levels of understanding and different learning strengths?
Unit 4: Understanding Procedures for Computing with Fractions, engages participants in building students' understanding of fraction computation. Different visual models to help students understand fraction operations will be considered.	1. How can students be introduced to operations with fractions in ways that form a good foundation for using fractions to solve problems? 2. How can you identify and address students' misconceptions about operations with fractions? 3. How can fair-sharing and number line activities be used to support an understanding of computations with fractions?
Unit 5: Wrap-up and Next Steps, will allow participants to reflect on, assess, and share the knowledge gained throughout the course, provide feedback for the work posted by colleagues and share what they have learned and their ideas for improving the MOOC-Ed.	1. How has your own understanding of teaching fractions changed? 2. What strategies/skills have you found most valuable? 3. What are your next steps after the course? 4. How can the *Fraction Foundations* MOOC-Ed be improved for future participants?

In order to register for the course, participants completed a survey that provided both demographic information and professional and learning goals. Once registered, participants were encouraged to take a pre-assessment created from MKT materials by the MOOC-Ed team for research purposes. As with all MOOC-Ed components, the assessment was not required for participants to engage in the course.

Figure 5 shows an example page from the *Fraction Foundations* MOOC-Ed, the introductory page to Unit 4 on *Understanding Procedures for Computing with Fractions*.

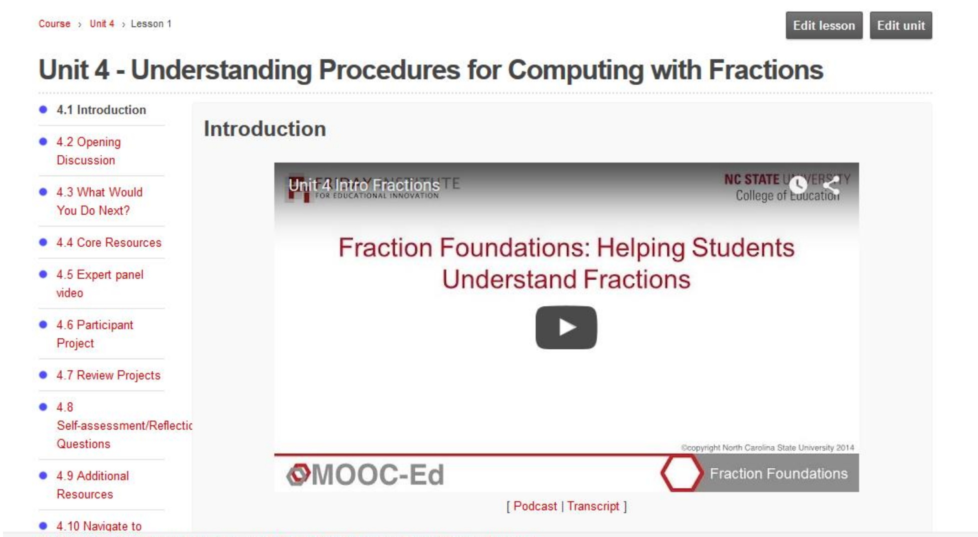


Figure 5. Introduction Page to Unit 4 of *Fraction Foundations*

Participants were free to navigate through options in each unit (like those pictured on the left pane of Figure 5) at their own pace and in any order they chose, thereby creating opportunities for self-directed learning by participants. Course components and activities came in various forms. Key components appeared in the same order in each unit for ease of use and consistency and are described in Table 2.

Table 2
Course Components and Activities (Friday Institute for Educational Innovation, n.d.)

Component	**Description**
Introduction	Opening video produced by the MOOC-Ed team introducing participants to the unit.
Discussion Forums	Discussion forums containing prompts where participants engaged in conversations on topics relevant to the unit.
"What Would You Do Next?" Videos	Videos produced by the MOOC-Ed team of students thinking aloud as they work on mathematical tasks. This component provided an opportunity for participants to analyze and discuss what they observed in the videos in a discussion forum.
Core Resources	Resources presented in the form of videos, articles and audio files relevant to the unit produced by external sources.
Expert Panel Video	Videos produced by the MOOC-Ed team of discussions with experts in the field on topics relevant to the unit.
Participant Project	Ideas and pacing for producing a project to apply what was learned during the course to practice. Participants were encouraged to try activities, analyze students' thinking and plan lessons for students, while receiving feedback from other participants in discussion forums.
Self-assessment and Reflection Questions	Multiple-choice questions modeled after and adapted from MKT questions (Hill et al., 2008) for participants' own use to assess and reflect on their own understanding of some key ideas in each unit. Detailed explanations of solutions were provided.
Additional Resources	Resources relevant to the unit and produced by external sources for participants' further exploration. Discussion boards were available for participants to engage in conversation related to the resources.

The course integrated a number of different technology resources to support the content. Google™ Course Builder was the platform on which the MOOC-Ed was created. Vanilla Forums© was used as the discussion forum platform and was integrated into the course platform, so that participants would be able to engage in discussion seamlessly. Survey Gizmo® was used for all surveys and the MKT assessment, and Google™ Hangouts was used to record all expert panels. Finally, YouTube™ was used to house all videos produced by the MOOC-Ed team.

Multiple strands of data were collected in quantitative and qualitative forms. Both forms were collected concurrently through closed and open-ended survey items. MOOC-Ed participants also took a pre- and post-assessment to measure growth in their mathematical knowledge for teaching. Qualitative data from a smaller sample of participants were also collected through personal interviews and in-person classroom observations. The interview and observation data were gathered both before and after the course and provide more detailed insight into the effectiveness and impact of the PD program on elementary educators' practice. As all course participants were invited to complete the surveys and assessments, the data gathered from these sources allow for analysis of a large sample, thereby providing opportunities to make generalizations to a population; data gathered from a smaller sample in the in-person component of the study allowed me to explore participants' pathways to making changes in knowledge, beliefs and practice at a deep level (Creswell & Clark, 2011). The sources and timing of data collection are summarized in Table 3 and will be described in detail in this chapter.

Table 3
Sources and Timing of Data Collection

Source	**Population**	**Sample used for study**	**Type**	**Timing**
MOOC-Ed Registration Form/Pre-survey	All registrants of the *Fraction Foundations* MOOC-Ed	Only participants who completed both registration and post-survey (N = 213)	Quantitative	Pre-course
Personal interviews and classroom observations	Participants in the MOOC-Ed who taught in schools local to the researcher's region.	Elementary mathematics teachers who participated in the MOOC-Ed (N = 4)	Qualitative	Pre-/mid- and post-course
MKT assessment	Participants in the MOOC-Ed who elected to take the assessment	Only participants who completed both the pre- and post-assessment (N = 196)	Quantitative	Pre- and post-course
Post-survey	Participants in the MOOC-Ed who elected to take the survey	Only participants who completed both registration and post-survey (N = 213)	Quantitative Qualitative	Post-course

Collecting and examining both qualitative and quantitative data provide the opportunity to validate whether and how a MOOC-Ed can serve as a viable, effective PD option for educators. Combining the approaches also allows for researchers from a variety of fields to understand and apply the findings. The results can then be used to develop and refine future PD opportunities for educators that are accessible, meaningful, and applicable. Descriptions of the methods used in this study to answer each research question are presented in the next sections.

Data Collection and Analysis for Research Question 1

Both quantitative and qualitative data were collected and analyzed to help answer whether and how the MOOC-Ed can change participants' knowledge and/or beliefs. Quantitative data came in two forms. First, data was gathered from a sample of MOOC-Ed participants through responses to closed-response questions on the post-surveys in the course. Results were used to help measure the extent to which the MOOC-Ed could serve as an effective form of PD for elementary educators, that is, whether the MOOC-Ed affected their knowledge, beliefs and/or practice. Second, a pre- and post-assessment on *Mathematical Knowledge for Teaching* was administered to all participants in the MOOC-Ed who elected to take it. Quantitative results from the assessment were intended to measure participants' growth in mathematical knowledge for teaching concepts related to fractions. Qualitative data came in the form of responses to an open-ended post-survey item addressing aspects of the course participants considered valuable.

Pre- and Post-Surveys

Sample/participants. The sample for this component of the study was gathered from participants enrolled in the first and second implementations of the *Fraction Foundations* MOOC-Ed who completed both the pre- and post-surveys administered in the course. According to data gathered from the registration forms, participants in the MOOC-Eds included classroom teachers, teacher leaders and coaches, PD designers, researchers, and administrators. Educators in the courses ranged from having zero to more than 20 years of experience in the field of education and were located in multiple states in the United States and a few countries outside of the U.S. Most of the participants were elementary mathematics educators, since the topic of the course was focused on the teaching and learning of fractions. Approximately 900 particiants enrolled in the first version of the course, and over 1700 participants enrolled in the second version. Participants were required to complete the pre-survey, as it was embedded in the registration form; however, none of the subsequent instruments were required to complete the course unless a certificate of completion was desired. Survey results in this study are reported for only participants who completed both the pre- and post-surveys.

A total of 213 participants (54 in FFI and 159 in FFII) completed both surveys.[3] Figure 6 shows the distribution of participants in each course by primary area of responsibility.

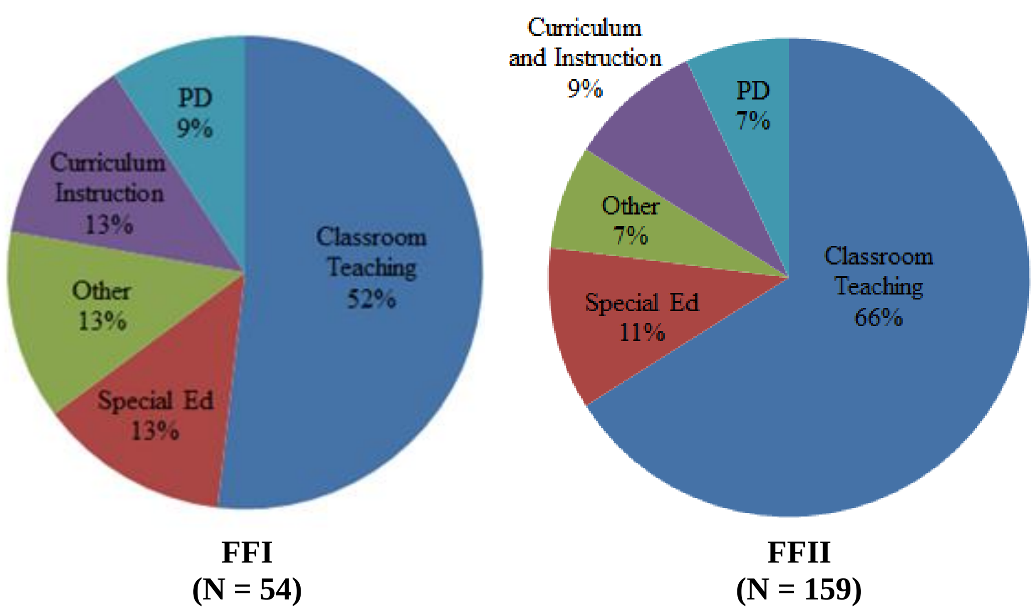


Figure 6. Participant Roles in FFI and FFII

[3] In FFII, 59 participants who completed both the pre- and post-surveys did not specify a primary area of responsibility. Therefore, for FFII, Figure 6 displays data from the remaining sample of 100 to obtain a description of the participants who responded.

Table 4 displays 1) the number of participants enrolled in the MOOC-Ed, 2) the number who responded to the post-survey as a whole; 3) the number who provided responses to the two open-ended items analyzed further; and 4) the number who requested and earned certificates of completion.

Table 4
Participant Enrollment and Survey Participation Totals

Course	Enrolled	Post-survey respondents	Respondents to "valuable aspects" question[4]	Respondents to "changes to practice" question[5]	Certificates of Completion
FFI	906	54	52	44	50
FFII	1749	159	147	147	157

Data collection instrumentation. In order to register for the course, participants submitted responses on an online registration form (see Appendix A for the forms in both the Fall and Spring implementations of the course). The page was an online survey in which participants answered sets of questions on different topics. The survey contained two primary sections. The first addressed demographic information, such as gender, years of experience in the field, district location, district type, and role in education. The second focused participants on their learning goals and motivations for enrolling in the course. Since there were no instruments available that would address all of the study goals, members of the research team developed the survey by first reviewing literature on survey development to measure success in online learning, self-efficacy, and goals for professional

[4] Responses to this survey item were analyzed to inform research question 1.
[5] Responses to this survey item were analyzed to inform research question 2.

development (e.g., Bandura, 2006; Mabe & West, 1982; Wang, Wang, & Shee, 2007). The initial survey was then revised by members of the research team based on conciseness, length and relevance to the study before being implemented in the course.

As noted earlier, research suggests that there are a number of characteristics of professional development that make it effective (Abdal-Haqq, 1996; Darling-Hammond et al., 2009). In an attempt to address a number of these characteristics as they related to the MOOC-Ed course, participants were invited to submit responses to a post-survey (see Appendix A) at the conclusion of the course. As before, since instruments addressing all of the study goals were not readily available, members of the research team developed the instrument by drawing from previous studies and validated instruments on teacher activity, quality and effectiveness of PD programs, and PD programs' impact on practice (e.g., Desimone, 2009; Garet, Porter, Desimone, Birman, & Yoon, 2001; Guskey, 2000). Details on the development of the survey are discussed in the *Reliability and validity* section later in this chapter.

The post-survey included three main components. One addressed participants' views of their experience in the PD program and the usefulness of particular components of the course (e.g., lesson videos, assignments). A second addressed participants' beliefs about the effectiveness of the course. A third component focused on participants' perceptions of the impact the course had and/or would have on their practice, which is discussed in the methods used to answer Research Question 2. Participants were also invited to enter responses to open-ended items addressing general reflections on the course, its usefulness, and suggestions for improvement to better serve participants' needs.

Data collection procedures. The pre-survey was embedded in the registration form that was required for participants to complete in order to enroll in the course. All survey results were collected and saved on a secure server accessible only by members of the research team and technical staff that supported the project.

The post-survey was first made available online to all course participants during the final two weeks of the course. Although participants were not required to complete the survey in order to complete the course unless they wanted a certificate of completion, they were encouraged to do so through a series of email announcements. All post-survey results were collected and saved on a secure server accessible only by members of the research team and technical staff that supported the project.

Appendix A includes all surveys that were used in this study. Though the data were separated into sections for analysis, the items on each survey were organized according to their connection to each other and the flow of the surveys.

Data analysis procedures. Basic descriptive statistics generated from the pre-survey were used to describe the sample of participants in this component of the study. Data included participants' primary area of responsibility, number of years of experience, and personal goals in taking the course.

Basic descriptive statistics were used to describe results from the post-survey. Three closed-response questions and one open-ended item were analyzed to help answer the first research question. The closed-response questions provided data on self-reported improvement in knowledge and skills related to teaching fractions, effectiveness of the implementation of design principles in the MOOC-Ed, and the effectiveness of components of the MOOC-Ed.

Spearman's rank correlation coefficient, r_s, was also used to measure associations between different elements of perceived effectiveness. For example, I explored the strength of association between perceptions of enhanced knowledge and skills in certain areas of practice and participants' beliefs about the extent to which certain components of the MOOC-Ed served effective by calculating rank-order correlations between individual responses to the post-survey questions that aligned with those elements. The data analyzed for this component of the study were in the form of Likert-scale responses, so this non-parametric measure of correlation is appropriate and robust (Clason & Dormody, 1994; Hauke & Kossowski, 2011; Spearman, 1904). I created a matrix to display the correlation coefficients associated with each pair of questions on the post-survey. I further investigated all coefficients that had an absolute value greater than 0.6, as they indicated at least moderately strong associations between the variables analyzed (Hinkle, Wiersma, & Jurs, 2003; Owen, Matthews, Petrie, Green, & Croft, n.d.). Associations between sub-items within items (e.g., items 7a) and 7b)) were not further investigated, as it was expected that participants would respond to sub-items similarly within any one question. Therefore, only associations between sub-items in distinct questions that corresponded to $|r_s|$ values greater than 0.6 are discussed. Caution is heeded to report the associations as correlational and not necessarily causal.

The open-ended survey item relevant to Research Question 1 addressed the most valuable aspect(s) of the MOOC-Ed ("What was the most valuable aspect of this MOOC-Ed in supporting your personal or professional learning goals?"). The responses were analyzed using a coding procedure that included grounded theory methods suggested by Charmaz (2006, as cited in deWaard et al., 2015, pp. 237-238). The procedure included multiple steps for each survey item:

1. *Initial coding*: A reading of all gathered data for each item was completed, and general codes were created and documented. A description of each code was then written to build an initial "codebook".
2. *Line-by-line coding*: Each survey response was read closely multiple times and coded using the initial codebook.
3. *Revision of codes*: Initial codes addressing similar categories were consolidated, and descriptions were revised.
4. *Second revision of codes*: A second iteration of revisions was performed with another member of the research team to refine and finalize the codes.
5. *Focused coding*: Focused coding began to synthesize results and identify qualitative differences in results between FFI and FFII. The final coded results were used to find patterns and identify themes across the two courses.

Reliability and validity. The initial pre-survey was developed by the MOOC-Ed design team in a collaborative effort to capture data that would be relevant to all of their research interests. Members of the team included veteran education researchers and members of the evaluation team who had developed surveys on effectiveness of PD for other projects. In order to address overall reliability of the surveys used in the course, the pre-survey was piloted in a previous course in the MOOC-Ed series, and reliability analyses (Cronbach's alpha) were conducted to inform revisions that were made for use in this study. After implementation in FFI, members of the evaluation team made revisions to the pre-survey for FFII in attempts to make the survey more concise and less time-intensive. The revisions were reviewed by other members of the team and mutually agreed upon.

In developing portions of the post-survey, I reviewed literature on effectiveness of professional development and its impact on practice. A number of validated instruments that were tested and used with populations of teachers of mathematics in previous studies (e.g., Garet et al., 2001) were identified, which served as the basis of the creation of a large preliminary list of items for this survey. Items were adapted from such scales as the *Description of One Professional Development Activity* and *Effectiveness of the Mathematics Related Professional Development Activity* subscales of the Mathematics Version of the Teacher Activity Survey (TAS) (Garet et al., 2001) as well as the Quality of Professional Learning Index: Measure of Outcomes of Professional Development (Ingvarson, 2003). Another member of the research team and I then met to identify the most relevant and essential items to keep. The reduced list was then shared with two other members of the research team, and all members met to finalize the order, wording and streamlining of the instrument. This process, along with revisions made by additional members of the research team, produced the surveys administered in FFI. After the first administration of the course, the evaluation team conducted interviews with participants to gather data to help inform revisions to components of the course and its evaluation. Revisions were made to the registration form and post-survey for the second administration of the course as a result.

In order to assess reasonableness of conclusions reached about the relationships identified in the data, conclusion validity was addressed. The relatively large sample of survey respondents helps strengthen this validity, as it provides for good *statistical power* (Austin, Boyle, & Lualhati, 1998). Furthermore, the process used to design reliable instruments described earlier adds to the strength. Finally, the research team designed the MOOC-Ed in this study using a research-based approach with careful attention to instructional design, relevance to participants and well-supported implementation. This, too, contributes to the strength of conclusion validity (Trochim, 2006).

In order to address face validity of results reported from the open-ended item on the post-survey, I met with another research team member to refine my original coding scheme. We revised the scheme together and independently coded all responses from FFI. We did not achieve an adequate level of agreement and subsequently met to establish agreement on the sample we both coded. In doing so, we further revised the coding scheme. We then independently coded a final sample of responses (from FFII) and achieved 80% agreement, ensuring inter-rater reliability. I then proceeded to code the small number of remaining responses that were not included in the samples provided to the research team member.

In order to address overall reliability of the surveys, the post-surveys were piloted in a previous course in the MOOC-Ed series, and reliability analyses (Cronbach's alpha) were conducted to inform revisions that needed to be made for use in this study.

MKT Assessment

As an additional quantitative measure of the extent to which participants experienced change in their knowledge as a result of their participation in the MOOC-Ed course, a pre- and post-assessment was made available to all participants on *Mathematical Knowledge for Teaching* (MKT) (Hill, Blunk, Charalambous, Lewis, Phelps, Sleep, & Ball, 2008; Learning

Mathematics for Teaching (LMT) Project, n.d.). Items on the assessment were related to teaching and learning fractions.

Sample/participants. Participants in the course were invited to take the assessment in the first week of the course through email announcements and placement in the first unit of the course. Participation was voluntary for MOOC-Ed participants, but it was required for those who sought to receive a certificate of completion. Fifty-four participants in the MOOC-Ed completed both the pre- and post-MKT assessment in FFI, and 142 completed both assessments in FFII.

Data collection instrumentation. Since no one intact MKT instrument that addressed the goals of this study was available, I designed the assessments based on a series of steps provided in training modules developed by the LMT Project team at the College of Education at the University of Michigan (Hill, Schilling, & Ball, 2004). Details on the development of the assessment are discussed in the *reliability and validity* section of this sub-section.

Data collection procedures. While the pre- and post-MKT assessments were voluntary (i.e., participants were not required to complete them in order to participate in the course), they were embedded within the course content. The pre-assessment served as one of the "tasks" that participants completed early in the course. The post-assessment was administered at the conclusion of the course and embedded as part of the final week's activities. All pre- and post-assessment results were collected and saved on a secure server accessible only by members of the research team and technical staff that supported the project.

Data analysis procedures. Basic descriptive statistics, scatterplots of pre- vs. post-assessment results, graphical representations of change scores, and effect size estimates (Cohen's *d*) were used to analyze results from the pre- and post-assessments.

Reliability and validity. While the original MKT instruments had been tested for validity and reliability by the LMT team, the assessments in this study were adapted from those instruments. As a result, multiple steps were taken to create assessments that included discriminating items and those with varying difficulty levels. First, all items that addressed fraction-related concepts were identified across all assessment forms provided by the LMT Project team. These included forms on Rational Numbers, Proportional Reasoning, and Elementary Number Concepts and Operations administered between 2001 and 2008. I removed all items that were duplicates of others on the list, and I sorted each item into three categories that aligned with the three major topics covered in the MOOC-Ed: 1) fractions in fair-sharing contexts; 2) fraction as measure; and 3) understanding operations with fractions. The categories were further expanded to include a category that better aligned with the first unit of the course. I grouped items that were similar in nature and concept assessed, and each item was considered for inclusion. I then selected only one item in each group, so as to limit the number of questions addressing similar concepts. My decisions were made based on two characteristics: slopes of each item's characteristic curves (ICC's), and the item's identified difficulty levels (the point at which an item's ICC has a maximum slope, or the point where a person with the associated ability level has a 50% chance of answering the item correctly). These values were provided by the LMT Project team and were based on results of research they performed to develop the MKT instruments. This research included participants who were primarily engaged in professional development on the topic of mathematics (Hill et al., 2004).

An item with a steeper slope in its ICC represents one that is more discriminating than an item with a less steep slope. A steeper slope indicates that the probability that a person with a specific “ability” level answers the item correctly is far different than that of a person with a different “ability” level, therefore differentiating the “ability” levels more precisely. With regard to item difficulty levels, I selected items based on the premise that the assessment should include questions that span a variety of difficulty levels, so as to reflect the anticipated sample population (diverse ability levels). After I created each draft of the assessment, I analyzed the strength of the assessment as a measurement tool based on its test information curve (see Figure 7).

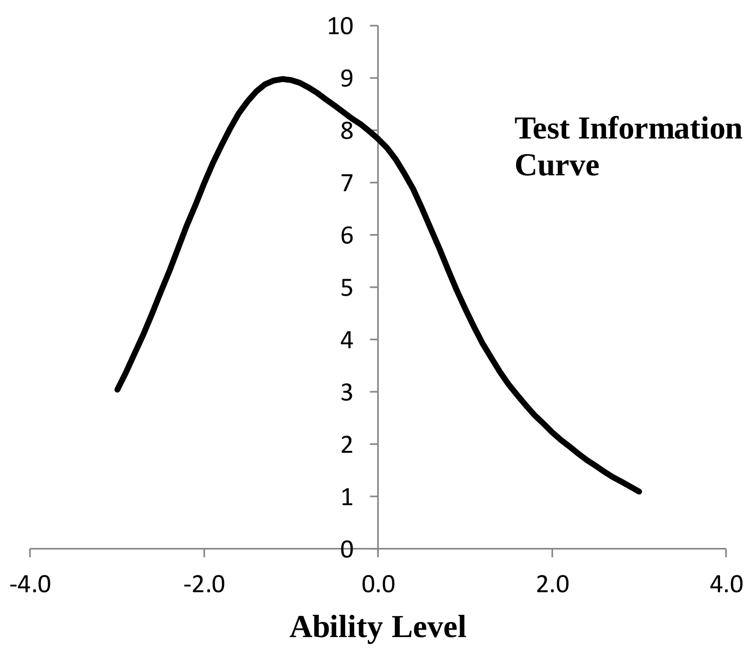


Figure 7. Test Information Curve for Final Pre- and Post-assessment

At the time of my development of the instrument, some members of the population of interest in this study (elementary educators) would have engaged in professional development that addressed mathematics-related topics before participating in the MOOC-Ed, while some would not have. In addition, based on pre-course personal interviews with participants, it was expected that many participants in the course would not have necessarily had experience learning mathematics for teaching in their teacher training or professional development programs. For this reason, it was expected that the population of interest would have “ability”, as defined by the LMT Project, lower than the mean ability determined by the LMT Project team. In order to design an assessment that targeted such a population, it was best for the test information curve to achieve its maximum value near the anticipated average ability level. This helped to ensure that the results from the assessments measured growth in participants’ MKT were less a matter of chance and more representative of actual knowledge growth.

The maximum of the curve occurred at -1.1, which represents an ability that is 1.1 standard deviations below the mean ability of all participants who have taken the MKT assessments as part of instrument development. This was approximately the anticipated ability level of much of the target population investigated in this study.

In developing the assessment, I was concerned about its length, as I expected that participation may be affected by the prospect of taking a long assessment before the course; in addition, I was concerned that participants would lack a feeling of satisfaction, since answers to the assessment would not be provided for them to check their results. However, as suggested by the LMT Project team, “longer tests tend to have higher reliability…[and]

tend to [provide] greater confidence in individual scores" (LMT Project, 2014a, slide 35). Consequently, I included the recommended 25-30 items in the final assessment.

As discussed earlier, I was trained to develop and administer a pre- and post-assessment on MKT, and I did so for this study. Since the assessments were not in intact form provided by the LMT Project, reliability and validity tests needed to be performed on the assessments. Items from the MKT item pool were first selected based on relevance to the mathematical content covered in the MOOC-Ed course. The selected items were then tested using slopes of the respective *item characteristic curves* (ICC) and difficulty scores to be considered for inclusion in the assessments (Baker, 2001). As is highlighted in the MKT training, validation was performed not on the instruments but on the "interpretation of scores" (LMT, 2014b, slide 2). This was done in a series of steps, as described in the MKT training module (c.f. Kane, 2006):

- Specify an interpretation for and planned use of scores
- Establish an interpretive argument, a network of linked assumptions that must be true in order to support the proposed interpretation and use of the test
- Investigate these assumptions using multiple sources of empirical evidence
- Prioritize most questionable assumptions
- Revise interpretation of scores as needed (LMT, 2014b, slide 3)

Data Collection and Analysis for Research Question 2

Both quantitative and qualitative data were collected and analyzed to examine whether and how the MOOC-Ed can impact participants' practice. Quantitative data came in the form of two closed-response post-survey items; qualitative data was gathered from one open-ended post-survey item.

Post-survey Items

The sample, data collection and data analysis performed to inform this component of the study were the same as that described in the section on Research Question 1 in this chapter.

Data collection instrumentation. The instrumentation used to answer this research question was the same as that reported for Research Question 1. As described earlier, one of the components of the post-survey focused on participants' perceptions of the impact the course had and/or would have on their practice. Two closed-response questions and one open-ended item on the post-survey were further analyzed to help answer this research question. One closed-response question only appeared on the post-survey in FFI and asked participants to report the extent to which they had attempted to make changes in specific areas of practice as a result of their participation in the MOOC-Ed. The second closed-response question appeared on the post-survey in both FFI and FFII and asked participants to rate the effectiveness of the MOOC-Ed in preparing them to make positive changes in their practice. The one open-ended item addressed the impact the MOOC-Ed had/would have on participants' practice ("Please describe any changes you have made to your practice, including how you have applied the knowledge, skills, and/or resources you gained in this course.").

Interconnections between Changes in Knowledge/Beliefs and Impact on Practice

As described in Chapter Two, this study was conducted under the theoretical perspective that changes in knowledge, beliefs and professional practice can be realized through multiple and diverse pathways. These pathways can highlight the interconnections between changes in knowledge/beliefs and changes in practice. In order to explore these interconnections, both quantitative and qualitative data was analyzed. Quantitative data came in the form of participants' responses to closed-response post-survey items described earlier in this chapter. Qualitative data came in the form of semi-structured personal interviews and classroom observations of four elementary mathematics teachers who had participated in the MOOC-Ed. All were conducted one time before the course, with the exception of one participant[6], and then again after the course had concluded.

Post-survey Items

The sample, data collection and data analysis performed to inform this component of the study were the same as that described in the section on Research Question 1 in this chapter.

Data analysis. Spearman's rank correlation coefficients were calculated to identify associations between the post-survey item on the MOOC-Ed's effectiveness in promoting changes in practice and each post-survey item referring to self-reported changes in knowledge/skills.

Observations/interviews

Sample/participants. While I attempted to recruit a diverse group of teachers for this component of the study (e.g., number of years of experience, instructional context, etc.), a

[6] One of the four teachers joined the study in the early weeks of FFII, so my first interview and observation occurred after she had already engaged with some of the content in the course.

convenience sample was used in order to obtain as much data as possible. Initially, 10 elementary teachers were recruited to participate in the study. Some were recruited prior to registration through referrals from PD providers. Others were recruited from the list of registrants in the MOOC-Ed prior to its launch. Since much of the qualitative data was to be collected in person through personal interviews and observations, participants were recruited from areas in the southeastern region of the United States. No requirements with regard to level of engagement in the course were specified as a condition of participation in the study. As the study progressed, a few of the original participants were no longer able to participate in the MOOC-Ed, due to both personal and professional reasons; they, therefore, did not complete their participation in this study.

Ultimately, four elementary teachers completed this component of the study. One of them participated in the first implementation of the *Fraction Foundations* MOOC-Ed (FFI); two of them participated in both the first and second implementations; and the fourth participated in the second implementation (FFII) only. Two of the teachers were contacted based on referrals from a PD provider who worked with them. The other two were first contacted through an email message sent to all enrolled participants who identified their location as being within the region desired. They each responded to the message indicating their interest in participating in the study.

