TWO-DIMENSIONAL TIME-DEPENDENT ANALYSIS OF PERFORATED END CAPS FOR NUCLEAR REACTOR PRESSURE VESSELS BY THE FINITE ELEMENT METHOD

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ABSTRACT

The design of prestressed concrete pressure vessels for nuclear reactors, and in particular, the design of the end caps to the cylinders increasingly call for more sophisticated and better analytical techniques. A method is proposed in this paper to analyse the mode of action of the end slabs, assuming constant elevated temperature and radial compression. These simplified assumptions represent reasonably realistic operating conditions. If required, more complex operating conditions can be incorporated. A special feature of the proposed analysis is the method by which an elastic solution can be modified to take account of the time-dependent effects such as creep and shrinkage at ambient and under elevated temperatures. The elastic solution is based on the two-dimensional finite element formulation using triangular plate elements. After obtaining the initial elastic state of stress, a complete elastic, thermoelastic and time-dependent solution is achieved in a step-by-step manner, based on the "Rate of Creep" approach, and using small time intervals during which the stresses are assumed constant.

The technique was used to analyse slab models tested and reported elsewhere. It was shown that the proposed method estimates the deformational behaviour of the models with an acceptable degree of accuracy. The redistribution of stress as a result of thermal creep predicted by the method also agreed well with the cracking pattern of the models. It is also seen that the proposed method which is quite general, takes into account loading history. In addition, it has the advantage of economising in computer time.

1. STATEMENT OF THE PROBLEM AND GENERAL PROCEDURE.

The analyses of Prestressed Concrete Pressure Vessels by the Finite Element method have been mostly performed on an elastic and two-dimensional basis, with computer programmes based on either axisymmetric or plane stress assumptions. The problem of the perforated end cap zones is a two-dimensional one in practice with time-dependent effects arising from the phenomena of bi-axial thermal creep and shrinkage of the material under the sustained load and temperature.

An attempt has been made in this work to investigate the actual behaviour of the perforated zone of the end caps of the vessel for the loading conditions of uniform elevated temperature and radial compression, by developing a method of analysis capable of forecasting not only the elastic stresses, but also the thermal, creep and shrinkage effects.

Two requirements are necessary for the proposed method of analysis to be successful. The first requirement is that we must have an elastic solution capable of being repeated at every stage at which the stress distribution is required for the mechanical or thermal loads. The second requirement is that the method should provide for the time-dependent thermal creep and shrinkage strains to be incorporated in the analysis.

The method of analysis adopted in this work is the finite element displacement method for two dimensional stress analysis. Using an appropriate matrix formulation for linear solutions, the method will supply an answer to the first requirement. The second requirement may also be satisfied by extending the finite element technique into the non-linear range, using a step-by-step procedure. The "Rate of Creep" is used to incorporate the laws governing the stress-strain-time relation of concrete in the analysis. Shrinkage strains are also incorporated in a similar manner.

The creep strains occurring at any point of the structure, may be considered proportional to the stress, and are affected by the temperature in such a manner as prescribed by a "specific thermal creep" law. The shrinkage is assumed uniform at all points of the structure.

To analyse models of the geometry shown in Fig. 1 the triangular two-dimensional element was adopted because of its ability to represent structures with irregular configurations.

The proposed method of analysis was used to examine the behaviour of three model specimens which were subjected to thermal and external load as reported elsewhere, G.D.STEFANOU [1], G.D.STEFANOU et al. [2]. The strain at selected points of the model specimens was measured at various stages and these were compared with those calculated by the proposed method of analysis. In this respect, the structural systems were sub-divided into an assemblage of discrete elements as shown in Fig. 2. The analysis was performed on a 90° symmetrical quadrant for each of the three loaded specimens. Accordingly, the structural idealization for the three specimens was as follows:- (i) Model 1-2. Ninety-four elements and 60 nodal points. (ii) Model 3-4. Two hundred and twenty-one elements and 146 nodal points Fig. 2. (iii) Model 9-10. Three hundred and three elements and 192 nodal points. Ninety-six of the elements represented the stand-pipe liners.

The radial load acting on the boundary of the loaded specimens was represented by statically equivalent concentrated forces acting at the nodal points of the boundary of the Finite Element system. The Cartesian co-ordinate system was used to represent the geometry of the elements in the orthogonal system adopted both for local and global axes of reference.

The development of the stiffness matrix for a typical triangular element is based on the principle of virtual displacements. From these considerations, the stiffness of the element is established resulting in a matrix equation for each element relating the nodal forces, F, to the nodal displacements, d, in terms of the stiffness coefficients k_{ij}, (i, j = 1,2,3).

$$F = k.d \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

The stiffness matrix K of the structural system was obtained from a direct stiffness approach by superimposing the stiffness contributions of all the elements to each nodal point of the system. From this operation an equilibrium equation results for the complete structure which relates the externally applied nodal forces R, and the nodal displacement d, resulting therefrom.

$$R = K \cdot d \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Known force and displacement boundary conditions were introduced as non-zero entries in the force and displacement vectors respectively.

The specimens under test were subjected to long-term radial compression and uniform elevated temperature. The temperature creates additional thermal strains in the structure which, under certain conditions, will cause thermal stresses to develop. In a similar manner, the sustained loading coupled with the additional thermal load, will create additional time-dependent strains and stresses. All these effects were incorporated in the analysis by calculating an additional load vector which was in turn, added to the externally applied loads to form the resultant load vector.

In the present development, the solution of the equations was performed by the direct methods of matrix inversion and direct elimination. The Gaussian elimination approach modified for the banded matrix was applied for the analysis of the perforated and composite specimens. In storing the matrix K, the terms within the band-width for a given row, were shifted to the extreme left, and a rectangular matrix resulted with rows equal in number to the rows of the original matrix, but with only as many columns as the terms in the band-width. The saving in storage capacity was substantial.

