STRESS CONCENTRATIONS AROUND A LARGE CIRCULAR OPENING CONSIDERING THE TENSILE FAILURE OF CONCRETE

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SUMMARY

Stress concentrations around a circular opening located in a flat plate whose material is elastic, linear, and homogeneous have been analyzed theoretically already in many studies. This theory will, however, be not reasonable to be applied directly to materials such as concrete, which have much smaller tensile strength compared with its compressive one and, moreover, is subject to tensile stress under reinforcement by bars. When concrete is subject to tensile stress, cracks occur in low tensile stress condition level and tensile failure follows. But in case of reinforced concrete, tensile stress after tensile cracks occurred is mainly imposed to reinforcing bars. That is, when the tensile strain of reinforced concrete member reaches to about 1.0×10^{-4}, tensile cracks begin to appear in concrete. As load increases, the number of cracks increases and strain is radically enlarged approaching to the strain of reinforcing bars only. This phenomenon is verified by uniaxial tensile tests.

In this paper, the above-mentioned phenomenon is taken into consideration for the plane problem, and stress concentrations and displacements by seismic forces, around a large circular opening located on the outer shield wall, which is a part of reactor containment facility for the PWR type nuclear power plant, are studied together with elastic cases.

The outer shield wall in this study is a reinforced concrete cylindrical shell structure, whose thickness and outside diameter are 90 cm and 44.3 m respectively, and the diameter of opening is 6 m. Since the diameter of the outer shield wall is large enough compared with the diameter of the opening, the analytical model used in this study is treated as a flat plate with an opening.

According to the result of this investigation, when large tensile stresses are yielded in concrete around opening under tensile or shear force due to seismic loads, it is fully understood how tensile stresses in concrete are transfered to reinforcing bars after tensile cracks occurred, and how different displacements are from elastic cases.

A finite element computer program is used in this study, which has been developed for evaluation of elastic-plastic problems. A finite element is composed of a concrete element representing idealised "bilinear" elasticity and a reinforcing steel element representing "linear" elasticity. These elements are laminated on same plane and each node is connected to each other so that equilibrium of displacements are satisfied.

The modulus of elasticity of concrete in reinforced concrete after tensile cracks occurred, has slightly minus value as verified in experimental results. However, the value is taken as zero in this study for convenience' sake of computer program.

This paper recommends that the analysis mentioned above shall be applied to design of reinforced concrete structures of nuclear power plant, etc., in which tensile stresses are severe and displacements are to be discussed.

1. Introduction

When stress concentrations and displacements around opening located on a reinforced concrete plate are discussed under a servere load condition, it will be necessary that the characteristics of concrete behavior after tensile failure are taken into consideration. How to estimate the rigidity of concrete after tensile failure is considered quite difficult. By uniaxial tensile tests [1] and [2], some of these characteristics are verified, but there are many problems left to apply these test results directly to actual structural design which is affected by many prameters such as concrete strength, quantity of reinforcing bars and friction between concrete and reinforcing bars, etc..

In this study, the rigidities are decided by modifying the stress-strain curves obtained by uniaxial tests while they are theoretically deficient in their aplications, and used in plane problems under the assumption that the tensile failures will occur only in one direction in this case. For the convenience of computer program, the rigidity of reinforced concrete is divided into the rigidity of concrete for compressive or tensile elastic regiens followed by tensile failure, and the rigidity of reinforcing bars which are shown in Fig.1 and Fig.2 schematically and their numerical values are shown in chapter 3.

In our analysis, the "non-linear" finite element program named CRC-FINENT is used for which the iterative method is specified. Brief analytical proceedure is shown in chapter 4. A finite element is composed of a concrete element representing modified "bilinear" rigidity and a reinforcing bars element representing "linear" rigidity. These elements are laminated in form of a plane and every nodes are connected one another so that equilibrium of displacements are satisfied.

For comparison, stress analyses are carried out for the two conditions of concrete; the one assumes elasticity followed by no failure (hereinafter Elastic) and the other assumes elasticity followed by the tensile failure (hereinafter Elastic-Plastic). In this study, the compressive failure and the effects by cyclic loadings are not considered.

A large opening taken here is located on the outer shield wall provided for the equipment hatch, and is subject to seismic loads and dead load. Three load conditions, Case 1, Case 2, and Case 3 are taken in accordance with seismic load directions.