Data collection instrumentation. Semi-structured interviews (Rubin & Rubin, 2011, as cited in DeCuir-Gunby, Marshall, & McCulloch, 2011) were conducted with the sample of participants so as to explore their specific paths in their professional development. Such interviews provided enough structure to address the same overarching topics with each participant but with the flexibility to ask follow-up questions relevant to the participants' individual responses. I developed the protocol by adapting questions from validated instruments used to study mathematics teachers' experience, instructional goals, and "access to and implementation of instructional resources" (Horizon Research, Inc., 2000a). These included the *Inside the Classroom Teacher Interview Protocol* (Horizon Research, Inc., 2000b), which has been tested and used with multiple populations of classroom teachers, and an interview protocol developed by Herrington et al. (2009) for their work on transfer of online PD to teachers' practice.

For classroom observations, the *Classroom Observation of Early Mathematics—Environment and Teaching* (COEMET) protocol[7] was adapted for use as a guide for classroom observations (Sarama & Clements, 2010). The COEMET protocol was developed by a team of researchers studying teaching and learning of early childhood mathematics; the instrument was based on research that focused on principles and strategies of effective teaching of early childhood mathematics and has been used for studies of populations of teachers similar to those in this proposed study (Clements & Sarama, 2008, 2011). The instrument was selected for purposes of this study because a number of its components were relevant to this study's research focus. The instrument is also comprehensive in its scope of observation. Indeed, it includes subscales on both overall *Classroom Culture*, where components of the "Environment and Interaction" as well as "Personal Attributes of the Teacher" are highlighted, and *Specific Math Activity*, where observations specific to the

[7] Access to the COEMET protocol was provided by D.H. Clements (personal communication, March 16, 2014). Permission was granted to adapt the protocol for purposes of this study.

lesson are made with regard to such things as *Eliciting Children's Solution Methods* and *Supporting Children's Conceptual Understanding* (Sarama & Clements, 2010). These were important aspects of classroom observations that I thought would be useful to include in participant profiles for analysis. Key indicators in the protocol for this study were also drawn from *Learning Trajectory-Based Instruction* (LTBI) interpretations (Sztajn et al., 2012) of effective mathematics classroom practices (Smith & Stein, 2011).

Appendix B includes the full instruments used in the interview/observation component of this study. Though the questions in the interview protocol are numbered and separated into sections, they were addressed in the order that was appropriate and relevant to the flow of conversation. As the interviews were semi-structured, additional questions not appearing in the protocol were asked in response to participants' dialogue.

Data collection procedures. I conducted three classroom observations for each of the three teachers who participated in FFI. The first was a pre-course observation in order to better support findings with regard to PD effectiveness. The second two observations were conducted after the course had concluded, within a one to three-month period. I observed the fourth participant, recruited during FFII, once during the course run and once after the course had concluded. Each observation was approximately one hour in length, depending on the length of the lesson.

I conducted a semi-structured interview of participants immediately following each classroom observation, and each lasted approximately 30 minutes. These interviews conducted after observation, or debriefs (Onwuegbuzie, Leech, & Collins, 2010), addressed what I observed and probed for participants to provide a *retrospective report* (Desimone,

2009) on their classroom activities. I focused on eliciting connections they could make between their lessons and their participation in the MOOC-Ed, particularly if those connections could be mapped to specific content or approaches made salient in the course. I also focused on the teachers' general views on the usefulness and effectiveness of the MOOC-Ed on their practice. All interviews and classroom observations were video taped, and interviews were transcribed for data analysis.

Data analysis procedures. LeCompte (2000) noted that qualitative data:

> Have no initial intrinsic organizational structure or meaning by which to explain the events under study; researchers must then create a structure and impose it on the data. The structure is created in stages, and forms the basis for assembling data into an explanation or solution. Creating the structure is analogous to the strategies used to assemble puzzle pieces. (p. 147)

In this light, interviews and observations conducted during the study were analyzed using thematic analysis (Boyatzis, 1998). I began data analysis with initial reflections and thoughts (Miles & Huberman, 1994) on what was observed during the lessons and heard through conversation in the interviews. For example, during observations, I recorded moments in which I observed the teacher emphasizing the concept of *fraction as a number* or *having value,* and not just "parts of a whole", since this was a theme made salient in the MOOC-Ed. As another example, I documented my thoughts on the questioning strategies and pedagogical moves that the teachers made as they engaged students in their activities.

I then engaged in thematic coding on transcripts of interviews to develop descriptive, or data-driven, codes (DeCuir-Gunby et al., 2011; Miles & Huberman, 1994), such as "external support for PD" and "pedagogical shifts". Statements from interviews and observations were used to illustrate each participant's profile in the case study, and a summary synthesis of results was performed in the interim of data analysis to support within-case analysis (Miles & Huberman, 1994; Paterson, 2010). The *Interconnected Model of Professional Growth* served as a framework for analysis of each teacher's growth as a result of the MOOC-Ed (Clarke & Hollingsworth, 2002)[8]. Finally, notes, codes and the theoretical framework were used to shape each participant's narrative.

Reliability and validity. Researchers warn that qualitative data obtained through self-report or self-evaluation can present serious risks to the validity of the results. However, some note that the risk is reduced and validity strengthened if participants are ensured that their responses will be kept strictly confidential and anonymous (e.g., Gordon & Petty, 1971; Sherwood, 1966; Sorenson, 1956, as cited in Mabe & West, 1982). In response to that, and per IRB standards, participants signed an informed consent form that included specific language ensuring that all data will be protected in confidence and only be associated with individuals by name within the research team. The participants were reminded of this during each interview as well. Results from classroom observations also serve as a measure of validity of self-reported data gathered during the interviews, as I was able to confirm whether the interview data aligned with what was observed. Interview and observation protocols were reviewed by at least one additional member of the larger research team, and revisions were based on these reviews. I conducted all interviews and observations, as opposed to sharing the duties, to ensure that questions asked and observations made were based in the same

[8] For more on this model, see the introduction to Chapter Two.

context. Two to three interview/observation visits were conducted to address validity and reliability of the protocols (Taylor, Pearson, Clark, & Walpole, 1999, as cited in Desimone, 2009). I also conducted each interview immediately following each observation, which helped strengthen participant-observer agreement (Desimone, 2009).

Summary

In this chapter, I presented the quantitative and qualitative methods used to address the research questions posed in this study along with interconnections between changes in knowledge/beliefs and changes in practice. I described the study's design, justified my choices of methods and analysis, and provided arguments supporting the reliability and validity of my results. Chapter Four will present findings that resulted from the use of these methods.

CHAPTER FOUR

The purpose of this study was to examine whether and how a free, open, online course for professional development in the form of a mathematics education MOOC-Ed can serve as a viable, effective option for educators in changing their knowledge and/or beliefs and impacting their practice. Specifically, I examined the extent to and ways in which the MOOC-Ed on *Fraction Foundations* changed participants' knowledge and/or beliefs and whether and how it impacted their practice. Pre- and post-surveys and assessments were administered to MOOC-Ed participants to gather both quantitative and qualitative data to inform the study, and semi-structured interviews and classroom observations were conducted with four elementary teachers who participated in the course. In this chapter, I present findings from the surveys and assessments and provide narratives for each teacher I observed and interviewed. Multiple sources of data provided results to help answer the research questions investigated in this study, and findings related to each question are presented in the next section.[9]

Research Question 1

The first question I explored in this study was as follows:

To what extent and in what ways can the Fraction Foundations MOOC-Ed change participants' knowledge and/or beliefs?

Results from the post-survey as well as the pre- and post-MKT assessment provide insight into the extent to which the MOOC-Ed was effective in changing participants' knowledge

[9] Results in this chapter will be presented with results from FFI and FFII combined, unless otherwise noted. Results are presented separately when notable differences were observed.

and/or beliefs.

Extent of Changes in Knowledge and/or Beliefs

Post-survey results. Results on the closed post-survey items regarding the MOOC-Ed's effectiveness in both FFI and FFII were overwhelmingly positive. Ninety-seven percent of respondents of the post-survey (FFI: N = 54; FFII: N = 159) found the MOOC-Ed to be somewhat effective, effective or very effective in supporting their personal and/or professional goals, which included growth in knowledge and/or skills in teaching. More specifically, at least 90% of respondents somewhat agreed, agreed or strongly agreed with statements that expressed the extent to which they believed that their knowledge and/or skills in content-specific factors of the course improved as a result of their participation in the MOOC-Ed (see Table 5).

Table 5
Post-survey Responses to Improvement in Knowledge/Skills

Question: To what extent do you agree with the following statements? As a result of my participation in this MOOC-Ed, I have improved my knowledge and/or skills related to…					
(N = 196, unless otherwise noted)	Strongly Disagree or Disagree	Somewhat Disagree	Neither Agree nor Disagree	Somewhat Agree	Agree or Strongly Agree
Fraction content standards[10]	3%	1%	7%	21%	70%
Common student difficulties and misconceptions about fractions	1%	1%	1%	12%	86%

[10] N = 195

Table 5 continued

Analysis of student thinking about fractions to inform instruction	1%	1%	1%	14%	83%
Effective use of fair-sharing activities to help students understand key fraction concepts	1%	1%	2%	10%	88%
Effective use of measurement and number line activities to help children understand key fraction concepts	1%	1%	3%	12%	84%
Effective use of fair-sharing and measurement activities to help students understand computations with fractions	1%	1%	2%	15%	81%
Addressing student learning differences when teaching fraction content and skills[11]	2%	1%	2%	21%	76%

These results suggest that the *Fraction Foundations* MOOC-Ed was effective in changing participants' beliefs about their knowledge and/or skills in teaching fractions. Of particular note is the percent of respondents who believed they improved in knowledge and/or skills related to common student difficulties and misconceptions about fractions. In the survey administered in the course registration form (i.e., pre-survey), the learning goal associated with the highest number of respondents in both MOOC-Eds was related to

[11] N = 195

understanding students' difficulties and misconceptions about fractions. Results from the post-survey then suggest that the course was successful in helping participants meet this goal.

Another significant result is participants' perceptions of their improvement in knowledge and/or skills associated with analysis of student thinking about fractions to inform instruction, as this was the learning goal associated with the second highest number of respondents to the pre-survey in both MOOC-Eds. As shown in Table 5, 83% of respondents either agreed or strongly agreed that their knowledge and/or skills in this area improved as a result of their participation in the MOOC-Ed.

Also notable is the percent of respondents in both courses whose perceived improvement in knowledge and/or skills related to effective use of fair-sharing, measurement and number line activities to help students understand key fraction concepts. Interestingly only 9% of the sample of participants in FFI chose “learn to effectively use fair-sharing activities to help students understand key concepts” as one of their primary learning goals for their participation in the pre- survey, and it was not rated among the goals of greatest importance by participants in FFII. Similarly, only 13% of the sample of participants in FFI chose “learn to effectively use measurement and number line activities to help students understand key concepts” as one of their primary learning goals, and participants in FFII also did not rate it among their most important goals. However, the results reported here suggest that the MOOC-Ed was effective in presenting those approaches in a way that supported participants' learning, whether they viewed improvement in those areas as a priority before participating in the course or not.

MKT results. As noted in Chapter Three, a pre- and post-assessment was made available to all participants on *Mathematical Knowledge for Teaching* (MKT) (Hill, Blunk, Charalambous, Lewis, Phelps, Sleep, & Ball, 2008; Learning Mathematics for Teaching (LMT) Project, n.d.) as an additional quantitative measure of the extent to which participants experienced change as a result of their participation in the MOOC-Ed. The results are intended to help describe the extent to which participants' mathematical knowledge for teaching fractions improved after having participated in the MOOC-Ed.

A total of 54 MOOC-Ed participants completed both the pre- and post-MKT assessment in FFI, and 140 participants completed both assessments in FFII[12]. The representation of each participant role for those who took both assessments is provided in Figure 8. The pre- and post-assessments were identical, included a total of 31 items, and were used in their exact form for both FFI and FFII.

[12] Participants whose scores decreased over 10 points were re-checked and eliminated from analysis if they did not answer more than 10 items. Participants whose scores increased over 10 points were re-checked to ensure that most of the questions in the pre-assessment were actually answered and incorrect or not answered at all. Those who did not answer most were eliminated, since I did not consider those participants having "completed" the pre-assessment. The sample values reported here include only participants after this reduction.

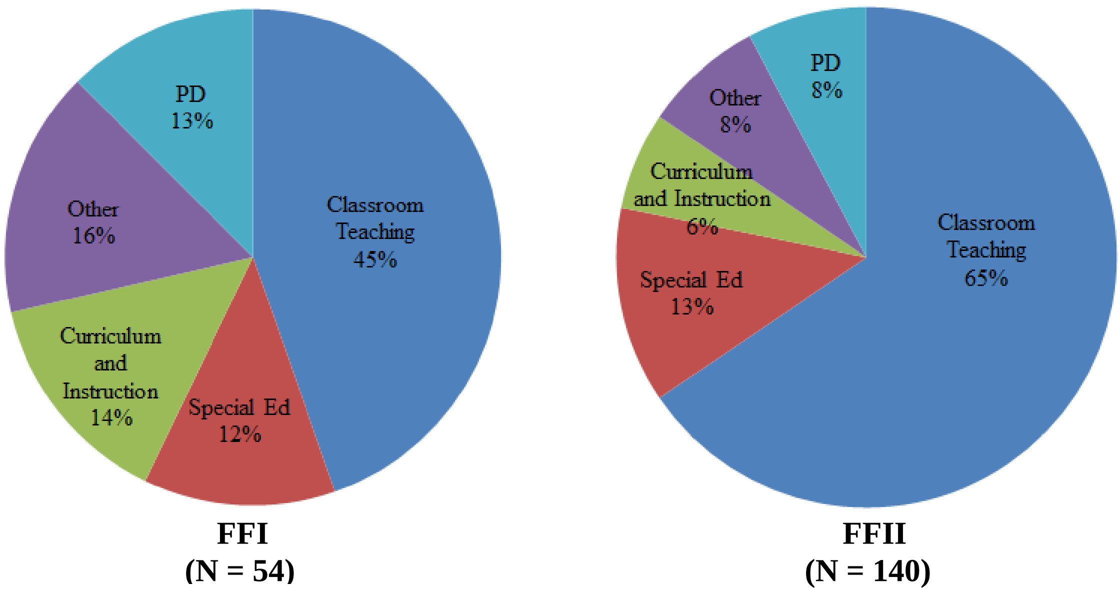


Figure 8. Pre- and Post-MKT Assessment Participant Roles

Table 6 displays the means results of the pre- and post-assessment for FFI and FFII, using a paired *t*-test to compare the differences. The difference in means on the pre- and post-assessment in FFI was not statistically significant, while the difference in means in FFII was significant with a small effect size.

Table 6
Summary of Means Results for FFI and FFII

	Pre-Assessment Mean	**Post-Assessment Mean**	**Increase in Means**	***p*-value**	**Cohen's *d***
FFI	63.98%	66.61%	2.63%	0.08	0.17
FFII	57.63%	61.98%	4.35%	<0.0001	0.26

Category Analysis. As described in Chapter Three, the MKT assessment was designed to include questions that addressed each of the four content units in the *Fraction Foundations* MOOC-Ed. In analyzing changes on content-specific items, I grouped items into four categories that aligned with the units presented in the course: 1) Introductory Fraction Concepts (7 items); 2) Fair-sharing Concepts and Activities (6 items); 3) Measurement and Number Line Activities (4 items); and 4) Operations with Fractions (14 items). Notable changes were observed between pre- and post-scores within only certain categories. However, it should also be acknowledged that the number of items in each category is rather small, so results are only marginally suggestive.

As shown in Table 7, none of the changes in scores within categories between pre- and post-assessments in FFI were statistically significant; however, the effect size of changes in scores on items in the Fair Sharing category did indicate a small effect. This suggests that using fair-sharing to teach fractions may be a new approach to developing students' understanding of fractions for some educators, and the MOOC-Ed was effective in helping participants strengthen their knowledge and skills in that area.

Table 7
MKT Assessment Results in FFI

FFI	**Pre-MKT Mean**	**Post-MKT Mean**	***p*-value**	**Statistically Significant**	**Cohen's *d***	**Effect Size**
Fraction Introduction	71%	75%	0.083	No	0.177	Too small
Fair Sharing	68%	73%	0.110	No	0.234	Small
Measurement	63%	64%	0.833	No	0.033	Too small
Operations	59%	60%	0.428	No	0.087	Too small

Most notable is the difference in means within content categories in FFII. As shown in Table 8, increases in mean scores in three of the four categories were statistically significant and showed small effect sizes. Interestingly, the category of items that did not show improvement, Fair Sharing, was the one that did show statistical improvement in FFI.

Table 8
MKT Assessment Results in FFII

FFII	**Pre-MKT Mean**	**Post-MKT Mean**	***p*-value**	**Statistically Significant**	**Cohen's *d***	**Effect Size**
Fraction Introduction	70%	74%	0.004	Yes	0.248	Small
Fair Sharing	63%	66%	0.209	No	0.112	Too small
Measurement	56%	63%	0.003	Yes	0.214	Small
Operations	50%	54%	0.002	Yes	0.214	Small

Ways in Which the MOOC-Ed Changed Knowledge and/or Beliefs

To examine the ways in which the MOOC-Ed changed participants' knowledge and/or beliefs, I investigated whether design principles implemented in the development of the course were related to participants' perceptions of change. As highlighted in earlier chapters, the design principles included 1) multiple voices; 2) self-directed learning; 3) peer-supported learning; and 4) job-connected learning. I first examined how the course exemplified these principles. Table 9 displays results from the post-survey items that addressed participants' level of agreement with statements related to the MOOC-Ed's success in implementing its design principles. I aligned each of the questions in the survey with the design principle it best represented (second column), and the items are grouped by these design principles.

Multiple voices (MV). The two sub-items I aligned to this principle were related to learning from a diverse group of people with varying levels of experience and expertise.

Self-directed learning (SDL). The four sub-items I aligned to this principle were related to opportunities for participants to choose their own path and engage in the MOOC-Ed in a way that served them best.

Peer-supported learning (PSL). The two sub-items I aligned to this principle were related to opportunities for participants to grow and gain knowledge through their supportive interactions with others in and outside of the MOOC-Ed.

Job-connected learning (JCL). The three sub-items I aligned to this principle were related to the use of practical examples from authentic experience to enhance learning in the MOOC-Ed.

Table 9
Post-survey Responses to Implementation of Design Principles

Question: To what extent do you agree with the following statements? This online professional development experience...						
(N = 210, unless otherwise noted)	*Design Principle Alignment*	Strongly Disagree or Disagree	Somewhat Disagree	Neutral	Somewhat Agree	Agree or Strongly Agree
provided meaningful opportunities to share ideas, resources, and experiences.	*MV PSL*	0%	0%	5%	13%	81%

Table 9 continued

(N = 210, unless otherwise noted)	*Design Principle Alignment*	Strongly Disagree or Disagree	Somewhat Disagree	Neutral	Somewhat Agree	Agree or Strongly Agree
was enhanced by the expertise of practitioners and/or leaders in the field.	*MV*	3%	1%	5%	17%	74%
enabled me to personalize my learning through differentiated resources and activities.	*SDL*	0%	1%	6%	10%	82%
provided activities and/or resources to help me gauge my learning (e.g. self-assessments, peer assessment or feedback)	*SDL*	1%	1%	3%	12%	82%
provided flexibility in the ways in which I could demonstrate my learning.[13]	*SDL*	0%	1%	5%	12%	82%
provided opportunities to investigate self-identified problems or areas of interests.	*SDL*	1%	1%	8%	9%	81%

[13] N = 209

Table 9 continued

(N = 210, unless otherwise noted)	*Design Principle Alignment*	Strongly Disagree or Disagree	Somewhat Disagree	Neutral	Somewhat Agree	Agree or Strongly Agree
provided supports for small-group work or collaborations around shared goals.	*PSL*	1%	3%	10%	16%	70%
provided models of effective practice (e.g. demonstrations, tools, resources) to support the application of course content.	*JCL*	1%	0%	2%	10%	87%
promoted critical-thinking and/or problem-solving through discussions and activities.	*JCL*	1%	0%	3%	13%	82%
provided projects or activities that can be embedded in my professional practice.	*JCL*	1%	0%	5%	9%	85%

Results from the closed post-survey items in this study suggest that the MOOC-Ed was highly successful in implementing the design principles. In order to investigate whether the principles were effective in promoting change in participants' knowledge and/or beliefs, I examined relationships between the MOOC-Ed aspects described in Table 9 and participants' perceptions of changes in their knowledge and/or skills. I discuss this in the next section.

Relationships between MOOC-Ed aspects and changes in knowledge/beliefs. As described in Chapter Three, in seeking to identify relationships between MOOC-Ed aspects and perceptions of change in certain knowledge and/or beliefs, I generated a matrix that displayed the Spearman's rank correlation coefficients associated with pairs of post-survey questions that related aspects of the course and perceived changes in knowledge and/or beliefs. While the analysis did not show strong relationships in FFI, a number of strong relationships were evident in FFII. Appendix E shows the full correlation tables for the post-survey questions relevant to the research questions in this study. Important results related to Research Question 1 are shared in Table 10, displaying correlation coefficients associated with the relevant items in FFII. Only associations between sub-items in distinct questions that corresponded to $|r_s|$ values greater than 0.6 are discussed.

Table 10
Correlation Matrix for Relevant Items in Post-survey (FFII) [14]

	Q7_2: Knowledge of student difficulties and misconceptions	Q7_3: Knowledge of analysis of student thinking about fractions	Q7_4: Knowledge of use of fair-sharing activities for teaching fractions	Q7_5: Knowledge of use of measurement and number line activities for teaching fractions	Q7_6: Knowledge of use of activities for students to learn computations of fractions
Q17_2: MOOC-Ed provided models of effective practice	0.55	0.56	**0.66**	**0.65**	**0.65**
Q17_3: MOOC-Ed provided projects or activities to embed into practice	**0.61**	**0.61**	**0.62**	**0.61**	**0.64**
Q17_5: MOOC-Ed promoted critical-thinking and/or problem-solving	0.59	**0.65**	**0.63**	0.59	**0.65**

[14] Correlation is significant at the 0.001 level (2-tailed)

Table 10 continued

Q17_7: MOOC-Ed enabled personalization of learning	0.58	**0.62**	**0.60**	0.58	0.55
Q17_8: MOOC-Ed provided opportunities to investigate self-identified problems or areas of interests	0.58	**0.60**	0.59	0.54	0.58

Key findings from this correlation analysis are presented here:

Finding 1: Participants' beliefs that the MOOC-Ed presented models of effective practice (e.g., demonstrations, tools, resources) were strongly associated with beliefs that their knowledge of effective use of fraction concept-specific activities (e.g., fair-sharing, measurement, computation) improved after participating in the MOOC-Ed.

Finding 2: Participants' beliefs that the MOOC-Ed provided projects or activities to embed into practice were strongly associated with beliefs that their knowledge of student difficulties and misconceptions about fractions, analysis of student thinking about fractions to inform instruction, and effective use of fraction concept-specific activities in the classroom improved after participating in the MOOC-Ed.

Finding 3: Participants' beliefs that the MOOC-Ed promoted critical-thinking and/or problem-solving (both about their practice and about fractions content specifically) through discussions and activities were strongly associated with beliefs that their knowledge of analysis of student thinking about fractions and effective use of fair-sharing activities to help children understand key fraction concepts improved after participating in the MOOC-Ed.

Finding 4: Participants' beliefs that the MOOC-Ed offered personalized learning through differentiated resources and activities were strongly associated with beliefs that their knowledge of analysis of student thinking about fractions to inform instruction and using fair-sharing activities effectively in the classroom improved after participating in the MOOC-Ed.

Finding 5: Related to finding 4, participants' beliefs that the MOOC-Ed provided opportunities to investigate self-identified problems or areas of interest were strongly associated with beliefs that their knowledge of analysis of student thinking about fractions to inform instruction improved after they participated in the MOOC-Ed.

MOOC-Ed components. In order to further investigate ways in which the MOOC-Ed promoted changes in participants' knowledge and/or beliefs, participants were asked to rate the effectiveness of certain components of the MOOC-Ed that they believed helped support their learning in the post-survey. Results from the post-survey related to this item were overwhelmingly positive. Table 11 shows that for each of the six components addressed in the survey, all were rated as somewhat effective, effective or very effective in supporting participants' professional learning by at least 90% of respondents across both courses.

Table 11
Post-survey Responses to Effective Components

Question: How effective were the following components of this MOOC-Ed in supporting your professional learning?:					
	Very Ineffective or Ineffective	Somewhat Ineffective	Neutral	Somewhat Effective	Effective or Very Effective
Course Videos (N = 213)	2%	2%	1%	8%	86%
Course Project (N = 204)	2%	1%	3%	9%	84%
Course Readings (N = 212)	2%	1%	5%	17%	75%
Practitioner Tools (e.g. rubrics, apps, websites, classroom aids, planning templates, etc.) (N = 210)	3%	1%	4%	10%	81%
Self-Assessment and/or Peer-Assessment Activities (N = 212)	2%	1%	4%	21%	72%
Discussion Forums and/or Online Chats (e.g. Twitter) (N = 210)	3%	1%	7%	22%	68%

In order to contextualize these results, participants were asked to describe “the most valuable aspect of this MOOC-Ed in supporting [their] personal or professional learning goals.” Ninety-six percent of respondents in FFI (N = 52) and 92% of respondents in FFII (N

= 147) provided descriptions. Several aspects of the course emerged as valuable in the open-ended responses participants provided. Most were related to particular components in the course (e.g., videos, project), while others related to effects of the course (e.g., impact on practice). Codes were created to categorize the responses, and each response was assigned codes that aligned with them (see Appendix F for full codebook). Brief descriptors of the codes are: 1) *Case videos*; 2) *Better conceptual understanding*; 3) *Resources*; 4) *Course project*; 5) *Impact on practice*; 6) *Videos (non-descript)*; 7) *Expert panels*; 8) *Peer support*; and, 9) *Discussion boards.* Results from the coding of responses are summarized in Table 12 and further discussed below[15]. All are presented together here in order to illustrate the diversity in participants' interpretation of the term "aspect" when related to effectiveness of the PD. Examples of participant responses are provided following each discussion.

Table 12
Results from Coding of "Most Valuable Aspect" Survey Question

Category/Code	**Percent of Responses in FFI (N = 52)**	**Percent of Responses in FFII (N = 147)**
Case Videos	40%	21%
Better conceptual understanding	33%	24%
Resources	19%	25%
Course project	15%	9%
Impact on practice	13%	12%
Videos (Non-descript)	13%	12%

[15] Results are provided separately for FFI and FFII because a number of codes had considerably different frequencies between the two courses.

Table 12 continued

Category/Code	Percent of Responses in FFI (N = 52)	Percent of Responses in FFII (N = 147)
Expert panels	12%	5%
Peer support	10%	16%
Discussion boards	4%	19%

Case videos. The qualitative responses suggest that the lesson videos, "What Would You Do Next" videos (student interviews or small group teacher videos), and/or the resource videos provided authentic images of students working through problems and presented valuable questioning strategies.

> *The videos that were taken of the students completing the activities were very helpful in seeing how students react and how the teacher responded or asked questions to prompt more learning.*
>
> *I thought watching the videos of actual students helped to understand misconceptions and learn how to address these misconceptions.*

Better conceptual understanding. Respondents cited improvement in knowledge and/or understanding of fractions and teaching fractions as the most valuable aspect of the course. This suggests that components of the MOOC-Ed contributed to participants' beliefs of deepened mathematical knowledge and confidence in their ability to teach the content to their students.

> *I think what was most valuable was clearing up some of the misconceptions about fractions by deepening my understanding so I can relate it better to my students.*
>
> *I believe that I understand fractions better myself after taking this course. This understanding will help me to be a better teacher to my students when it comes to working with fractions.*

I had a deeper look into what fractions mean and it improved my understanding (and my teaching) of fractions.

I was able to stretch my own thinking while reasoning through the various methods of teaching fractions.

Resources. Additional resources provided in the MOOC-Ed served as one of the most valuable aspects of the course to participants. Some referred to the articles and other readings as being relevant and applicable to their practice. Others appreciated the classroom tasks, some accompanied by teacher guides, that participants could incorporate into their teaching.

The most valuable aspect of the course was the classroom activities suggested along the way. These activities were real eye-openers for me and gave me many new ideas for how to approach fractions in a more organic way with my students. Instead of just teaching an algorithm and thinking of word problems, these activities really showed me that understanding fractions requires students to draw, manipulate and explain their thinking about fractions. I couldn't have come up with the suggested activities on my own before taking this course, and now I feel like I have new tools that I use as springboards for my own ideas and am much more effective at teaching fractions.

Course project. Participants seemed to value the opportunity to choose and create useful materials for their classrooms and to collaborate with others to refine those materials.

I appreciated the opportunity to do the project as it really focused my professional development as a staff developer and coach.

I feel that having the classroom project is a great way to use real world application to the objectives that were outlined in this course.

I found that the final project was most valuable because I had to think through, plan, and formally write up a lesson based on new knowledge from this course.

Impact on practice. Some participants believed that the most valuable aspect of the MOOC-Ed was that it had an impact on their current practice. Responses included references to different approaches or enhanced activities used in the classroom, conducting student interviews, implementation of new strategies, and introduction of new activities in providing professional development.

> *I will adjust my own lessons in the future to include more activities where my students will use manipulatives, number lines, understanding the unit, fair shares, and various fractional representations before moving onto teaching the algorithms.*

> *This MOOC-Ed led me to really question my students and listen more carefully to what they said. By doing this, I feel that I have become a better teacher by following their leads as to what they do and do not understand.*

> *My big 'aha' was to have students draw and build fractions when solving problems. I have seen wonderful results with these strategies.*

> *I now realize how important it is for students to do the talking and explaining. Students need understanding instead of the traditional algorithm. This course helped me change my teaching strategies to better go with the Common Core Standards.*

Videos (non-descript). Some respondents referenced videos in their answer to this survey item without specifying whether they were lesson videos, expert panels, "What Would You Do Next" videos, or other resources with video content[16]. This adds to the finding of the great value the course videos held for participants in the course.

> *The videos were very helpful for me, especially since I have poor conceptual reasoning when it comes to fractions.*

> *I really liked all of the videos so I could really see someone else work with fractions.*

Expert panels. Participants referred specifically to the value of the expert panel videos. They appreciated being presented with insights, ideas and advice from experienced

[16] In order to avoid inflating the results reported for the case videos and expert panel videos, since I did not want to assume meanings from responses that weren't made explicit, such responses are reported in this separate category.

researchers and other practitioners in the field.

> *I really enjoyed the expert panel. I thought they gave a good history to explain what was going on in the classrooms and how to fix it.*

> *The panel interviews gave me insight on other points students may find difficult, along with ways to help with different lessons and resources.*

Peer support. As one of the key design principles incorporated in the development of the MOOC-Ed, peer support was well-referenced in the aspects of the course participants found most valuable. References to peer support included assignment feedback from other participants, recommendations or suggestions from others, and working with a group at a local institution.

