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ON THE FRACTURE ANALYSIS OF CONCRETE STRUCTURES TAKING INTO CONSIDERATION SIZE EFFECTS

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ABSTRACT

The authors (Rios & Riera, 2004) determined numerically the response of geometrically similar reinforced concrete beams built in four different sizes, tested to rupture by Leonhart & Walter (1965) and later reproduced by Ramallo *et al* (1993), to quantify the size effect in reinforced concrete beams. For such purpose the so-called Discrete Element Method (DEM) was employed, modelling the nonhomogeneous character of concrete and steel by assuming that their stiffness and specific fracture energies are random fields in 3D-space. The constitutive criteria adopted by the authors, based on Hillerborg´s model (1978), presented improvements in the consideration of the cross-correlation between relevant variables. The discrete numerical model was also used to reproduce experimental results due to Viet *et al* (2000) on the influence of sample size on the tensile strength of concrete.
In all cases, in order to represent the stress-strain relation for concrete in tension, a bi-linear diagram was resorted to, which has proved to be adequate when the size of the elements is sufficiently small and the nonhomogeneous properties of the material are properly accounted for. These conditions require large DEM or FEM models, which cannot be usually employed in engineering practice due to cost-effectiveness considerations. It is shown in the paper that accurate predictions of the failure load of concrete structures, that account for size effects, can be made using larger elements and therefore reduced computational costs, if the appropriate stress-strain relations are adopted. These relations depend both on the size of the element and on the correlation lengths of the random fields that describe the relevant material properties, such as the specific fracture energy. The proposed equations were determined by Monte Carlo simulation, using, for the various sizes of structural samples, the same level of discretization.

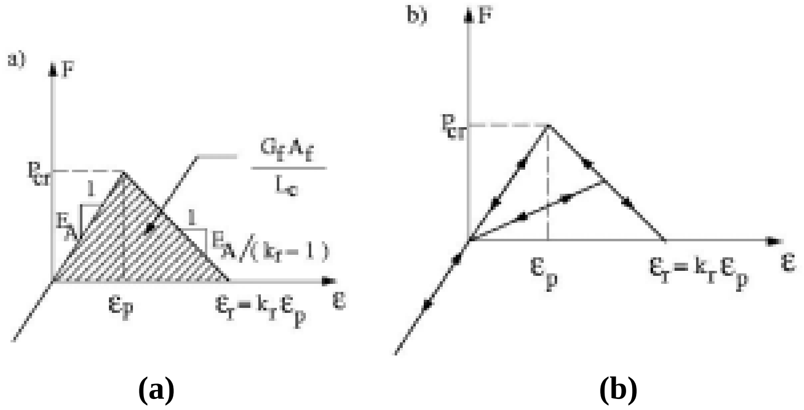
1. INTRODUCTION

The authors (Rios & Riera, 2004) determined numerically the response of geometrically similar reinforced concrete beams built in four different sizes, tested to rupture by Leonhart & Walter (1965) and later reproduced by Ramallo et al (1993), to quantify the size effect in reinforced concrete beams. For such purpose the so-called Discrete Element

Method (DEM) was employed, modelling concrete and steel by assuming that their stiffness and specific fracture energies are random fields in 3D-space. The constitutive criteria adopted by the authors, based on Hillerborg´s model (1978), presented improvements in the consideration of the cross-correlation between relevant variables. The same discrete numerical model was also used to reproduce experimental results due to Viet et al (2000) on the influence of sample size on the tensile strength of concrete. Note that the approach allows the simultaneous consideration of the size effect due to fracture (Bazant, 1976, 1993) and the influence of material nonhomogeneities (Weibull, 1949). In fact, in the present formulation, the DEM requires that the random distributions of relevant material properties, such as the specific fracture energy of the material, be properly simulated in the numerical model. It has been shown that when the discrete elements mesh is dense, the shape of the strain softening branch, a straight line in Hillerborg´s model, has marginal influence on the results (Rocha and Riera, 1990; Iturrioz and Riera, 1998; Rios and Riera, 2004). However, for structural components subjected to tension or shear or in case of large discrete elements, the effective stress-strain softening branch tends to present the convex shape measured in concrete and mortar samples and numerically simulated by Wittmann et al (1987) and Roelfstra and Wittmann (1987). Both authors contributed significantly to the understanding of the fracture process in concrete, but did not address the issue of size effects or model invariance. It is clear that the constitutive criteria adopted for an individual element represents a sort of average over a material volume and hence depends on the size of this volume, in other words, on the size of the element. The notion that the parameters needed to describe the behavior of engineering materials under load, actually depend on the size of the elements adopted in the numerical model, is now widely acknowledged. However, the approach has not yet found a place in routine engineering practice on account of several factors, among which the most important is the lack of a procedure to evaluate those parameters for elements of arbitrary size. It is the purpose of this paper to provide a practical method, applicable in the nonlinear fracture analysis of concrete structures of arbitrary dimensions that fail mainly by tension, that is, structures in which sliding with friction does not play a significant role in the failure process.