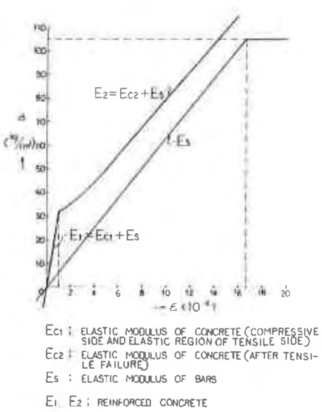

Fig. 1 Elastic modulus of reinforced concrete in tensile side

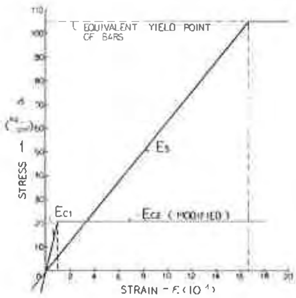

Fig. 2 Elastic moduli of concrete (modified) and reinforcing bars

The finite element model used in this study and load conditions are shown in Fig.3 and Fig.4 respectively. As shown in Fig.3, the model is divided into A and B zones in accordance with quantitities of reinforcing bars estimated by an elastic theory without counting tensile strength of concrete.

2. Analytical model and loads

The dimensions of the reinforced concrete plate used in this study are shown in Fig.3, which is 24 meters wide, 18 meters high and 0.9 meters thick. The plate is reinforced with SD 35 (deformed bar, yield point fsy = 3,500 kg/cm^2), and quantity of reinforcing bars decided by elastic analysis is slightly varied in accordance with the difference of those directions at any point, but on the average Pt = 3% for A zone and 1.2% for B zone are used in all directions in the analytical model. Finite elements are shown in Fig.3. An element is composed of a concrete element and a reinforcing bars element which are laminated in form of a plane as related in Introduction.

Load conditions are two vertical and one horizontal load cases as shown in Fig.4. Uniformly distributed loads are applied along the top side of analytical model, and uniformly distributed horizontal and vertical supports are provided along the bottom side of the analytical model. Applied loads for three cases are as follows which are obtained by the dynamic analysis, etc..

Case 1 Combined vertical load due to bending moment caused by earthquake and dead load of the outer shield wall (downward)

$P_1 = 498t/m$

Case 2 Combined vertical load due to bending moment caused by earthquake and dead load of the outer shield wall (upward)

$P_2 = 217t/m$

Case 3 Idealized horizontal load due to shear force by earthquake

$P_3 = 138t/m$

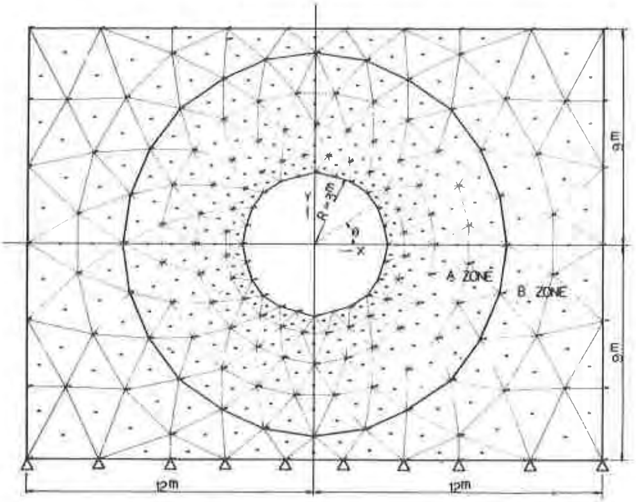

Fig. 3 Analytical model

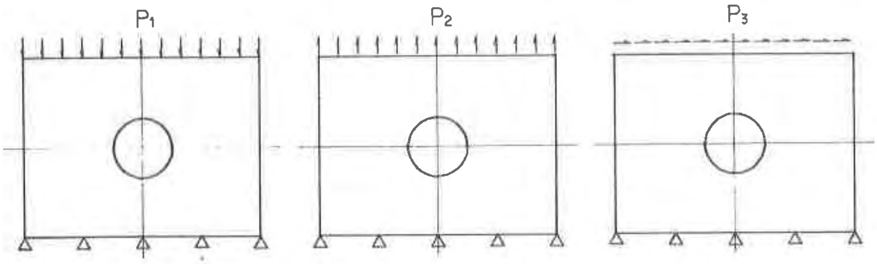

Fig. 4 Load conditions

3. Rigidities

As referred in chapter 1 and 2, the rigidity of reinforced concrete plate is evaluated from rigidities of concrete and reinforcing bars. Elastic moduli of concrete for elastic region and after tensile failure, equivalent elastic moduli of reinforcing bars for A and B zones, strength [4] and Poisson's ratio of each member are as follows, where elastic modulus of compressive side of concrete are of the same value as used in elastic region of tensile side, and compressive failure is not considered.