> *The classroom activity effectively produced a lesson which we felt confident to share, and we were grateful for all feedback.*

> *The most valuable aspect of this course was to work closely with my colleagues. We provided one another with suggestions to better help our students.*

Discussion boards. Interestingly, engaging in discussion boards in the MOOC-Ed was far more frequently referenced in FFII than in FFI. This may be explained by the fact that there was a much larger number of participants in FFII than in FFI, increasing the likelihood that participants would engage in continued exchanges of dialogue and with more diverse responses. Participants valued reading other practitioners' ideas and strategies as well as challenges in the classroom.

> *I really felt that the discussions were most beneficial. It was very interesting to hear what other professionals had to say and their rationales behind it. It heavily influenced my thinking and my future planning.*

> *Hearing from colleagues was helpful in terms of gathering new ideas and strategies. It was also nice to hear how others are implementing strategies similar to mine.*

Research Question 2

As described earlier, the second component of this research explored the impact the MOOC-Ed had on participants' practice. This component was guided by the following research question:

To what extent and in what ways can the Fraction Foundations MOOC-Ed impact participants' practice?

Extent of Impact on Practice

An overwhelming majority of respondents in both courses believed that the MOOC-Ed was effective in preparing them to make positive changes in their practice. Specifically, 81% of respondents across both courses believed that the MOOC-Ed was either effective or very effective in preparing them to make such changes. When asked to rate the extent to which they had attempted to make changes in specific areas of practice as a result of their participation in the course, respondents in FFI rated the extent decidedly high in all areas.[17] Table 13 shows the percent of respondents who indicated having made a moderate or significant change in each of the areas addressed in the survey.

[17] This question was removed from the post-survey in FFII, so results are only reported for FFI.

Table 13
Percent of Respondents Indicating Moderate or Significant Attempts to Change Practice

Question: To what extent have you attempted to make changes in each of the following areas of practice as a result of participating in the MOOC-Ed? (N = 47, unless otherwise noted)	
Approaches to address the diversity of student mathematical proficiency	89%
Approaches to assessing students' mathematical proficiency	85%
Mathematical language I use	83%
Mathematics curriculum content[18]	78%
Collaboration on instructional planning	74%

Ways in Which the MOOC-Ed Impacted Practice

In seeking to identify relationships between elements of perceived effectiveness in the MOOC-Ed and its impact on participants' practice, I generated a matrix that displayed the Spearman's rank correlation coefficients associated with pairs of post-survey items that related aspects of the course and the MOOC-Ed's potential to impact practice. Appendix E shows the full correlation tables for the post-survey questions relevant to the research questions in this study. Important results related to Research Question 2 are shared in Table 14 below. The table displays correlation coefficients associated with items relating MOOC-Ed aspects and the course's potential impact on practice that showed strong relationships in at least one of the course implementations. Only associations between sub-items in distinct questions that had absolute values greater than 0.6 are discussed.

[18] N = 46

Table 14
Correlation Matrix Relating MOOC-Ed Aspects and Its Potential to Impact Practice (Q11[19])[20]

Post-survey item	FFI	FFII
Q3: Effectiveness in supporting goals	**0.67**	**0.62**
Q17_1: MOOC-Ed was enhanced by expertise of practitioners and/or leaders in the field	0.36	**0.61**
Q17_2: MOOC-Ed provided models of effective practice	0.53	**0.70**
Q17_3: MOOC-Ed provided projects or activities to embed into practice	**0.67**	**0.62**
Q17_5: MOOC-Ed promoted critical-thinking and/or problem-solving	**0.62**	**0.70**
Q17_10: MOOC-Ed provided activities and/or resources to help me gauge my learning	0.41	**0.64**
Q17_11: MOOC-Ed clearly communicated purpose, goals and expectations of course	0.56	**0.64**

The results indicate that participants' beliefs that the MOOC-Ed was effective in preparing them to make positive changes to their practice were strongly associated with their beliefs that the MOOC-Ed:

- was effective in supporting their professional goals.
- was enhanced by the expertise of practitioners and/or leaders in the field.
- included models of effective practice (e.g., instructional decisions, approaches to assessment, questioning strategies) to support the application of course content.

[19] Q11: "Overall, how effective do you feel this MOOC-Ed was in preparing you to make positive changes in your professional practice?"
[20] Correlation is significant at the 0.001 level (2-tailed)

- provided projects and/or activities that can be embedded in their practice.
- promoted critical-thinking and/or problem-solving through discussions and activities.
- provided activities and/or resources to them gauge their learning (e.g., self-assessments, peer feedback).
- clearly communicated the purpose, goals, and expectations of the course.

Descriptions of Changes to Practice. In order to contextualize the quantitative results that help answer this research question, participants were asked in an open-ended post-survey question to describe specific changes they had made in their practice as a result of their participation in the MOOC-Ed. Over 80% of the survey respondents in FFI and over 90% of respondents in FFII provided qualitative answers to the prompt "Please describe any changes you have made to your practice, including how you have applied the knowledge, skills, and/or resources you gained in this course". Brief descriptors of the codes used are: 1) *Changes to approach*; 2) *Use of manipulatives/tools*; 3) *Attention to student thinking*; 4) *Serving as a resource*; 5) *Increased focus on concepts*; and 6) *Personal improvement in knowledge.* Results from coding of responses are summarized in Table 15 and further discussed below.[21] These results are reported as the percent of survey question responses that related to the code category, and those responses included reference to the category at least once.[22] Examples of participant responses are provided following each discussion.

[21] Results are provided separately for FFI and FFII because two codes had notably different frequencies between the two courses.

[22] Some participant responses received more than one code.

Table 15
Results from Coding of "Changes to Practice" Survey Question

Category/Code	Percent of Responses in FFI (N = 44)	Percent of Responses in FFII (N = 147)
Changes to approach	61%	63%
Use of manipulatives/tools	30%	33%
Attention to student thinking	27%	18%
Serving as a resource	16%	12%
Increased focus on concepts	14%	12%
Personal improvement in knowledge	9%	4%

Changes to approach. The impact most frequently described by respondents in both implementations of the MOOC-Ed was that of a change in teaching approach. Responses were diverse in their descriptions. Some described changes that had already been implemented, and others described anticipated changes going forward.

> *I have used the videos practices to help me in the engagement of my students in sharing their ideas more freely in solving fractional problems independently working through the steps.*

> *I have started asking more probing questions and have allowed my students to take the time to explore and "play" with fractions using models, strips, and number lines. Two very important changes occurred: I have spent more time listening to my students discuss their ideas about fractions and fraction problems, and I have allowed my students more time to discuss their ideas with each other in groups.*

> *This course encouraged me to not only back up when the students don't seem to grasp the concept, but to think of alternative methods of presentation more than I had been.*

Use of manipulatives/tools. Respondents referenced a change in the use of new or more manipulatives, tools, or models (e.g., fraction bars; the number line) in their instruction. This suggests that the MOOC-Ed was effective in providing participants with practical and useful tools for instruction that would help deepen students' understanding of fractions.

> *I have been allowing more time for my students to figure out the problem, using manipulatives in a more effective way based on the resources found in this fraction course.*
>
> *I have successfully implemented fractions strips and number lines into my 3rd grade remedial math group. They are really understanding "like fractions".*
>
> *I have been using number lines a lot more, giving my students another representation of fractional amounts. This helped a lot when transitioning to decimals after fractions.*

Attention to student thinking. Respondents referenced their improvement in their attention to student thinking in the classroom. Responses included improved knowledge of how students make sense of concepts, better questioning strategies based on student responses, and addressing student misconceptions and challenges. This provides additional evidence that the MOOC-Ed was effective in providing resources for participants to strengthen their understanding and analysis of student thinking.

> *I have changed my practice in that I look more deeply at what children know about fractions and then build off their knowledge to help them clear up their misconceptions.*
>
> *I have made changes in the way that I have been interviewing my students on what they are thinking when they are figuring out a math problem. Letting them share the way they are thinking has helped me adjust my lessons according to their needs.*
>
> *I now really talk to the students and have interviews so I can assess better. I also look closer at student answers I once would have considered incorrect.*
>
> *I have started using some of the more open-ended fraction tasks with my students. I am finding they offer a deeper insight into what the student*

understands, and where areas of uncertainty or misunderstanding lay.

Serving as a resource. Respondents referenced an impact on their ability to serve as a resource to other educators. Some participants referred to their improved mentoring abilities, while some others referred to their roles as members of a professional learning community. This suggests that the MOOC-Ed was successful in encouraging and empowering participants to share what they learned with their peers, one of the goals of the MOOC-Ed.

> *Because of this course, I have been able to be a better resource for the math teachers at my school. For example, when my 5th grade began their study of fractions, I had a better understanding of the misconceptions that would possibly arise. My coworkers thought I was a "genius" when the students exhibited the misconceptions that I had prepared the teachers to address.*
>
> *I am a staff developer for my district, and I am working as a coach for 5 school sites as well as developing and providing the professional development for all third grade teachers in the district. This course has helped me focus on what is important for teachers to know in their upcoming professional development.*
>
> *I lead secondary mathematics teachers and as our standards have called for a greater demand of teaching through application, I have used examples from the class such as the "what would you do next" scenarios with my team to guide their work as they coach teachers and in our professional development sessions. Moreover, using fraction models have been an area of opportunity and I have been able to support the teaching of this more effectively by sharing the work with models, the number line and having teachers seek to truly understand the students' thinking by creating multiple opportunities for students to make their thinking explicit.*

Increased focus on concepts. Respondents referenced an impact on their focus on conceptual understanding in their instruction. This suggests that the MOOC-Ed was effective in providing the information and resources participants needed to implement changes in their practice that emphasized deeper thinking and more conceptual understanding of fractions.

> *I have used the knowledge, skills, and resources in planning my instruction. I now focus on understanding/quality instead of quantity. I am trying to provide problems that are meaningful to my students-problems where they can demonstrate understanding or lack of in order to facilitate instruction.*

> *I have looked more at fractions and their relationship to the whole than earlier. I have learned that students don't necessarily understand this, whereas before, I just sort of assumed that they did. As a result, the meaning of numerators and denominators is not grasped as fully as needed to progress with their understanding of calculations involving fractions.*

> *As I taught fractions this year, I tried to stay away from simply teaching the rules of working with fractions, and made sure that students had a more conceptual and concrete understanding of them.*

Personal improvement in fraction understanding. Respondents referenced an impact on their own fraction understanding. This suggests that the MOOC-Ed was effective in providing resources and information that strengthened participants' knowledge of fractions, thereby improving their ability to teach the subject to their students.

> *This course has helped me hone my conceptual skills.*

> *The changes that I have made to my practice were the applying my knowledge and understanding of fractions in my classroom, and it gave my students a better understanding also.*

Interconnections between Changes in Knowledge/Beliefs and Impact on Practice

As Clarke and Hollingsworth's (2002) *Interconnected Model of Professional Growth* suggests, it is reasonable to expect changes in knowledge and/or beliefs to be associated with changes in practice. In order to investigate this, I was interested in identifying relationships between participants' perceptions of changes in their knowledge and/or beliefs and the impact the MOOC-Ed could have on their practice. Correlation analysis showed a number of strong relationships. Important results related to the relationship between changes in knowledge and/or beliefs and the MOOC-Ed's potential to impact practice are shared in Table 16 below. The table displays correlation coefficients associated with items that showed strong relationships in at least one of the course implementations. Only associations between sub-items in distinct questions that corresponded to absolute values of correlation coefficients greater than 0.6 are discussed. Appendix E shows the full correlation tables for the post-survey questions relevant to this study.

Table 16
Correlations between Responses to Post-Survey Questions on "Preparation to Make Changes in Practice" (Q11) and "Improvement in Knowledge/Skills" (Q7)[23]

	FFI	**FFII**
Q7_1: Knowledge of fraction content standards	0.44	**0.65**
Q7_2: Knowledge of student difficulties and misconceptions	0.52	**0.63**
Q7_3: Knowledge of analysis of student thinking about fractions	0.54	**0.60**
Q7_4: Knowledge of use of fair-sharing activities for teaching fractions	0.57	**0.64**
Q7_5: Knowledge of use of measurement and number line activities for teaching fractions	**0.68**	0.58
Q7_6: Knowledge of use of activities for students to learn computations of fractions	**0.63**	**0.61**

[23] Correlation is significant at the 0.001 level (2-tailed)

These results indicate that participants' beliefs that the MOOC-Ed was effective in preparing them to make positive changes to their practice were strongly associated with their perceptions of improvement in their knowledge of each of the following:

- fraction content standards.
- common student difficulties and misconceptions about fractions.
- analysis of student thinking about fractions to inform instruction.
- using fair-sharing, measurement and number line activities to help students understand key fraction concepts.

The results presented in this section suggest a strong relationship between participants' beliefs about their improved knowledge after their participation in the MOOC-Ed and the MOOC-Ed's potential to impact their practice.

Participant Narratives

In order to further explore how and in what ways participation in the *Fraction Foundations* MOOC-Ed could promote changes in knowledge and/or beliefs and impact practice, four elementary teachers who participated in the course were interviewed and observed. Using the *Interconnected Model of Professional Growth* (Clarke & Hollingsworth, 2002) as a framework for this discussion (see Figure 1), as I describe each of these teachers and the themes that emerged in our conversations, I also note the path I interpreted each teacher to take in their "change environment" as they experienced changes in their knowledge, beliefs and practice during and after participating in the MOOC-Ed.

As described in Chapter Three, the model divides a teacher's "change environment" into four domains: the *personal domain* (knowledge, beliefs and attitudes), the *external domain* (external sources of information, professional development), the *domain of practice* (implementation of new approaches/activities), and the *domain of consequence* (outcomes and conclusions) (Clarke & Hollingsworth, 2002, p. 950). In my analysis, the MOOC-Ed serves as part of the *external domain.* The model illustrates movement between the respective domains occurs as a result of *enactment* and *reflection* (indicated by the arrows in the figure).

Within each narrative, I identify domains or transitions between domains that describe the teacher's path through their "change environment" during and after participating in the MOOC-Ed. For example, if a teacher moved from their *personal domain* to the *external domain*, I notate that as "*personal* → *external*" in parentheses following the description of that transition. Domain identifiers are as follows:

Table 17
Domains in Teachers' "Change Environment"

Domain	**Identifier**
Personal Domain	*personal*
External Domain	*external*
Domain of Practice	*practice*
Domain of Consequence	*consequence*

At the conclusion of each narrative, I provide a summary of the teacher's path using a "growth network" based on the Clarke and Hollingsworth model.

Marilyn

At the time of my first visit, Marilyn had been teaching for 21 years and had taught kindergarten, 2nd and 3rd grades. It was her 15th year teaching 3rd grade. Marilyn was initially trained as an educator in her pre-service teacher program in college and received her degree and certification in K-6 education. She has National Board Certification as a Generalist and master's degrees in both Education and Administration. Both advanced degrees were obtained through online programs, so Marilyn was comfortable with and successful at learning online. She teaches in a rural county in central North Carolina that serves students between pre-kindergarten and eighth grade. The school had received STEM (Science, Technology, Engineering and Math) certification and promotes discovery and investigation in the classroom.

Professional development is quite readily available to Marilyn at her school. She teaches in an environment in which meaningful and relevant professional development is valued, particularly by the administration. "Our principal is all for innovation." The elementary teachers also have the support of a mathematics content specialist who visits the school approximately twice a month to offer professional development that fits their needs. "When the specialist comes, she reminds us to stay out of that comfort zone--to push, to push, the [students] and yourself."

At her school, while Marilyn and her colleagues work in vertical teams, where teachers from across grades meet to discuss their approaches, curricula and best practices, they meet just twice a year. Marilyn values those meetings, as they provide opportunities for her to learn what her students come to her knowing and what they are expected to know after they leave. She noted that the vertical team meetings:

> give us a chance to intermingle, because we are three halls, and sometimes we never get to see each other. I always know what's ahead of me, but sometimes I'm not very sure about what the 2nd and 1st grade kids [have learned].

It is clear that Marilyn is passionate about what she does. She is thoughtful and reflective about her practice and cherishes the opportunities she has to affect her students' growth. Marilyn recalls working with one of her students, Albert, who struggled through a lesson on measuring perimeter using non-standard units:

> When I sat down with Albert, who struggles and I knew he didn't know, but just to get him to repeat my words. You know, that gives me goose bumps, because tomorrow I'm going to kneel right down beside his desk, and I'll say, 'Yesterday, remember when we had those magnifying glasses?' He has those tangible things to remember and that's what keeps me coming back to work every day.

Marilyn credits her own learning difficulties for some of her strength as a teacher, as they encouraged her to find ways to learn that helped her meet her own needs:

> A little bit about me--I did not read until I was in 3rd grade. I am identified LD [Learning Disabled] reading and math across the board, so I had to come up with lots of strategies in order to do just some of the basic things.

She found that at times, her PD experiences validated what she was already doing. Some of the strategies she learned in PD workshops were ones she had adopted in order to strengthen her own learning:

> I was overwhelmed, all these mathematical minds. You know, "What am I doing

> here?" But then from that I was able to take my disability and turn it into something that I could use. A lot of the strategies that I learned in that workshop, I was already doing because I had such a hard time. I teach gifted kids but I always say the learning disabled are the most gifted because they have to adapt and come up with strategies in order just to conform.

Views on professional development. Marilyn is an experienced teacher who exhibits great passion for her work and improving upon her practice. She is active in seeking opportunities for professional development and enjoys diversity in learning experiences. She believes that "you have to be [open to learning] because the [students are] getting younger and younger, and I'm getting older and older…and you have to bend and sway with them and the way that they think." She is reflective about her practice and adjusts her plans from previous years based on what she learns through both formal and informal methods of professional development. For example, in preparing a lesson on perimeter, Marilyn noted that:

> I usually jump right into perimeter giving them the polygon, telling them to add the sides; but I had a lot of success with 'non-standard' teaching area getting that baseline in…I attended [a week-long, residential workshop] for two summers, and that was one of the things using the non-standard units that I had done with them. I happened to be digging through some of those things, and that activity was in there with area, so I said, 'I'll just try it with perimeter.'

Having had diverse professional development experiences, Marilyn recognizes elements that she finds make that PD effective for her. Marilyn values PD that is situated, or provides activities that are grounded and modeled in the classroom, such as engaging with the actual tasks students would be asked to do. She recalled an experience she had in a PD program that worked well for her:

> We went through all of the strands, and they would take us through what was new--the newer thinking. Then [they would] have us do some of the activities. So you kind of went back and forth; you were the professional student first, and then they brought you back down to your grade level to see…well, 'this is what this looks like to a 3rd grader.'

Marilyn believes that reflection and opportunities to discuss approaches and ideas with other practitioners are important elements of a PD experience. She finds that the exchange of thoughts and perspectives promotes positive, "step-by-step" change that allows her to build upon what she learns to make incremental and evolving adjustments to her practice. Marilyn also finds that opportunities for continuing these exchanges after a PD experience are valuable in making that PD effective. Related to that, Marilyn appreciates PD that can impact her practice during the school year, or "when you're in the throes of it." She finds that PD that is relevant to what she is teaching at the time it aligns with content in the curriculum is highly effective. Even further, Marilyn values PD that she can engage in at a level that fits her own professional needs, offering opportunities for personalization.

Teaching approach. Marilyn cited how PD has impacted her teaching as evidence that a PD experience is effective. She spends more time developing ideas with her students than she did as a younger teacher:

> You know, maybe ten years [ago], area and perimeter would all be in one week. This

> is my third week, and I just finished area. I took 2 ½ weeks on area, and so, today was day one of perimeter, which I'll probably take another week or so on. Before, it would all be pushed together, five days, quiz on the next week and we're done.

Marilyn arranges her students in the classroom in groups of 3 to 4 students. She incorporates activities that are structured around group work and investigation, and promotes frequent justification by word and in writing. She seats students according to a research-based model that she justified as being effective for her students. The students are "strategically placed with one high person at each table and one low; everyone else filters in between." Students with "opposite abilities" are seated across from one another, and students with "similar" abilities are seated next to each other. When Marilyn facilitates discussion on new topics, she has students engage with those seated opposite them:

> When they're sharing, then sometimes I can hear my stronger ones do that formative assessment and kind of tweak their thinking. Most of the time, if it's a shoulder partner share, I know they have it, and so I don't need to intervene as much; so when they do share out, I either have to stop, you know, and correct everyone's thinking or that immediately lets me know I can go on.

Marilyn maintains a culture of rigor and high expectations in her classroom. Her students understand that they are expected to "defend" their reasoning and express their challenges as they learn. She believes that this strengthens her students' understanding and success:

> We talk a whole lot. They learn more from each other, and I know if I can teach you how to do it, then I understand it. So we use the word 'coaching' a lot in here; you

> know, I need for you to coach someone, and they just want to tell them the answer; but I want you to give them hints. If you can hint, you have it.

Marilyn uses her students' thinking to inform her instruction. As she plans her lessons, she anticipates students' misconceptions and the conclusions they might make. "I try to think like them when I'm planning." She builds in frequent opportunities for students to share their thinking as a form of assessment:

> It's almost like that quick formative assessment, when they share their thinking; if they're on the wrong path, then I have to immediately correct that. And sometimes if they're sharing, sometimes I purposefully choose the ones that I know are struggling; so when they're sharing, then I will most likely have a student that 'saved' them--so that it's not coming from me, it's coming from them.

Marilyn is also mindful of how her students' behavior can influence how her lessons could go. She varies students' tasks in the classroom to keep them engaged and active:

> You just have to think about their behaviors, not so much as, you know, being a good child or not so good child, but…what they would do with the information and the lesson you present.

In her teaching, Marilyn emphasizes precision in mathematical language (e.g., "not 'times', but 'multiply'). She puts great emphasis on students' explaining their strategies and their choice of those strategies, and she promotes students' reasoning both verbally and in writing. Students understand that they are expected to be prepared to "defend" their answers, and when helping other students, they are to serve as a "coach". Marilyn also engages students in discussion about their "confessions" or problems they faced as they worked on assignments:

> We all confess; everybody has a confession, everybody has something wrong. And so, they're very comfortable with [that]. So, when I say, 'Well, what was he or she thinking?' they are very comfortable with going in and helping.

Marilyn encourages students to develop their ideas and choose strategies that work best for them. For example, during a lesson on perimeter and area, Marilyn reminded her students that they have choices when they solve problems. "If you don't like a number, decompose it--break it down. I'm a best friend of 2, 5 and 10." The effect is that her students are used to these expectations and seem to understand that they are classroom norms.

Classroom visits. The class I observed before Marilyn started the MOOC-Ed included 21 students with diverse ability levels. Six students were identified as AG (Academically Gifted); one was identified as ESL (English as a Second Language); and one was identified as LD (Learning Disabled) in reading. The remaining students were classified as "regular ed".

On the day of this visit, Marilyn introduced the concept of perimeter, following 2 ½ weeks of instruction on area. For the lesson, Marilyn's students were instructed to measure the perimeter of a desk using non-standard units, such as paper cups and straws. She required students to write their answers in complete sentences and ultimately do a "gallery walk", so that the students could tour each group's results. Following this activity, Marilyn brought the students back together and engaged in a discussion on observations and generalizations.

MARILYN: Now, after you did your tour, thinking about the perimeter of everyone else's desk, thinking about their units they used, what was something interesting you saw on someone else's desk, and their perimeter and their units?

STUDENT: I realized that the smaller your units are, that's how many you're going to have a whole lot more.

MARILYN: Okay, so the smaller the unit, the more of them you have to use?

As a parting exercise, or "exit ticket", Marilyn had her students answer the question, "If you had to coach someone on perimeter, what would you tell them?"

Marilyn's approach and overall classroom atmosphere during the following two visits were consistent with the observations made in the first visit. These visits occurred 4 and 6 months after the MOOC-Ed had concluded, respectively. In the second visit, Marilyn conducted a lesson on area models and their connection to the distributive property. The lesson was framed around a hands-on activity with crackers to build intuition for the concept of area. She was attentive to precise mathematical language, monitored student work closely, and asked probing questions as students "defended" their thinking. Students were frequently asked "How do you know?", and fellow classmates were asked to analyze other students' work as they "confessed" their challenges in solving problems. In this visit, students seemed to talk more than Marilyn did during the lesson. Indeed, her students expect it:

> They know that when you go up there, you better be ready. They try their best for me not to ask them any questions. So, when they come up there, they have all their bases covered.

In my third visit, Marilyn conducted a lesson on ordering unit fractions. She had selected an activity she found on a free online teacher resource that developed students' understanding of how unit fractions compare in value. The activity was grounded in the common misconception that dividing by a bigger number, and holding the numerator constant, results in a greater value. At the conclusion of the activity, instead of providing students with a conclusion herself, she called on a student to explain why they thought a fraction with a larger denominator is actually smaller than one with a smaller denominator (and like numerators). The student explained that "the more [an object] is partitioned, the smaller each piece is going to get" and followed it with an illustration in front of the class. Marilyn concluded the lesson by having students write a note to a second grade student in their journals: "Tell them about partitioning and that pesky denominator. Tell them about what happens when that denominator gets bigger."

Marilyn's MOOC-Ed experience. Marilyn participated in FFI and was referred to the course by the mathematics content specialist she and her colleagues were working with at her school. As a third-grade teacher, Marilyn found that PD on fractions was high need, since she would be focusing quite a bit of time on the topic in her curriculum. Part of her motivation for taking the MOOC-Ed and considering a different approach to teaching fractions was fueled by new standard assessments (*personal* → *external*):

> I saw how much fractions played a part last year on the new EOG, and I knew that we had to do something different. The [students] needed something different to be able to do that problem solving, and the number line was such an intricate part of the fractions. No more is it 'here's the shape and it's partitioned'; it's 'where does it fall [on the number line].' So I know we all needed, you know, something different.

While Marilyn had a number of goals going in to the course, her primary one was to learn new approaches to teaching fractions, as new standards had been adopted:

> I was hoping that it would give me a chance to learn something different to teach fractions a little easier; [with] the new [standards], we have so many things added that I haven't taught before. So I was looking for more strategies on how to teach those.

In working toward that goal, Marilyn recognized that, with over 20 years of teaching experience, what she was learning in the MOOC-Ed was taking her "a little bit out of [her] comfort zone." For example, when she viewed one of the expert panel videos in the course, she learned that it may be more effective for students to create their own manipulatives. She recognized that this would give students more control over their own learning and increase their retention:

> I have always given my children plastic manipulatives for fractions; and in one of the course resources, one of the gentlemen was talking about letting the children make their own manipulatives, and that would kind of get them to see what a whole was. When they [the students] are doing it, it makes it a little more meaningful, versus me passing out all those plastic pieces, and the [students] don't have a point of reference. And that's so much out of my comfort zone because I want their pieces to look, you know, unified. But if they're cutting their own whole up, then they have a perfect point of reference. So I said that was my 'aha' moment, that I had to let that part go, you know, and let [the students] have it. (*external* → *personal*)

Marilyn's work toward accomplishing her goal of implementing new approaches to teaching fractions was evident in her lesson on comparing and ordering unit fractions in my last visit to her classroom. She partially credited the MOOC-Ed for motivating her to reflect on her practice and implement activities she hadn't used to teach fractions before (*external* → *personal* → *practice)*. She described these activities as being more "hands-on" and "not so much using the old-fashioned fraction models." For example, in one of her activities, Marilyn had her students place cards with the numbers 0, 1/8, ¼, ½, and 1 in order on a string taped to the board, mimicking a number line. As the activity progressed, she realized her students' growth in understanding how the fractions related to each other:

> I could see how all of this is coming, you know, back full circle when you hang those number tents up there, and the students have to go back to making that half fair share when they're labeling, the zero, the 1 and the ½. So that was kind of my---okay, 'this does make sense' kind of thing. I knew I wanted them to label the zero, ½ and the whole, but I didn't realize how much that one activity only partitioning giving them the half was their reference. I didn't put it all together until they were putting the whole numbers back up there, and I was like, 'Yeah!'. (*practice* → *personal*)

Marilyn found two main elements that contributed to the MOOC-Ed's effectiveness. One was the MKT pre-assessment, which she found "eye-opening". She noted that the activity put her in the position of a student and gave her perspective as to how her students may feel when learning fractions. The rigor of the pre-assessment motivated her to put focused effort into the course:

> Once I got to where it got confusing, and I didn't really know where to go, I almost felt like my students when I'm giving them something, and they don't know where to go. A lot of that I didn't really realize you needed fractions to solve it until I was sitting there kind of scribing, going 'yeah, you need a lot of fraction knowledge to

> solve these', and they were quite difficult. It made me…pay close attention during the actual lessons in the MOOC, so that I can do a point of reference, and I'll know what's going on with those other questions.

Marilyn also found the case videos with students in the classroom, as well as the expert panels, to be particularly effective elements of the MOOC-Ed. They prompted her to reflect on her own practice and consider what she could do differently (*external* → *personal*):

> I am an auditory, visual learner, and just going through those resources and listening to those professors and listening to those experts…I would think, 'well last year I taught it this way, so this year I need to at least try a different way.'

In reflecting on how she would define "success" in the course, Marilyn referred to her own sense of comfort in teaching fractions to her students, or "being able to relax teaching fractions." Success would include her implementing some of the lessons she was exposed to in the course and feeling confident about addressing students' challenges:

> Success would be for me to be able to do those [lessons], and then be able to help the [students]. I'm really good at finding the mistake; but the MOOC kind of gave you the language to get them out of it, you know, to help them. And I think that was a lot of what I was missing. Sometimes with fractions you just restate what you just said, instead of giving [students] a complete different strategy. I need to be able to get you out of this without completely rescuing you. And fractions are just…I rescue a lot with fractions you know.

Impact on practice. With over 20 years of teaching experience and a strong teaching approach, significant changes to Marilyn's practice following PD may not have been expected. However, Marilyn reflected on how her practice had changed since having participated in the MOOC-Ed and credited the course as having taught her to give her students more ownership in their learning (*external* → *personal*):

> Let them have it--let *them* have it. It doesn't have to be perfect; they have to talk to each other, and they need manipulatives. They have to have them, no matter how easy I think the skill is. It just never hurts to have those manipulatives--that point of reference…that is personal for them.

In designing the lesson I observed in my final classroom visit, Marilyn explained that the MOOC-Ed motivated her to avoid reverting to using the activities she had used in the past to teach fractions and consider implementing a new one. She remarked that the MOOC-Ed encouraged her to:

> [try] to do, purposefully, [try] to do something different on purpose. Trying not to drag out my old fraction folder and dig from it. Trying to do something other than I would normally have done.

Marilyn also learned to take more time when teaching fractions and to let her students develop their own ideas. She understands that such pacing is important for students of all levels of ability:

> I always thought that manipulatives and that type of pacing would be geared more for your middle way to low students, and we just take for granted that your high achievers understand. It has impacted me to just slow down. It's not a race. You have to strategically take your time and give them a chance to develop that knowledge, to start small and then let it blossom (*external* → *personal* → *practice*).