1.1 Matrix formulation.

The theoretical basis and technique for two-dimensional analysis has been described elsewhere. However, a brief description of the procedure is presented here with special reference to this work.

a) Type of stress: Plain stress

The basic equations for the plain stress-strain relations for isotropic materials are stated here to facilitate subsequent discussions.

b) Strains:

$$\begin{aligned} e_x &= \frac{1}{E}(\sigma_x - v\,\sigma_y) && \text{(a)} \\ e_y &= \frac{1}{E}(\sigma_y - v\,\sigma_x) && \text{(b)} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3) \\ e_{xy} &= 2\,\frac{(1+v)}{E}\,\sigma_{xy} && \text{(c)} \end{aligned}$$

And in matrix form: $e = D^{-1} . \sigma$ .... (4)

c) Stresses: Solving equations (3) for stress we obtain:

$$\sigma = D . e \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Where D = the elasticity matrix $= \dfrac{E}{(1-v^2)} \begin{bmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & \frac{1-v}{2} \end{bmatrix}$

A typical element is shown in Fig. 3 with local co-ordinates (x, y), and corresponding displacements (u, v), for each of the three nodes of the triangle. The global co-ordinates and corresponding displacements are represented by (X, Y), and (U, V) respectively. Assuming linear displacements functions within the element and constant stress distribution the element strains may be obtained as a function of nodal point displacements d.

$$e = B \cdot d \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (6)$$

The stress-strain relation for an isotropic Hookean material may be written as:-

$$\sigma = D\,(e - e_o) \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (7)$$

Or, $$\sigma = D \cdot B \cdot d - D \cdot e_o \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (8)$$

Where e_o is the initial strain, and B is the displacement transformation matrix.

2. ELASTIC SOLUTION.

An initial calculation was carried out and the elastic stresses and strains resulting from the instantaneous applied radial compression were obtained. The results obtained from this solution formed the starting point for the step-by-step solution in the creep phase of the problem.

2.1 Sign convention: Tension loads are assumed to be positive, and therefore, their displacements, stresses and strains are positive. Strains produced from a rise in temperature are assumed to be positive, and those produced from a fall in temperature are negative. Shrinkage strains are negative.

2.2 Mechanical loads: The radial load P_i applied at the boundary of the loaded specimens was replaced by concentrated forces Ri, acting at the nodal points in the boundary.

2.3 Matrix equations: The following equations were needed for the elastic solution:-

a) Element strains: $e = B \cdot D$

b) Element stresses: $\sigma = D \cdot B \cdot d - D \cdot e_o$

c) The element stiffness matrix: $k = B^T \cdot D \cdot B \cdot t \cdot A \quad \ldots \ldots \ldots \ldots (9)$

Where t = thickness of the element, and A = area of element.

d) The equilibrium equations for the element: $F = k \cdot d$

e) The equilibrium equations for the structure: $R = K \cdot d$

3. THERMOELASTIC SOLUTION FOR TWO-DIMENSIONAL STRESS ANALYSIS.

If a structure is operating at elevated temperatures, the determination of the thermal stresses plays an important part in the design, and the stress-strain equations must therefore, include the effects of temperature.

For the purposes of this investigation the concrete was assumed to be an isotropic and elastically homogenous material. If an isotropic body is subjected to a uniform temperature change, it undergoes a uniform expansion without any angular distortions. In an unrestrained element, the thermal strains may be expressed as:-

$$e_{t_{ij}} = d_{ij} \cdot a \cdot d \cdot T \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (10)$$

Where i, j = x, y, z, d_{ij} = The Kronecker delta, dT = temperature rise, and a = the coefficient of linear thermal expansion.

The total strain at each point of a heated element consists of two parts: a) the thermal strain $e_{t_{ij}}$ due to uniform thermal expansion, and b) an elastic strain e_{ij} which is required to maintain the displacement continuity of the element subjected to non-uniform temperature distribution. If the element is subjected to a system of external loads, the elastic strain e_{ij} described above will also include strains arising from such loads. Therefore, the total strain, e_T, in the element can be expressed as:-

$$e_T = e_{ij} + e_{tij} \quad \ldots\ldots(11)$$

Substituting equation (10) in equation (11) we obtain:-

$$e_T = e_{ij} + a.dT_{ij} \quad \ldots\ldots(12)$$

Substituting now equations (3) into equation (12) the total strains for the element (elastic and thermal) is obtained.

$$e_T = D^{-1}.\ \sigma + DT \quad \ldots\ldots(13)$$

Similarly solving for stresses we obtain:-

$$\sigma_T = D\ .\ e_T - \underline{DT} \quad \ldots\ldots(14)$$

Where DT and $\underline{DT}$ are the thermal strain and stress matrices respectively.

$$DT = a.dT\begin{bmatrix}1\\1\\0\end{bmatrix},\ \underline{DT} = \frac{E.a.dT}{1-v}\begin{bmatrix}1\\1\\0\end{bmatrix}$$

The effects of temperature may be included in the analysis in two ways:- a) By the method of "restraint", also known as the "freezing technique", or b) By the "direct formulation" of the thermal load vector.

In this paper both methods were used. The two methods are identical in principle, the only difference being in the steps followed:-

3.1. The method of restraint.

The steps of this method may be summarized as follows:

a) assume the structure to be restrained against temperature movements by a suitable force system, and evaluate the necessary forces. b) Apply to the same structure without any temperature change the reverse of the system of restraining forces found in step (a).

c) Superpose the displacements and stresses obtained from step (b), on step (a).

3.2 APPLICATION OF THE METHOD TO THE FINITE ELEMENT TECHNIQUE.

Consider a triangular element subjected to a temperature rise dT. (dT = positive for an increase and negative for a decrease in temperature). The free thermal strain in the element is equal to:- $e_t^f = +DT$.

Step 1: Freezing stage:-

a) For the element:

Strain in the element. $e_{tf} = -DT$

Stress. $\sigma_{tf} = D.e_{tf}$

Displacement. $d_{tf} = 0$

Nodal forces on the element. $F_{tf} = (B^T.D.e_{tf}.t.A)$

b) For the structure:

Thermal load vector. $F_{tfs} = \sum (B^T.D.e_{tf}.t.A)$

Where σ_{tf} = thermal stress in the freezing stage. e_{tf} = thermal strain in the freezing stage. d_{tf} = thermal displacement in the freezing stage. t = thickness of element. A = area of element. B^T,D,e_{tf} = as previously defined.

Step 2: Releasing stage:-

a) For the element:

Nodal forces to balance freezing forces: $F_{tr} = -F_{tf}$

$\therefore F_{tr} = -(B^T.D.e_{tf}.t.A)$

b) For the structure:

Thermal load vector. $F_{trs} = \Sigma-(B^T.D.e_{tf}.t.A)$ ....(15)

Equilibrium equations. $F_{trs} = K.d_{tr}$ ....(16)

Displacements. $d_{tr} = K^{-1}.F_{trs}$ ....(17)

Strains. $e_{tr} = B.d_{tr}$ ....(18)

Stresses. $\sigma_{tr} = D.e_{tr} = D.B.d_{tr}$ ....(19)

Step 3: Releasing + Freezing stage (Superposition).