2. DESCRIPTION OF PROPOSED MODEL

In previous studies (Rocha and Riera, 1990; Iturrioz and Riera, 1998; Rios and Riera, 2004), the authors resorted to the so-called Hillerborg´s model, in which the constitutive law for the cracked element consists of a bi-linear diagram, as shown in Figure 2.1. In the figure, F represents the tensile load in the element, which depends on the strain ε.



(a) **(b)**

Figure 2.1 : Bi-linear constitutive law for fragile material

In the following, the bi-linear diagram will be substituted by a continuous function, modeled using the following expression:

$$f = a\,\varepsilon\; exp(-\,b\,\varepsilon^{c}) \qquad (0 < \varepsilon) \qquad (2.1)$$

in which f denotes the stress and a, b and c are constants. The stress times an influence area yields the force in the element. The slope at the origin, which defines the initial stiffness of the element, is obtained from the derivative, given by:

$$df / d\varepsilon = a\,(1 - b\,c\,\varepsilon^{c})\; exp(-\,b\,\varepsilon^{c}) \qquad (2.2)$$

when $\varepsilon = 0$, the stiffness is:

$$df / d\varepsilon \,\big|_{\varepsilon=0} = a \qquad (2.3)$$

The strain at maximum stress ε_{max} and the peak stress f_{max} are given by:

$$\varepsilon_{max} = (b\,c)^{-1/c} \qquad (2.4)$$

$$f_{max} = a\,(b\,c)^{-1/c}\; exp\,[-b\,(b\,c)^{-1/c}] \qquad (2.5)$$

The unloading branch remains a straight line, as schematically shown in Fig.2.1(b). Finally the area enveloped by the curve for positive values of ε, which defines the total work W required for fracturing the element, is given by:

$$W/L_c = a\,c^{-1}\,b^{2/c}\,\Gamma(2/c) \qquad (2.6)$$

In which Γ denotes the Gamma function and L_c the element's length. The proposed function (eq. 2.1) presents several attractive features: over a wide range of parameters it differs little from Hillerborg´s bi-linear law, it is continuous and has a continuous first derivative. The leading parameter a denotes the initial stiffness and, last but not least, it resembles the global stress-strain curves measured in concrete samples subjected to tension under constant strain rates (Roelfstra and Wittmann, 1987). Now, setting the work W/L_c equal to the area in the diagram of Fig.(2.1):

$$G_f\,A_f / L_c = a\,c^{-1}\,b^{2/c}\,\Gamma(2/c) \qquad (2.7)$$

where A_f represents the equivalent fracture area of one bar. If the initial stiffness a, *i.e.* the slope of the stress-strain curve at the origin, the specific fracture energy G_f and the strain at peak load ε_{fmax} are specified, then eqs.(2.4) and (2.7) define the remaining parameters b and c in eq.(2.1). It is convenient, however, to specify the initial stiffness a and then select the parameters b and c by fitting the function (2.1) to the bi-linear diagram, which leads to:

$$b = 0.327\,[\,1.264 - exp\,(-\,0.106\,k_r)] \qquad (2.8)$$

$$c = 1.156 / [\,1 - 1.330\,exp\,(-0.288\,k_r)] \qquad (2.9)$$

If the parameters of the bi-linear model *or* of function (2.1), as defined by eqs. (2.8-9), are specified for a given element size, the parameters for larger elements may be determined by simulation, as described in next section. At the same time, the properties of the corresponding random fields will change.