Marilyn had observed the effects of changes in her practice on both her and her students. The effects appear to be cyclical. The MOOC-Ed motivated her to give her students more ownership, which helped them learn more effectively and gain confidence (*practice* → *personal* → *consequence*). This then further encouraged Marilyn to continue with that approach (*consequence* → *personal*):

> The [students] enjoy asking each other questions, because some of my lower students, if they get a concept, they all of a sudden become the expert. And so, now they are the ones that are helping, which builds that self-esteem. So it has impacted me to kind of 'let go'. That's how it's changed my thinking.

The MOOC-Ed promoted internal changes in Marilyn as well. It gave her a chance to reflect and consider what changes to her practice might look like. These changes would result in making the content she teaches more "meaningful" to her students:

> If something is working well, it doesn't necessarily mean it's right. This is what I've done in the past and it has worked, but the MOOC took it to another level that says, 'You can get the same results by going this path *and* is more meaningful.' The more meaningful part is where the MOOC kind of hit. A lot of times we just go through it and teach it, and it's not as meaningful. The resources in the MOOC started you from the bottom to the top and made it more meaningful for the children (*external* → *personal* → *practice*).

Marilyn recognizes that change is ongoing and that she will continue to work on implementing what she learned from the MOOC-Ed:

> So I think that's where I'm going to struggle with fractions is slowing it down;

building it up; making it more meaningful. That's going to be my challenge, you know, not to want to rush through it, we're getting close to the end of the year. I'm going to have to take my time. (*practice* → *personal*)

Marilyn's path in her "change environment". In analyzing Marilyn's path among the four domains in her "change environment" through her MOOC-Ed experience (see Figure 9), I describe her growth as beginning and ending[24] in the *personal domain*, with her "growth network" as follows (Clarke & Hollingsworth, 2002):

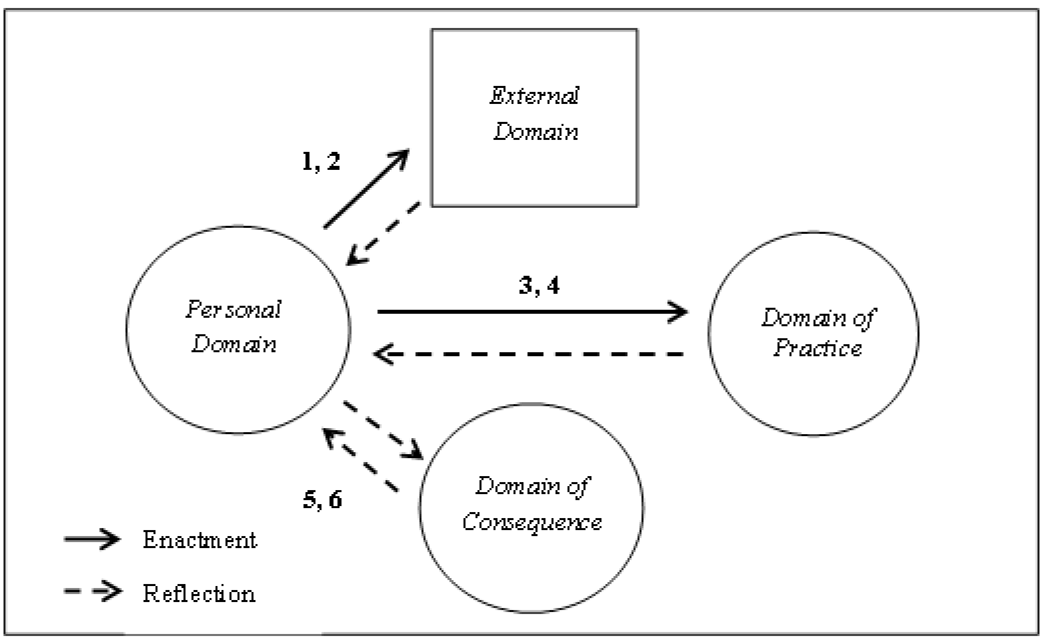


Figure 9. Marilyn's "Growth Network" (Clarke & Hollingsworth, 2002). Numbers are provided in order of edges in the network, with *x*, *y* indicating order from top arrow to bottom arrow, respectively.

[24] The words "beginning" and "ending" are used here only to indicate the domains in which the participant is observed as being at the beginning and conclusion of the study. It is assumed that the participant continues their path as they continue their growth.

Marilyn entered the MOOC-Ed with knowledge and beliefs about how to teach children and how they learn that aligned with much of the course. She frequently asked students to share their thinking, "defend" their answers with sound reasoning, and use precise mathematical language. Marilyn had also frequently participated in PD that provided opportunities to engage in "meaningful" tasks that students could do in the classrooms themselves. In the figure, the solid arrow pointing from the *personal domain* to the *external domain* indicates Marilyn's "enacting" her knowledge and beliefs as she engaged in the MOOC-Ed (edge #1). The dotted arrow pointing back to Marilyn's *personal domain* indicates Marilyn's "reflection" and enhancement of that knowledge and belief after having engaged in the MOOC-Ed, or the *external domain* (edge #2). With the knowledge she gained, Marilyn then enacted what she learned and her enhanced perspective in her *domain of practice* in the form of new activities in the classroom, refined pedagogical methods to elicit student thinking, and increased focus on precise language (edge #3). After observing results of this enactment, Marilyn reflected on what both she and her students learned in the process in order to further shape her knowledge of what her students know and her beliefs on whether the approaches she was now taking were effective for her students' success (edge #4). With this knowledge, Marilyn drew conclusions about how to serve her students better in the *domain of consequence*, based on what she learned from the MOOC-Ed and from enacting that knowledge in the classroom (edge #5). Marilyn is consistently mindful and reflective about her practice, and with her conclusions, she embraced the new strategies and knowledge she had learned (edge #6).

Marilyn's growth network is fluid and cyclical, and it is clear that her professional path has and will continue to traverse each of the edges of this network.

Jill

At the time of my first visit, Jill had been teaching for 28 years, most of which were 4^{th} grade. Jill began her career as a pre-service teacher in college. She teaches in the same school as Marilyn does; however, they did not participate in the MOOC-Ed together. Jill's school is in a rural county in central North Carolina that serves students between pre-kindergarten and eighth grade. The school has received STEM (Science, Technology, Engineering and Math) certification and promotes discovery and investigation in the classroom.

As described in Marilyn's school, professional development is readily available to Jill at her school. The principal was new this year, and Jill described her as "very supportive of anything we try, especially if it's research based." The elementary teachers also have the support of a mathematics content specialist who visits the school approximately twice a month to offer professional development that fits their needs. Jill values that time with the specialist and appreciates her personalization of the support she gives. Jill even noted that between her visits to the school, the specialist makes herself available even at a moment's notice. "We can call her…[and say] 'I need help with this, what can you give me?'"

Jill also has a grade-level professional learning community (PLC) that she works with to plan and discuss issues specific to teaching 4^{th} grade at her school. Although they are rare, Jill also greatly values the opportunities she has to engage in PD that spans the 3-5 grade band. She finds it helpful to learn about what and how her students learned the previous year before they enter her classroom and what they need to know going forward:

> In 4th grade I've always told them, 'We take what you've learned and we have to think about it and use it in different ways…You know that perimeter is the distance around…you learned that in 3rd grade. I've seen [the teachers] teach that to you'; but then [the students] come to me and some of them [feel] that even though we've done it…they're still [saying] 'How do I do this?' …So it's neat to see where the [students] take it when we pick up what we do with it and then, what the next grade level expects, so they can go on.

With her significant teaching experience, Jill is reflective about her practice, and her teaching philosophy acknowledges student learning differences. She remarked that to her:

> effective instruction is whatever helps that child understand that concept. It might be drawing those circles; it might be bringing in a box of powdered sugar or a loaf of bread. I don't think you can do it one way. We don't all like our hamburger the same way.

Jill also acknowledges that the way she teaches now differs markedly from how she was taught herself, which motivates her to participate in PD opportunities that will strengthen her own understanding:

> In all honesty, there are things I was taught to do as a student growing up that I never understood the reason behind it. So as a teacher, I've been working really hard, especially within the last five years to try to make sure [the students] understand why we do it this way.

Views on professional development. Jill is a highly experienced teacher who is increasingly open to new and varied PD experiences to grow as an educator. She believes that in order to improve, she needs to embrace and implement ideas of other practitioners who have proven effective methods. Jill noted that in seeking such PD opportunities:

> I'm trying to go with research-based things. I know if [the content specialist says], 'Let's look at this…let's think about it this way,' it's not been always easy; but it's opened me up to thinking and realizing, 'You know what? There are people that have a better understanding of how things should be taught and what works best, and I need to try it.

In defining effective professional development, Jill pointed to two particular, though related, characteristics: 1) hands-on experiences, and 2) opportunities to model how students engage in learning:

> Effective professional development…gives you opportunities to experience what your children experience. It allows you to ask questions and be responded to in a way that's helpful…to get you where you need to be so you can help those kids.

Jill described an ongoing PD program at her school that does this effectively and what she learns from it:

> [The PD staff] comes in with activities, and we sit and do them; [the staff] won't say, 'this is right' or 'this is wrong.' They model for you what you should be modeling for your kids. You understand how the kids feel, and you understand that it's okay not to say, 'this is right, this is wrong, this is what you should do.' Let [the students]

> have more wait time; let them have more development, time to develop and determine their answer.

Jill added, though, that such opportunities and experiences should be coupled with an understanding that not all participants will learn and/or change at the same pace or to the same extent:

> I think also, [effective PD] allows you to…accept the fact that just like with kids, we're not going to all get there at the same pace. Even as adults we've got to understand we're not going to all get there at the same pace.

Teaching approach. In observing Jill's classroom, it became clear that Jill is keenly attuned to her students' needs. She is mindful about how they may feel as they learn certain new concepts, and she anticipates their behavior. Jill noted that teaching "keeps me on my toes to make sure you give each [student] what they need without appearing to do more for one than the other." Students in her classrooms ranged in abilities, from those with learning disabilities (LD) to those identified as AG (academically gifted). Her students were seated in small groups, and she designs her lessons to include a mix of direct instruction and hands-on group activities.

It also became clear through our conversations that Jill had not focused on giving her students frequent opportunities to explain their thinking or communicate their ideas in the classroom in the past. A couple of months prior to participating in the MOOC-Ed, Jill had begun to shift her approach after having observed her student teacher. "Knowing that I liked the response that he was getting, the understanding…I just felt like it was a better way to go." In my observations, it became clear that Jill had fully embraced the approach. Taped on one of the walls in her classroom was a poster with the statement "*Take Time to Listen*", which illustrated her increased focus on giving her students more time and opportunities to share their thinking. Words of guidance during her lessons included "The only way we know what's going on up here (*points to head*) is if we talk," and "I learn a whole lot more from what I hear than what I say." Moreover, Jill seemed to value having her students offer multiple approaches to solving problems, which became evident in my classroom observations.

Classroom visits. In my observation prior to the start of the MOOC-Ed, Jill was teaching a lesson on weights and measures to her 4th grade class of 20 students. She began the lesson by passing around actual objects for students to get a sense of what the objects' weights felt like. In her approach, she incorporated wait time, emphasized the importance of "thinking" before sharing thoughts, and engaged her students in a conversation about the topic.

> JILL: If ½ a ton is 1000 pounds, how many pounds would be in a whole ton?...Let the thinkers think. *Waits a few seconds.* How many halves does it take to make a whole?
>
> STUDENTS: Two.
>
> JILL: So, one half plus one half would be two halves, which is equal to a whole. So, if this half is a thousand, this half is going to be what?…
>
> STUDENT: 500
>
> JILL: What? You said that without thinking. (*Laughs gently*). A thousand.   So, what could I do with these?
>
> ALL: Add them.

JILL: So, when I add them, I get 2000. So, one ton is how many pounds?

STUDENTS: 2000.

JILL: 2000 pounds.

Through the lesson, Jill asked her students to document their reasoning on paper and addressed actual student work in both the full class discussion and as they worked in small groups. In a full class discussion, Jill probed for her students' thinking and encouraged them to try their ideas out, as opposed to giving them the answer: "What's going on (*points to head*)? What's your thinking? What's one thing you could do?..Try it!" After giving them time to confirm or refute their initial ideas, she asks "Why did you choose to do that?"

As her students worked in small groups, Jill further emphasized the importance of them thinking deeply about the concepts and communicating them. As she worked with one particular group, she further encouraged her students to express their reasoning, whether it was orally or in writing.

JILL: If we know that there are 16 ounces in 1 pound and 2000 pounds in 1 ton, then how many ounces are in 1 ton? You tell me, so that I know what you're thinking…[*Waits a few seconds.*] If I know there are 16 ounces in 1 pound, then how many ounces are in 2 pounds? What would you do? Show what that would look like.

As Jill brought the class back together, she invited students to offer their solution strategies to problems they worked on in their groups. She intentionally selected students to share their solutions and highlighted how she values multiple approaches to problems and varied levels of understanding. It became clear that this was a classroom norm for Jill and her students; they seemed comfortable thinking of different ways to solve problems. In closing the lesson, Jill remarked to her students:

> I have observed a lot of cool thinking going on. 'How can you observe thinking, Ms. J?' I observe it through what you write, what you say, and just listening to some of you reason things out. We're all at different places. Is that alright? *Students say 'yes'.* Yes."

In my post-MOOC-Ed observations, as it was then a new school year, Jill was teaching a class her school identified as a "transition class". It was comprised of students who had not passed the district reading test in the previous school year. Jill described this group of students as "active" and challenging to teach due to some of their behaviors.

I witnessed a number of subtle changes in Jill's approach in these post-MOOC-Ed observations. In my first visit, Jill opened the lesson with the question, "Is fraction a number? Thumbs up or thumbs down." In the course of the lesson, she encouraged her students to view fractions in different ways (e.g., part-whole, a number on a number line, a ratio). It became clear through our conversation following the lesson that this was a different approach to teaching fractions than Jill had employed in the past. In my second observation after the MOOC-Ed a few weeks later, Jill had continued with this approach by focusing on the concept of "fraction as number", a point made salient in the course.

In these visits, Jill was very attentive to the language she used to introduce math concepts, such as "numerator", "denominator", and "commutative property". As an example, during one of her lessons, Jill asked, "What can we infer about fractions that are equivalent or equal to ½? What can we deduce or make an inference about?" Jill emphasized the importance of being precise with using such language to her students, and this became clear as I heard students share their thinking, using words such as “multiples” in their explanations.

As she did in her lesson in my pre-observation, Jill continued to challenge her students to explain their thinking in words. For example, after asking her students to decide whether 7/14 is equivalent to ½, Jill playfully remarked “This is what Ms. J. wants you to be able to come out and say…Hey, Ms. J., I know it’s this *because…*” In my post-observations, however, I did witness a significant change in Jill’s approach as she circulated to groups of students as they worked on tasks. In one instance, Jill observed one of her students struggling with a task in which the students were asked to represent equivalent fractions by shading rectangles partitioned into various numbered parts. As she approached the student, Jill asked him a few questions to better understand what he was doing. She avoided guiding him to an answer but instead conducted somewhat of an impromptu “interview” to dig deeper into his thinking. Her questions were more of the nature of “Can you tell me why you shaded this figure that way?” and “Why do you think these are different?” This approach was also highlighted in the MOOC-Ed, and it became clear through our conversation following the observation that Jill began conducting such interviews more after having participated in the MOOC-Ed.

In my second post-observation, Jill opened the lesson by reviewing a quiz on adding and subtracting mixed numbers. Instead of working through each of the problems, Jill demonstrated an error she observed a number of students make on two particular problems, adding $3\frac{2}{3}$ and $1\frac{1}{3}$ and subtracting $10\frac{1}{4}$ from $10\frac{3}{4}$. She asked her students to identify the errors and called on them to share their thinking. This was a different approach than I'd observed Jill use in the past and one that was addressed in the MOOC-Ed.

In this lesson, I again observed Jill digging deeper into her students' thinking. The lesson was on multiplication with fractions, and she framed it by first building intuition into the operation. She first asked her students if they thought it was possible to multiply fractions.

> JILL: I just want to see if you think, 'Yes, we can multiply fractions', or 'No, we can't multiply fractions.' Using what we know about repeated addition and whole numbers, and multiplication of whole numbers, can that same idea be related to fractions? *Waits for students to give the "thumbs up" or "down".* You have to tell me what your thought is. Then I know from there what I need to do. *Waits*. So, all of you who said 'Yes, we can', tell me what it would look like. Think about it. I'm pulling *your* brain. I know that workbook's going to show you wonderful things. But, I want you to tell me what do you think? How would we show multiplication with fractions with [repeated addition]? What do you think it might look like? Take a minute to write it on your paper what you think it might look like. And you (*pointing to student who said it can't be done*) write down *why* you think it won't work. Show me why.

In our conversation following the lesson, it became clear that this approach to introducing her students to fraction multiplication was new to her and inspired by the MOOC-Ed (external → practice):

I would never have started a lesson…last year I would never have started that lesson, 'Can you multiply fractions that way?' I would have said, 'Okay, they were going to multiply fractions, get your book, this is what we're going to do.' I would never have started it that way.

Jill later asked her students to represent $\frac{1}{3}$ times 4 as an equation and had one come to the board to share her result. The student wrote $\frac{1}{3}x\ 4 = \frac{4}{3}$, and Jill probed for explanation.

JILL: Now, Karen, tell us what you did.

KAREN: I wrote $\frac{1}{3}$ times 4 equals $\frac{4}{3}$.

JILL: Can you explain to me *why* you wrote the 4 as a whole number? Why did you use it with the numerator and not the denominator? I'm guessing that you multiplied it with the numerator and not the denominator. Can you explain to us why?

KAYLA: Because there's 4 one-thirds, so $\frac{1}{3}$ times 4, because there's 4 one-thirds, and I count it 4 times. So, $\frac{1}{3}$ times 4 equals $\frac{4}{3}$.

JILL: (*To the class*) What do you think? Do you think it's conceivable? (*One student says 'No'.*) What's throwing you off? You gotta talk to me.

This exemplifies Jill's increasing focus on drawing out her students' thinking and using that thinking to generate rich discussion around fraction concepts.

Jill's MOOC-Ed experience. After having learned about the MOOC-Ed, Jill was motivated to enroll for a few reasons. One reason related to her service to her students:

> When I went through [college], I did a math class for teachers, but we didn't work with fractions. For fractions to be such an in-depth thing now and such an important part [of education], if I don't push myself and take advantage of these things and say, you know, 'Jill, you gotta change, you've got to become more effective at helping these kids,' then I'm doing them a disservice.

Another reason Jill was motivated to participate in the MOOC-Ed was to refine her own fraction understanding and expand her vision of how fractions could be represented:

> I grew up; you learned the algorithm; you didn't understand necessarily what it meant. And models had to be that one certain way; a half has to look this way or it's not a half. There was no variation. I think that's the biggest thing with math now…getting us out of that it's got to be a very narrow-minded way. So I felt like [the MOOC-Ed] was a great opportunity to learn more myself.

Jill enrolled in FFI but wasn't able to engage at the level she wanted to due largely to time constraints and other work obligations. She was eager, then, to take the course in the Spring, as she had more time to devote to it, and her fractions curriculum aligned with the timing of the course. Jill appreciated the opportunity to take the course a second time:

> Some of those things the second time around, it's like 'Now I see, this is a better way; this is what I need to be doing. I still struggle some with how to make sure they understand and…how to plan for that, but I know that's just going to be a work in progress. Maybe I needed that first time around just to kind of see, and then the second time around I had had a sampling, so I was more prepared for what was going

> to be shown. The second time around I had a better understanding of what it meant to understand a fraction and to be able to prove that you understood what a fraction was.

In outlining her goals in participating in the MOOC-Ed, Jill alluded to her primary motivations for taking the course: gaining a deeper understanding of fractions and how to facilitate that understanding in her students:

> My biggest [goal] was to understand how fractions work, how to explain to children what a fraction was because when I came through school, I was not taught the way I would learn to present it to the children. But how can I teach them to understand if I don't understand? So, I just wanted a better understanding of what a fraction was, what it meant to partition…and then to see [the students] today to be able to, 'well you've got four more models, that's your four wholes that are represented, but you've got a fourth of each one or a third of each one'. I would never have looked for that conversation from them before.

I asked Jill how she would define success in taking the MOOC-Ed, and while her answers focused primarily on her effect on her students, they also addressed her own personal beliefs of confidence in teaching:

> Seeing more of the students begin to question and be able to explain what they're thinking and say, 'I did this because I needed to see it to make sense of all of it.' I guess [success would mean] for them to be more comfortable in taking the time that it takes for them to explain it and know that it's okay to explain it that way. And success for me too, is to realize it's okay if I don't quite get it the first go round, or I don't come at it from the perspective that maybe another adult does.

In later conversations with Jill, we revisited the idea of what success in the MOOC-Ed meant to her, after she had completed the course. She described achievements related to shifts in her approach to teaching in general and presenting fractions to her students in particular (*external* → *practice*):

> Success for me was truly understanding how to present the concept of a fraction…watching other educators work with children, and even though they could have easily said, 'But wait a minute!'…watching them sit back, rephrase, rethink, "Okay, where am I going to go next; what do I need to do to bring that understanding?" and being able to do that and seeing a kid get up and say, 'Well, this is what I did and this is why'. (*practice* → *personal*)

Jill believed she had achieved the goals she set out in taking the MOOC-Ed. She pointed to specific changes in her use of precise mathematical language, her ability to communicate the ideas she presents, and her ability to probe her students' thinking to better address their needs (*personal* → *practice*):

> I've worked towards trying to make sure I use vocabulary and make sure the children are understanding the concepts as we go; and I'm getting a better understanding and better resources to use to help me do a better job of explaining to [the students], or pushing them, to see what they can come up with. [The MOOC-Ed] has made me more comfortable and educated me on what I needed to look for in the kids for understanding (*practice* → *personal* → *practice*).

While her responses were overwhelmingly positive, Jill recognized that the work continues:

> Will [the students] all be able to put it in words and all have a true understanding of it when they leave me? No, even though they'll be able to do the stuff; but at least there's a foundation that 5th grade can build upon.

Jill identified a number of elements of the MOOC-Ed that contributed to its effectiveness that closely aligned with the design principles implemented in the course: 1) content focus; 2) peer support and discussion; and 3) case-specific videos.

As Jill explained in her motivation to participate in the MOOC-Ed, she was seeking to gain a deeper understanding of fractions so that she could teach her students more effectively:

> I think [the MOOC-Ed was] very effective in challenging you as an adult to look at and think, it almost makes me feel like I'm that little 4th grader or that 5th grader, and I'm looking at it new for the first time because it's not the kind of problems I grew up with; and it's not necessarily the problems you're going to see or did see in a textbook.

Jill appreciated the opportunity to share ideas and challenges with other practitioners through the discussion forums:

> It helps you look at things from different perspectives, and just hearing other people's strategies, their struggles, what they try. I had commented on one student [in the case videos] that I thought had an understanding--then somebody came back and replied…they didn't really think that you could tell if that child had an understanding. I realize that, too, it's going to be dependent on where you're coming from, what you're used to seeing…It was just really neat to see perspective, but also, with that

> perspective what you could learn even if their group was not the same makeup as your group, or their grade level was not the same, it still all blended and worked together.

In exchanging feedback on project ideas, Jill was also able to refine her own activities for use in the classroom. She greatly valued the comments and support she received from other participants on her course project, which was on equivalent fractions on a number line. Jill recalled, "Some folks pointed out…'Do you realize how long your number lines would be?'", which encouraged her to adjust the activities in her project accordingly.

Jill found that the "What Would You Do Next" videos were highly effective in their illustration of approaches to teaching fractions she had begun to adopt as a result of the MOOC-Ed. Jill appreciated the opportunity to observe questioning strategies and specific tasks played out with actual students in the clinical interviews, as they modeled what she could do with her own students. Jill also greatly valued the videos that showed teachers "in action" as they used tools and approaches to teaching fractions that were somewhat new to her. These videos gave her a window into classrooms that she may not have had otherwise and provided authentic examples of how certain approaches to teaching fractions could play out in the classroom. The videos effectively linked theory to practice:

> There was one video where the teacher did fractions with the number line…And then there was another one where she had a game where she just used the whole and then the half and the fourths and had the students putting different [fraction strips to make a whole], like you might have a half and you might have a fourth…I mean to me, that was like, 'Oh I never would have done that.' I wasn't taught, even in college, I

> wasn't taught to do it that way. So to see that and like, 'Oh yeah, that's where we're headed', that will be an activity we do before the year is over with. (*external* → *practice*)

Impact on practice. Some might expect that an educator with almost 30 years of teaching experience may not be open to making changes in their practice. Jill contradicts that assumption and continues to be active in her growth as a teacher. Jill reflected on how her practice has changed since having participated in the MOOC-Ed and largely credits the course as being the impetus of that change (*external* → *practice*). Not only did Jill's teaching practice evolve as a result of the MOOC-Ed, but so did her teaching philosophy.

Jill recognizes that there has been a shift in mathematics education in recent years and believes that students need to be exposed to different approaches and encouraged to explore different models of mathematical constructs. She attributes the MOOC-Ed as helping make that salient for her. In recalling a video she watched in the course, Jill described her thinking as she observed the teacher with three students working on a problem, allowing the students to approach the problem in different ways:

> The teacher had the [students] putting different [fraction] pieces on there, like you might have a half and you might have a fourth, and I mean to me, that was like, 'Oh I never would have done that.' I wasn't taught, even in college, I wasn't taught to do it that way…I want them to see, we don't all fit in a mold…and I think that's the biggest thing with math now is getting us out of that it's got to be a very narrow minded way. [The MOOC-Ed] has helped me to close my mouth sometimes and just listen. I've always been one to jump in and you know, I want them to get it…and it's

> helped me realize, 'Okay, just stop, stop and let them talk more about what's going on' (*external* → *practice* → *personal*).

Jill's change was evident as she worked with a student, Elijah, who was struggling with an activity on fractions. Jill provided her students with a sheet of rectangles, each one of which was partitioned into equal numbers of pieces. The students were instructed to shade the portion of each rectangle that represented ½ of the whole shape and express that part of the shape using different names for ½ (e.g., 2/4, 3/6). Elijah was having difficulty shading the rectangle that didn't contain any partitions. She reflected on her discussion about the problem with him:

> I kept going, 'Where are you going with that?' Where…why…and I finally thought to myself, you've got to shut up Jill, you've got to close your mouth and just let him take it. I would never have thought that he needed to see that whole and work through that whole process to be able to see what the half would be. It didn't make any sense to me.

Jill recognized that he had an understanding that made sense to him, and she identified questions that would help guide him based on his understanding. She attributed this shift in her practice to the MOOC-Ed (*personal* → *consequence* → *personal*).

Jill was also more mindful of the vocabulary she used in teaching, being sure to use precise mathematical language and emphasize the importance to students. In reflecting about her increased focus on vocabulary and its effect on her students' learning, Jill recalled:

> When we did this activity Monday, one little boy [said], 'Ms. J, it's like you're using multiples!' I mean, this is a little fellow who would not have come up at the

beginning of the year, and even raised his hand to say that; so he's beginning to feel comfortable in voicing that (*practice* → *personal* → *consequence* → *personal*).

As a result of the MOOC-Ed, Jill had also begun to conduct interviews with her students to better understand their thinking:

> I did the first one with the fair sharing, with the two cakes divided between eight people. The [students] eventually, with questioning and answering, could get there; so I know that I've really got to make sure that they understand when we're looking at a 1/4th, looking at the fraction, this is a fourth, this is a fourth…That's from the MOOC as well…making sure they understand the piece, that part, that it is a number, it does represent something and that's knowing that if there's 3/4 , it's three ¼ pieces, which I was guilty of never, ever having taught it that way.

While Jill was motivated and successful at making changes to her practice as a result of the MOOC-Ed, she was also mindful of the challenges she faces in implementing that change. She recognized that she needs to be "organized" with her new materials and lessons to ensure that she provides her students with meaningful tasks that further their thinking. With these challenges, however, Jill seemed committed to continuing on her path.

Jill's path in her "change environment". In analyzing Jill's path among the four domains in her "change environment" through her MOOC-Ed experience (see Figure 10), I describe her growth as beginning in the *external domain* and ending[25] in the *domain of consequence*, with her "growth network" as follows (Clarke & Hollingsworth, 2002):

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[25] The words "beginning" and "ending" are used here only to indicate the domains in which the participant is observed as being at the beginning and conclusion of the study. It is assumed that the participant continues their path as they continue their growth.

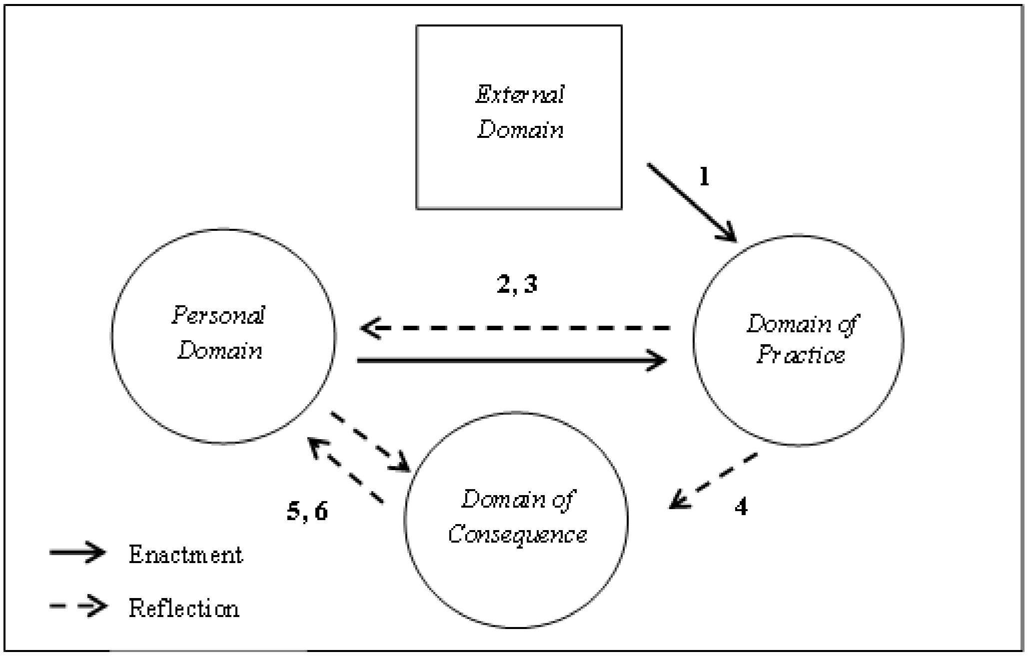

Figure 10. Jill's "Growth Network" (Clarke & Hollingsworth, 2002). Numbers are provided in order of edges in the network, with *x*, *y* indicating order from top arrow to bottom arrow, respectively.