Total displacements. $d_T = d_{tf} + d_{tr}$

Total strains. $e_T = e_t^f + e_{tf} + e_{tr}$

Total stresses. $\sigma_T = \sigma_{tf} + \sigma_{tr}$

3.3. The "direct formulation".

In this approach the thermoelastic effects can be handled by introducing fictitious body and boundary forces at the nodal points of the finite element system. The fictitious nodal point loads for the free element are calculated from the normal stresses required to give the same displacements as the known temperature gradient. These loads constitute the thermal load vector for the element.

Having defined the elemental thermal load vector, the next step is to form the thermal load vector for the entire structure. This is done by adding the contributions of all elements to the nodes which are directly connected. For these loads, the structure is analysed, and the actual displacements and strains obtained. With regard to the stress distribution, the normal stresses used to simulate the thermal effects should be subtracted from the calculated stresses in the elements.

The thermoelastic equations are:-

Nodal forces for the element. $F_{te} = B^T.D.e_t^f.t.A$ ....(20)

Load vector for the structure. $F_{ts} = \sum (B^T.D.e_t^f.t.A)$ ....(21)

Equilibrium equations. $F_{ts} = K.d_t$ ....(22)

Displacements. $d_t = K^{-1}.F_{ts}$ ....(23)

Strains. $e_t = B.d_t$ ....(24)

Stresses. $\sigma_t = D.B.e_t - D.e_t^f$ ....(25)

4. TIME-DEPENDENT SOLUTION.

In this section the time-dependent effects arising from the creep and shrinkage characteristics of the concrete are incorporated in the analysis.

4.1 Creep effects:

The rate of creep method was applied to the problem of this investigation. The time-dependent solution is non-linear and requires either an iterative or a step-by-step approach. The method of solution adopted here is a step-by-step technique, which has been programmed for general elastic and time-dependent analyses. In the present work the non-linearity is incorporated in the analysis by introducing the governing laws which are based upon the appropriate characteristic properties of the material and in particular by utilizing the creep laws which govern the stress-strain-time behaviour of the concrete. If the creep strains obtained from loaded and uniformly-heated specimens are made specific with respect to stress and temperature, and plotted against a time base, a single curve is obtained known as the "specific thermal creep" curve. The rate of creep strain for uniaxial stress is given by equation

$$\frac{de_c}{dt} = \sigma.\phi\,(T).\frac{dc}{dt} \qquad \text{.......(26)}$$

This equation indicates that for an increment in the rate of specific thermal creep during a time-interval dt, the rate of free creep strain can be calculated provided that the stress σ at the start of the time-interval is known.

Let e_c be the increase in free creep strain during the time— interval t, σ the stress at the start of the time-interval, and c, the specific thermal creep strain increment during the same interval of time. From the foregoing the following relation must hold:-

$$e_c = \sigma.\phi\,(T).c \qquad \text{.......(27)}$$

The incremental creep strain so obtained at the end of the time-interval is considered as "initial strain", and an elastic solution carried out for the interval will give the state of stress σ at the end of this interval. For biaxial state of stress the incremental free creep strains for the element are:-

$$\frac{de_{cx}}{dt} = (\sigma_x - v_c\sigma_y)\,\phi\,(T)\,\frac{dc}{dt} \qquad (a)$$

$$\frac{de_{cy}}{dt} = (\sigma_y - v_c\sigma_x)\,\phi\,(T)\,\frac{dc}{dt} \qquad (b) \qquad \text{.......(28)}$$

$$\frac{de_{cxy}}{dt} = 2(1 + v_c)\,\sigma_{xy}\,\phi\,(T)\,\frac{dc}{dt} \qquad (c)$$

In these equations v_c is the creep Poisson's ratio.

The resultant creep strain at the end of the interval is therefore given by:-

$$e_c = DC.\sigma \qquad \text{.......(29)}$$

Where $e_c = (e_{cx}, e_{cy}, e_{cxy})$ = The creep strain vector in the element.

σ = total stresses at the start of the time- interval, and

$$DC = \phi(T)\ dc \begin{bmatrix} 1 & -v_c & 0 \\ -v_c & 1 & 0 \\ 0 & 0 & 2(1+v_c) \end{bmatrix}$$

= the specific thermal creep matrix.

The total strain e_T in the free element will be the sum of the elastic e_e, thermal e_t and creep e_c strains.

$$e_T = e_e + e_t + e_c$$

and in matrix form $e_T = D^{-1}.\sigma_e - DT + DC.\sigma_e$ ....(30)

With the stress, strain and the creep matrix defined for the free element, it is now possible to proceed with the evaluation of the creep load vector for the elements and for the entire structure. The creep effects may be incorporated in the finite element matrix equations in a similar manner as described for the thermoelastic effects, using the "Method of restraint" or the "Direct formulation" procedures.

4.2. Matrix equations for creep effects by the "Method of restraint".

Free creep strain $e_c^f = + DC.\sigma$ (Stress with proper sign)

Step 1: Freezing stage:-

a) For the element:

Creep strain. $e_{cf} = - DC.\sigma$

Stress. $\sigma_{cf} = D.e_{cf}$

Displacement. $d_{cf} = 0$

Nodal forces on element. $F_{cf} = B^T.D.e_{cf}.t.A$

b) For the structure:

Creep load vector. $F_{cfs} = \sum(B^T.D.e_{cf}.t.A)$

Step 2: Releasing stage:-

a) For the element:

Nodal forces to balance freezing forces: $F_{cr} = -F_{cf}$

$\therefore F_{cr} = -(B^T.D.e_{cf}.t.A)$

b) For the structure:

Creep load vector F_{crs}: $F_{crs} = \sum -(B^T.D.e_{cf}.t.A)$ . . . . (31)

Where B^T, D, t and A as previously defined.

Equilibrium equations. $F_{crs} = K.d_{cr}$ . . . . (32)

Displacements. $d_{cr} = K^{-1}.F_{crs}$ . . . . (33)

Strains. $e_{cr} = B.d_{cr}$ . . . . (34)

Stresses. $\sigma_{cr} = D.e_{cr} = D.B.d_{cr}$ . . . . (35)

Step 3: Releasing + Freezing stage (superposition).