3. DESCRIPTION OF NUMERICAL MODEL

A concrete cubic body subjected to monotonically increasing uniform displacements on its upper and lower faces will be first analyzed, simulating constant strain rate, displacement controlled tensile tests. The array of discrete elements shown in Fig. 3.1 was adopted in the study. It presents 1200 DOF, being the smallest array that would lead to satisfactory simulations of the total displacement, thus limiting the time needed to process all samples. Table 3.1 shows the basic dimensions of the three sample sizes analyzed, denoted as models DEM1 to DEM3. The material properties of concrete are indicated in Table 3.2

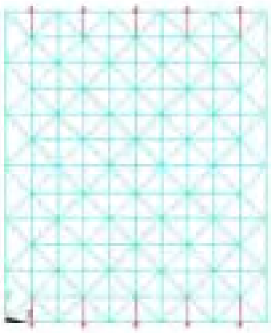


Figure 3.1 Frontal view of the discrete elements array used in modelling the cubic concrete specimen.

Table 3.1: Basic dimensions of samples

	L_c[m]	L[m]
DEM1	0.01	0.05
DEM2	0.05	0.25
DEM3	0.25	1.25

Table 3.2: Material properties of concrete

E (Young Modulus)	3.5E10[N/m^2]
ρ (specific mass)	2400 [Kg/m^3]
G_c (toughness)	100[N/M]
$CV(E)$ (coefficient of variation of E)	30 [%]
$CV(G_c)$(coefficient of variation of G_c)	30 [%]
ε_p (critical strain)	1.28E-4
L_{corr}	0.01 [m]

Figure (3.2) shows the proposed equation (2.1) fitted to Hillerborg´s bi-linear diagram. Note that both stress and strain are normalized. The resulting curve corresponds to model DEM0, and is used as the constitutive law for the elements of model DEM1 defined above. Twelve simulations were carried out, of which the six plotted in Figure (3.3) were selected by inspection, eliminating those curves that presented unloading branches with sharp unloading steps. This is due to the rather coarse mesh employed in the analysis for reasons of economy.

Next, the mean stress-strain curve for model DEM1, also shown in Fig. (3.3) is assumed to define the effective stress-strain curve associated to the size of model DEM1. The proposed law (2.1) is fitted to the mean previously decribed, as shown in Fig. (3.4). This law is now the constitutive criterium for model DEM2.

Results of six simulations for model DEM2 are presented in Fig. (3.5), which includes the mean curve as well. The law (2.1) fitted to the mean of all six simulations for model DEM2 is shown in Fig. (3.6). The procedure is repeated for model DEM3. The effective stress-strain curves for this case are shown in Fig. (3.7). Note that for this size unloading occurs suddenly, clearly illustrating a more fragile behavior. It was decided to censor the law, suddenly interrupting it at the abcissa 2.4, as indicated in Fig. (3.8). The results reported herein suggest that for very large sizes, eq.(2.1) may not present a good fit to the data.