Elastic modulus of concrete	Compressive side and elastic region of tensile side	$E_{c1} = 2.3 \times 10^5 kg/cm^2$
	After tensile failure	$E_{c2} = 0$
Elastic modulus of reinforcing bars		$E_{s*} = 2.1 \times 10^6 kg/cm^2$
Equivalent elastic modulus of reinforcing bars	A zone	$E_s = P_t\ E_{s*} = 0.63 \times 10^5 kg/cm^2$
	B zone	$E_s = P_t\ E_{s*} = 0.252 \times 10^5 kg/cm^2$
Compressive strength of concrete		$f_{cc} = 250 kg/cm^2$
Tensile strength of concrete		$f_{ct} = 25 kg/cm^2$
Modified tensile strength of concrete (See Fig.2.)		$f_{ct}' = 21 kg/cm^2$
Poisson's ratio of concrete		$V_c = 1/6$
Yield point strength of reinforcing bars		$f_{sy} = 3{,}500 kg/cm^2$
Poisson's ratio of equivalent reinforcing bars		$V_s = 0$

4. Analytical procedure

The computer program used in this analysis is so called "material non-linear program" which was made by revising and improving of a finite element program which was previously developed for analysis of two dimensional stress problems.

As the analytical procedure, an iterative method is used, when lager tensile stresses than modified tensile strength occur in any element, loads composed by the exceeding tensile stresses are re-acted in the opposite directions at nodal points of the element, the stresses are recalculated, and these analytical sequences are repeated until "no exceeding tensile stress condition of concrete" is gained. Flow chart of procedure mentioned above is as follows;

$[\mathbb{K}]$ Generate stiffness matrix.

↓

$\{\mathbb{R}\}$ Generate load vector.

↓

$[\mathbb{K}]\{\mathbb{r}\} = \{\mathbb{R}\}$ Solve simultaneous equation.

↓

$\{\mathbb{r}\}$ Calculate displacement,

where $\mathbb{r} = \Sigma \mathbb{r}_i$.

↓

$\{\sigma\}$ Calculate stress,

where $\sigma = \Sigma \sigma_i$.

↓

$\{\sigma\}$ Calculate principal stresses (σ_1, σ_2) and there angle (θ).

↓

$\{\mathbb{R}\}$ Generate load vector for exceeding tensile stress, iterate these sequences and converge to modified tensile strength of concrete (fct').

where

$[\mathbb{K}]$	;	Stiffness matrix
$\{\mathbb{R}\}$	;	Load vector
$\{\mathbb{r}\}$	;	Displacement vector
$\mathbb{r}_i$	;	Displacement for i-th iterative step
$\{\sigma\}$	;	Stress vector
σ_i	;	Stress of i-th iterative step
σ_1, σ_2	;	Principal stress of element
θ	;	Angle of principal stress

In this study, 19 iterations for analyses of Case 1 and Case 2, and 39 iterations for analysis of Case 3 were required to obtain the resonable results.

5. Results

Stresses and displacements obtained by analysis are shown in Fig.5 Fig.16. Fig.5 Fig.7 show principal stresses for Case 1, Case 2, and Case 3 respectively in which Elastic-plastic are considered.

Fig.8, Fig.10, and Fig.12 show circumferential stresses, Fig.9, Fig.11, and Fig.13 show radial stresses and Fig.14 Fig.16 show displacements under three load conditions for Elastic and Elastic-plastic studies.

Case 1;

In case 1, tensile stress of concrete larger than modified tensile strength in Elastic study occur only around opening on $\theta = 90^\circ$ in circumferential direction, which is observed in Fig.8 and Fig.9. Differences of stresses of concrete and reinforcing bars (hereinafter rebars) between Elastic and Elastic-plastic studies are not significant except mentioned above. Therefore, in both studies displacements on $\theta = 0^\circ$ is almost same.