Total displacements $d_c = d_{cf} + d_{cr}$

Total strains $e_c = e_c^f + e_{cf} + e_{cr}$

Total stresses $\sigma_c = \sigma_{cf} + \sigma_{cr}$

4.3. Matrix equations for the creep effects by the "Direct formulation" approach.

a) Nodal forces for the element. $F_{ce} = B^T.D.e_c^f.t.A$

b) Load vector for the structure. $F_{cs} = \sum(B^T.D.e_c^f.t.A)$ . . . . (36)

Equilibrium equations. $F_{cs} = K.d_c$ . . . . (37)

Displacements. $d_c = K^{-1}.F_{cs}$ . . . . (38)

Strains. $e_c = B.d_c$ . . . . (39)

Stresses. $\sigma_c = D.B.e_c - D.e_c^f$ . . . . (40)

4.4. Shrinkage effects.

It is well established that shrinkage effects are usually ignored for structures operating at ambient temperatures.

For structures operating at elevated temperatures, such as the prestressed concrete pressure vessels, shrinkage strains must be treated as an additional loading.

Although it was impossible to determine the distribution of shrinkage throughout the model structures it was possible to assess the effects of shrinkage from unstressed models of identical proportions. The shrinkage data obtained were incorporated into the analyses in the form of initial strains in a similar manner to the inclusion of thermal and creep effects.

Let e_s represent the shrinkage strains. The strains for the typical element due to shrinkage are:

$$e_s = \begin{bmatrix} e_{sx} \\ e_{xy} \\ e_{sxy} \end{bmatrix} = f(T).ds \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \quad \ldots\ldots\ldots\ldots (41)$$

Where $f(T)\ ds \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = DS$ = the shrinkage strain matrix, $f(T)\ ds$ = the shrinkage function and ds = The incremental shrinkage strain for the time- interval considered, and is obtained from the shrinkage time-strain graphs (ds with proper sign).

Only normal shrinkage strains exist, and the shear strains are zero.

For the free element the total strains due to mechanical loads, thermal, creep and shrinkage effects are:-

$$e_T = e_e + e_c + e_t + e_s$$

or $$e_T = D^{-1}.\sigma_e + DT + DC.\sigma_e + DS \quad \ldots\ldots\ldots\ldots (42)$$

4.5. Matrix equations for shrinkage effects by the "Method of Restraint".

Free shrinkage strain e_s^f: $e_s^f = DS$

Step 1: Freezing stage.

a) For the element.

Frozen shrinkage strain. $e_{sf} = -DS$

Stress. $\sigma_{sf} = D.e_{sf}$

Displacement. $d_{sf} = 0$

Nodal forces on element. $F_{sf} = B^T.D.e_{sf}.T.A$

b) For the structure.

Shrinkage load vector. $F_{sfs} = \sum (B^T.D.e_{sf}.t.A)$

Step 2: Releasing stage.

a) For the element.

Nodal forces to balance freezing forces. $F_{sr} = -F_{sf}$

$\therefore F_{sr} = -(B^T.d.e_{sf}.t.A)$

b) For the structure.

Shrinkage load vector. $$F_{sfs} = \sum - (B^T.D.e_{sf}.t.A) \quad \ldots\ldots\ldots\ldots (43)$$

Equilibrium equations for the structure. $$F_{srs} = K.d_{sr} \quad \ldots\ldots\ldots\ldots (44)$$

Displacements. $$d_{sr} = K^{-1}.F_{srs} \quad \ldots\ldots\ldots\ldots (45)$$

Strains. $$e_{sr} = B.d_{sr} \quad \ldots\ldots\ldots\ldots (46)$$

Stresses. $$\sigma_{sr} = D.e_{sr} = D.B.d_{sr} \quad \ldots\ldots\ldots\ldots (47)$$

Step 3: Releasing + freezing stage (superposition).

Total displacements $d_s = d_{sf} + d_{sr}$

Total strains $e_s = e_s^f + e_{sf} + e_{sr}$

Total stresses $\sigma_s = \sigma_{sf} + \sigma_{sr}$

4.6 Matrix equations for shrinkage effects by the "Direct formulation" approach.

Nodal forces for the element. $F_{se} = B^T.D.e_s^f.t.A$

Load vector for the structure. $F_{ss} = \sum(B^T.D.e_s^f.t.A)$ ....(48)

Equilibrium equations. $F_{ss} = K.d_s$ ....(49)

Displacements. $d_s = K^{-1}.F_{ss}$ ....(50)

Strains. $e_s = B.d_s$ ....(51)

Stresses. $\sigma_s = D.B.e_s - D.e_s^f$ ....(52)

5. APPLICATION OF THE PROPOSED METHOD IN ANALYSING THE TEST SPECIMENS.

The proposed method of analysis was used to predict the behaviour of the test specimens in the following stages:-

i) At ambient temperatures: a) Elastic solution. b) Sustained loads with creep and shrinkage effects.

ii) At elevated temperatures: a) Sustained loads with thermal, creep and shrinkage effects.

iii) At ambient temperatures (after cooling). a) Sustained loads with creep and shrinkage effects. b) Loads gradually removed (unloading stage).

The step-by-step procedure, was adopted and total solutions were obtained at each step of the analysis using both "the method of restraint" and "the direct method". Only the direct method is described here. Accordingly, the relevant equations to be set up and solved for the three stages are as follows:-

At time $t = t_0$: The external loads R were applied and an elastic solution obtained for displacements d_e, strains e_e, and stresses σ_e, using eqs.(2), (6) and (8). The initial strain e_o in eq.(8) is zero.

At time $t = t_1$: (time-interval dt_1) The shrinkage strains e_{os1}, and creep strains e_{oc1}, were calculated for the incremental shrinkage, d_{s1}, and specific creep, d_{c1}, strains occurring in the interval. The specific creep, d_{c1}, and shrinkage, d_{s1}, data were obtained from experimental curves, Fig. 4 and the creep and shrinkage strains e_{oc1}, and e_{os1}, so obtained are treated as initial strains at t_1, (end of interval dt_1, start of interval dt_2), and the fictitious nodal forces F_{oc1}, and F_{os1}, are calculated for each element from equations for creep and shrinkage.

Summing up the nodal forces of all the elements connected to a nodal point, the load vector for the structure is formed. Denoting F_{cs}, and F_{ss}, the creep and shrinkage load vectors respectively, and adding them to the external loads R, equation (2) becomes:-

$$R + F_{cs1} + F_{ss1} = K.d_1 \quad \text{.......................(53)}$$

The solution of the above equation gives the total displacements d_1, strains e_1, and stresses σ_1, at time t_1. In calculating the stresses σ_1, the effects of the initial strains must be included.