Jill began the MOOC-Ed having been inspired by the approach she observed a student teacher implement in his teaching and motivated to learn more about how to help her students build a better understanding of the mathematics she taught. She was eager to participate in the course, as it would give her opportunities to improve her methods of teaching fractions and to strengthen her own understanding of fractions. In the figure, the solid arrow pointing from the *external domain* to the *domain of practice* indicates Jill's "enacting" what she learned from the MOOC-Ed in her teaching (edge #1). The dotted arrow pointing to Jill's *personal domain* indicates Jill's frequent "reflection" on how changes in her approach were affecting both her and her students' success (edge #2). With changes in her *personal domain*, Jill enacted her new knowledge and beliefs about how to teach and learn fractions into her practice (e.g., new questioning strategies, focus on language, implementing tasks to build intuition and meaning), represented by the solid arrow back to Jill's *domain of practice* (edge #3). With this enactment, Jill observed "salient outcomes" in her students, illustrated by the solid arrow from her *domain of practice* to her *domain of consequence* (edge #4). She witnessed their increased precision in language, comfort with her expectations of them to explain their thinking, and more nuanced understanding of fractions. With this, Jill drew conclusions about how to serve her students better in the *domain of consequence*, based on what she learned from the MOOC-Ed and from enacting that knowledge in the classroom (edge #5). With reflection and continued motivation to grow, Jill embraced what she learned both in the MOOC-Ed and in her classroom to make long-lasting changes and help ensure positive outcomes (edge #6).

Jill's growth network is strong and deliberate, and her professional path will continue to traverse each of the edges of this network.

Helene

At the time of my first visit, Helene had been teaching elementary school for 18 years, including special education, and it was the first year she was teaching 5th grade. It became clear very early on in our conversations that the environment she teaches in brings special challenges. Helene teaches in a high-poverty area, where approximately 70% of the students are on free or reduced lunch. She explained that her school was recently designated a "focus school", for which the school is observed and reviewed by the district over three years and required to show growth.

I visited Helene's classroom once before the MOOC-Ed and twice after it concluded. Helene explained in my first visit that her particular students that year had experienced several challenges in the previous year with regard to teacher retention. She believed that their performance on end-of-year testing was greatly affected by those challenges. Only a small number of students had passed the mathematics exam in the previous year, and Helene was eager to find ways to improve her practice to help her students move forward. She referred to herself as "passionate about what I do" and often asks herself "'What can I do better?' because it's like anything else--you can't necessarily change other things, but you can change yourself and do better every time."

Professional development is readily available to Helene, and her district supports it, with a director of PD that provides opportunities for teachers. She appreciates diverse methods of PD, so that she can learn from multiple perspectives. In my first year visiting Helene, in addition to activities during the year, Helene's school engaged in three days of schoolwide PD ("Transformation Inspiration") after the school year was over. Helene had engaged in a number of online PD programs, including *I-Innovator* and another MOOC, and she met regularly with a PLC at her school.

In that year, although Helene had options for PD, the quality of the opportunities was not always strong. For example, while Helene met with her PLC, the group did not seem to serve its best purpose:

> I don't know that our PLC's are the way they are supposed to be. From what I understand, they're supposed to be like a collaborative group, and ours just end up being a mini-staff meeting. We don't actually sit down and talk about our

> students…that's done in the halls, at lunch, and maybe on a text. It's not the way it's supposed to be--a collaborative effort to see what's going on in each of your classrooms and how you can work together to move that student…and what this teacher's doing that may or may not have worked with a similar student, and what you may want to try...No, our PLC's are 'sit and get' from the administration; it's just another staff meeting.

The coordinated PD activities at the end of the year were also intended to serve teachers better than they did. Helene explained that teachers would be allowed to sign up for activities that they felt would serve their needs, thereby attempting to personalize their PD. However, "what ended up happening was the principals dictating what we were going to take; and so, then, it kind of loses the point."

In my second visit with Helene, it seemed that her district's program made positive changes toward providing opportunities to engage in effective PD. She explained that the activities were now more focused on how to improve students' academic proficiency. "Now we do have a wonderful math coach…and she is fantastic. She gives us a whole lot of resources, and we have monthly meetings…That's a good way to share." Her district had also changed its requirements for PD to now include proof of use of what was learned:

> I think you have to do things with [what you learn] too because what they've referenced is it's not just a 'sit-n-get'; you're going to have to do something with it and then show it if you want credit.

It was clear that Helene was motivated to learn and grow professionally so that she can give her students the support they need to succeed. She believed that having a strong command of the concepts she teaches makes her a more effective teacher:

> I think the students always benefit when you have a deeper knowledge. I tell them, ' I could just teach you the steps to do this, and quite frankly that's how I was taught, but I made those connections a lot on my own; then as a teacher I'm making deeper connections, and that's what I want you to get out of this.' I try to make them understand that there's a difference between knowing how to do something and understanding how to do it. I'm trying to get them to understand that they need this deep understanding so that they can then build on it. They're not just doing math as a process or how you manipulate numbers.

Not only was Helene eager to engage in professional development activities, but she was committed to changing her practice based on what she learned. She recognized that "once you consume that professional development, it's your responsibility to implement it." Along with her motivation, however, Helene saw many obstacles to her students' ability to succeed. One obstacle was their lack of strong mathematical preparation for the 5^{th} grade. For example, Helene noted that:

> Subtraction is not good for [the students]. They don't understanding regrouping. So when you're trying to regroup with tenths and hundreds, and they're not really sure what tenths and hundreds are, you've got to kind of go slow.

Another significant obstacle was the difficult, sometimes dangerous, environment in which her students live:

> I mean they have big dreams, but the day to day, how to manage your life, how to manage your emotions, how to…you know, when you're feeling bad about something at home, you know, how to overcome that or not enough sleep or not enough food or whatever it is; or gun shots in the middle of the night, because you know, we have gangs in this town. We might be a small town, but we've got a lot of challenges. And these kids are right in the middle of the projects. [You've] got a lot of kids that are not living with parents for one reason or another…have always been raised by grandma. See mom on just the rarest of occasions, maybe you know, don't even know dad…there are just a lot of them overwhelmed as well and only have a certain amount of resources to deal with it.

Helene saw the effects of her students' home life in the behavior of her students, making it sometimes challenging to implement what she learns in her PD activities.

Views on professional development. In conversations with Helene, I learned what she values in PD and what she believed makes it effective. She appreciates personalization and choice in her PD activities, as well as opportunities to reflect on approaches she has used in the past to teach certain topics. Helene is also always thirsty for a challenge when it comes to PD activities. She appreciates the moments when:

> I have to kind of reach down in myself to do something, even if it's something that I don't have time to do…I'm gonna do my best on it, and I like things that are a little bit challenging…[T]here's so much further that I can go, and there's so much more knowledge that I could have. You know I'm not going to get my PhD in math, but I can take these kinds of courses when they're offered and glean from them what I can.

In recalling an activity she considered particularly effective, Helene highlighted that she believed the PD designers were treating her as a professional:

> I remember, you know, doing some work, whether they were articles to read or…and I was like, 'Okay, these people are treating me like a grownup…you know, because sometimes you don't get treated as a grownup when you're a teacher. You get told what to do, you get told how to do it, and it's a little discouraging. One of the reasons I liked [the PD activity] is because I could use some creativity; I can use my brain. And I always want to grow; I always want to try to do better.

Teaching approach. Helene draws upon her years of diverse teaching experience to shape her approach in the classroom. She finds that it is critical for her to reinforce concepts throughout the year, so that her students can make connections between related topics and to increase their retention:

> Whenever I teach something, I don't assume anything. The other thing that I've learned from my special ed background is, these kids, whether they are 'identified' or not, because they don't have a lot of support at home, you can't just teach something and forget about it; you've got to reintroduce it.

Most of Helene's students sat in small groups of 3-5 students, and her lessons included a mix of collaborative group tasks, direct instruction and individual work. Group tasks included activities that involved handouts, discussion and sometimes manipulatives. Helene often used technology during direct instruction, primarily through a smartboard, where she projected activities and assignments. At times, she selected students to share their ideas with the class at the board during class discussions. Helene was mindful about designing her lessons to provide enough differentiation to encourage her students to remain focused and engaged, which often proved to be a challenge.

Classroom visits. In my first visit of Helene's classroom, before she had participated in the MOOC-Ed, the lesson was focused on adding decimals. She began the lesson with a class discussion on the previous night's homework and then led into an activity on adding decimals using grids. Helene exhibited some attention to using precise mathematical language, and as she circulated around the room to monitor her students' work, she intentionally selected students to share their solutions with the class.

In the full class discussion, Helene attended to some misconceptions she noticed were common among her students as she monitored their work. For example, in a discussion on adding the decimals 0.36 and 0.45 vertically, Helene emphasized both the mechanics of the operation and the reasoning behind it.

> HELENE: Once we don't have the grid to model this in any way, what do you notice about the decimals that will get you the correct answer?
>
> STUDENT: It should be lined up.
>
> HELENE: Can anybody tell me *why* the decimal needs to be lined up?
>
> STUDENT: Cause it makes it easier to add?
>
> HELENE: Okay.
>
> STUDENT: Plus, you can't make it crooked, cause there's a zero behind it.
>
> HELENE: What if I made it nice and straight, but I still didn't have the decimals lined up?

.36
<u>.450</u>

What if I had 253 and I wanted to add 167, why can I not do it like that:

253
<u>167</u>

STUDENT: Because you won’t be adding the right numbers.

HELENE: Tell me a little bit more specifically what you mean by that.

STUDENT: It won’t be the same place value.

HELENE: Yes, it won’t have the same place value. If I have a dime and I have a penny, and I want to add them together, can I say that I have 2?

STUDENT: No!

HELENE: No, because I don’t have the same value. I still just have one dime and one penny! I cannot add them together because they are not the same value. When I have 6 tens and 2 hundreds, I cannot add them together, because I still have 260. I don’t have 8 anything. I don’t have 8 tens; I don’t have 8 hundreds. I just have 6 tens and 2 hundreds.

In my second visit, the first after Helene had participated in the MOOC-Ed, the lesson was focused on operations with fractions. Helene organized “centers” for her students to work in, in which each group of students engaged with a different task related to fractions. Some of the centers involved resources Helene found on the internet (e.g., worksheets, interactive websites), which indicates her openness and desire for prepared materials that she can use directly in the classroom.

One of the centers in this lesson was referred to as the "Teacher Group". In this center, Helene engaged 5 students in a discussion with her on multiplying fractions. She emphasized four "ways" to multiply fractions and connected the idea to multiplying whole numbers: 1) repeated addition; 2) array/visual depiction; 3) on a number line; and 4) memorize (a fact we know). When talking about fractions, Helene highlighted the idea that "fractions have value, too. The difference is that a fraction of something would be less than 1; so we're talking about a 'part'." A number of these ideas were made salient in the MOOC-Ed. In addressing the idea of multiplication of fractions as repeated addition, Helene seemed to be even more attentive to precision in the mathematical language she used. For example, she emphasized using the words *unit, numerator,* and *denominator*:

HELENE: How do I multiply $\frac{2}{3}$ and 3 using repeated addition? I can add $\frac{2}{3}$ three times. And because this is addition, I have to have a common denominator. And my rule for adding when I have a common denominator is I simply add the numerators—because my *unit* is $\frac{1}{3}$. I have $\frac{2}{3}$ plus $\frac{2}{3}$ plus $\frac{2}{3}$, which means I have one-third how many times?

STUDENT: Three

HELENE: *One-third* how many times?

STUDENT: Oh, uh, six.

HELENE: Right. And I call it $\frac{6}{3}$, but it means $\frac{1}{3}$ how many times? How many $\frac{1}{3}$rds do I have all together?

STUDENT: Six.

HELENE: Right, six $\frac{1}{3}$ pieces.

In representing the operation with a visual model, Helene drew a rectangle and split it into three equal pieces, coloring two of the three. She then drew copies of that same rectangle, reminding students that the unit was $\frac{1}{3}$.

As Helene discussed representing fractions on a number line, she again highlighted the idea of "fraction as number" and that they have a value, another point made explicit in the MOOC-Ed. In this discussion, Helene also gave her students more time to think about their answers than I had observed in my first visit.

HELENE: Fractions live on the number line just like whole numbers do. It's just that fractions live in between whole numbers, except for the ones that equal whole numbers. How could we show $\frac{2}{3}$ times 4? What would we do with this number line to represent $\frac{2}{3}$ times 4?

KYRIE: Split between the numbers.

HELENE: You want to come up and do it? (*Student comes up to the board*). Kyrie says split it three times. And I said 'split what'? And he said, 'split between 0 and 1 three times'.

Kyrie split the interval between 0 and 1 by making 3 tick marks. Helene attends to his misconception by asking him to count his parts. She reminded students that only n-1 marks are needed to make n parts.

HELENE: Let's name these fractions…$\frac{4}{3}$, $\frac{5}{3}$, $\frac{6}{3}$. And what is 6 divided by 3? 2! And isn't the fraction bar a division symbol? Yes, it is! What are we multiplying? We're multiplying the *value* of $\frac{2}{3}$ to a number! This is no different from when we multiplied whole numbers. $\frac{2}{3}$ is a value just like 2 is a value.

Following this discussion and briefly addressing the algorithm to solve the problem of multiplying $\frac{2}{3}$ and 4, Helene had her students work on multiplying $\frac{9}{7}$ and 2 using 1) repeated addition; 2) fraction strips; and 3) the algorithm. As they worked, Helene circulated among the students in the group and engaged a few in one-on-one conversations with her on their approach. In these conversations, I observed Helene probing her students' thinking more than "telling" them how to proceed, another key point emphasized in the MOOC-Ed. Her instruction was characterized more by questions than statements.

HELENE: (One-on-one with student) How much is $\frac{9}{7}$ compared to 1…bigger or smaller than 1? What is one whole when you're talking about 7ths? Think about that when you're thinking about adding $\frac{9}{7}$ ths two times.

As she approached a student representing $\frac{9}{7}$ by splitting a rectangle into 9 parts and coloring 7 of those parts, she asked:

HELENE: What does this fraction show right here? How many parts do you have [your rectangle] divided into?

STUDENT: Nine parts.

HELENE: Nine parts. And how many are colored?

STUDENT: Seven.

HELENE: So is that $\frac{9}{7}$ or $\frac{7}{9}$?

STUDENT: $\frac{7}{9}$

HELENE: Okay. Now let me ask you another question. When you think about $\frac{9}{7}$, is that bigger than 1 or smaller than 1?

STUDENT: Smaller than 1? Bigger than 1.

HELENE: Yes, it's an improper fraction. So, if it's bigger than 1, are you going to draw 1 whole or more than 1 whole?

STUDENT: More than 1 whole.

Following the centers, Helene brought the class together and led a discussion on a review worksheet. In this discussion, Helene guided the dialogue more than I had observed in the small group discussions, but she still focused the discussion on the reasoning behind the concepts.

HELENE: Mia has 3 candy bars. She gave her brother and her sister each ¾ of a candy bar. So, is this going to be a one-step problem or a 2-step problem?

STUDENTS: Two!

HELENE: Because what do I have to do first?

STUDENTS: Add!

HELENE: I've got to add the ¾ two times, right? What amount of the candy bars does Mia have left?

STUDENT A: *Student comes to the board*. I'm going to divide these [rectangular bars on the board] into four pieces.

HELENE: Can you tell me why?

STUDENT A: Because the denominator is 4, and 3 is the total.

HELENE: That means that the total is 4? Or is that each unit is…What is that?

STUDENT A: One-fourth.

HELENE: It means the unit is one-fourth. Four pieces is the same as saying that the unit is ¼.

Student colors in ¾ on two of the bars in yellow.

HELENE: So, so far, she's colored in what?

STUDENT B: How much was taken away.

HELENE: So, what's not colored is what?

STUDENT B: How much she has left.

HELENE: Good.

STUDENT A: Then I'm going to shade in what's left [in a different color].

HELENE: Good. Now show it numerically.

STUDENT A: We know that we have 1 whole, because this bar shows 4 fourths colored in. And then, this is ¼ because we have ¼ colored in. And so we have 1 and $\frac{2}{4}$. And, we know that $\frac{2}{4}$ can also be written as ½.

In closing the discussion, Helene emphasized that different methods of multiplying fractions may be more appropriate than others, depending on the problem, and welcomed students to use the method(s) that make most sense to them:

> You do the way that makes sense to your brain. Now, sometimes, there are going to be problems that are just too big to draw, like 96 times ¼; are you going to want to draw that? It would be a lot. So, there are times when you're going to want to use the regular algorithm or repeated addition. If that's the way your brain works, then that's okay!

In my second observation after Helene had participated in the MOOC-Ed, the topic of the lesson was ratios. After a brief introduction to the topic connecting ratios to real-world contexts, Helene had her students work in groups on a task she had adapted from a teacher resource website. Following the task, she brought the class together for a discussion of the relationship between ratios and fractions, based on the activity they had just engaged in.

> HELENE: I would like to talk about how ratios are alike and how they're different from fractions—cause you all saw that one of the ways that we can write a ratio is like a fraction, right? But, it does not always describe the same thing as a fraction, okay? So, when we compared fruit loops [from the activity], we did a lot of part-part comparisons. Who can tell me two of the part-part ratios that we showed in our fruit loop ratios?
>
> STUDENT: Blue and purple?

HELENE: Blue and purple! We compared blue and purple, and that was a part-part, right? *Writes "blue-fraction bar-purple" on the board.* Everybody agree? *Students nod.* Okay. Calvin, what was another part-part?

CALVIN: Yellow and green?

HELENE: Yes, we compared yellow and green. *Writes "yellow-fraction bar-green" on the board*. Okay, let me ask you a question. When we have a fraction bar, let's say it looks like this (*draws a fraction bar and splits it into 8 equal pieces*). And let's say that we have 3 purple that are colored in. What would be the fraction of the purple parts? Calvin?

CALVIN: 3 out of 8?

HELENE: Okay, so we have 3…and what's our unit?

CALVIN: Uh…

HELENE: What is each one of these [parts]?

CALVIN: Oh, one.

HELENE: One?

OTHER STUDENTS: One-third! Three-eighths! One-eighth!

HELENE: One-eighth! They're each one-eighth, right? They're each one-eighth. So, the fraction shows three-eighths, okay? So, when we write a fraction, we're talking about a particular unit. That's not what we're necessarily talking about when we do a ratio. Now, since we used all fruit loops in the last [exercise] that we did, when we compared our favorite to the total, what were some of the ones you all had for that? Ethan?

ETHAN: Blue.

HELENE: Blue was your favorite? And you compared it to what?

ETHAN: All of my fruit loops.

HELENE: So, that was the…

ETHAN: Total.

HELENE: Total. And those were all different colors, weren't they?

SADIE: And that's part-total!

HELENE: And that's part-total. And isn't that what we're looking at? When we do part-whole…when we do this kind of fraction, isn't that what we're talking about? So, in some instances, a fraction and a ratio do show the same thing, but not in every instance. Ratios can be used to show a variety of things. We can talk about, in ratios, girls to boys (*draws girls-fraction bar-boys*). Those are two different things, right? So, we don't have to be comparing the same unit. A ratio is a little different from a fraction.

This discussion showed further evidence that Helene had embraced a number of approaches inspired by the MOOC-Ed, such as focusing on the *unit* when thinking about fractions, presenting different conceptions of fraction (as number, as ratio, etc.), and questioning as opposed to "telling".

Helene's MOOC-Ed experience. Helene registered for FFI but was only able to participate in a portion of it. She then participated in FFII in attempts to complete more of it. She had participated in a previous MOOC-Ed and received notification of the *Fraction Foundations* course by being a part of the mailing list. Helene found the previous MOOC-Ed useful, and she believed that a course in teaching and learning fractions would help serve her needs, since she was teaching 5th grade for the first time in regular education. Helene was therefore very interested in participating in the *Fraction Foundations* MOOC-Ed "for the purposes of making sure that I make that transition to the Common Core language and developing the new techniques and strategies to help…Plus the EOG [End-of-Grade Test] for 5th grade, it's about 47% fractions." She included it as part of her *Individual Growth Plan* (IGP), a requirement for teachers at her school.

Helene had a number of goals in taking the *Fraction Foundations* course. She wanted to increase her knowledge of ways to represent fractions (e.g., numbers on a number line) as well as refine the language she used to describe them. "We just tend to use fraction strips and circles and sometimes we use sets…I guess I just wanted more examples of everyday kind of situations…to incorporate more [fractions]." Helene also had some more implicit goals that she hadn't realized inspired her to take the MOOC-Ed until we first talked. She was thirsty for understanding the theory behind how students learn fractions and how she could adjust her approach to better foster that learning. This became clear as we talked about how she would define success in participating in the MOOC-Ed:

> Success should be…a change in your instruction. Not just a change, but obviously a change that leads to more student engagement and student success. I guess its ways to implement theory--that would be definitely the next step.

In another conversation, Helene went further to state that:

> Success is, for me, just incremental change…in myself, and applying the knowledge that I've learned, because it's great to learn stuff; it's another thing to implement it. I

> don't expect it to happen overnight, but I do expect that I will reflect on where I want to go and where I am based on what I've learned. (*external* → *personal*)

Helene's "incremental change" was evident in my observations of her classroom before and after the MOOC-Ed, as described earlier. She had begun to employ more probing questioning strategies, was more attentive to precision in the mathematical language she used, and she attended to student thinking by addressing common misconceptions. Helene had also emphasized the different representations of fractions both in her class discussions and the tasks she chose for her students to engage in. (*external* → *personal* → *practice*)

Helene found a number of elements that contributed to the MOOC-Ed's effectiveness. She greatly valued its content focus, as it highlighted the different representations of fractions, and emphasized the idea of fractions as having value and the importance of identifying "the unit" and "the whole". She remarked that "I think the MOOC just goes in more depth with your different models and things that are really helpful."

Helene also found the discussion forums useful. She appreciated the opportunity to exchange ideas and read others' perspectives. "I like to be able to post to other people…I also just feel like the level of professionalism is there." In talking about the case videos presented in the MOOC-Ed, Helene reflected on what she took away from the discussions among participants:

> Seeing the comments, and the discussion about differentiation between how much help you give students, you know, how directed you are; that helps me think about, 'Okay, where do I fall in that continuum and where do I want to fall?' (*external* → *personal*)

Helene believed that the resources and research articles provided in the course were useful and reliable, since she trusted that they were chosen for meaningful reasons:

> These people that put these things together, they actually know what articles you should read, and they actually know what they're talking about. If you take the time and really do a good job on it, you can get a lot out of it.

With regard to scheduling the PD into her day, Helene also appreciated the flexibility that the MOOC-Ed presented. She appreciated that she could engage with it on her own time and at her own pace, as well as the opportunity to take the course a second time.

Impact on practice. Having participated in both *Fraction Foundations* MOOC-Eds, Helene reflected on how the experiences had an impact on her practice. She described a number of changes specific to her approach to teaching fractions. One change was in implementing tasks that give her students opportunities to represent fractions differently and use models to develop understanding:

> One thing the MOOC talks about a lot is the different representations of fractions, and so I've tried to bring that in. When we first started fractions, some of [the students] just want to cut to the chase; they just want the easy thing. But I would give them… the fraction pieces, and I would just have them build improper fractions. I would just give them the number of pieces and say, 'Okay, how do you put this together?' Because the pieces fit together in a whole, it was a natural thing…that was kind of a discovery thing. I don't think I would have done as much of that before. (*external* → *personal* → *practice*)

Another change Helene attributed to the MOOC-Ed is a more conceptual approach to teaching fractions (e.g., increased focus on "the unit"; fractions having a "value"; connection to other mathematical concepts):

> Definitely attending to *unit*, definitely asking,…'How does this compare to a whole?', or 'What would a whole look like?'; 'Where might this be on the number line?' Definitely doing a lot more [of that]. I'm really trying to bring in the inter-connectedness of fractions as a value compared to the values of a ten, the values of a hundred, the values of a one…We might have a one in the tens column, but it represents ten, or you know a five in the millions column represents millions; so you're talking about it as a unit and then, therefore, it makes more sense to manipulate it as a number, as a value. (*external* → *personal* → *practice*)

This was evident in my observations of Helene's lessons after she had participated in the MOOC-Ed. Through our conversations, it became clear, too, that the MOOC-Ed inspired Helene to be even more aware of her approach in the classroom:

> It really is a very subtle difference that you can kind of catch yourself referring back to the old ways and realizing that you need to be consistent about the language and consistent about how you're representing this. (*external* → *personal* → *practice*)

Helene considered the effects her changes were having on her students. Although she identified continued struggles in their understanding of fractions, she acknowledged their growth in certain areas:

> They were trying to use the strategies but not completely successful…[To] look at them now and how they are making that connection…I think they're understanding

fractions as a value, but they don't understand the numerator, the denominator and the whole number and how it relates to the number line. They're not completely there, [but] they're way better than they would have been; so I count that as success. (*personal* → *consequence* → *personal)*

Obstacles to change. With certain successes in both changing her practice and observing the effect of those changes in her students, Helene also recognizes significant obstacles to change. One is the limitation imposed by the brisk pacing of the math and science curriculum at her school and the time it would take to implement certain changes:

> Sometimes I think my students tend to sit back and wait for me to do it, and I'm really trying to be more of a 'No, you can do this---How does this make sense?' It's very time consuming, so I don't do as much of it as I would like. We have so much [material] to get through…so much science content that I'm just never going to get through [it].

Helene also believed that she would have implemented more changes in her fractions instruction if the timing of the course better aligned with when she was actually teaching the content in her classroom.

Through our conversations, however, the most significant obstacle to change became clearer and was supported by my observations of Helene's classroom. The environment in which she was teaching and which her students were learning was affecting her students' ability to learn. Much of this was due to the behavior of a few students in her classes, which limited the time and attention she could give to students who needed additional help:

> My challenges this whole year have been small group instruction and working with my most needy kids; giving them the time, while my more capable kids ideally would be working. It's because of behavior. There might be times where the whole class is engaged, and I can go and really work with a specific student. But they just were not ready…I had too many [acting out] for them to really be ready for me to give my whole attention to a group and be able to rely on them to do what they're supposed to do without throwing crayons...

Helene's class sizes and lack of instructional support also served as obstacles to change. She found it difficult to plan new activities for the classroom because of other professional obligations during the day:

> I think [class size] makes a tremendous difference in how you're able to really get to know and relate to the kids. And I think it's also very tough to have two tested subjects; we're the only teachers in all of elementary that have two tested subjects; and we have a 45-minute planning period that I would say a good 30% of the time we don't get to utilize for planning because of meetings. And then, unlike some schools, we have lunch duty every day.

Helene was successful in implementing changes in her practice based on the MOOC-Ed, and she observed some positive effects of her efforts. It was clear, however, that Helene faced a number of pervasive challenges to making changes. Nonetheless, Helene was committed to finding ways to engage in learning and improve her practice.

Helene's path in her "change environment". In analyzing Helene's path among the four domains in her "change environment" through her MOOC-Ed experience (see Figure

11), I describe her growth as beginning in the *external domain* and ending[26] in the *personal domain*, with her "growth network" as follows (Clarke & Hollingsworth, 2002):

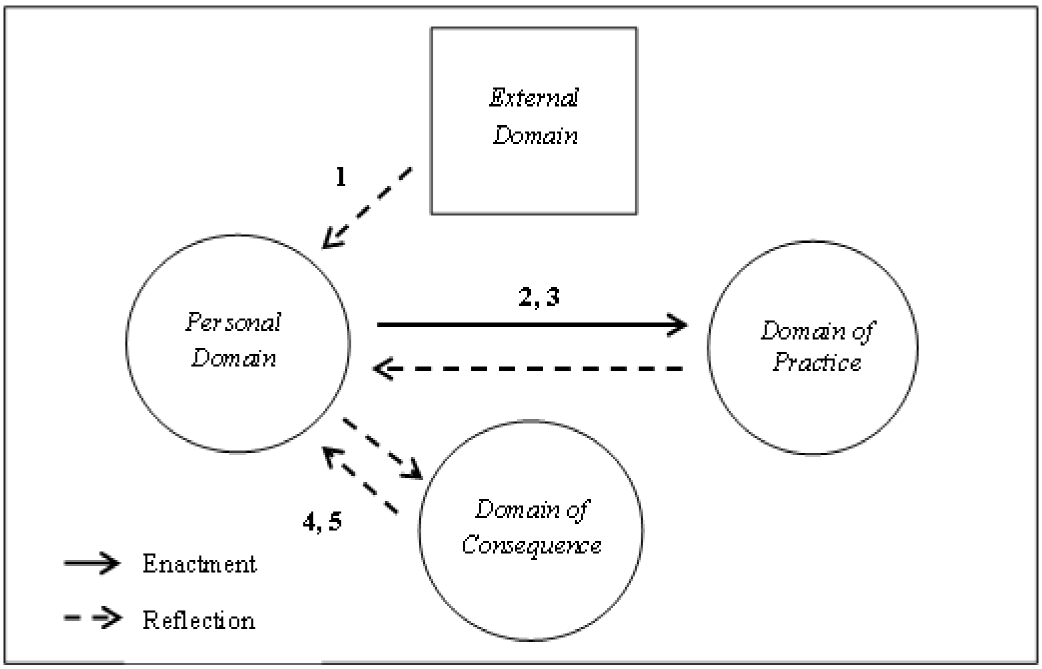

Figure 11. Helene's "Growth Network" (Clarke & Hollingsworth, 2002). Numbers are provided in order of edges in the network, with *x*, *y* indicating order from top arrow to bottom arrow, respectively.

Helene entered the MOOC-Ed with a desire to gain a deeper knowledge of fractions, how to teach them to students, and how students conceptualize them. She engaged in the course with intentions to implement change in her instruction and foster more conceptual understanding in her students. In the figure, the dotted arrow pointing from the *external domain* to the *personal domain* indicates Helene's "reflecting" on what she was learned in

[26] The words "beginning" and "ending" are used here only to indicate the domains in which the participant is observed as being at the beginning and conclusion of the study. It is assumed that the participant continues their path as they continue their growth.

the MOOC-Ed that affected her knowledge and beliefs about teaching and learning fractions (edge #1). The solid arrow pointing to Helene's *domain of practice* indicates Helene's "enactment" of her new knowledge and refined approach in her classroom in the form of new instructional tasks, increased attention to using precise mathematical language, and presenting the different representations of fractions (edge #2). After observing results of this enactment, Helene reflected on what both she and her students learned in the process and whether implementing the new knowledge had an effect on her students' understanding and success (edge #3). With this knowledge, Helene drew conclusions about how she could serve her students better in the *domain of consequence*, based on what she learned from the MOOC-Ed (edge #4). Helene continues to reflect on her practice and seek opportunities to grow by engaging in PD activities that are relevant to her practice (edge #5). It is important to note that I did not view Helene's *domain of practice* and *domain of consequence* as directly linked. It was clear that Helene truly believed that what she learned from the MOOC-Ed would serve her students well and foster her professional growth. However, I believe that her conclusions were made based more on her reflections than on what she observed in her classrooms. This may have been affected by the obstacles she faced in implementing some of the changes she would have liked.

Sara

At the time of my first visit, Sara had been teaching for approximately 20 years and was teaching 4th grade for the first time. She was licensed in K-6 general education and had a master's degree in education. The class that I observed was comprised of students who struggled in reading, most of whom did not pass the end of grade reading exam the previous year.