Maximum circumferential compressive stress in concrete and rebars around the opening of concrete and rebars on $\theta = 0^\circ$ is $\sigma = -124.0 - 16.4 = -140.4$ kg/cm^2, and that at the outside of the model (x = 12.0m) is $\sigma = -47.1 - 1.8 = 48.9$ kg/cm^2 in all. Therefore, stress concentration ratio around opening is $\alpha = -140.4/-48.9 = 2.9$, which is almost equal to $\alpha = 3.0$ [3] obtained by an elastic theory.

Case 2;

As the circumferential stresses of concrete in Elastic study everywhere on $\theta = 0^{o}$, and around opening on $\theta = 45^{o}$ are larger than modified tensile strength ($fct' = 21.0 kg/cm^2$), it is understood that the tensile stresses of concrete exceeding tensile strength in Elastic study are transfered to rebars in Elastic-plastic study and consequently the stress of rebars become larger than that of concrete, and circumferential stresses of concrete on $\theta = 0^{o}$ are converged to $fct' = 21.0$ kg/cm^2 being angles of principal stresses (σ_1) are nearly $\theta = 90^{o}$ everywhere as shown in Fig.10.

As the equivalent maximum tensile stress of rebars is $\sigma_{es} = 35.7$ kg/cm^2, actual stress becomes $\sigma_s = 35.7/pt = 35.7/0.03 = 1{,}190$ kg/cm^2 which is smaller than yield point strength of rebars $fsy = 3{,}500$ kg/cm^2 and accompanied with many cracks are predicted by Fig.1 and Fig.2.

Radial stresses are less than tensile strength everywhere, and differences of stresses between Elastic and Elastic-plastic studies are not significant as shown in Fig.11.

Displacements in Elastic-plastic study are about two times as large as those in Elastic study on $\theta = 0^{o}$ as shown in Fig.15.

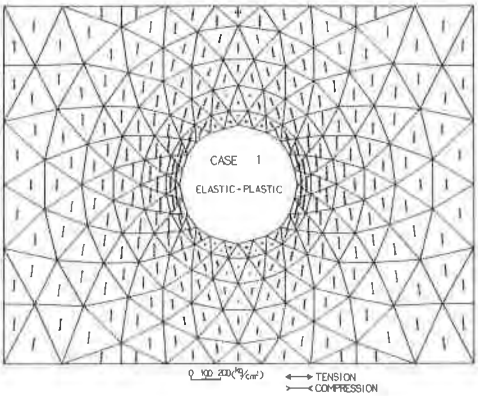

Fig. 5 Principal stresses in Case 1 by Elastic-plastic study

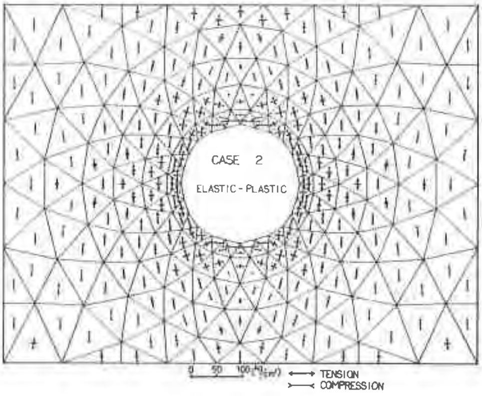

Fig. 6 Principal stresses in Case 2 by Elastic-plastic study

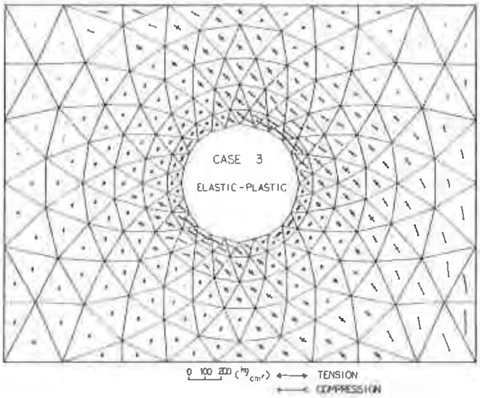

Fig. 7 Principal stresses in Case 3 by Elastic-plastic study

Case 3;