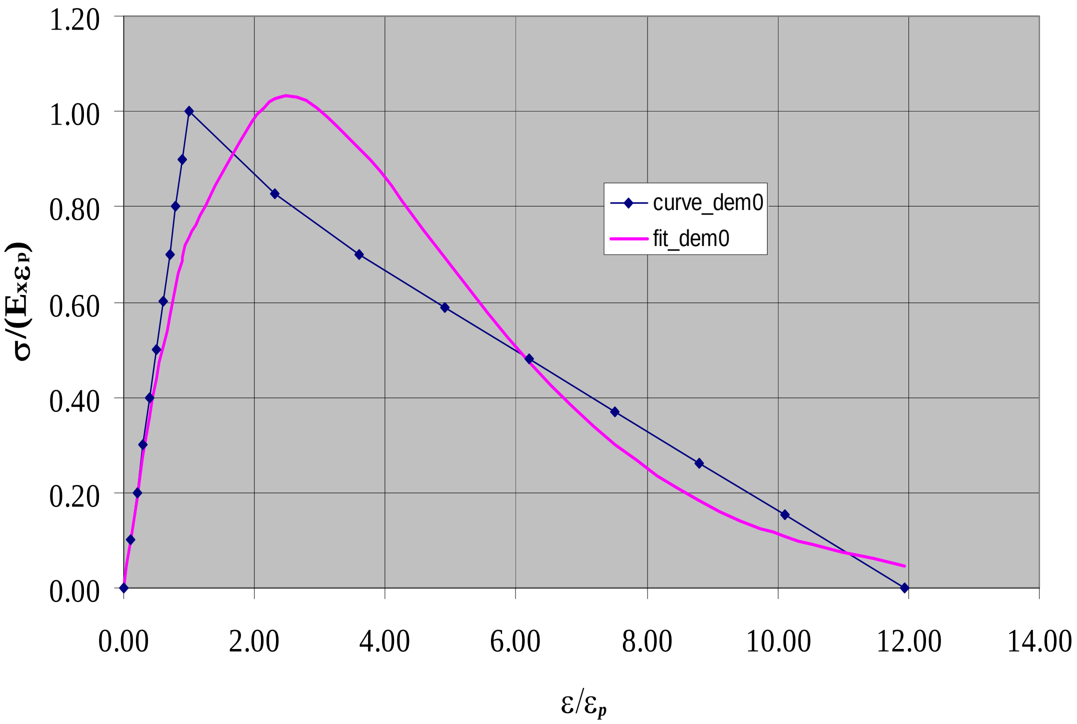


Figure 3.2: Proposed law (eq.2.1), with the coefficients determined to fit Hillerborg´s bilinear law. Note that normalized strains and stress are plotted in the horizontal and vertical axes.

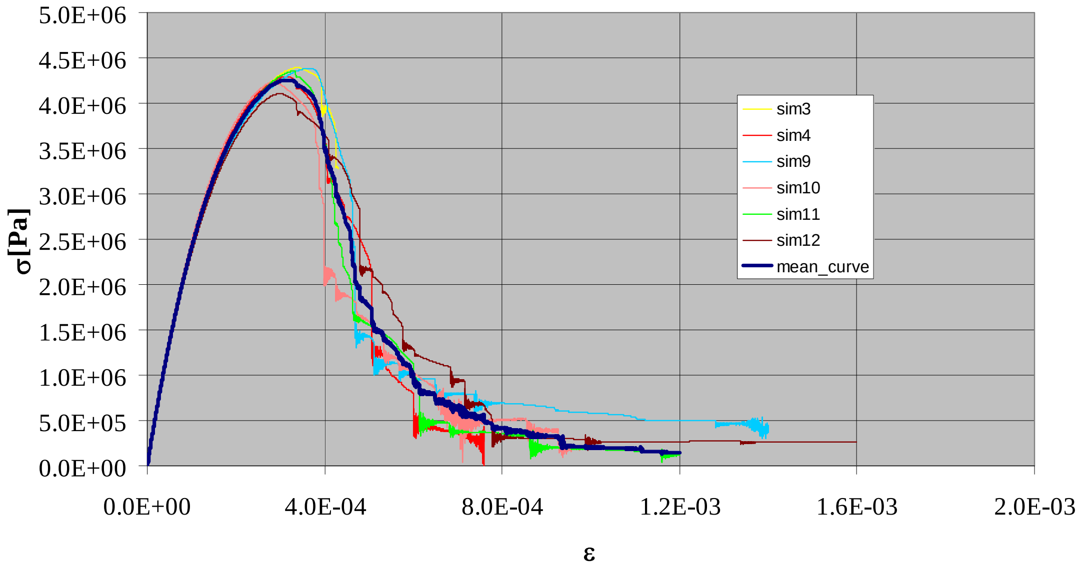


Figure 3.3: Global stress-strain curves for Serie DEM1 and mean curve for all simulations