Sara works in an environment that encourages and fosters professional growth, and she takes advantage of opportunities that are presented to her. PD is readily available to her, and she credits her county for the access and her assistant principal for useful resources:

> I always have more than I need. A lot of [the PD] comes to us for the most part. I have that great support system…Our assistant principal taught math for a long time and actually wrote some of the state curriculum. So she brought in [math PD] and gave me [resource] books.

Sara teaches in a collaborative and supportive environment where she feels welcome to sharing classroom activities and materials:

> I have a great manipulative kit and the [previous] math teacher left all their stuff. It's a great school so when I needed the pennies, dimes, and nickels and dollars, I just went down to a 2nd grade classroom and said, 'Can I borrow yours?' I had a fabulous teammate that teaches AG and has taught just math for a long time that has said 'You need to teach it conceptually; and she had written a lesson that she shared with people across the state…She just kind of said, 'So I've written a lesson plan; if you follow this you'll develop the conceptual understanding, and [the students] develop it.

Despite the availability, access and support Sara receives for professional development, she acknowledges challenges to engaging in such activities:

> I think it's hard with the way the United States has schools set up, since I have kids from 8 o'clock in the morning until after 3 in the afternoons. By the time buses get

> here, it would be hard to do [the activities]. I think we find ourselves in staff development in the summers or after hours, and you're tired and maybe not 100% engaged.

This speaks to Sara's view of the value a MOOC-Ed can bring in a teacher's professional growth, as it allows them to engage at their own pace and on their own time.

Views on professional development. In conversations with Sara about PD in general, I learned about other elements of PD that she feels are effective. One is the opportunity to implement what was learned and be able to return with reflections and questions. She had had experience with this in her master's program and appreciated the "continuous" professional development that the experience afforded. Relatedly, Sara values PD that which is relevant to and aligns with the curriculum she is currently teaching, since it "at least it gives us some time to go back [to the classroom] to practice and then go back and talk about it."

Teaching approach. Sara is an experienced teacher who exhibits strong qualities of a patient and confident practitioner. She has always been attuned to precision in both language and mathematical notation and holds her students responsible for communicating their understanding orally and in writing accurately. Her classroom is adorned with posters containing inspirational goals and useful strategies, which she had posted before participating in the MOOC-Ed. For example, posted on one side of her room is "My Gaudy Goal: Every student will be able to support and prove their answers with equations and math language." On another wall, Sara has a concise list of "problem solving strategies" (Figure 12) that students can refer to as they work together:

Problem Solving Strategies:

- Make a picture or diagram
- Look for a pattern
- Act it out or use objects
- Solve a simpler problem
- Make an organized list
- Make a table or chart
- Work backwards
- Use logical reasoning
- Guess and check
- Brainstorm
- Write an equation

Figure 12. List Posted in Sara’s Classroom

It was clear that Sara’s approach to teaching fractions aligned closely with the ideas and principles presented in the MOOC-Ed (*practice* → *external* → *practice*). In particular, she had a number of posters in the front of her room that illustrated different representations of fractions (Figure 13):

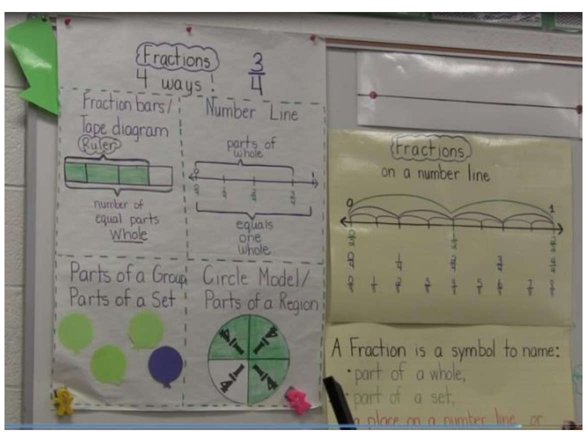

Figure 13. Picture of Posters on Sara’s Classroom Wall

Sara's students sit in groups of 4 to 5, and she engages them in a diverse mix of activities during her lessons, including class discussions, collaborative group work on tasks, and centers for differentiated activities. It was clear that Sara values having her students communicate about mathematics frequently and be active in their learning.

Classroom visits. At the time of my first visit to Sara's classroom, the MOOC-Ed had already begun, and she had participated in a portion of it. During my visit, her lesson was focused on fractions, and it was clear that Sara was an experienced teacher who valued the use of manipulatives and technology in her teaching. Her students were seated in groups of 4 to 5 students, and they seemed to understand that rich discussion and justification of solutions were classroom norms. As students sat down at the start of class, Sara opened with a fraction problem where students were to choose 2 ways to solve it. Students used fraction kits, magic squares and other tools in their process. Sara encouraged them to justify their claims and "build" their fractions. As she conducted a classroom discussion, Sara exhibited a strong conceptual understanding of fractions and was attentive to her students' thinking, asking them follow-up questions to probe their understanding. Sara also emphasized use of precise mathematical language as she conducted the class discussion as well as using multiple strategies to solve problems (e.g., decomposition, drawing pictures, using manipulatives). As students worked in groups, Sara continued to circulate around the room, monitoring their work and challenging students to communicate their ideas to each other and documenting their explanations in complete sentences.

Specific to fractions, Sara related them to time and how we can use fractions to describe time (e.g., "a quarter after 5"). She was consistent in referring to "the whole" and engaged the students in an activity placing fractions on a number line, an approach made salient in the MOOC-Ed.

In my second observation, this one after the MOOC-Ed had concluded, Sara conducted a lesson on decimals. Her opening problem was to build 75 hundredths using 10-blocks and then writing that decimal as a fraction, reinforcing the relationship between fractions and decimals. After this, Sara set up centers for her students to work through together as the day's activity. Through this, Sara connected the importance of precision to real-world contexts (e.g., engineers building a bridge) and reemphasized the use of accurate mathematical language:

STUDENT: "Six-hundred and fifty-seven POINT…"

SARA: "Not POINT…How do you read that number?"

STUDENT: "Six-hundred and fifty-seven AND eighty-nine."

SARA: "Eighty-nine whats?"

STUDENT: "Eighty-nine hundredths"

Through my observations, it became clear that Sara had embraced many of the approaches presented in the MOOC-Ed before having participated in the course. As she shared during our interviews together, the MOOC-Ed reinforced many of the strategies Sara had already employed, and she was grateful for the validation.

Sara's MOOC-Ed experience. Sara enrolled in the Spring offering of the *Fraction Foundations* MOOC-Ed, and, while she was disappointed that she was not able to complete the project component, she believed that she engaged with most of the rest of the course. There were a number of features of the MOOC-Ed that she found particularly effective.

Sara appreciated the personalized nature of the course, as she had the choice to engage with components that were most relevant to her and her practice:

> I like that I could pick and choose what I wanted because some of it was geared towards 4th, 5th, 6th grade. I really felt like that's what it hit; but, if I got into a video and realized this really isn't applicable to me, I could just kind of skip it; so I had a lot of selection.

Sara also found the "What Would You Do Next" and other resource videos particularly intriguing, as they challenged her to think about her practice and consider alternate approaches. She appreciated the opportunities to reflect on her methods and observe other teachers in real-life contexts with students:

> The videos that I watched with the MOOC instructors that were working with students-- almost every single one of those had [the students] using manipulatives. It was very effective to make me realize I needed to go back, especially when we started decimals to the manipulatives--to fall back to the manipulatives for place value and to start there--to start with the concrete to develop the conceptual understanding. I think that that kind of brought to the forefront that 4th graders especially need to still use manipulatives for everything. (*external* → *practice*)

While Sara felt "disappointed" with herself that she didn't "finish" the MOOC-Ed, she believed that she had made progress in accomplishing the goals she set out for her participation in the MOOC-Ed:

> I think it's really solidified and backed up…if I ever had any inkling of 'I'm not doing it right', 'I don't need to teach conceptually, we just need to do more problem solving and more test prep stuff,' I think, ['no'].

Impact on practice. Though Sara had seemingly already implemented a number of the ideas presented in the MOOC-Ed before she had participated in the course, she described a few impacts the course had on her practice. In particular, she credited the MOOC-Ed in providing resources [e.g., case videos, teacher documents with sample probing questions] to strengthen her questioning strategies with her students, both to probe for understanding and challenge her students to think deeply (*external* → *practice*):

> I hope [the MOOC-Ed is] starting to change how I'm questioning children as far as probing them. So that's really what I want to try to focus on is pulling some smaller groups of kids and asking those kinds of questions, because I think I probably lead them to the water more often than letting them search for it themselves. I think that's something I definitely need to work on. And the MOOC-Ed I think gave me the things that I need to do to do it. Whoever was being videoed [in the What "Would You Do Next" videos] didn't lead the students to the answer. It was more like 'show me what you did, think about what you did, is there another way to do it.' I hope I'm doing that more because I kind of thought, 'Hum, I would have just said, 'Yesterday we talked about that halves and fourths were in the same family and that you can't add those two together. You need to make them equivalent,' rather than making them do it on their own.

Sara described a specific example of how she implemented this change in a recent lesson. She added that the change showed evidence of success:

> I did that with a couple of my kids yesterday with some word problems on line plots that were also fraction work problems. They came to me with nothing and said, ‘I don’t understand’…I sent them back and said, ‘When you have some thinking, show me…write some equations or write some numbers to show me your thinking and then give me some place to start from and then I can help you’, rather than just before, I think I would have just jumped right in and said, ‘Oh, you do this, this, and this.’ So, hopefully, that made them think and work some on their own. One child in particular came back, and I think he understood what I was asking for. (*practice* → *consequence* → *personal*)

Sara also credited the MOOC-Ed for giving her ideas on how to refine the tasks she assigns her students and the manipulatives they use to foster their understanding of fractions:

> I think [the MOOC-Ed] made me realize that I could choose the pattern blocks as manipulatives. I really hadn’t thought of using the pattern blocks as much other than how much is a whole. I think it really helped a lot and hearing other people use the pattern blocks when they were working problems and talking about what they were doing in their class made me think, “Well, then I just really need to use those for [my class]”…[Also], a lot of the [tasks] where they had to show that equivalent is not always equal. Those are great extensions for me. (*practice* → *consequence* → *personal*)

Sara noted that the MOOC-Ed made her more mindful of the specific problems she has her students work on, based on the tools she plans for them to use, indicating a change in how she designs her lessons:

> I think the MOOC-Ed made me more aware. Usually I just pull [problems] out of the air, but yesterday I was like, 'No, if I'm using the fraction blocks, I don't want to change and try to make 1/5th's out of them…that's not what we're trying to do". So I had to make sure that I was doing halves, third's, and sixth's.

In thinking about the effects of the changes she's implemented in her practice, Sara believes that her students have benefited from her efforts, touting the impact of the MOOC-Ed as a success:

> I think, that the "corrective" group [has gotten] more confident. I think they're much more confident with, 'Yes, you can break this up and decompose this fraction.' And last year, I did not use manipulatives so much to teach mixed number. I tried to just show them the algorithm more, and I don't think they got it. I think the group has a more concrete understanding of mixed numbers [now] because I [made changes]. (*consequence* → *personal*)

Sara's path in her "change environment". In analyzing Sara's path among the four domains in her "change environment" through her MOOC-Ed experience (see Figure 14), I describe her growth as beginning and ending[27] in the *personal domain*, with her "growth network" as follows (Clarke & Hollingsworth, 2002):

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[27] The words "beginning" and "ending" are used here only to indicate the domains in which the participant is observed as being at the beginning and conclusion of the study. It is assumed that the participant continues their path as they continue their growth.

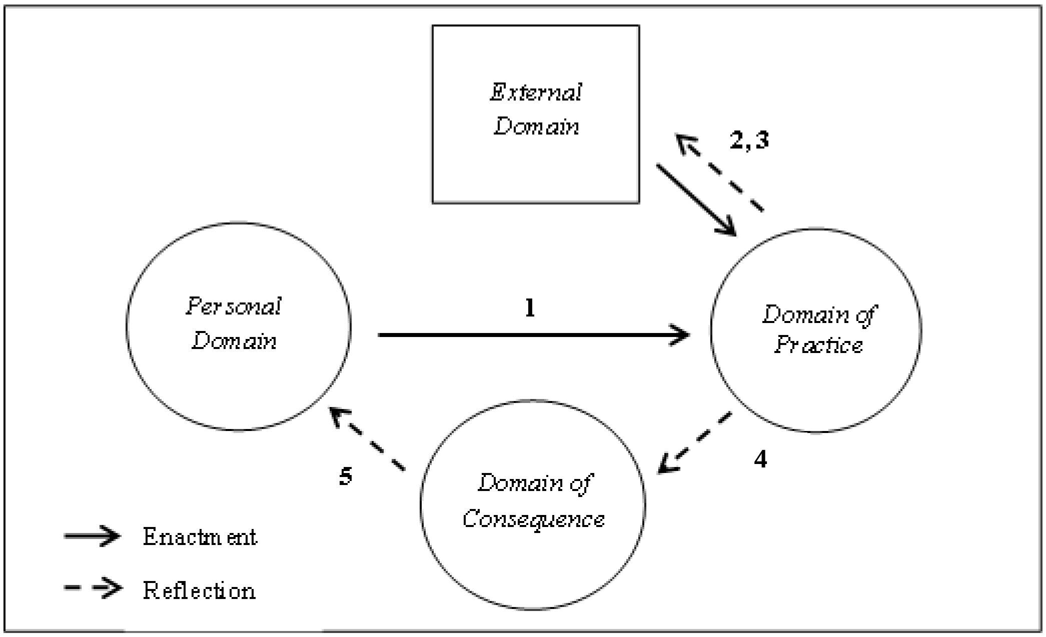

Figure 14. Sara's "Growth Network" (Clarke & Hollingsworth, 2002). Numbers are provided in order of edges in the network, with *x*, *y* indicating order from top arrow to bottom arrow, respectively.

As with Marilyn, Sara entered the MOOC-Ed with knowledge and beliefs about how to teach children and how they learn that aligned with much of the course. She emphasized the use of precise mathematical language and was attentive to her students' thinking. Her tasks were designed to incorporate the use of manipulatives, and she stressed the importance of representing fractions in different ways. In the figure, the solid arrow pointing from the *personal domain* to the *domain of practice* indicates Sara's "enacting" her knowledge and beliefs in her classroom (edge #1). The dotted arrow pointing from her *domain of practice* to the *external domain* indicates Sara' "reflection" on her practice after implementing strategies that reflect her knowledge and beliefs about teaching strategies and connecting them to what she learned in the MOOC-Ed (edge #2). With the knowledge she gained from the MOOC-Ed, Sara then enacted what she learned and her enhanced perspective in her *domain of practice* in the form of new activities in the classroom, providing students with more time and room to think, and refining her questioning strategies (edge #3). After observing results of this enactment, Sara reflected on what both she and her students learned in the process and drew conclusions about how to serve her students better in the *domain of consequence* (edge #4). Sara is consistently mindful and reflective about her practice, and with her conclusions, she embraced the new strategies and knowledge she had learned (edge #5).

Summary

In this chapter, I presented findings from multiple data sources to help answer the research questions that guided this study. Analysis of both quantitative and qualitative data was performed to identify the extent to and ways in which the *Fraction Foundations* MOOC-Ed was effective in changing participants' knowledge, beliefs and/or practice. Observations and interviews of a small sample of participants provided enhanced perspective into how the MOOC-Ed promoted those changes as well as evidence of those changes. Analysis of each teacher's path along the *Interconnected Model of Professional Growth* revealed the complexity of teacher professional growth. In Chapter Five, I discuss the findings and the significance, implications, limitations and conclusions the results suggest.

CHAPTER FIVE

The purpose of this study was to examine whether and how a MOOC-Ed in mathematics education can serve as a viable, effective option for educators in changing their knowledge and/or beliefs and impacting their practice. In Chapter Four, I analyzed results from multiple data sources to help answer the research questions that guided this study. In this chapter, I discuss those findings, examine the significance and implications of the work, describe study limitations, and offer suggestions for future research.

Discussion of Study Findings

The goal of this study was to answer the following general research question:

What is the impact of the Fraction Foundations MOOC-Ed on elementary mathematics educators?

In order to help answer this question, this study was guided by the following sub-questions:

1. To what extent and in what ways can the *Fraction Foundations* MOOC-Ed change participants' knowledge and/or beliefs?
2. To what extent and in what ways can the *Fraction Foundations* MOOC-Ed impact participants' practice?

Answers to these research questions were informed by findings presented in Chapter Four and are discussed here, along with connections between the findings and research.

Research Question 1

Participants in the *Fraction Foundations* MOOC-Ed overwhelmingly believed that their knowledge and/or skills improved in multiple areas as a result of their participation. These areas include: 1) fraction content standards; 2) common student difficulties and misconceptions about fractions; 3) analysis of student thinking to inform instruction; 4) effective use of various content-specific activities to help students understand key fraction concepts; and 5) addressing student learning differences when teaching fractions. Results from the pre- and post-MKT assessment suggest that the *Fraction Foundations* MOOC-Ed can have an effect on participants' mathematical knowledge for teaching fractions, but not to a consistently significant extent. Indeed, results on the MKT pre- and post-assessment varied slightly between the two implementations of the course, with those from FFII showing greater overall improvement than those from FFI. The greater increase in average scores and performance on content categories in FFII as compared to the changes in FFI may be explained by the fact that traditional elementary school curricula do not include fractions content until the Spring. As a result, participants in FFI, which was administered in the Fall, may not have had the opportunity to apply their learning directly to their practice during the run of the course. In contrast, participants in FFII, which was administered in the Spring, may have had more opportunities to apply what they were learning (i.e., the "abstract") to their practice (i.e., the "concrete") and gain more foundational and practical knowledge in teaching fractions through that application, a perspective shared by researchers on teacher PD (e.g., Clarke, 1994; Guskey, 1986).

Implementation of the four design principles that guided the development of the MOOC-Ed contributed to participants' perceptions of changes in their knowledge and/or skills. Participants believed that the course promoted those changes by providing the following elements:

1) Rich opportunities for participants to learn from a diverse group of people with

varying levels of experience and expertise. (*multiple voices*)

Discussion: Results showed that participants overwhelmingly believed that the MOOC-Ed was enhanced by the expertise of practitioners and leaders in the field and provided meaningful opportunities to share ideas, resources and experiences. Participants found that opportunities to learn from a diverse group of people were some of the most valuable aspects of the course, in which course videos and discussion forums were specifically effective in supporting their professional growth.

Through videos depicting student interviews, video lessons with teachers in the classroom, and exchanges of thoughts, insights and feedback both on discussion boards and through collaboration with local colleagues, participants have the opportunity to hear from diverse groups of people with different perspectives, an affordance some researchers contend is critical for effective PD (e.g., Herrington & Oliver, 2000; Lave & Wenger, 1991). Expert panel videos, for example, give participants the opportunity to listen to conversations between experienced researchers and practitioners around approaches to teaching and how students think. These conversations provide practical and meaningful suggestions for participants that they could then incorporate into their practice, thereby enhancing their knowledge and skills.

2) Flexible opportunities for participants to choose their own path and engage in the MOOC-Ed in a way that serves them best. (*self-directed learning*)

Discussion: Results showed that participants overwhelmingly believed that the MOOC-Ed enabled them to personalize their learning and provided flexibility for them in how and with what they engaged in the course. Correlation analysis showed that participants' beliefs that the MOOC-Ed offered such opportunities were strongly associated with beliefs that their knowledge of analysis of student thinking about fractions and effective use of content-specific activities for student learning improved after participating in the MOOC-Ed.

With opportunities to personalize learning, participants can choose resources and the order in which they engage with them, and at their own pace, to best fit their needs (Ginsburg et al., 2004; Ingvarson et al., 2005; Knowles, 1975; Sparks & Loucks-Horsley, 1989). With this, participants can identify areas they believe they need improvement in, and therefore choose to engage with components of the PD that offer resources. For example, a classroom teacher who would like to learn more about how to address student difficulties in adding fractions with unlike denominators may choose to engage deeply in the discussion around a clinical interview in the computations unit. Beliefs that the PD offers such choice and personalization can promote beliefs that knowledge was gained, since the activity engaged in was chosen for the purpose of learning something new. This is supported by research on PD that offers personalization of learning (Guskey, 2000; Renninger et al., 2011; Yang & Liu, 2004).

Diversity of resources provided in the course (e.g., practitioner articles, classroom activities and tasks, and teacher guides) can motivate participants to engage with them, as they have the opportunity to determine the usefulness and relevancy of those resources to their practice. With freedom to choose among diverse resources, participants can feel more ownership in their learning, thereby motivating them to continue engaging with the materials they find relevant to their needs.

3) Opportunities for participants to grow and gain knowledge through their supportive interactions with others in and outside of the MOOC-Ed. (*peer-supported learning*)

Discussion: Results showed that participants overwhelmingly believed that the MOOC-Ed provided supports for meaningful and collaborative work among peers. Participants found that opportunities to engage in peer-support activities (e.g., giving and receiving feedback from others) were some of the most valuable aspects of the course, in which discussion forums were specifically effective in supporting their professional growth.

Through exchanges of insights, common student misconceptions, and feedback both on discussion boards in the course and through collaboration with local colleagues, participants have the opportunity to be a part of a supportive environment that fosters learning of teaching strategies, understanding of student thinking, and creation of practical tools for the classroom. With this support, participants gain confidence in their skills and are encouraged to continue their growth, thereby promoting further changes in their self-efficacy, knowledge and skills. This is supported by research on effects of PD that includes peer support elements (Joyce & Showers, 2002; Kellogg et al., 2014; Simoneau, 2007).

4) Practical examples from authentic experience and job-connected activities that can be embedded in participants' practice. (*job-connected learning*)

Discussion: Results showed that participants overwhelmingly believed that the MOOC-Ed provided models of effective practice to support the application of the content they were learning in the course as well as authentic projects and activities that was relevant and applicable to their practice. Correlation analysis showed that participants' beliefs that the MOOC-Ed provided such resources were strongly associated with beliefs that their knowledge of analysis of student thinking about fractions, including challenges and misconceptions, and effective use of content-specific activities for student learning improved after participating in the MOOC-Ed. Even further, participants found that job-connected resources were among the most valuable aspects of the course, in which the course project, readings and practitioner tools were specifically effective in supporting their professional growth.

Models of effective practice provide authentic, relevant and practical resources that are meaningful to practitioners; they contribute to the effectiveness of any PD program (Clarke, 1994; Elmore, 2002; Herrington & Oliver, 2000). For example, video episodes of teachers and students and associated discussion prompts to promote critical thought give participants opportunities to think deeply about what they have viewed and observe students struggling or exhibiting common misconceptions in real time. Expert panel videos provide a resource for participants to listen to discussions from experienced educators around anticipated student difficulties and effective teaching approaches, thereby enhancing their own knowledge. Video lessons, student interviews, small group teacher videos and resource videos provide authentic images of students working through problems, revealing their thinking, and models of questioning strategies. Such resources provide opportunities for participants to think deeply about their approach and consider alternative strategies.

With opportunities for critical thinking and problem-solving within the MOOC-Ed, participants have chances to "vet" activities for use in their classroom and how to use them effectively (e.g., questions to ask and when to ask them). This can translate to improved self-efficacy and knowledge of using those activities through discussion with other educators. In addition, with opportunities to think through content-specific activities with unfamiliar or non-traditional elements (e.g., MKT-style self-assessment questions), participants believe that they understand the activities at a deeper level, thereby making them feel more knowledgeable about using them effectively in the classroom.

Including projects and activities in PD that are relevant for use in participants' practice, such as creation of usable materials for the classroom (e.g., course project) can motivate participants to implement what they learned from engaging in those activities into their practice. Personally developing the materials gives participants an opportunity to think critically about what they want their students to learn. Through implementation of those materials, participants have an opportunity to observe and better understand their students' reasoning and consider teaching approaches that would address learning differences, thereby enhancing their knowledge of student thinking (Clarke, 1994; Herrington & Oliver, 2002; Webster-Wright, 2009).

Research Question 2

Participants overwhelmingly believed that the MOOC-Ed was effective in preparing them to make positive changes in their practice. These changes include: 1) approaches to teaching; 2) approaches to assessment; 3) use of mathematical language; 4) use of manipulatives/tools; 5) attention to student thinking and learning differences; 6) focus on concepts as opposed to "rules"; and 7) collaboration on instructional planning and/or serving as a resource to others.

Specific aspects of the MOOC-Ed were found to be strongly associated with its potential to impact participants' practice. Through analysis of these associations, a number of implications emerged, which are consistent with research on aspects of PD that contribute to its impact on participants' practice (e.g., Clarke, 1994; Elmore, 2002; Herrington & Oliver, 2000). They are discussed here:

Implication 1: A MOOC-Ed that includes models of effective practice (e.g., instructional decisions, approaches to assessment, questioning strategies) can be effective in preparing participants to make positive changes to their practice.

Discussion: Models of effective practice (e.g., instructional decisions, approaches to assessment, questioning strategies) can provide realistic, relevant, and practical examples of practice that are meaningful to participants, thereby motivating them to make changes to their own practice. This is consistent with research on the impact of PD on teachers' practice (Desimone et al., 2002; Ingvarson et al., 2005).

Implication 2: A MOOC-Ed that provides projects and/or activities that can be embedded in their practice can be effective in preparing participants to make positive changes to their practice.

Discussion: With opportunities to engage with, implement and experiment with those activities, participants may gain confidence in making changes to their practice. This is consistent with research on the impact of PD on participants' practice (e.g., Vrasidas & Zembylas, 2004). By engaging in answering challenging questions in the MOOC-Ed, analyzing both content-specific material and student work, and communicating their thinking through written discussions with other educators, participants are better equipped to implement activities that help their students strengthen their knowledge and to do so with attention to, analysis of and anticipation of student thinking (Borko, 2004; Monk, 1994).

Implication 3: A MOOC-Ed that promotes critical-thinking and/or problem-solving through discussions can be effective in preparing participants to make positive changes to their practice.

Discussion: Through engagement in answering challenging questions posed within activities, analyzing both content-specific material and student work, and communicating their thinking through written discussions, participants are given opportunities to think critically and solve problems through those activities. With this active learning, participants' confidence in their abilities to address student difficulties, assess their students' understanding, probe for deeper thinking, and connect student reasoning to others' can be strengthened. Increased self-efficacy can then promote changes in practice (e.g., incorporating new activities; asking higher-order thinking questions) (Ingvarson et al., 2005; Learning Forward, 2011).

Implication 4: A MOOC-Ed that provides activities and resources to help participants gauge their learning (e.g., self-assessments, peer assessment and/or feedback) can be effective in preparing participants to make positive changes to their practice.

Discussion: As discussed in Chapter Two, research suggests that opportunities for participants to reflect on their practice can be effective in promoting changes in participants' practice (e.g., Garet et al., 2001; Ingvarson et al., 2005; Thompson, 1992). Reflecting on practice can provide those who engage in it with opportunities to take a critical look at their work. These opportunities can be used, for example, to refine practices by considering alternate approaches to teaching certain topics, asking better-defined questions, and analyzing interactions with students.

Further, the exercise of sharing ideas, experiences and challenges with peers provides participants with opportunities to learn new approaches, validate some of their own practice, and build a sense of community populated by others who share similar struggles and successes, an implication that is supported by research on teacher PD (e.g., Ball & Cohen, 1999; Hawley & Valli, 1999; Simoneau, 2007). Educators who are supported by colleagues who share common experiences and a sense of camaraderie may be more likely to make changes in their practice because they feel more confident to test new approaches in the classroom. Indeed, they would engage in analysis and reflection about those approaches together, with a focus on improvement as opposed to criticism.

Implication 5: A MOOC-Ed can impact participants' practice in diverse ways.

Discussion: As the findings in this study suggest, participants' practice can be affected in multiple ways. Changes to teaching approaches can include both curricular and pedagogical shifts. A curricular change may be characterized by a teacher planning to help students develop intuition into concepts before teaching associated rules or algorithms. Through this, they would design tasks that emphasize the core meanings of the topics studied and ask more rigorous questions to help students gain deeper conceptual understandings. This may be done, too, with increased use of manipulatives and tools to further understanding.

Pedagogical changes may involve giving students more time to think and explore concepts, asking probing questions during class discussions, listening to students' reasoning, and challenging students to communicate their thinking using precise mathematical language.

Changes to practice could also occur in participants' roles as resources to other practitioners. With enhanced knowledge and experience through their participation in the PD experience, participants can serve as resources to their colleagues by sharing what they learned. Teacher leaders, PD providers and other staff developers could enhance their roles by refining their plans for those they serve.

Interconnection between Changes in Knowledge/Beliefs and Impact on Practice

As one of his "Ten Important Principles of Professional Development", Clarke (1994) wrote that PD should:

> Recognize that changes in teachers' beliefs about teaching and learning are derived largely from classroom practice; as a result, such changes will follow the opportunity to validate, through observing positive student learning, information supplied by professional development programs. (p. 43)

This perspective is shared by others (e.g., Guskey, 1986, 2002) but does not address the converse relationship—namely that changes in practice are derived from changes in teachers' beliefs about teaching and learning. In their analysis of results from the national teacher survey they administered, Garet et al. (2001) found that "enhanced knowledge and skills have a substantial influence on change in teaching practice" (p. 934). In this light, I was interested in exploring the interplay between changes in teachers' knowledge and beliefs and those made in their practice, as the respective changes inform each other. This perspective is shared by others in the field of PD research and is one of the core elements of the theoretical framework used in this study (Clarke & Hollingsworth, 2002).

The analysis showed a number of strong relationships between participants' perceived changes in knowledge and their beliefs that the MOOC-Ed prepared them to make positive changes to their practice. These relationships offer a number of implications, which are discussed here.

Implication 1: Participants' perceptions of improvement in their knowledge of fraction content standards can be associated with beliefs that the MOOC-Ed prepared them to make positive changes to their practice.

Discussion: With perceptions of enhanced knowledge of content-specific standards, educators may believe they are better equipped to refine their approaches, lessons and activities to address those standards, thereby feeling prepared to make positive changes to their practice. For example, with improved knowledge of the importance of understanding fractions as numbers, participants may then plan to implement new tasks that emphasize this concept, such as representing fractions on the number line and comparing them to whole numbers. Further, with a better understanding of the sequence of standards and how they relate to each other, teachers may feel prepared to ask more probing questions of their students that will further their students' thinking toward related standards. For example, when teaching students to compare fractions based on their size, building to the idea of comparing fractions with different denominators, teachers can challenge their students to express the importance of comparing those fractions only when they refer to the same whole.

Implication 2: Participants' perceptions of improvement in their knowledge of common student difficulties and misconceptions can be associated with beliefs that the MOOC-Ed prepared them to make positive changes to their practice.

Discussion: With perceptions of increased knowledge of student difficulties and misconceptions, educators may feel more confident in their ability to make changes in their practice to better address those misconceptions. Teachers may be reluctant to make changes because they are uncomfortable with the risk it brings if they are not wholly confident with their knowledge of the material. With increased confidence in their knowledge of student difficulties and misconceptions, they can address them in advance, thereby making more positive changes in their practice.