As shown in Fig.12 and Fig.13, circumferential stresses of both concrete and rebars on $\theta = 90^{o}$, 135^{o}, 180^{o}, and 315^{o} have specific differences between Elastic and Elastic-plastic studies. The circumferential stresses of concrete around opening in Elastic-plastic study on $\theta = 135^{o}$, and 315^{o} are converged to rather less values than the modified tensile strength of concrete, because directions of principal tensile stresses slightly deviate from circumferential direction. Equivalent maximum circumferential tensile stress of rebars is

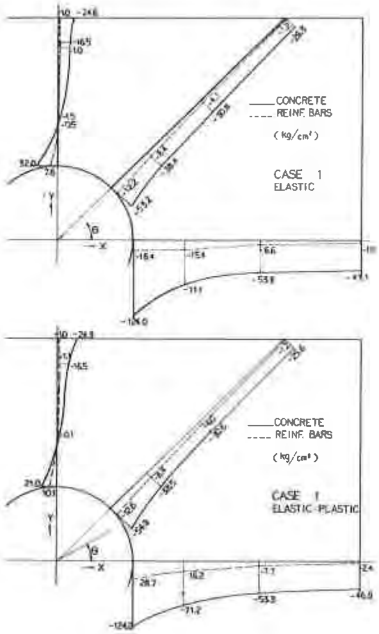

Fig. 8 Circumferential stresses in Case 1

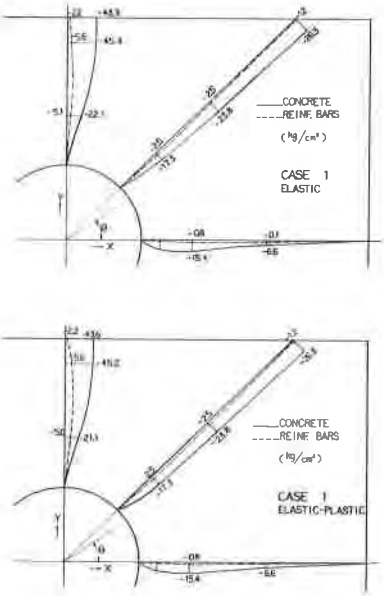

Fig. 9 Radial stresses in Case 1

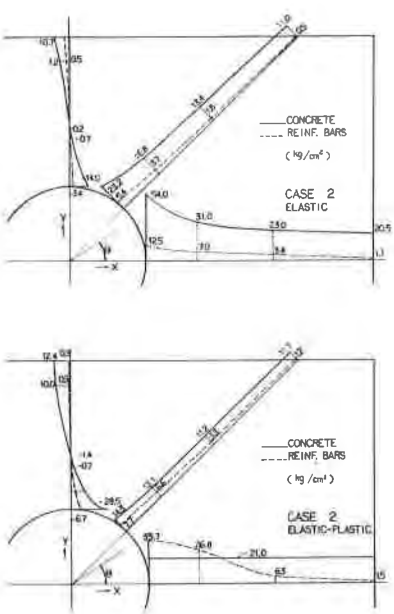

Fig.10 Circumferential stresses in Case 2

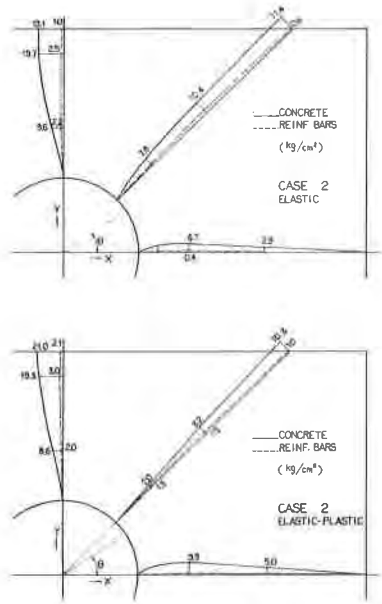

Fig.11 Radial stresses in Case 2

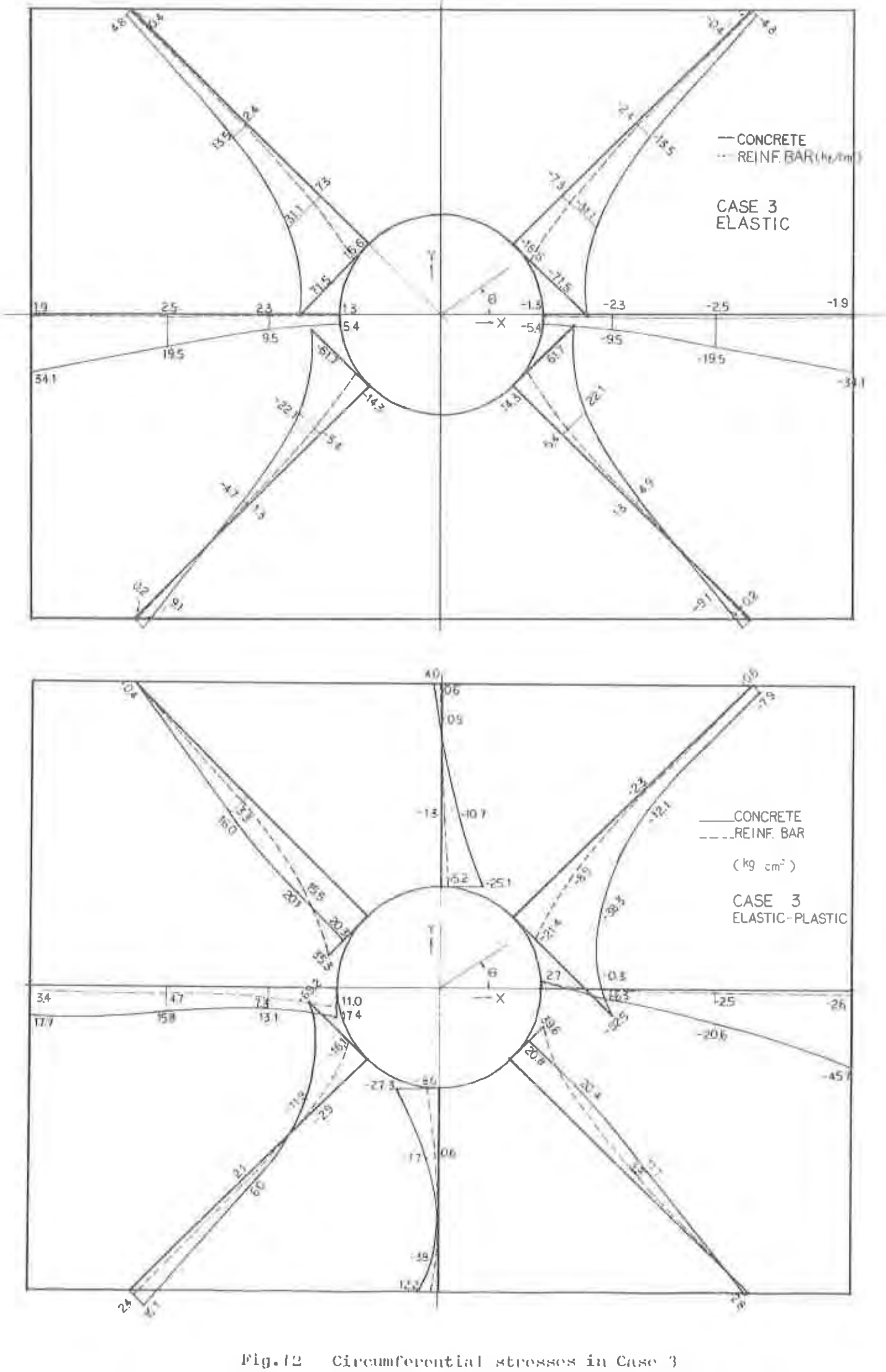

Fig. 12 Circumferential stresses in Case 3

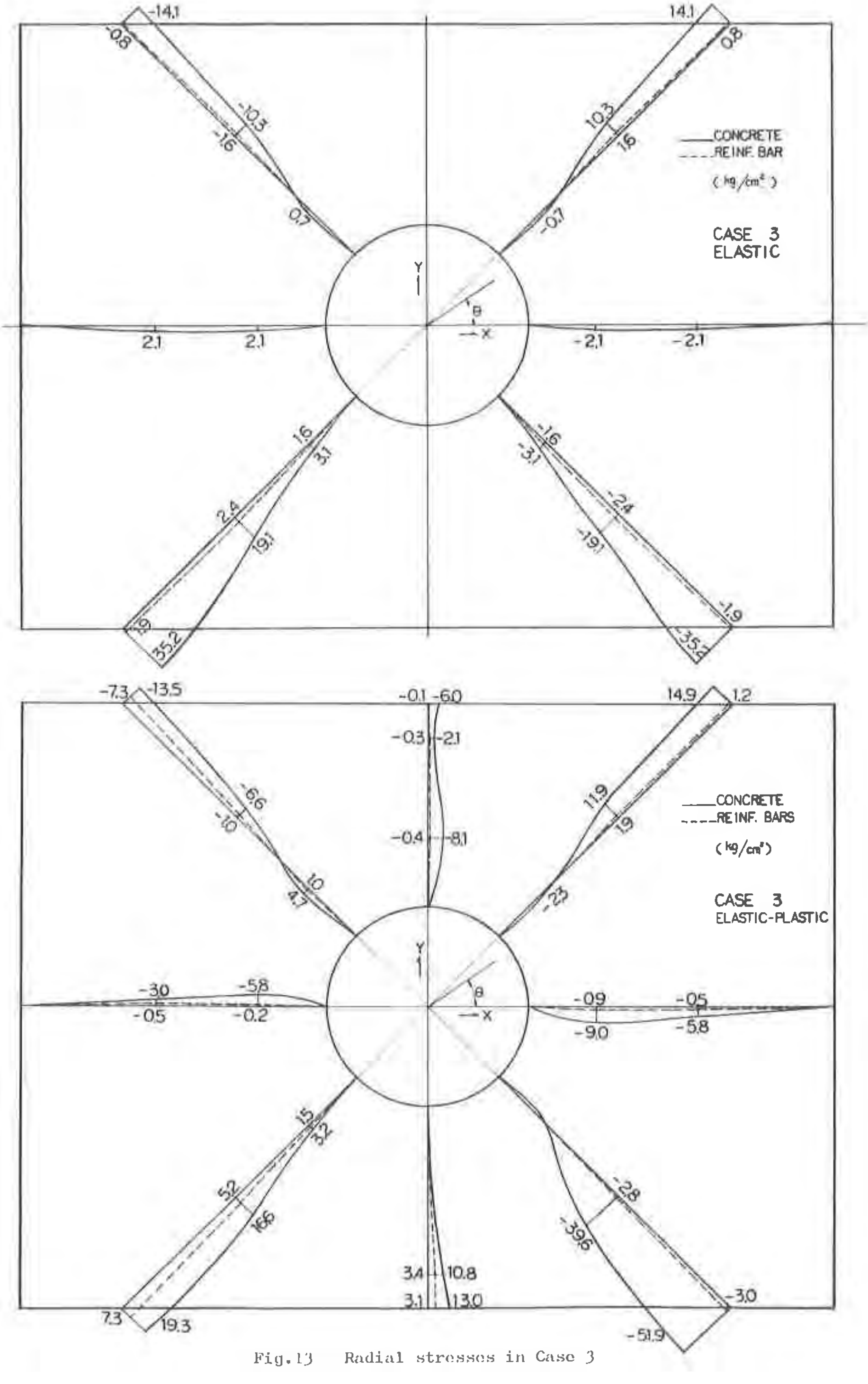

Fig.13 Radial stresses in Case 3

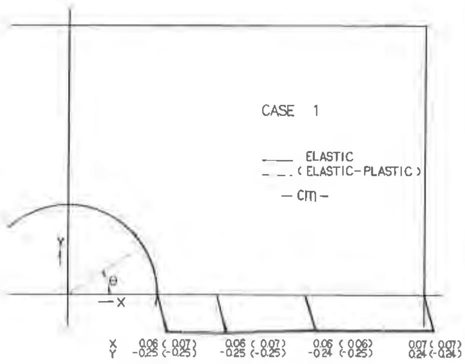

Fig.14 Displacements in Case 1

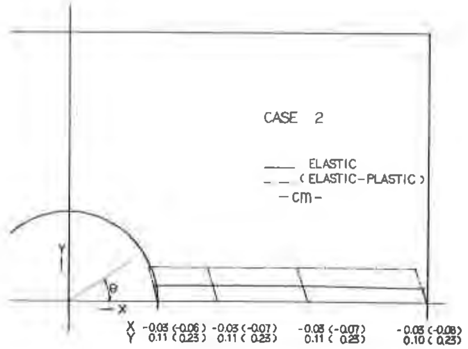

Fig.15 Displacement in Case 2