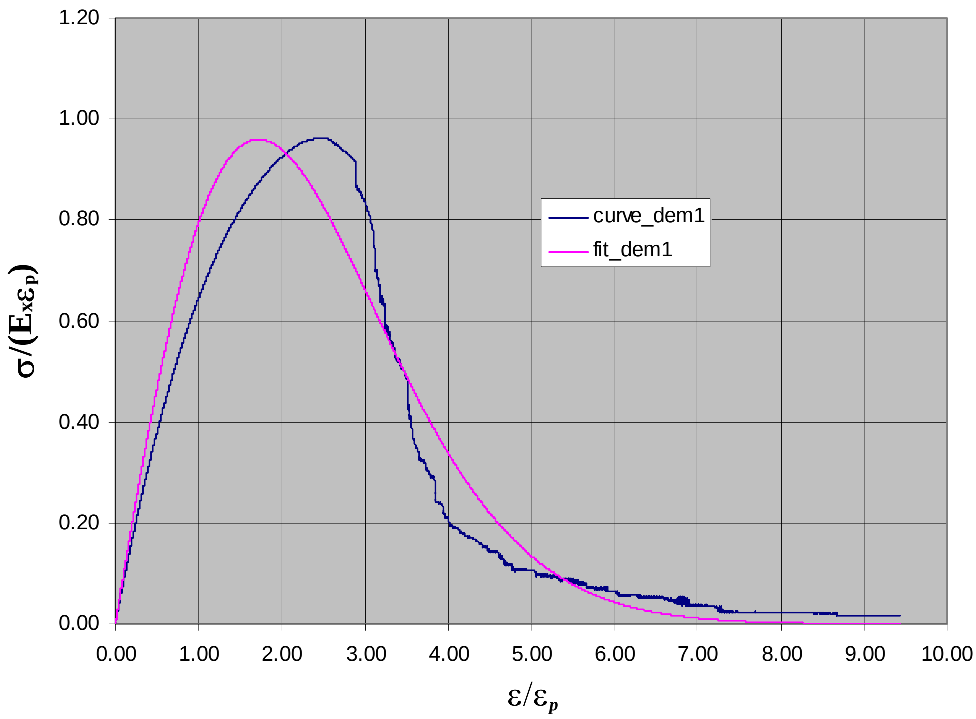


Figure 3.4: Fit of the generic law (eq.2.1) to the normailized mean stress-strain curve computed by simulation for model DEM1. (b=0.2722, c=1.5139)

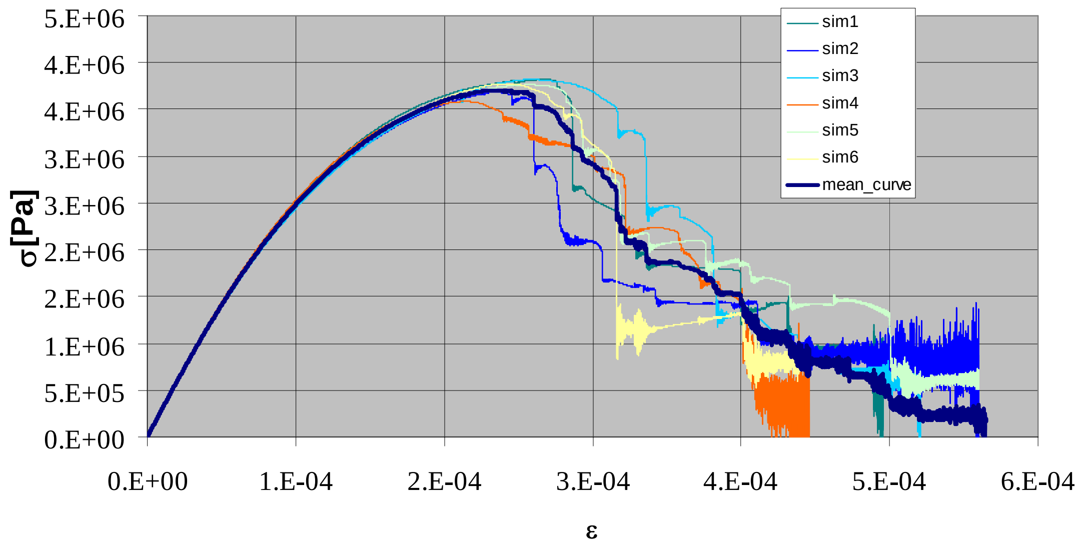


Figure 3.5: Global stress-strain curves for Serie DEM2 and mean curve for all simulations.