Implication 3: Participants' perceptions of improvement in their knowledge of analysis of student thinking can be associated with beliefs that the MOOC-Ed prepared them to make positive changes to their practice.

Discussion: As a similar result to that earlier, a perceived increase in knowledge of student thinking to inform instruction may bring confidence to then make changes in practice. Teachers may feel more confident to implement new activities and use different approaches that include the analysis of student thinking (e.g., student interviews) with increased knowledge, because they find value in it as part of supporting their growth.

Implication 4: Participants' perceptions of improvement in their knowledge of effective use of activities in the classroom can be associated with beliefs that the MOOC-Ed prepared them to make positive changes to their practice.

Discussion: A perceived improvement in one's knowledge of using concept-specific activities effectively can encourage participants to implement those activities into their practice, because they feel empowered to implement activities that have been "tested" and "proven" to be effective. They would support the use of those activities, because they understand their purpose.

Analyses of the data gathered through the interviews and observations of four elementary mathematics teachers who participated in the MOOC-Ed also showed interconnections between changes in knowledge and/or beliefs and the MOOC-Ed's impact on practice. Using the *Interconnected Model of Professional Growth* (Clarke & Hollingsworth, 2002) as a framework for analysis, this study was conducted with the belief that educators move along diverse pathways in their journey through professional growth, transitioning between and among four domains: 1) *personal domain*; 2) *external domain*; 3) *domain of practice*; and, 4) *domain of consequence*. Results from my analyses provide rich evidence that supports this theory. Indeed, in exploring the paths that the four teachers took in their "change environment", I found that each was unique and effective in different ways. Marilyn's path illustrated one in which the *personal domain* was seemingly revisited after each of the other domains, highlighting her ongoing and valued self-reflection. Jill's path reflected growth that relied more on implementation and evidence of success. With perceptions of limited opportunities to implement changes in the classroom, Helene's path involved changes related more to personal beliefs and professional goals. Sara's path was characterized more by growth as a result of implementation of changes to practice and observation of results in order to support a firm teaching philosophy. The uniqueness in these paths suggests that the MOOC-Ed offered participants the opportunity to personalize their learning and navigate their way through the PD in order to experience changes that promoted their own growth.

Conclusion

This study supports the conclusion that a MOOC-Ed can serve as an effective form of professional development in changing educators' knowledge and/or beliefs and having an impact on their practice. It highlights the diversity of ways that such PD can affect educators' professional growth, when it provides them with opportunities to hear from and communicate with *multiple voices* and engage in *self-directed*, *peer-supported*, and *job-connected learning*. In this light, it is also important to recognize that development of online PD in the form of a MOOC is unique from traditional PD, in that a significant percentage of it must be designed and executed in a number of rigid ways that cannot be revised after launch (e.g., static webpages, layout of discussion boards). Evidence of the diverse pathways through which participants experienced changes in this study underscores the idea that online PD should not set a rigid or prescribed path for its participants. Indeed, as Clarke and Hollingsworth (2002) argued, in engaging in professional learning, educators traverse a path that reflects a complex network of domains that represent what and how they learn in a way that can impact their practice and contribute to their professional growth. Designing online PD in a way to maximize the number of participants for which it is effective must be done with this understanding, as it was in the case of the MOOC-Ed in this study.

Significance and Implications of the Work

This study will have an impact in a number of ways and for diverse groups, including PD developers, education researchers, school administrators, PD providers, and classroom teachers. For PD developers, results can be used to help develop and refine future PD opportunities for mathematics educators that are effective in changing participants' knowledge and beliefs and have a meaningful impact on their practice. The design principles implemented in the development of the *Fraction Foundations* MOOC-Ed were effective in promoting those changes with practice-based aspects (e.g., providing models of effective practice) and specific course components (e.g., videos, discussion boards) that recognized the diverse needs of its participants.

For education researchers, this study will serve to fill this gap in the literature on OPD as a resource for professional learning that can inspire and support new initiatives for professional development. While there is a focus in the research community on design and evaluation of OPD, there is limited empirical research conducted on measuring its effectiveness on participants' knowledge and beliefs and the impact the PD experiences have on participants' practice (Dede, Ketelhut, Whitehouse, Breit, & McCloskey, 2009). Indeed, the NRC (2007) highlighted the great need for research on the actual effects of OPD on educators, as "we know very little about what characterizes effective teacher professional development" (Hilda Borko, as cited in NRC, 2007, p. 24). Further, since MOOCs only began to emerge in the last few years, the field of MOOC research is still quite new. Much of the research that has been performed on MOOCs has centered on university-level courses focused on a particular field of study, such as engineering and computer science. As noted in Chapter Two, MOOC research has explored topics such as user experience, participant retention, and social network analysis; further, much of the data analysis has been quantitative, due to the large volume of data available through the technological affordances of the online delivery platform (e.g., analytics) (Liyanagunawardena, Adams, & Williams, 2013). This study will add to the literature on MOOCs as a form of PD with an added focus on qualitative data analysis. Moreover, through its pragmatist lens, this study will seek to identify connections between the effectiveness of a PD program and its impact on teachers' practice, a connection that needs to be further studied using carefully designed methods (Desimone, 2009).

For school administrators and PD providers, this study provides strong evidence that the *Fraction Foundations* MOOC-Ed can serve as a viable, cost-effective and quality solution to satisfy the urgent need for professional development to support elementary teachers in their continuing mathematics education for teaching, a need that is well-documented (e.g., Ball et al., 2008; CBMS, 2001, 2012; Hill et al., 2008). This implies, too, that future mathematics education MOOC-Eds can serve as similarly strong options for such PD.

For classroom teachers, this study provides strong evidence that the *Fraction Foundations* MOOC-Ed can help promote changes in their knowledge and beliefs about teaching and learning fractions as well as have diverse impact on their practice. As a result, with multiple options for PD to serve their needs in mathematics education, teachers can consider MOOC-Eds as a viable and effective form of PD, supported by both quantitative and qualitative evidence.

Study Limitations

It is important to recognize several limitations of this study. With any study on PD effectiveness, the PD program (e.g., the MOOC-Ed in this study) may be viewed as a *treatment* in an experiment as part of an intervention into participants' practice. Effects of treatments are traditionally studied using a control group to assess whether changes can be attributable to the treatment. This study, however, did not include a control group.

Another limitation of this study is that all quantitative data gathered, with the exception of the MKT assessments, was self-reported. The survey instruments were targeted toward participants' behaviors (some future anticipated behaviors) and beliefs about their understanding and their practice, which threatens validity (Mabe & West, 1982).

Since the sample of participants in the qualitative component of the study was a convenience sample, participants in the study did not necessarily teach at the same school, work in the same instructional context or follow the same curriculum. This signals a lack of "consistency in the sample across situation and time" (Mabe & West, 1982, p. 287). Furthermore, while attempts were made to gather data from participants with varied educational experience, grades taught, school location, and other differentiating factors, this study design did not establish that as a requirement for consideration of participation. Therefore, it was a challenge to ensure diversity of participants in the study, which limits the generalizability of the findings.

In addition, since I served as the primary instrument (Creswell, 2007) in this component, I came to the role with my own biases, such as my perspectives, beliefs, knowledge, and experience, which can have an effect on how I conducted the interviews and made observations (Onwuegbuzie, Leech, & Collins, 2010). Despite following an observation protocol, I may have had biases that affected what I noticed as I observed teachers in the classroom. During the semi-structured interview process, I may have chosen to ask certain questions in response to participants' answers based on my biases.

Another limitation of this study is that observed behaviors of teachers, their choices, and their implementation of certain tasks or lessons may only be evident during classroom observations *because* they were being observed. Though each teacher was questioned by me about the authenticity of the impact of the MOOC-Ed on their practice, this can serve as a limitation.

Finally, it is important to recognize that observations performed to find evidence of the impact of the MOOC-Ed on teachers' practice would ideally need to be performed over a long period of time. A longitudinal study would allow a researcher to follow up with participants more frequently and better assess whether their practice has been impacted (e.g., new lessons they implement, different mathematical language they use in their teaching). Also, since observations were not necessarily performed when the curriculum aligned with the content presented in the MOOC-Ed, it was an additional challenge to assess whether the course had an impact on teachers' practice. A long-term study would provide more opportunity for me to observe effects of the program when the curriculum does align with the content.

Suggestions for Future Research

As discussed in the Study Limitations, there are a number of interesting opportunities for future research related to this study. First, as the focus of this study was on the MOOC-Ed itself and its general impact on practice, it did not explore deeply the impact the MOOC-Ed may have had on specific pedagogical issues related to teaching mathematics, such as task design and questioning strategies.

Second, this study explored ways in which the MOOC-Ed was effective and not on elements that did not serve as effective. Future study could focus on features of the MOOC-

Ed that participants perceived as limiting their growth or elements that contributed to participants' decisions to leaving the course, thereby informing development of future online PD. This can involve, for example, further analysis of features that showed little or no correlation with perceived changes in knowledge or practice.

Third, since the focus of this study was on the effectiveness and impact of the MOOC-Ed from the educators' perspective, it would be interesting to explore the effect their participation in the MOOC-Ed had on their students.

Fourth, it would be a rich study to explore the longer-term effects of the MOOC-Ed on teachers' practice and whether and how their participation furthered their professional growth and impact on both their students and other colleagues.

Finally, as blended learning programs increase in availability and demand, there is much to be explored about the effectiveness and impact a MOOC-Ed can have on educators who engage in the PD as a collaborative group.

It is clear that there is much to be studied and understood about the effectiveness and impact of MOOC-Eds for its participants. Rich data emerged from this study that can be further analyzed and used to refine future MOOC-Eds, and more globally, online PD programs for educators. This inspires motivation to continue the work toward making MOOC-Eds and other online PD for educators effective in ways that serve educators well in promoting changes in their knowledge and beliefs and to have an actual, true impact on their practice.

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APPENDICES

Appendix A: Survey Instruments

Registration/Pre-Course Survey

FFI

1. Gender:

- ☐ Male
- ☐ Female

2. Is your work primarily based at:

- ☐ School
- ☐ School District
- ☐ Other Organization

3. School name:

4. District name:

5. Organization name:

6. Location of School/District/Organization:

7. What is your primary area of responsibility?

8. I specialize in the following grade levels (check all that apply):

- ☐ Kindergarten
- ☐ Elementary
- ☐ Middle Grades
- ☐ High School
- ☐ Post-Secondary
- ☐ N/A

9. My highest level of education completed is:

10. Years of experience as an educator (round to nearest whole number):

11. Do you plan to participate with a peer group outside of this MOOC-Ed? (e.g. a school-based PLC or informal group of colleagues)

- ☐ Yes
- ☐ No

12. How much time do you expect to have available to spend on this MOOC-Ed?:

13. Do you receive any incentives for participating in this MOOC-Ed from your school, district or other sources?

- ☐ Yes
- ☐ No

14. Were you required to participate in this MOOC-Ed by your school or organization?

- ☐ Yes
- ☐ No

15. How did you hear about the MOOC-Ed?

16. To what extent do you agree with the following statements?

During my participation in the Learning Differences MOOC-Ed, it is important to my professional development that I...

	Strongly Disagree	Disagree	Moderately Disagree	Neither Agree Nor Disagree	Moderately Agree	Agree	Strongly Agree
Deepen my understanding of the fraction content standards.							
Deepen my understanding of students' difficulties and misconceptions about fractions.							
Improve my ability to analyze student thinking about fractions to inform instruction.							
Improve my use of							

fair-sharing activities to help students understand fraction concepts.							
Improve my use of measurement and number line activities to help children understand fraction concepts.							
Improve my ability to help students understand computations with fractions (e.g. addition, multiplication).							
Engage in fun and inspiring activities							
Exchange ideas and experiences with other educators							
Collaborate on joint projects							
Collect new resources or tools							
Experience learning in a MOOC-Ed							
Make changes to my professional practice							
Earn a certificate of completion							

17. Please indicate your current level of understanding regarding the following subjects:

	Unfamiliar	Little understanding	Moderate understanding	Strong understanding	Complete understanding
Mathematical content covered in this MOOC-Ed					
Students' thinking and reasoning about the topics in this MOOC-Ed					
Links between the Common Core State Standards and student learning					
Students' strategies in solving tasks					

18. Please indicate your current degree of confidence in incorporating the following concepts into your practice:

	No confidence	Low confidence	Moderate confidence	High confidence	Complete confidence
Mathematical content					

covered in this MOOC-Ed					
Students' thinking and reasoning about the topics in this MOOC-Ed					
Links between the Common Core State Standards and student learning					
Students' strategies in solving tasks					

FFII

1. Gender:

- ☐ Male
- ☐ Female

2. Is your work primarily based at:

- ☐ School
- ☐ School District
- ☐ Other Organization

3. School name:

4. District name:

5. Organization name:

6. Location of School/District/Organization:

7. What is your primary area of responsibility?

8. I specialize in the following grade levels (check all that apply):

- ☐ Kindergarten
- ☐ Elementary
- ☐ Middle Grades
- ☐ High School
- ☐ Post-Secondary
- ☐ N/A

9. My highest level of education completed is:

10. Years of experience as an educator (round to nearest whole number):

11. Do you plan to participate with a peer group outside of this MOOC-Ed? (e.g. a school-based PLC or informal group of colleagues)

- ☐ Yes
- ☐ No

12. How much time do you expect to have available to spend on this MOOC-Ed?:

13. Do you receive any incentives for participating in this MOOC-Ed from your school, district or other sources?

- ☐ Yes
- ☐ No

14. Were you required to participate in this MOOC-Ed by your school or organization?

- ☐ Yes
- ☐ No

15. How did you hear about the MOOC-Ed?

16. To what extent do you agree with the following statements?

During my participation in the *Fraction Foundations* MOOC-Ed, it is important to my professional development that I...

	Strongly Disagree	Disagree	Moderately Disagree	Neither Agree Nor Disagree	Moderately Agree	Agree	Strongly Agree
Deepen my understanding of the fraction content standards.							
Deepen my understanding of students' difficulties and misconceptions about fractions.							
Improve my ability to analyze student thinking about fractions to inform instruction.							
Improve my use of fair-sharing activities to help students understand fraction concepts.							
Improve my use of measurement and number line activities to help children understand fraction concepts.							
Improve my ability to							

help students understand computations with fractions (e.g. addition, multiplication).							
Engage in fun and inspiring activities							
Exchange ideas and experiences with other educators							
Collaborate on joint projects							
Collect new resources or tools							
Experience learning in a MOOC-Ed							
Make changes to my professional practice							
Earn a certificate of completion							

Post-Survey

1. Thank you for taking the time to participate in our post-survey. The information obtained in this survey will help us evaluate the effectiveness and impact of this MOOC-Ed on your practice and be valuable in improving our future course offerings. This survey is designed to take no more than 15 minutes of your time.

2. Please specify the MOOC-Ed in which you participated.

3. As a whole, how effective was this MOOC-Ed in supporting your personal and/or professional learning goals?

- ☐ Very Ineffective
- ☐ Ineffective
- ☐ Somewhat Ineffective
- ☐ Neutral
- ☐ Somewhat Effective
- ☐ Effective
- ☐ Very Effective

4. How effective were the following components of this MOOC-Ed in supporting your professional learning?

	Very Ineffective	Ineffective	Somewhat Ineffective	Neutral	Somewhat Effective	Effective	Very Effective
Course videos							
Course Readings							
Discussion							

Forums and/or Online Chats (e.g., Twitter)							
Practitioner Tools (e.g., rubrics, apps, websites, classroom aids, planning templates, etc.)							
Self-Assessments and/or Peer-Assessment Activities							
Course project							

5. What was the most valuable aspect of this MOOC-Ed in supporting your personal or professional learning goals?

6. What recommendations do you have for making this course more valuable to future participants? Please explain.

7. To what extent do you agree with the following statements?

As a result of my participation in this MOOC-Ed, I have improved my knowledge and/or skills related to…

	Strongly Disagree	Disagree	Somewhat Disagree	Neither Agree or Disagree	Somewhat Agree	Agree	Strongly Agree

fraction content standards.							
common student difficulties and misconceptions about fractions.							
analysis of student thinking about fractions to inform instruction.							
effective use of fair-sharing activities to help students understand key fraction concepts.							
effective use of measurement and number line activities to help children understand key fraction concepts.							

effective use of fair-sharing and measurement activities to help students understand computations with fractions.							
addressing student learning differences when teaching fraction content and skills.							

8. *Question asked for participants in a different MOOC-Ed and not asked for participants in the Fraction Foundations MOOC-Ed.*

9. *Question asked for participants in a different MOOC-Ed and not asked for participants in the Fraction Foundations MOOC-Ed*.

10. *Question asked for participants in a different MOOC-Ed and not asked for participants in the Fraction Foundations MOOC-Ed*.

11. Overall, how effective do you feel this MOOC-Ed was in preparing you to make positive changes in your professional practice?

- ☐ Very Ineffective
- ☐ Ineffective
- ☐ Somewhat Ineffective
- ☐ Neutral

- ☐ Somewhat Effective
- ☐ Effective
- ☐ Very Effective

12. Have you attempted to make changes in your own practice because of your participation in the MOOC-Ed?

- ☐ Yes
- ☐ No

13. Please describe any changes you have made to your practice, including how you have applied knowledge, skills, and/or resources you gained in this course.

14. In what way, if any, do you anticipate applying the knowledge, skills, and/or resources you acquired from this course to your professional practice?

15. To what extent have you attempted to make changes in each of the following areas of practice as a result of participating in the MOOC-Ed?

	No Change	Minor Change	Moderate Change	Significant Change	Not Applicable
Mathematics curriculum content					
Mathematical language I use					
Collaboration on instructional planning					

Approaches to assessing students' mathematical proficiency					
Approaches to address the diversity of student mathematical proficiency					

16. When introducing changes in your practice based on your experience in the MOOC-Ed, to what extent have any of the following issues arisen?

	Not at all	Small extent	Moderate extent	Fairly great extent	Great extent
Insufficient planning time					
Resistance from other teachers					
Resistance from administrators					
Curricular or pacing guide constraints					
Other (please describe)					

17. To what extent do you agree with the following statements?

This online professional development experience…

	Strongly Disagree or Disagree	Somewhat Disagree	Neutral	Somewhat Agree	Agree or Strongly Agree
was enhanced by the expertise of practitioners and/or leaders in the field.					
provided models of effective practice (e.g. demonstrations, tools, resources) to support the application of course content.					
provided projects or activities that can be embedded in my professional practice.					
provided meaningful opportunities to share ideas, resources, and experiences.					
promoted critical-thinking and/or problem-solving through discussions and activities.					
provided supports for small-group work or collaborations around shared goals.					
enabled me to personalize my learning through differentiated resources and activities.					
provided opportunities to investigate self-identified problems or areas of interests.					

provided flexibility in the ways in which I could demonstrate my learning.					
provided activities and/or resources to help me gauge my learning (e.g. self-assessments, peer assessment or feedback)					
clearly communicated the purpose, goals, and expectations of this course					

18. What additional course features, activities, and/or resources would have helped you utilize this MOOC-Ed more effectively?

19. Were you able to complete all of the activities that you wanted to complete in this course?

- ☐ Yes
- ☐ No

20. On average, how many hours per unit did you spend on the MOOC-Ed activities?

- ☐ 1-2 hours per unit
- ☐ 3-4 hours per unit
- ☐ 5-6 hours per unit
- ☐ 7-8 hours per unit
- ☐ more than 8 hours per unit

21. Have you discussed or shared what you learned in this MOOC-Ed with the following:

	Yes	No	Not

			Applicable
Other teachers in your school or department who participated in the MOOC-Ed			
Other teachers in your school or department who did not participate in the MOOC-Ed			
Administrators (e.g., principal or department chair)			
Participants who teach in other schools outside of formal meetings held as part of the MOOC-Ed			
Other (please describe)			

22. MOOC-Eds are often used as part of a professional development blended learning model. Which of the following best describes any peer groups with which you participated and/or collaborated outside of this MOOC-Ed?

- ☐ School or district-based professional learning team
- ☐ Online professional learning network (e.g., Twitter, Facebook group)
- ☐ Informal group of colleagues
- ☐ Other (please describe)
- ☐ I did not participate and/or collaborate with anyone outside of the MOOC--Ed

Appendix B: Semi-Structured Interview and Observation Protocols

Interview Protocol

Pre-Observation Interview

I appreciate your letting me observe your class. I have some questions I'd like to ask you related to this lesson. Would you mind if I taped the interview? It will help me stay focused on our conversation, and it will ensure I have an accurate record of what we discussed. Please know, however, that your responses are kept confidential, and any reference to them will be made strictly anonymously.

1. How long have you been teaching? What are you teaching this year? Describe the students in your classrooms (e.g., ability levels, behavioral characteristics, demographics)

2. How long have you been teaching this lesson? (*Lesson plan will be provided to interviewer by the teacher.)*
   If taught before, ask: How have you taught this lesson in the past?

3. How would you describe effective/quality PD? Describe a PD experience that was particularly effective. Describe one that was not.

4. In what ways do you feel that quality PD is readily available to you? What are the reasons it is or isn't?
   If participant doesn't mention PLC, ask: Do you work with a PLC? If so, what, if any, are typical activities of your PLC? How has it affected your professional growth?

5. How would you describe "success" in your anticipated participation in this MOOC-Ed?

Interview Protocol (during/post MOOC-Ed)

A. Goals of PD/Impact on Practice

1. In what ways do/did you find the MOOC to be effective/ineffective? Include both affordances and constraints.

2. *To be asked after the course has concluded:* Are you engaging in any follow-up PD activities or learning to continue your progress after this course?

3. How would you describe "success" in your participation in this MOOC-Ed now that you've participated?

4. What progress have you made toward the goals that you outlined in the MOOC? (Show original goals from registration).

5. What aspects of the MOOC have you found to inform or affect your practice? Specifically, how would you describe that impact?
    a. Have you applied something you learned from the MOOC to your practice? If so, how?

    b. What do you plan to do that applies what you learned in this MOOC?

    c. What materials will you use, or have you used, from the MOOC in your practice? Why did you choose to use them?

    If participant does not specifically address the following or needs further probing, ask the following questions

    d. In what ways has the MOOC affected the way you address students' strategies?

    e. In what ways has the MOOC affected your approach to assessing students' mathematical proficiency?

    f. In what ways has the MOOC affected the mathematical language you use?

    g. In what ways has the MOOC affected the way you think about student learning?

B. Student Learning Goals

6. Where does this lesson fit in the sequence of the unit you are working on? What have the students experienced prior to today's lesson?

7. What is/are the specific purpose/learning goals in today's lesson?

8. What, if any, elements of the MOOC did you use to help you design the lesson?
   If additional probing is needed, ask: How did you design/adapt the tasks you'll be implementing in the lesson? How did the MOOC contribute to your design/adaptation?

9. What is the next step for this class in this unit?

Post-Observation Interview

1. How do you think the lesson went today?
   a. What do you think went particularly well?
   b. What would you do differently?

2. How was it different from how you've taught it previously?

3. Is there anything you can attribute to the MOOC as having made a difference?
   If participant doesn't provide specific parts of the lesson, ask: Can you point to a specific during your lesson when you used knowledge, insights, or skills you gained from the MOOC?

4. I noticed during the lesson that you did (…). Could you explain what you were thinking at the time?

5. How do you think student thinking influenced/influences your instruction?

6. Is there anything else you'd like to share about today's lesson, your experience in the MOOC, etc.?

Observation Protocol (for pre, during and post-MOOC-Ed)[28]

Pre-Observation Data

Teacher's Name: ________________________ Date: _____________________

Setting of observation: _________________________ Grade: __________________

Time of observation: __________________________

Number of students in classroom: ________________________

Topic(s) discussed/covered: ____

Purpose (objectives) of class as defined by teacher: ____

Section I: Classroom Culture

Part I: Teacher-Student Interaction	SD	D	N	A	SA	NA[29]
1. The teacher actively interacted with and was responsive to students. • responded verbally or nonverbally to students' questions or request for attention • initiated interaction with students about their activity						
Evidence/Notes:						
2. The teacher used moments as they occurred to develop mathematical ideas.						

[28] Adapted from Classroom Observation (COEMET), Version 4 (© Julie Sarama and Douglas H. Clements) Modified Feb 9, 2010

[29] SD = Strongly disagree; D = Disagree; N = Neither agree nor disagree; A = Agree; SA = Strongly agree; NA = Not applicable

• explored mathematical ideas as they occurred, even if they appeared beyond usual expectations • took opportunity to discuss and build upon students' incorrect but understandable responses						
Evidence/Notes:						
Part II: Teacher Attributes						
3. The teacher appeared to be knowledgeable and confident about mathematics. • demonstrated accurate knowledge of mathematical ideas and procedures • demonstrated knowledge of connections between or sequences of mathematical ideas						
Evidence/Notes:						
4. The teacher showed curiosity about and/or enthusiasm for math ideas and/or connections to other ideas or real-world situations. • commented on or discussed mathematical ideas in reading a story • showed interest in the mathematics that emerged in students' play, construction, or discussions • asked students how the math ideas connect to their real lives						
Evidence/Notes:						

Section II: Mathematical Activity

Part I: Mathematical Focus	SD	D	N	A	SA	NA
5. The teacher displayed an understanding of mathematics concepts.						
Evidence/Notes:						
6. The mathematical content was appropriate for the developmental levels of the students in this class (as described by the teacher). • used tasks at level of difficulty consistent with students' level of thinking and learning • used tasks in sequence corresponding to students' growing level of thinking						
Evidence/Notes:						
Part II: Organization, Teaching Approaches, Interactions						
Check organization(s) used: ☐ Whole Group ☐ Individual Activity with Teacher ☐ Small Group (Teacher) ☐ Centers (Structured) ☐ Game ☐ Play (Unstructured) ☐ Other: Describe organization of activity if not listed, such as**,** transition, etc.						
7. The teacher began by engaging and focusing students' *mathematical thinking*. • directed students' attention to, invited them to consider, a mathematical question, problem, or idea						
Evidence/Notes:						
8. The *pace* of the activity was appropriate for the developmental levels/needs of the students and the purposes of the activity.						

Evidence/Notes:						
9. The teacher's management strategies enhanced the quality of the activity. • prepared materials ahead of time • organized students effectively • orchestrated interaction to maintain students' involvement						
Evidence/Notes:						
10. The teacher was actively involved in the activity for what percentage of time (*beyond* setup or introduction)?						
Evidence/Notes:						
11. The *teaching strategies* used were appropriate for the development levels/needs of the students and purposes of the activity. • strategies matched instructional goals • strategies provided appropriate level of support • strategies maintained all students' engagement with the mathematical ideas						
Evidence/Notes:						
Part III: Expectations						
12. The teacher had high but realistic mathematical expectations of students. • the teacher asked all students to try to solve problems and attempt various solution strategies. • the teacher gave students opportunities to engage in mathematical practices.						
Evidence/Notes:						
13. The teacher acknowledged and/or reinforced students' effort, persistence, and/or concentration. • recognized students' actions, implicitly or explicitly, verbally or nonverbally						

Evidence/Notes:						
Part IV: Eliciting students' solution methods						
14. The teacher asked students to share, clarify, and/or justify their ideas. • used a range of question types to probe and challenge students' thinking • encouraged students to explain their mathematical thinking; e.g., asked "why?", "how did you…?", "could you…?" questions.						
Evidence/Notes:						
15. The teacher facilitated students' responding. • elicited many solution methods for one problem • encouraged elaboration of students' responses • waited for and listened attentively to individual students • responded to errors as learning opportunities						
Evidence/Notes:						
16. The teacher encouraged students to listen to and evaluate others' thinking/ideas. • actively elicited communication between students • stated and reinforced the expectation that students would listen to each other						
Evidence/Notes:						
Part V: Supporting students' conceptual understanding						
17. The teacher supported the describer's thinking. • reminded students of conceptually similar problem situations • provided background knowledge • directed group help for an individual student • assisted individual students in clarifying their own solution methods						
Evidence/Notes:						

18. The teacher supported the listener's understanding. • asked a different student to explain a peer's method • encouraged the student to put the explanation in their own words or provide an alternate explanation						
Evidence/Notes:						
19. The teacher provided "just enough" support. • facilitated students' actions at appropriate level, providing adequate (not too little) but not too much (e.g., doing the task for the student) help or information						
Evidence/Notes:						
20. The teacher selected student work to discuss and based the sequence of the presentation on paths along progressively more sophisticated strategies.[30]						
Evidence/Notes:						
21. The teacher anticipated and/or addressed students' misconceptions.[31]						
Evidence/Notes:						
22. The teacher emphasized and presented choices of representations. [32] • The teacher drew out and dealt with students' representations.						
Evidence/Notes:						
Part VI: Extending students' mathematical thinking						
23. The teacher built on and/or elaborated students' mathematical ideas and strategies.						

[30] Sztajn et al. (2012)
[31] Sztajn et al. (2012)
[32] Sztajn et al. (2012)

• re-described students' ideas and strategies, adding mathematics content and vocabulary						
Evidence/Notes:						
24. The teacher encouraged mathematical reflection. • drew out key math ideas during and/or towards the end of the activity • helped students make connections to math ideas from other activities and/or real-life experiences						
Evidence/Notes:						
25. The teacher observed and listened to students, taking notes as appropriate (only need notes in small groups)						
Evidence/Notes:						
26. The teacher adapted tasks and discussions to accommodate the range of students' abilities and development.						
Evidence/Notes:						

Additional Field Notes

Based on time spent, the focus of this lesson is best described as:

Teaching aids/materials used in lesson:

Instructional strategies used:

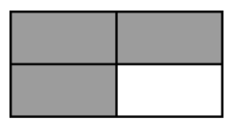
Assessment strategies used:

Appendix C: Pre-Post MKT Assessment

1. During a unit on fractions, Ms. Hendrickson introduced the following definition for fraction.

> We will write $\frac{1}{d}$ for one of the parts where some whole is divided into *d* equal parts.
>
> Then, $d \bullet (\frac{1}{d}) = 1$, and $\frac{n}{d}$ is then defined as $n \bullet (\frac{1}{d})$, or *n* copies of $\frac{1}{d}$.

She then asked students to *use this definition* to explain what fraction of the figure below is shaded.



Students gave several explanations. Of the following, which most clearly uses the proposed definition to explain what fraction of the figure is shaded? (Circle ONE answer.)

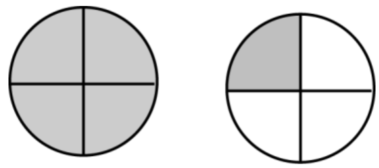
a) The big rectangle is divided into 4 equal pieces. So that means that 3 out of 4 pieces, or $\frac{3}{4}$ of the big rectangle, is shaded.

b) The small rectangle is $\frac{1}{4}$ of the big rectangle because there are four equal small rectangles in the big one. Three small rectangles are shaded, so that means $\frac{3}{4}$ of the big rectangle is shaded.

c) There are four equal pieces in the whole. One of those pieces, or $\frac{1}{4}$, is not shaded. Because the whole is $\frac{4}{4}$, that means that the remaining shaded part must be $\frac{3}{4}$ of the whole.

d) All of the explanations make direct use of the proposed definition.