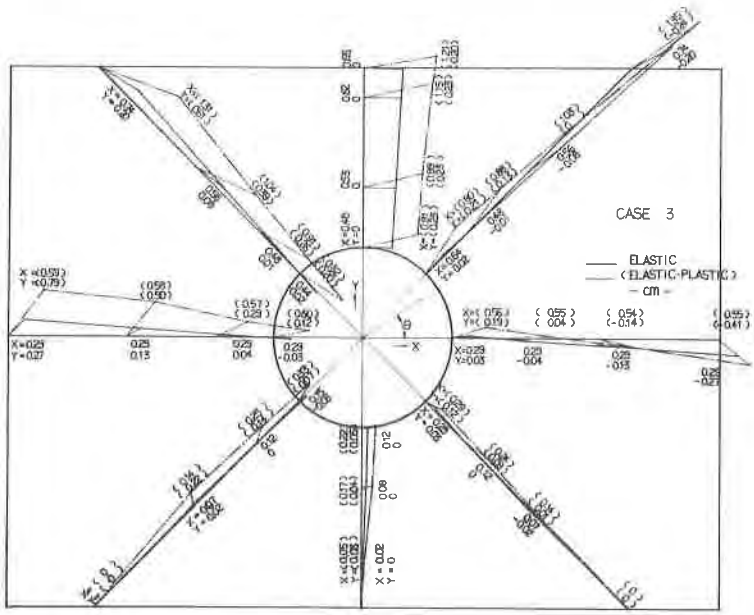

Fig.16 Displacement in Case 3

$\sigma_{es} = 39.6$ kg/cm^2 around opening on $\theta = 315^\circ$.

Compressive stresses of concrete in circumferential directions on $\theta = 0^\circ$, and 45°, and in radial direction on $\theta = 315^\circ$ both in Elastic-plastic study are larger than those of Elastic study. It is probably because of transition of stress transferring mechanism by tensile failures of concrete and bending moment due to horizontal load. Above fact is recognized from the principal stresses drawn by ploter in Fig.7. Maximum compressive stress around opening is $\sigma_c = -92.5$ kg/cm^2 on $\theta = 45^\circ$, therefore stress concentration ratio is further increased compared with that in Elastic study.

As shown in Fig.16, the displacements in Elastic-plastic study are two three times as large as those in Elastic study on $\theta = 135^{\circ}$, and 180° in which directions tensile failures of concrete occur, and on $\theta = 90^{\circ}$ in which direction large compressive principal stresses are found to be derived in Elastic-plastic study. Displacements on $\theta = 0^{\circ}$, and 45° in Elastic-plastic study in which compressive stresses are large, are almost same as those in Elastic study.