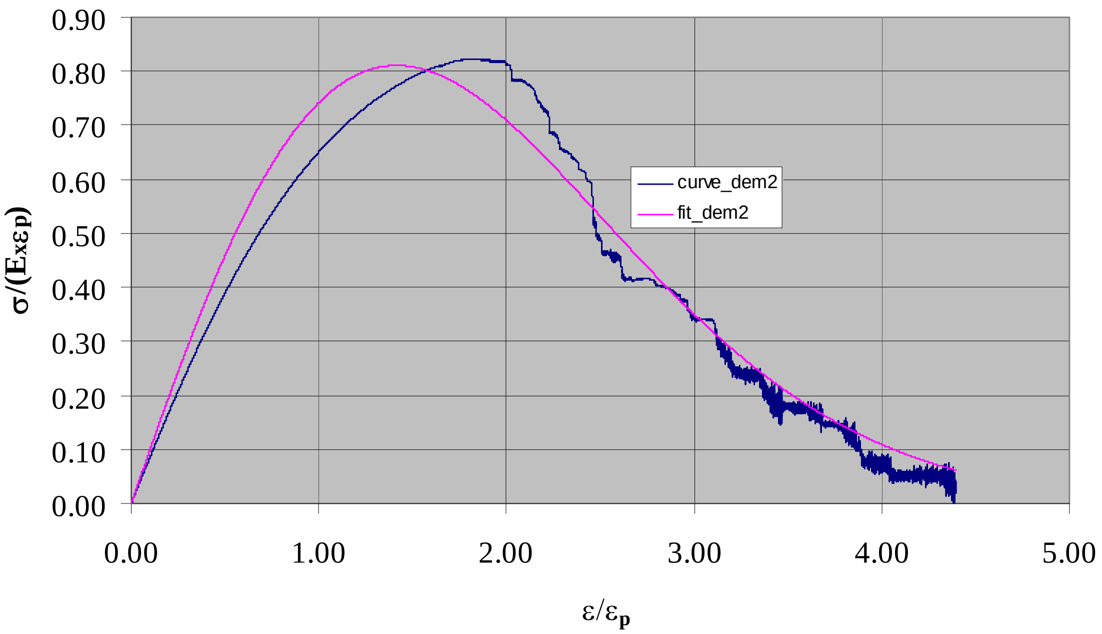


Figure 3.6: Fit of the generic law (eq.2.1) to the normalized mean stress-strain curve computed by simulation for model DEM2.(b=0.2985, c=1.7)

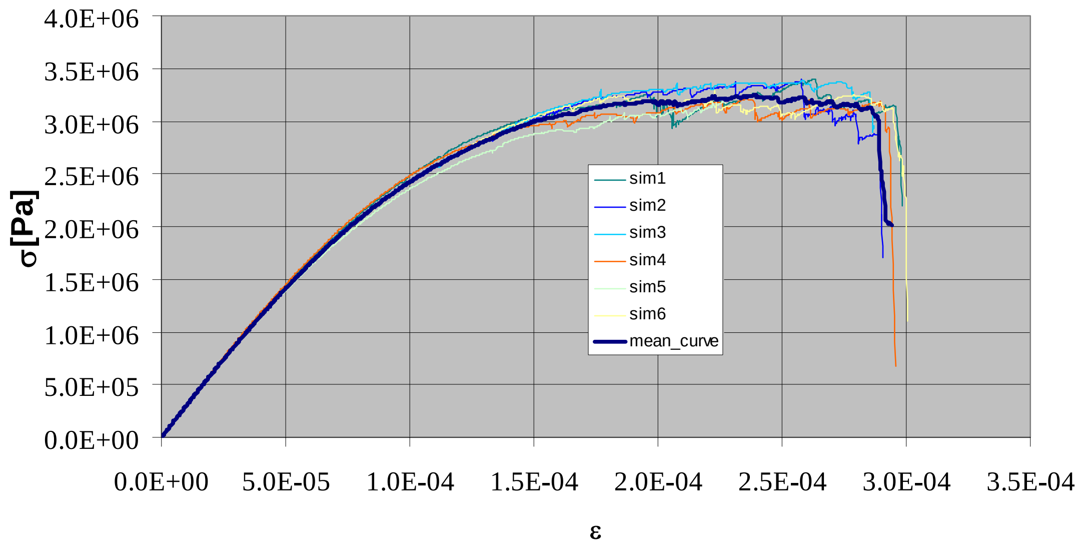


Figure 3.7: Global stress-strain curves for Serie DEM3 and mean curve for all simulations