At time $t = t_2$: (time interval dt_2) With the stresses known at t_1, we may now proceed to the next time interval dt_2, following the same procedure as for t_1. When a solution is to be obtained at elevated temperatures, the thermal effects must be included in the analysis. This is accomplished by adopting the same procedure used to incorporate the shrinkage and creep effects. Denoting F_{ts}, the thermal load vector for the structure, the equilibrium equation (2) becomes:-

$$R + F_{cs} + F_{ss} + F_{ts} = K.d \quad \ldots\ldots (54)$$

The solution of this equation gives the total displacements, strains and stresses for the external loads creep, shrinkage and temperature effects.

6. COMPARISON OF EXPERIMENTAL RESULTS WITH ANALYTICAL VALUES.

Experimental work has been carried out on three loaded specimens of which two were perforated with steel lining and one solid as shown in Fig. 1. The schedule of testing when radial external load and uniform elevated temperature loads were applied is given in Fig. 5. To illustrate the validity of the proposed method, the observed and calculated strains at various points of the specimens through the loaded period were plotted in Fig. 6 to Fig. 9. The shrinkage and creep data used in the calculation was obtained from two control specimens of the experimental programme.

7. CONCLUSIONS.

a) From Fig. 6 to Fig. 9 it is evident that for the loading and temperature conditions of the experimental programme, the proposed method of analysis predicts satisfactorily, within practical engineering tolerance, the behaviour of perforated slabs. Distribution of stresses can be calculated from the strain in the usual manner if desired.

b) Although the proposed method was developed primarily for uniform temperature and radial loading, it can be seen that slabs with temperature cross fall and transversed loading can easily be catered for by further development of the computer programme.

8. APPENDIX.

a) Material constants used in the calculations

Crushing cube strength of concrete in psi. 5370, 6400, 7760 and 8890, at 14, 28, 56 and 90 days respectively.

Tensile strength of concrete in psi. 370 and 424 at 28 and 90 days respectively.

Modulus of elasticity of concrete at ambient temperature on commencement of the test 5.17×10^6 psi. At elevated temperature, 4.88×10^6 psi. At ambient temperature after removal of heat 4.84×10^6 psi.

Coefficient of linear thermal expansion of concrete 13.2×10^{-6} and 13.6×10^{-6} per °C at commencement and on removal of heat respectively.

Poisson's ratio of concrete $v_c = 0.18$

Modulus of Elasticity of steel $E_s = 30.0 \times 10^6$ psi. and the Poisson's ratio $v_s = 0.30$.

9. REFERENCES.

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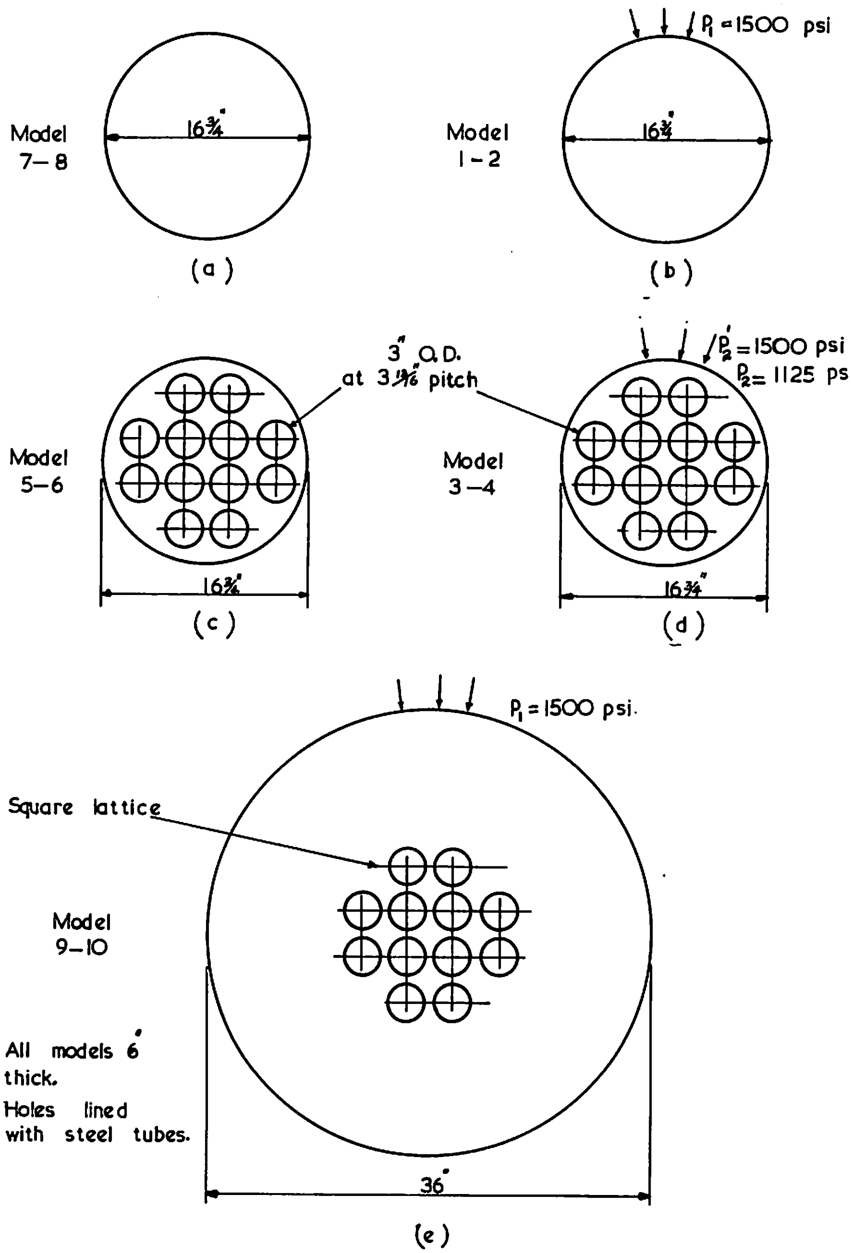