Among three cases, stress tranferring pattern of Case 3 in Elastic-plastic study is most comspicuous. By the reason that the load of Case 3 is not symmetry, the process of convergence computation is not to tend to be smoothed out and iterative steps and computing time are required about two times as much as those of other two cases (Case 1 and Case 2), as stated in chapter 4

6. Conclusion

Under the load conditions of Case 2 and Case 3, it is confirmed that displacements of major portions in Elastic-plastic study are about two times as large as those in Elastic study, and the stress distribution of concrete and rebars in Elastic-plastic study are remarkably changed from those in Elastic study.

All stresses are imposed to concrete and rebars in our computation, but in practical design they shall be imposed to rebars only at the crak points where tensile failures occur. Consequently, the maximum actual stress of rebars becomes $\sum \sigma_s = (21.1 + 39.6)/0.03 = 2{,}020$ kg/cm^2 in Case 3. As this numerical result is less than the yield point strength of rebars, this study substantiate more or less for the estimation of the quantities of rebars by the tensile stresses derived from an elastic theory.

The probable crack conditions for facilities of nuclear power plant where the displacements and crack widths shall be severely discussed will be predicted by the comparison between crack widths or numbers based on test results, and the stresses or displacements derived from Elastic-plastic study. Unfortunately we could not collect the sufficient test results of these matters for this study.

In this study uniaxial test results are applied directly to the plane problems by the assumption that tensile failure of concrete occurs only in one direction. If there would be possibility occurring tensile failures in any more directions, evaluations of tensile failure mechanism in plane problems will be necessary.

As the reinforcing bars can transfer only axial stresses, studies of shearing stress transferring mechanism in the reinfored concrete shall be taken into account in these analyses.

As much time is required in the iterative computations compared with the time in normal elastic analysis, the procedures of much more time saving in computation shall be devised.

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