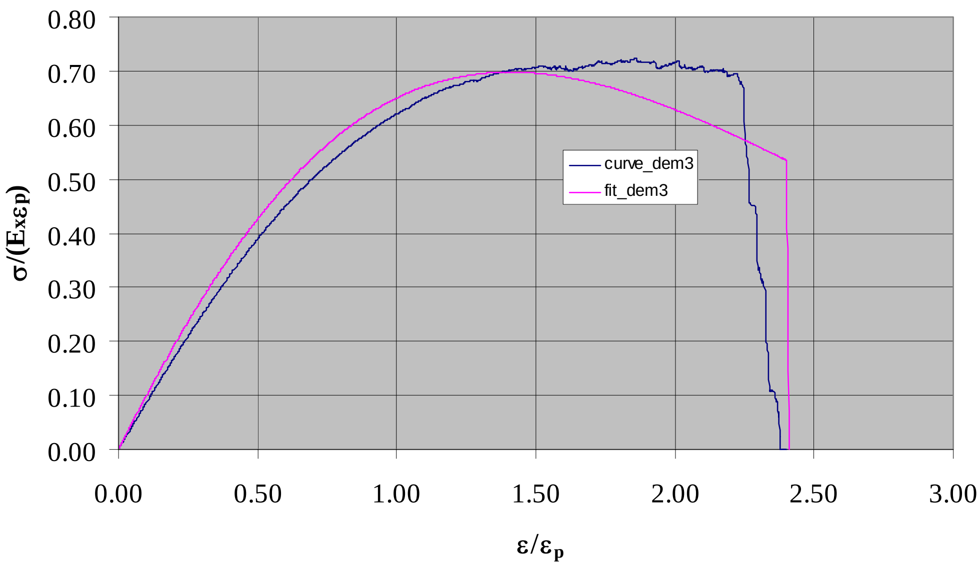


Figure 3.8: Fit of the generic law (eq.2.1) censored at normalized strain 2.4, to the normalized mean stress-strain curve computed by simulation for model DEM3.

4. RESULTS OF NUMERICAL ANALYSIS

Fig.(4.1) shows a plot of the normalized stress-strain laws fitted to the mean curves computed by simulation for the three sizes of models DEM1 and DEM2, as well as the original law, herein denoted as DEM0. The parameters corresponding to these curves are summarized in Table 4.1, which also indicates the values of the maximum stress and the strain at which it occurs, for all cases. It may be seen that the global behavior of the material is correctly reproduced: the maximum stress, that would correspond to the *strength* of the element, decreases with its size. The area under the curve and the maximum strain also decrease with size, indicating that a more fragile behavior should be expected as the size of the element (structure) increases.

Table 4.1: Coeficients of size-dependent stress-strain law and values of maximum stress

	lc	B	c	ε_{fmax}	σ_{max}[Pa]
dem0(L=0.01m)	-	0.3047	1.169	2.94e-4	4.4e6
dem1(L=0.05m)	0.01	0.273	1.51	2.21e-4	4.18e6
dem2(L=0.25m)	0.05	0.2985	1.7967	1.77e-4	3.56e6
Dem3(L=1.25m)	0.25	(0.43096)	(1.4254)	1.68e-4	3.08e6

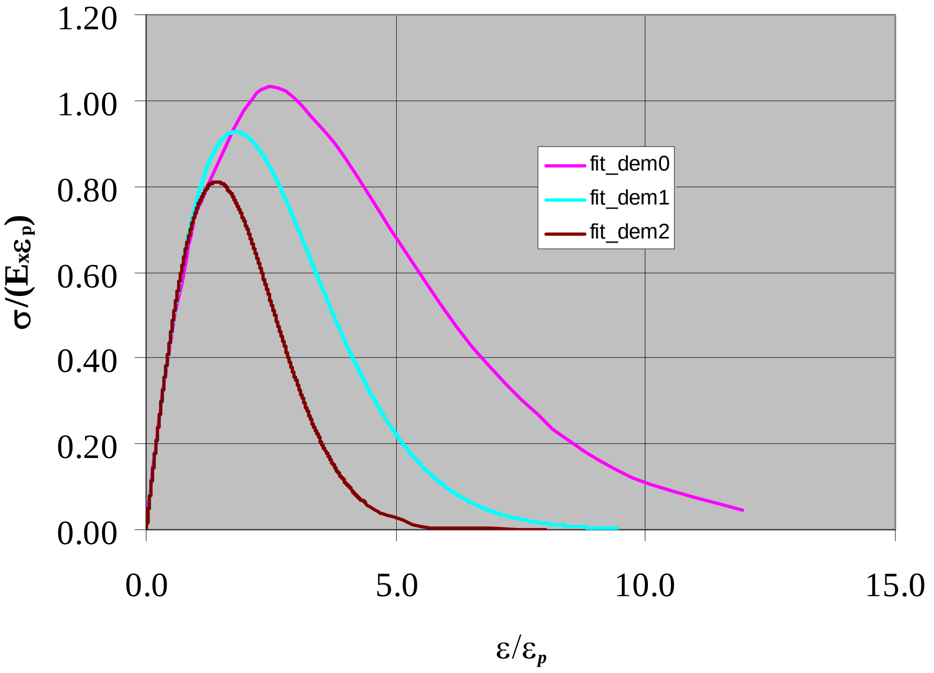


Figure 4.1: Comparison of the original generic [law (eq.2.1] with the functions fitted to the normalized mean stress-strain curve computed by simulation for models DEM0, DEM1, and DEM2.