FIG. 1. TEST SPECIMENS

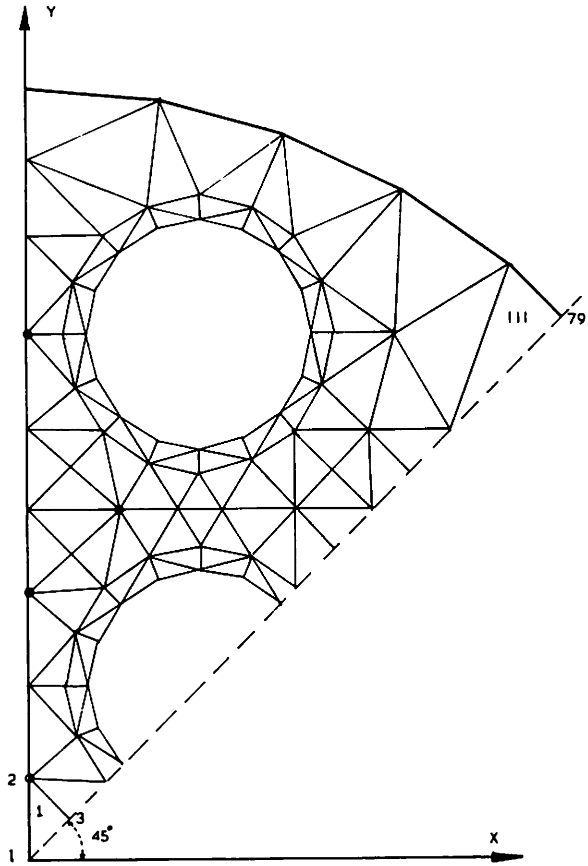

FIG.2. STRUCTURAL IDEALIZATION
FOR THE 16 3/4 DIAMETER SPECIMEN MODEL 3-4
45° SECTOR

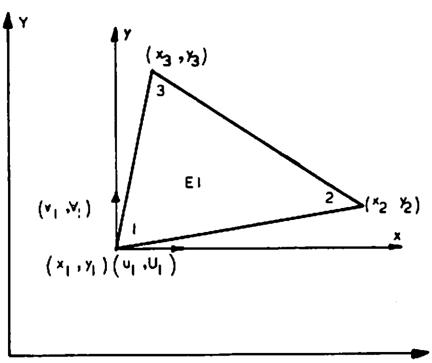

FIG.3. TYPICAL TRIANGULAR ELEMENT E1

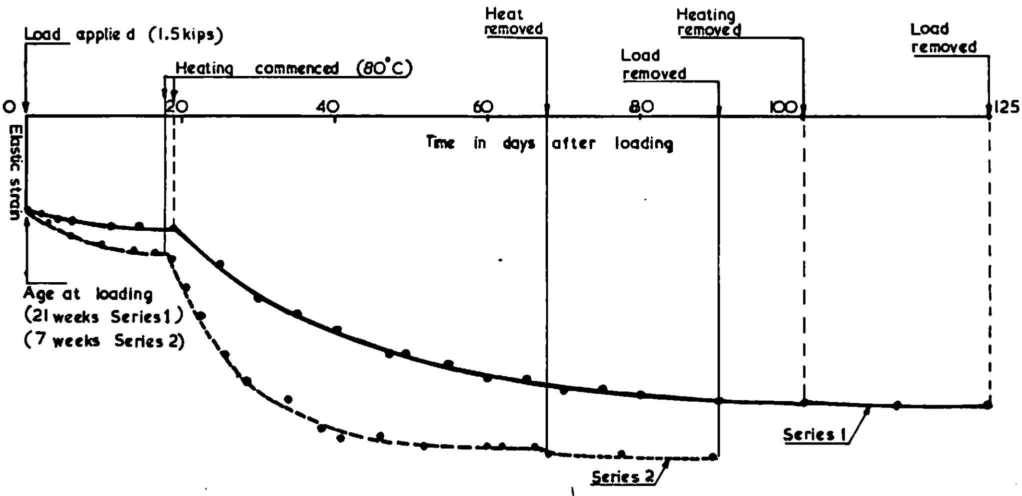

FIG. 4. SPECIFIC CREEP CURVE

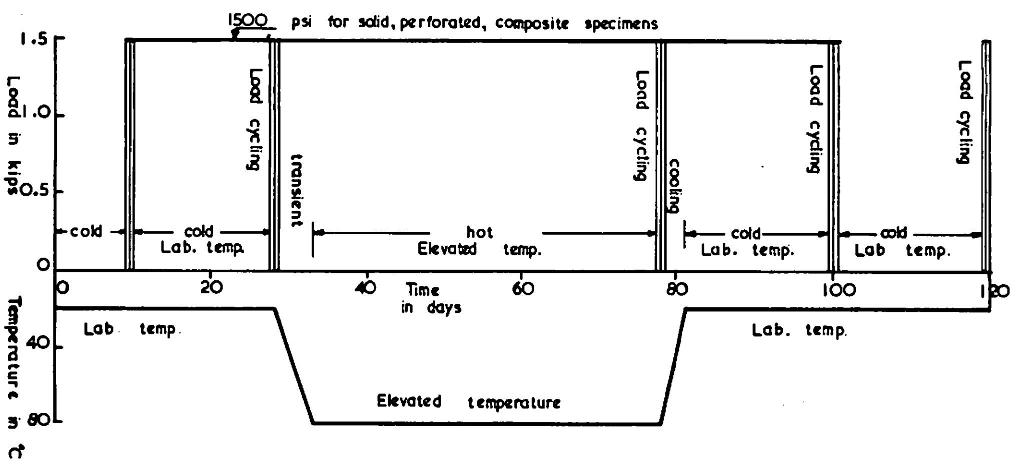

FIG. 5. SERIES 2 TEST PROGRAMME

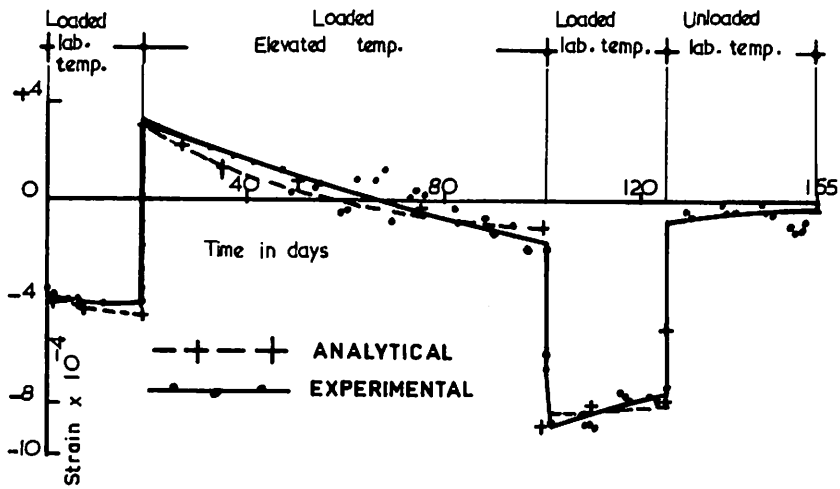

FIG. 6. SERIES I. RADIAL GROSS STRAINS MODEL 3–4

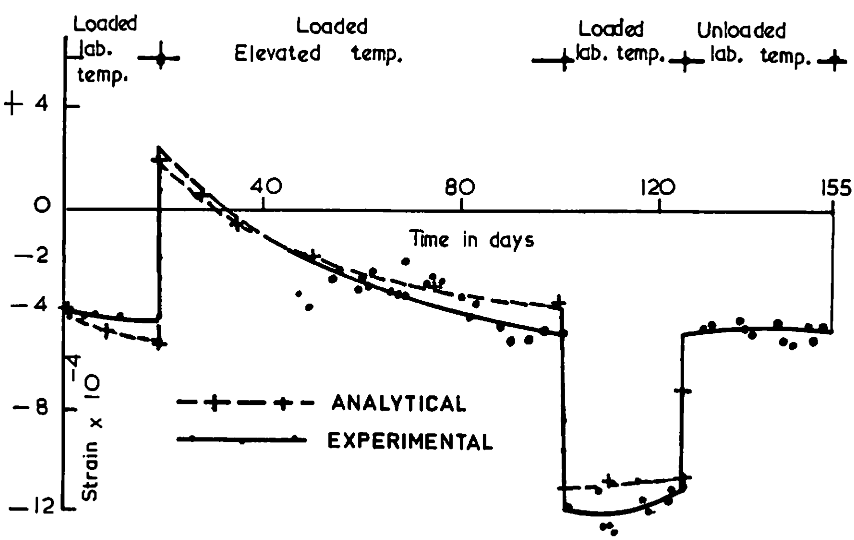

FIG. 7. SERIES I. RADIAL GROSS STRAINS
PERFORATED ZONE MODEL 9—10

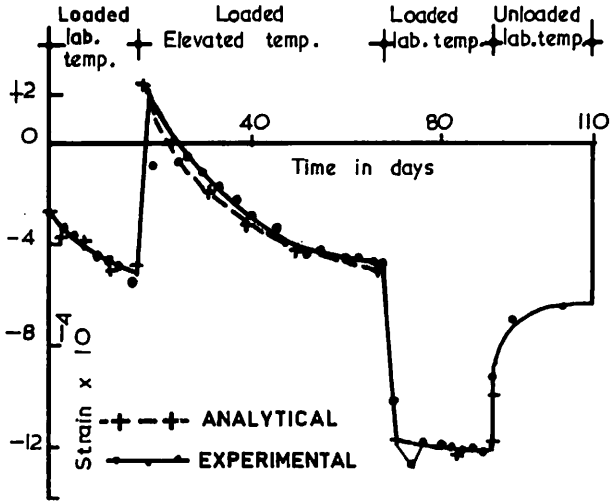

FIG. 8. SERIES 2. TANGENTIAL GROSS STRAINS
MODEL 9–10 PERFORATED ZONE

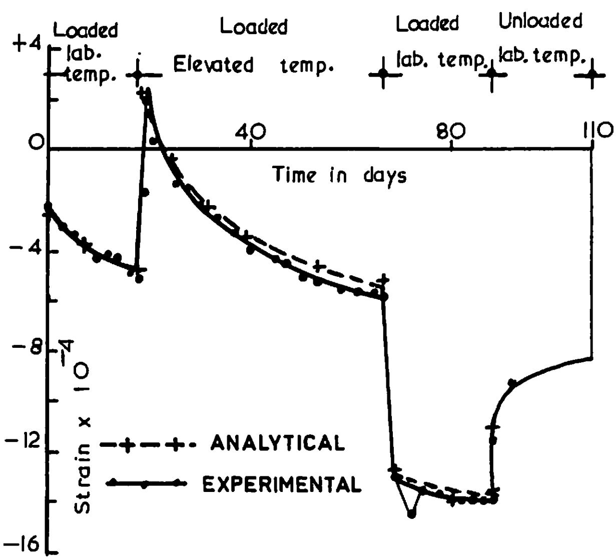

FIG. 9. SERIES 2. TANGENTIAL GROSS STRAINS
MODEL 9–10 OUTER SATELITE

DISCUSSION

C G. D. STEFANOU, U. K.

1. The mathematical formulation of the analytical procedure presented in this paper should be clear enough not to warrant any further amplification.
However we should like to give a few comments on the data used in working out various total strain-versus-time curves as mentioned in the paper when we used the proposed method to compare with the observed behaviour. You will observe that the shrinkage strains used, have been taken as uniform for all elements, at any particular time interval. This assumption seems to be justifiable, considering that the temperature distribution in the specimens tested was reasonably uniform, as seen from Figs. P1 and P2. Furthermore, the actual shrinkage strains worked out for various gauge lengths have also been found to be very similar. In view of these two facts it seems reasonable to assume uniform shrinkage distribution. The actual shrinkage for the specimens, plotted with respect to time, is shown in Figs. P3 and P4.
From the above discussion, it is implied that at any interval of time, the shrinkage strains used in the analysis were obtained from shrinkage strain-time curves drawn from direct readings on the control specimens.