2. Mrs. Johnson thinks it is important to vary the whole when she teaches fractions. For example, she might use five dollars to be the whole, or ten students, or a single rectangle. On one particular day, she uses as the whole a picture of two pizzas. What fraction of the two pizzas is she illustrating below? (Mark ONE answer.)



a) 5/4

b) 5/3

c) 5/8

d) 1/4

3. Ms. West's class was working on simplifying fractions. She had taught them to simplify fractions by dividing the numerator and the denominator by a common factor. One of her students asked, "Why doesn't the fraction's value get smaller when we divide the numerator and the denominator by the same number?" Below are responses to the question from other students in this class.

Although none of these is a complete explanation, which of the following provides the best evidence that the student understands why simplifying a fraction produces an equivalent fraction? (Circle ONE answer.)

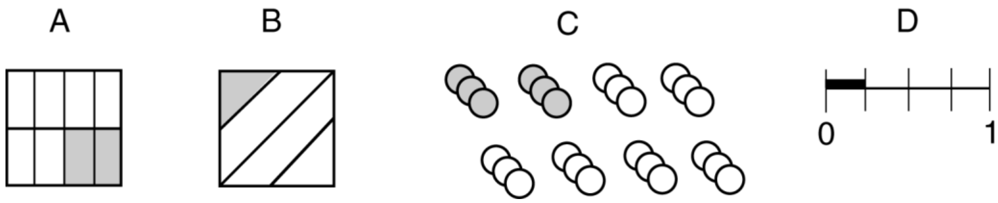
a) This works because you divide the top and bottom by the same number, so the new fraction has to be the same amount.

b) This works because you are really just dividing the fraction by 1, so the new fraction is the same amount.

c) This works because you are making the numerator and denominator smaller by the same amount.

d) This works because, for example, $\frac{3}{4}$ is the same amount as $\frac{12}{16}$, only with smaller numbers.

4. Mr. Ives is using a new mathematics text to teach fractions this year. The textbook recommends having students create their own representations for fractions. Here are several student representations of $\frac{1}{4}$:



Which representation(s) correctly show 1/4? (Mark YES, NO, or I'M NOT SURE for each choice.)

	Yes	No	I'm not sure
a) A	1	2	3
b) B	1	2	3
c) C	1	2	3
d) D	1	2	3

5. Three children were working together on the following problem:

Geoffrey has 3 dozen pencils. He wants to distribute as many as possible equally among 5 containers. Does he have enough to put 6 pencils in each container?

"That's easy," says Eric. "We just have to multiply 3 times 12, which is 36. Then divide 36 by 5. That's $7\frac{1}{5}$. So, yes, he would have enough."

"You are making too much work, Eric," replies Heather. "You can see right away that he has enough, because the problem is 3 dozen divided by 5, which is $\frac{3}{5}$ of a dozen. Then all we have to do is figure out whether $\frac{3}{5}$ of a dozen is more or less than 6 pencils. And since $\frac{3}{5}$ is more than one half, he has enough, because 6 is half of twelve."

Delena jumps in, "Well $\frac{3}{5}$ of a dozen is obviously less than 6 pencils, because 3 is less than 6, and so is 5."

Which student's response shows the best understanding of the meaning of fractions? (Circle ONE answer.)

a) Eric

b) Heather

c) Delena

d) They all understand the meaning of fractions equally well.

6. One of Ms. Hines' students was comparing $\frac{2}{5}$ and $\frac{3}{7}$ and made the following argument:

$\frac{2}{5}$ is one-half of a fifth less than $\frac{1}{2}$ and $\frac{3}{7}$ is one-half of a seventh less than $\frac{1}{2}$, and half a fifth is bigger than half a seventh, so I conclude that $\frac{2}{5}$ is bigger than $\frac{3}{7}$.

What is the best assessment of the student's argument? (Circle ONE answer.)

a) The student has a clever idea, but it does not work for these particular fractions.

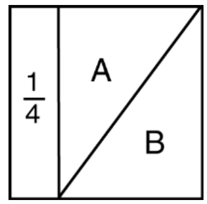
b) The student has a clever idea, but drew the wrong conclusion.

c) The student does not understand how to compare fractions, because to compare fractions you need to find a common denominator.

d) The student does not understand $\frac{1}{2}$ because the student tries to think about one-half of one-seventh and one-half of one-fifth, which do not make sense.

7. Mr. Sanchez attended a district-sponsored professional development session at which teachers were working on the following problem:

Given that the rectangular strip is 1/4 of the total square, what fractional part of the square is the triangle marked A?



An observer sitting next to Mr. Sanchez was confused about how to do the problem, and asked what Mr. Sanchez had found. What should he respond? (Mark ONE answer.)

a) 1/2

b) 1/3

c) 3/8

d) 3/2

8. A group of Ms. Lester's students was following a set of directions to move a paper frog along a number line. Their last direction took them to $\frac{1}{2}$. The next direction says:

Go $\frac{1}{3}$ of the way to $\frac{3}{4}$. What number will the frog land on?

The students disagreed about where the frog would land. Which answer should Ms. Lester accept as correct? (Circle ONE answer.)

a) $\frac{1}{12}$

b) $\frac{2}{3}$

c) $\frac{7}{12}$

d) $\frac{5}{6}$

e) $\frac{1}{4}$

9. Mr. Nessbaum was teaching division with fractions to his class. “If we purchase 8 big chocolate bars from the school candy sale,” he said, “and we want everyone in the class to have at least $\frac{2}{5}$ of a chocolate bar, do we have enough for our 25 students?” Mr. Nessbaum expected his students to write 8 divided by $\frac{2}{5}$ = 20. Instead the students came up with several different approaches. Which of the students’ approaches is valid? (Mark APPROACH IS VALID, APPROACH IS NOT VALID, or I’M NOT SURE for each.)

	Approach is valid	Approach is not valid	I’m not sure
a) After I cut each bar into five pieces, there are 40 pieces and everybody gets two. 40 divided by 2 equals 20, so we don’t have enough.	1	2	3
b) There are 25 students. 25 times $\frac{2}{5}$ is 10 so we don’t have enough.	1	2	3
c) You have to add $\frac{2}{5}$ twenty times to equal 8 whole chocolate bars. But that is not 25 times, so there are not enough pieces for everyone in our class.	1	2	3
d) 8 bars divided by 2 is 4 which is less than 5, so each student can’t get $\frac{2}{5}$ of a bar each.	1	2	3
e) There aren’t enough because if I start with 8 and keep subtracting $\frac{2}{5}$, I get to zero before I have subtracted it 25 times.	1	2	3

10. Mr. Grimes asked his students what they knew about finding the lowest common denominator (LCD) in order to add fractions. His students came up with a variety of ideas. Which of the following is true? (Mark ONE answer.)

a) You cannot add fractions without first finding the LCD.

b) If you use the LCD, you will never have to simplify your answer.

c) You can always find the LCD by multiplying the two denominators.

d) None of these is true.

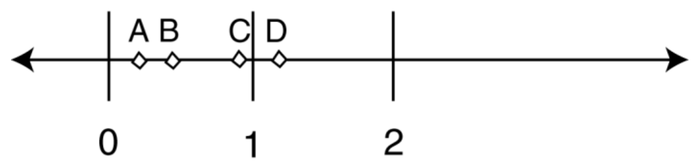
e) I'm not sure.

11. Mrs. Wise wants to include some word problems on her fractions quiz. Which of the following problem(s) could she use as a word problem for $\frac{1}{2} - \frac{1}{3}$? (Mark YES, NO, or I'M NOT SURE for each one.)

	Yes	No	I'm not sure
a) I have 1/2 of a pizza left. My brother comes in and eats 1/3 of my leftover pizza. How much pizza is left?	1	2	3
b) Farmer Brown has plowed up $\frac{1}{2}$ of a field. He wants to plant $\frac{1}{3}$ of that half in corn. What fraction of the entire field will be planted in corn?	1	2	3
c) Mom has $\frac{1}{2}$ of a cup of sugar. She needs to use $\frac{1}{3}$ of a cup of sugar to make some brownies. How much sugar will Mom have left?	1	2	3

12. Mr. Stone is looking through his mathematics materials for problems relating fractions to number lines. He comes across the following problem:

Which point is closest to $\frac{7}{16}$ x $\frac{1}{2}$?



He has not used a number line for this kind of problem before and he wants to make sure he is using it correctly. Which of the following is the intended answer to this problem? (Circle ONE answer.)

a) A

b) B

c) C

d) D

13. Ms. Lawrence is making up word problems for her students. She wants to write a word problem for $3 \div \frac{1}{2}$. Which of the following word problem(s) can she include? (Circle YES, NO, or I'M NOT SURE for each.)

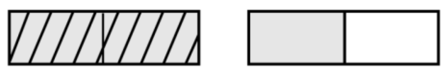
	Yes	No	I'm not sure
a) Melissa has 3 pizzas and she wants to give half of them to her friend. How much pizza will her friend get?	1	2	3
b) Dan has 3 cups of chocolate chips. He wants to bake cookies, and each batch requires $\frac{1}{2}$ cup of chocolate chips. How many batches of cookies can Dan make if he uses all of the chocolate chips?	1	2	3
c) Three friends each have half of a cookie. How many cookies would they have if they put them all together?	1	2	3

d) Jacquie has 3 dollars. If she has half the amount needed to purchase a discount phone card, how much does a phone card cost?	1	2	3

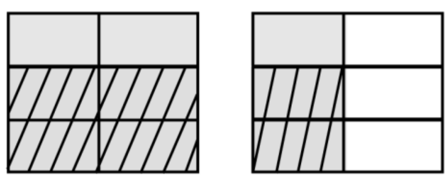
14. At a professional development workshop, teachers were learning about different ways to represent multiplication of fractions problems. The leader also helped them to become aware of examples that do <u>not</u> represent multiplication of fractions appropriately.

Which model below <u>cannot</u> be used to show that $1\frac{1}{2}$ x $\frac{2}{3} = 1$? (Mark ONE answer.)

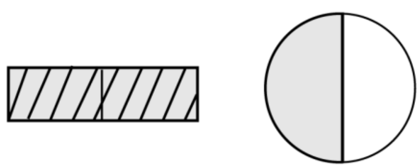
A)



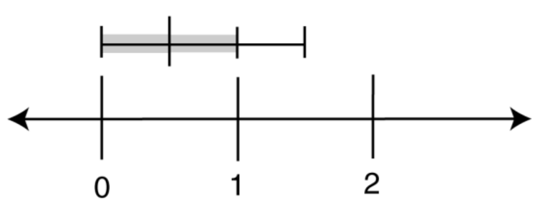
B)



C)



D)



15. Ms. Peterson's class was working on understanding the meaning of division using fractions. She asked her students to interpret $\frac{1}{6} \div \frac{2}{3}$.

For each of the questions below, decide whether it can be a mathematically valid interpretation of $\frac{1}{6} \div \frac{2}{3}$. (Circle VALID INTERPRETATION, NOT A VALID INTERPRETATION, or I'M NOT SURE for each.)

	Valid interpretation	NOT a valid interpretation	I'm not sure
a) How many $\frac{2}{3}$s are in $\frac{1}{6}$?	1	2	3
b) How much is $\frac{1}{6}$ of $\frac{2}{3}$?	1	2	3
c) What number is $\frac{1}{6}$ two-thirds of?	1	2	3
d) How many $\frac{2}{3}$s can you subtract from $\frac{1}{6}$ before reaching 0?	1	2	3

16. Mr. Lewis asked his students to divide $\frac{6}{8}$ by $\frac{1}{2}$. Charlie said, "I have an easy method, Mr. Lewis. I just divide numerators and denominators. I get $\frac{6}{4}$, which is correct." Mr. Lewis was not surprised by this as he had seen students do this before. What did he know? (Circle ONE answer.)

a) He knew that Charlie's <u>method</u> was wrong, even though he happened to get the right <u>answer</u> for this problem.

b) He knew that Charlie's <u>answer</u> was actually wrong.

c) He knew that Charlie's <u>method</u> was right, but that for many numbers this would produce a messy <u>answer</u>.

d) He knew that Charlie's <u>method</u> only works for some fractions.

e) I'm not sure.

Appendix D: Student-Parent and Interview/Observation Consent Forms

North Carolina State University

INFORMED CONSENT FORM for RESEARCH (Student/Parent- Classroom observation)

This consent information is valid May 13, 2014 through May 13, 2015

Title of Study: The Friday Institute MOOC Education Project

Principal Investigator: Glenn Kleiman Faculty Sponsor (if applicable): N/A

What are some general things you should know about research studies?
Your child's teacher has agreed to participate in our study. Your child's participation in this study is voluntary: they have the right to be a part of this study, to choose not to participate, or to stop participating at any time. The purpose of research studies is to gain a better understanding of a certain topic or issue. Your child is not guaranteed any personal benefits from being in a study. Some research studies also may pose risks to those that participate. In this consent form you will find specific details about the research in which your child is being asked to participate. If you do not understand something in this form it is your right to ask the researcher for clarification or more information. A copy of this consent form will be provided to you. If at any time you or your child have questions about your child's participation, do not hesitate to contact the researcher(s) named above.

What is the purpose of this study?
This research project aims to support across a broad cross-section of the education population for interpreting the Common Core State Standards (CCSS). To attain this aim we will examine how participants use the MOOC-Ed course and its resources in implementing the CCSS.

What will happen if you or your child are involved in the study?
Our research team will conduct classroom observations that will be video recorded. Teachers will be encouraged to conduct their classes as normal as possible with few, if any, interruptions caused by the project. During videos, students will be addressed by first name only, and no other identifying information will be recorded. If names are needed to describe students' or participants' contributions, pseudonyms will be used. If you do not consent to your child being video-recorded, they will be placed in the classroom out of view of the video camera, maintaining continuity of instruction. Exact duration will vary, depending on the instructional plan. Each classroom observation will typically last 1 to 2 hours, and, if schedules permit, teachers will be observed on more than one occasion.

Risks
We do not foresee any risks to this research for your child. None of the students' work on this project will have any consequence for their regular classes or grades. Assessment items they engage with will have no bearing on their grades. The school principal or other responsible administrator has approved student participation in this research, as well.

Benefits
There are not necessarily any direct benefits for your child from participating in this research. However, there are potential indirect benefits. Your student will be part of research that, we hope, will directly benefit future students and teachers in learning and teaching mathematics, by supporting better teaching and assessing that gives important feedback on student progress and helping teachers improve their teaching of mathematics for each student.

Confidentiality
The information in the study records will be kept strictly confidential. The research team will retain all digital data and non-digital work products the subjects generate (drawing, paper-and-pencil solutions/attempts, and other artifacts). All data (including video recordings) will be stored securely in our offices and on secure servers in the Friday Institute. With parental consent (signature below) particular clips from the video recordings may be shared with people beyond our project team (for instance, professional meetings and teacher professional development classes (held face to face or online) or materials for teachers.). Project reports may include individual student responses. Children will not be identified by name in such presentations or materials, or in any written or oral reports of the research.

Compensation
None.

What if you have questions about this study?
If you have questions at any time about the study or the procedures, you may contact the directors of the research, Dr. Glenn Kleiman (919.513.8509, glenn_kleiman@ncsu.edu) at the Friday Institute for Educational Innovation, North Carolina State University, 1890 Main Campus Road, Campus Box 7249, Raleigh, NC 27606.

What if you have questions about your rights as a research participant?
If you feel you have not been treated according to the descriptions in this form, or your rights as a participant in research have been violated during the course of this project, you may contact Deb Paxton, Regulatory Compliance Administrator, Box 7514, NCSU Campus (919/515-4514).

Consent To Participate
I have read and understand the above information. I have received a copy of this form. I agree to participate in this study with the understanding that I may withdraw at any time.

[Parent or guardian]: I consent to my child's participation in this study, including videotaping of my child for research purposes. I affirm that I have discussed, with my child, my child's role in this study. I affirm that my child understands this, and has agreed to participate.

___________________________	______________________________
Student Signature (Grades 6-8 only)	Student Name (please print)

____
Grade

______________________________	______________________________
Parent/Guardian Signature	Parent/Guardian Name (please print)

____
Date

North Carolina State University

INFORMED CONSENT FORM for RESEARCH (MOOC-Ed Observations and Personal Interviews)

This consent information is valid May 13, 2014 through May 13, 2015

Title of Study: The Friday Institute MOOC Education Project

Principal Investigator: Glenn Kleiman Faculty Sponsor (if applicable): N/A

What are some general things you should know about research studies?

You are being asked to take part in a research study. Your participation in this study is voluntary. You have the right to be a part of this study, to choose not to participate or to stop participating at any time without penalty. The purpose of research studies is to gain a better understanding of a certain topic or issue. You are not guaranteed any personal benefits from being in a study. Research studies also may pose risks to those that participate. In this consent form you will find specific details about the research in which you are being asked to participate. If you do not understand something in this form it is your right to ask the researcher for clarification or more information. A copy of this consent form will be provided to you. If at any time you have questions about your participation, do not hesitate to contact the researcher(s) named above.

What is the purpose of this study?

This research project aims to support across a broad cross-section of the education population for interpreting the Common Core State Standards (CCSS). To attain this aim we will examine how participants use the MOOC-Ed course and its resources in implementing the CCSS.

What will happen if you take part in the study?

If you agree to participate in this component of the study, you will be invited to take part in personal interviews and/or observations. If you choose to participate, you will communicate with the research team to set up the date, time, and location of the observations and interviews. You have the right to withdraw from the study at any time. Sessions will be video and audio-recorded, and the recordings will be collected and retained for analysis. The results of the research, including video excerpts or conversational exchanges, may be used in research presentations, publications, and professional development presentations by the research team (including webinars, courses (both face to face or online), or presentations that may be made available online, provided as professional development activities or made available to other professional development providers).

Interviews will be conducted as open-ended, seeded with semi-structured questions. You will be asked about various issues, including your perceptions of the MOOC-Ed course experience and your evaluation of how you will use what you learned in your practice. Exact duration will vary, depending on the format of interactions. Each classroom observation will typically last 1 to 2 hours, and personal interviews will each last approximately 1 hour.

Risks

As for any discussion among professionals, participants could conceivably experience discomfort or uncertainty relating to topics or questions raised. This however is no larger a risk to any individual than any routine online or personal discussion such professionals could encounter in their daily professional lives, so does not represent any risk particular or unique to this project. There may be professional risk to you from the observations if confidentiality were inadvertently breached, as your teaching behavior will be recorded and evaluated.

Benefits

No specific direct benefits are predicted from participation in the IRB-approved research. However, consistent with the intent of The Friday Institute MOOC Education Project, you may gain your own insight into particular mathematical tasks, strategies, and knowledge that you would not have gained in typical professional development courses, may become more engaged with mathematics as a topic of interest, and may broaden your understanding of student learning and mathematical instruction, become more observant of mathematical activity in the classroom, and become more confident in your educational practice.

Confidentiality

Records of observations and interviews will be digitally recorded in both video and audio format. They will be stored securely on Friday Institute or NCSU-contracted servers. Data will be accessible to project staff, videography staff, and NCSU technical support staff only.

Compensation

You will not receive anything for participating.

What if you are a NCSU student?

Participation in this study is not a course requirement and your participation or lack thereof, will not affect your class standing or grades at NC State.

What if you are a NCSU employee?

Participation in this study is not a requirement of your employment at NCSU, and your participation or lack thereof, will not affect your job.

What if you have questions about this study?

If you have questions at any time about the study or the procedures, you may contact the directors of the research, Dr. Glenn Kleiman (919.513.8509, glenn_kleiman@ncsu.edu) at the Friday Institute for Educational Innovation, North Carolina State University, 1890 Main Campus Road, Campus Box 7249, Raleigh, NC 27606.

<u>What if you have questions about your rights as a research participant?</u>
If you feel you have not been treated according to the descriptions in this form, or your rights as a participant in research have been violated during the course of this project, you may contact Deb Paxton, Regulatory Compliance Administrator, Box 7514, NCSU Campus (919/515-4514).

<u>Consent To Participate</u>
I have read and understand the above information. I have received a copy of this form. I agree to participate in this study with the understanding that I may choose not to participate or to stop participating at any time without penalty or loss of benefits to which I am otherwise entitled.

____
Signature   Name (please print)

____
Date

Appendix E: Correlation Analysis Tables

FFI	Q3: Effectiveness in supporting goals	Q7_1: Knowledge of fraction content standards	Q7_2: Knowledge of student difficulties and misconceptions	Q7_3: Knowledge of analysis of student thinking about fractions	Q7_4: Knowledge of use of fair-sharing activities for teaching fractions	Q7_5: Knowledge of use of measurement and number line activities for teaching fractions	Q7_6: Knowledge of use of activities for students to learn computations of fractions	Q7_7: Knowledge of student learning differences	Q11: Impact on practice
Q7_6: Knowledge of use of activities for students to learn computations of fractions	0.633704	0.615757	0.580837	0.661602	0.754567	0.676468	1	0.49533	0.633776
Q11: Impact on practice	0.674693	0.437727	0.524513	0.53735	0.566335	0.682433	0.633776	0.484328	1
Q12: Attempts to make changes to practice	0.281972	0.179145	0.103256	0.133892	0.172969	0.181863	0.114077	0.2426	0.336002
Q17_1: MOOC-Ed was enhanced by expertise of others.	0.480857	0.413476	0.365122	0.29391	0.347428	0.353057	0.306993	0.465264	0.364651
Q17_2: MOOC-Ed provided models of effective	0.508332	0.465123	0.491496	0.429052	0.515255	0.486444	0.441687	0.466374	0.52687

practice.									
Q17_3: MOOC-Ed provided projects or activities to embed into practice.	0.528782	0.346957	0.503355	0.567041	0.525319	0.504728	0.478458	0.590617	0.669672
Q17_4: MOOC-Ed provided meaningful opportunities to share ideas, resources and experiences.	0.38499	0.236195	0.299364	0.345152	0.389098	0.423896	0.273791	0.499602	0.529618
Q17_5: MOOC-Ed promoted critical-thinking and/or problem-solving	0.415555	0.348535	0.423714	0.523853	0.472559	0.556302	0.414676	0.594311	0.619682
Q17_6: MOOC-Ed provided supports for collaborative group work.	0.289562	0.504252	0.227533	0.354251	0.234012	0.356048	0.380493	0.48263	0.336087
Q17_7: MOOC-Ed enabled personalization of learning	0.391788	0.190943	0.243684	0.422844	0.402686	0.375419	0.33021	0.580503	0.471003

Q17_8: MOOC-Ed provided opportunities to investigate self-identified problems or areas of interests.	0.359395	0.371301	0.444305	0.391885	0.477058	0.226544	0.445556	0.465904	0.364547
Q17_9: MOOC-Ed allowed for valuable flexibility for demonstration of learning.	0.460129	0.455351	0.304705	0.214228	0.404608	0.279466	0.365468	0.471891	0.379535
Q17_10: MOOC-Ed provided activities and/or resources to help me gauge my learning	0.415844	0.443351	0.432062	0.368153	0.468971	0.400351	0.420546	0.520801	0.408269
Q17_11: MOOC-Ed clearly communicated purpose, goals and expectations of course.	0.547782	0.316495	0.278079	0.277315	0.425554	0.512052	0.409312	0.285177	0.557468

FFII	Q3: Effectiveness in supporting goals	Q7_1: Knowledge of fraction content standards	Q7_2: Knowledge of student difficulties and misconceptions	Q7_3: Knowledge of analysis of student thinking about fractions	Q7_4: Knowledge of use of fair-sharing activities for teaching fractions	Q7_5: Knowledge of use of measurement and number line activities for teaching fractions	Q7_6: Knowledge of use of activities for students to learn computations of fractions	Q7_7: Knowledge of student learning differences	Q11: Impact on practice
Q11: Impact on practice	0.624288	0.650066	0.625997	0.599859	0.641297	0.584285	0.614157	0.569679	1
Q12: Attempts to make changes to practice	0.129164	0.346486	0.313574	0.272853	0.213234	0.26225	0.274441	0.357158	0.262932
Q17_1: MOOC-Ed was enhanced by expertise of others.	0.47916	0.487037	0.519449	0.44794	0.497308	0.440903	0.488021	0.455701	0.608853
Q17_2: MOOC-Ed provided models of effective practice.	0.587072	0.529796	0.546763	0.562933	0.657479	0.648342	0.654453	0.581963	0.69957
Q17_3: MOOC-Ed provided projects or activities to embed into practice.	0.557027	0.542207	0.605107	0.60983	0.615644	0.609277	0.635494	0.554933	0.623218

Q17_4: MOOC-Ed provided meaningful opportunities to share ideas, resources and experiences.	0.497483	0.513365	0.554983	0.494656	0.576956	0.536248	0.557625	0.514939	0.569264
Q17_5: MOOC-Ed promoted critical-thinking and/or problem-solving	0.566094	0.537912	0.586079	0.646732	0.628376	0.591706	0.645659	0.56121	0.702671
Q17_6: MOOC-Ed provided supports for collaborative group work.	0.425233	0.533552	0.463023	0.545348	0.559002	0.481636	0.511985	0.516242	0.539377
Q17_7: MOOC-Ed enabled personalization of learning	0.489683	0.511604	0.581074	0.619243	0.60183	0.580549	0.551509	0.569808	0.60912
Q17_8: MOOC-Ed provided opportunities to investigate self-identified problems or areas of	0.480951	0.492325	0.576779	0.599502	0.590517	0.544326	0.582434	0.587869	0.563635

interests.									
Q17_9: MOOC-Ed allowed for valuable flexibility for demonstration of learning.	0.525724	0.588682	0.550697	0.528834	0.549659	0.532649	0.533876	0.54142	0.584418
Q17_10: MOOC-Ed provided activities and/or resources to help me gauge my learning	0.517229	0.572381	0.563998	0.553683	0.589622	0.571648	0.562243	0.521941	0.637479
Q17_11: MOOC-Ed clearly communicated purpose, goals and expectations of course.	0.445429	0.517125	0.500826	0.517673	0.589256	0.563583	0.524088	0.514624	0.64202

Appendix F: Codebook for Qualitative Post-Survey Responses

Q4: Most Valuable Aspect			
Code	**Brief Title**	**Description/References**	**Example**
W	Case videos	Identifies responses that reference the lesson videos, WWYDN videos, student interviews, small group teacher videos, etc. This is distinct from the expert panel videos. There is a separate code for that.	"The activities with students that asked me to reflect on my practice, then discuss my next steps." "Student interviews."
V	Videos (non-descript)	Identifies responses that included a reference to "videos" but did not specify what type of video (to distinguish between codes W and EX). To avoid over-coding one or the other, this code was created to count the references to videos but avoid breaches of reliability.	
EX	Expert panels	Identifies responses that reference professors in the videos, expert panel, etc. This is distinct from the "case videos" code.	"Exposure to the three panelists and their thoughts about fraction instructions were very valuable." " I found the readings and what the experts thought about fractions informative."
U	Better conceptual understanding	Identifies responses that reference enhancement of personal mathematical understanding, understanding of student thinking, anticipation of student misconceptions, knowledge, etc.	"I learnt new things in this course. I can say that I understand fraction better than before."
PR	Project	Identifies responses that reference the participant project, final project, development of project with colleagues, etc.	"Working with my colleagues to write a project"

DB	Discussion Boards	Identifies responses that reference discussion board dialogue/exchanges, exchange of strategies/ideas for the classroom	"The discussion forum interactions were also very valuable." "Discussions with other teachers who teach my content." "Peer interaction and shared ideas"
PS	Peer-support	Identifies responses that reference peer support in the form of assignment feedback from other participants, participant recommendations/suggestions, working with a group at a local institution. This code addresses any support that comes from other participants in the MOOC-Ed or in a blended program where the MOOC-Ed is a resource.	"The collaboration aspect was excellent and it emphasize certain aspects to be watchful for when teaching fractions."
IM	Impact on practice	This code refers to responses that identify actions that will be taken or changes to current practice as a result of the MOOC-Ed. Identifies responses that reference different approaches used in the classroom, more/enhanced activities to bring into the classroom, conducting student interviews, implementation of new strategies, introduction of new activities in PD/mentoring	"Being able to use the images and videos in class as math talks with my students." "This MOOC-Ed led me to really question my students and listen more carefully to what they said. By doing this, I feel that I have become a better teacher by following their leads as to what they do and do not understand."
R	Resources	Identifies responses that reference collection of activities, value of readings, new lessons/lesson ideas, etc. This code refers to items/artifacts that the participant can take from the	"Reading materials, links to websites and videos"

		MOOC-Ed and use in their practice.	

Q11: Impact on Practice			
Code	**Brief Title**	**Description/References**	**Examples**
ST	Student thinking	Responses address the participant's improvement in their attention to student thinking. Responses would include improved knowledge of how students make sense of concepts, better questioning strategies based on student responses (i.e. probing), etc. Responses could address participant's improvement in addressing student misconceptions and challenges. These improvements may be related to improved understanding of how and why students have challenges grasping certain concepts.	"Have learned to listen to student thinking and expressing themselves." "I have implemented student interviews to better help me understand my students thinking." "I am more conscious at trying to prevent common misconceptions by questioning students on prior concepts." "I am better prepared to help students work through common fraction errors."
T	Tools	Responses specifically address/state the use of more manipulatives, tools, models (e.g., the number line), etc. These may reference curriculum changes (including introducing more applications) but specifically refer to use of those tools, manipulatives, etc.	"I have included a number line at the front of my class with whole and fractional numbers." "I am making a more deliberate effort to use a variety of representations; I am having students use more representations in their work."
RES	Resource	Responses address the participant's increased ability to serve as a resource to other teachers, improved mentoring strategies, ability to collaborate with other teachers. This code identifies any references to the participant's ability to work with	"I have been able to use many of the resources in discussions with teachers." "I am sharing these resources as a math coach."

		other practitioners.	
CON	Concept-focused	Responses specifically address an increased focus on concepts (as opposed to algorithms or rules) in the classroom. If a statement included this kind of reference, then it is coded CON only. If the response included more words that addressed APP, then code the response both CON and APP.	"As I taught fractions this year, I tried to stay away from simply teaching the rules of working with fractions, and made sure that students had a more conceptual and concrete understanding of them."
U	Fraction understanding	Responses address participant's personal improvement in fraction understanding/knowledge.	"I look at fractions in a whole different way." "The changes that I have made to my practice were the applying my knowledge and understanding of fractions in my classroom."
APP	Teaching Approach	Responses reference a change in teaching approach/pedagogy/methods (e.g., groupwork, increase in student voice in the classroom). This code is used for "actions" or "plans for action" in the classroom as opposed to "things".	"I learned that it's important to give the students a chance to think before speaking, because it takes them time to process questions you are asking them." "I have been using the different methods to teach and having a more open mind."