A inspection of the preceding results shows that coefficient *b* does not present a well defined trend. Until additional results become available, it may be admitted that it is not affected by size. The mean value of all simulations is b = 0.29, which will be adopted in the following. The third coeficient, c, clearly increases with size, as shown in Fig. (4.2). The following expression were fitted to the limited data so far avaliable:

$$b = 0.292 \tag{4.1}$$

$$c = 2.17(L_c)^{0.13} \tag{4.2}$$

The results given above allow the analysis of fracture and fracture propagation in concrete structures taking into due consideration the influence of size in applications of both the DEM and FEM. The smooth curves in Fig. 4.1 should lead to improved results, requiring less elements than the bi-linear model. Additional experimentation is needed for larger sizes, in which case the proposed model (2.1) is no longer applicable, on account of the usual trend of the sample to become more fragile. Moreover, the properties of the random fields that describe the material properties, such as E or G_f, also change with size. Presently, it is suggested to assume that the expected values of both quantities remain constant, while their variance decreases, for the 0.25m element length, to half the values adopted in Table 3.2.

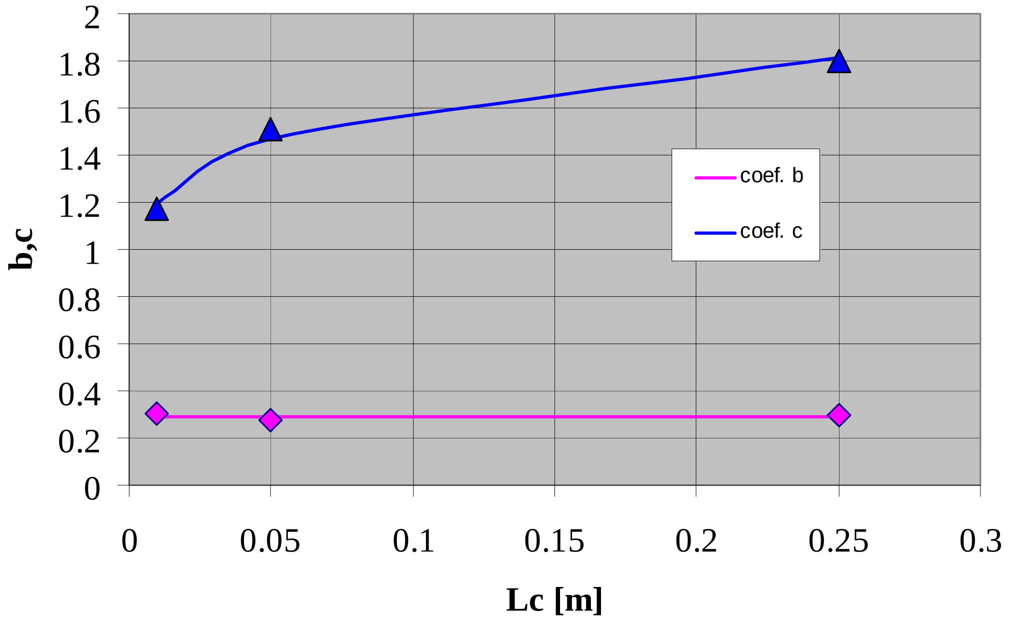


Figure 4.2. Variation of coeficients b and c in terms of size Lc: points obtained by simulation for lengths equal to 0.01, 0.05 and 0.25m, and curves fitted to simulated results.

5. CONCLUSIONS

The adoption of the continuous effective stress-strain law (2.1) to replace the bi-linear law introduced by Hillerborg or the three-linear expression suggested later by Whittmann and Roelfstra, is herein proposed as a basic expression for the numerical analysis of concrete and reinforced concrete structures subjected to fracture. It is shown that this constitutive law is size dependent. If a single mathematical expression is adopted for concrete, as herein proposed, then only its coeficients will vary with size. Results obtained by simulation on the evolution with size of the coeficients in the proposed law are reported in the paper, for element sizes smaller than 0.5m, typically used in numerical studies of fracture. It should be noted that the results reported depend on the *basic* curve, denoted as DEM0, and therefore they should still be *calibrated* by comparison with experimental results for a given sample size, or various sample sizes, if available. Further research is also required to extend the results to element lengths above 0.25m, range in which the behavior is not closely predicted by the proposed model and to characterize the random fields that define the relevant material properties.

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