Although it is recognised that in the full-scale structure shrinkage will be restricted to a skin effect only, in the cap of a reactor vessel, in the analysis of this paper proper recognition of shrinkage had to be given because of the relatively small size of the specimens tested.
2. The rate of creep method has some deficiencies when used in predicting structural behaviour. It can be used to advantage, in many instances, when adequate data are available. It is often criticized that errors will be involved in the temperature transient states. In the present case where the application of temperature took place after much of the initial creep due to prestress, had taken place, the actual error involved can be expected to be comparatively small, when specific creep data corresponding to the actual temperature variations are taken into account. (Fig. 4). This was done in the analysis of this paper. The experimental results obtained from the tested specimens compared well with the theoretical predictions.
3. The distribution of stresses at various selected points in the composite specimen have been calculated and plotted against a time basis as shown in Figs. P5 and P6. The tensile stresses indicated, on unloading were verified by the observed cracked pattern as shown in Fig. P8.
4. The experimental work referred to in the paper was carried out initially for the UKAEA at Imperial College and was continued at the City University on models with different stand-pipe wall thickness and test programme.
The results of part of the investigation had been reported in a paper by the first two authors in the conference on model techniques on PCPV in London in July 1969. The analytical approach presented in this paper concurred with the experimental findings mentioned above and partly illustrated. The set up of the experimental work is shown in Fig. P9.

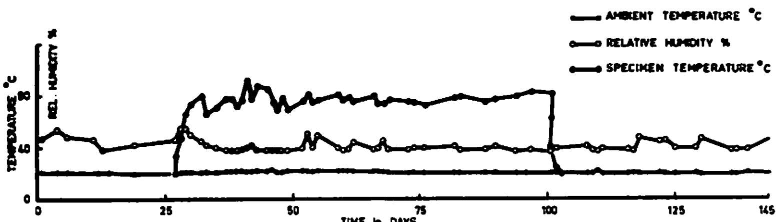

FIG. P1. SERIES 1
MODEL 5/6 TEMPERATURE & RELATIVE HUMIDITY

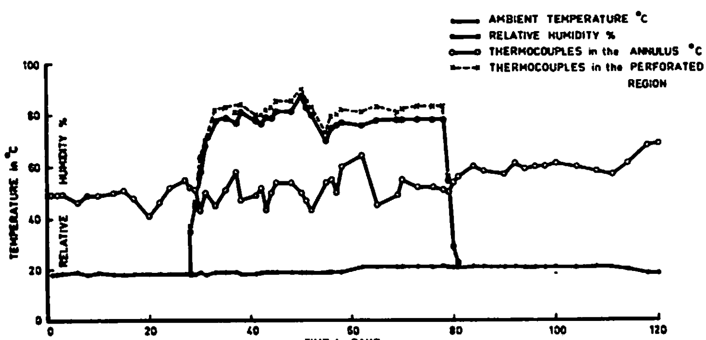

FIG. P2 SERIES 2
MODEL 9/10 COMPOSITE CREEP SPECIMEN

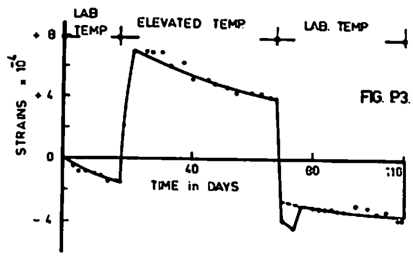

FIG. P3. SERIES 2
TANGENTIAL
SHRINKAGE &
THERMAL STRAINS
MODEL 5/6

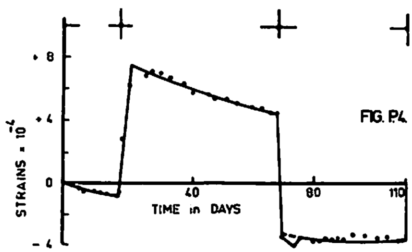

FIG. P4. SERIES 2
DIAMETER
SHRINKAGE &
THERMAL STRAINS
MODEL 7/8

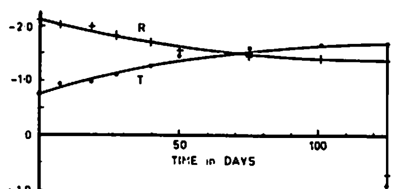

FIG. P5. SERIES 1:
TANGENTIAL & RADIAL STRESSES - R.C. METHOD
POSITION 4 - COMPOSITE SPECIMEN

COMPOSITE SPECIMEN

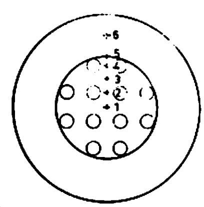

FIG. P7.

This FIGURE INDICATES POSITIONS where the TANGENTIAL and RADIAL STRESSES were drawn as PREDICTED by the RATE of CREEP METHOD.

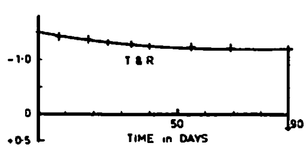

FIG. P6. SERIES 2:
TANGENTIAL & RADIAL STRESSES - R.C. METHOD
POSITION 1 - COMPOSITE SPECIMEN

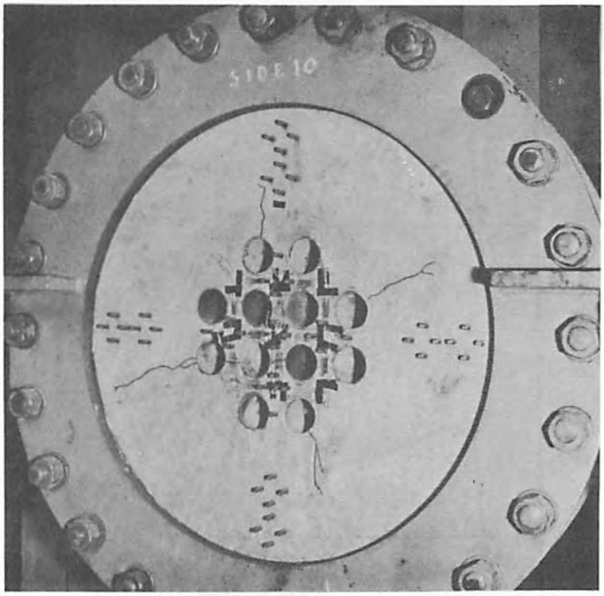

FIG. P 8

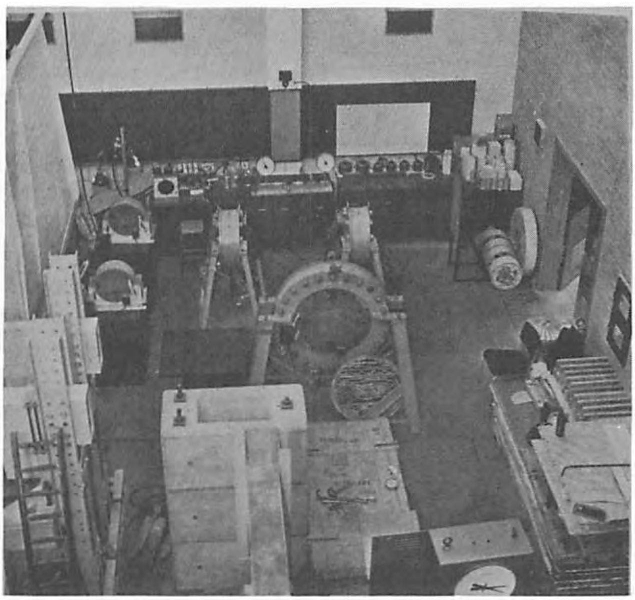

FIG. P 9