Seismic Analysis of Cantilever Retaining Walls

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ABSTRACT

A linear analysis approach for seismic response of retaining walls (Scott, 1973) has been generalized by incorporating wall flexibility in a direct yet simple manner. The cantilever wall is treated as an Euler-Bernoulli beam connected to the soil backfill, modelled by a shear beam, through Winkler springs. The response is obtained by modal superposition. An example problem has been worked out.

INTRODUCTION

Seismic response of earth retaining structures, e.g., basement walls of nuclear reactor facilities, involves a complex soil - wall interaction problem. However, retaining walls are usually designed by the static limit state approach of Mononobe-Okabe. This method has several shortcomings (e.g., Hadjian, 1978; Nazarian and Hadjian, 1979). For instance, it does not consider dynamic characteristics of the wall, backfill and the ground motion. As an alternative, linear elastic solutions to the dynamic earth pressure problem have been proposed (e.g., Matuo and Ohara, 1960; Wood, 1973; Scott, 1973; Tajimi, 1973; and Yeh, 1976). However, these solutions treat the wall as a rigid body. Scott (1973) treats the soil as a one - dimensional shear beam which is attached to the rigid wall by Winkler springs representing soil - wall interaction. This results in a fairly simple analysis approach. Solution is also given for a particular soil model while incorporating rigid body rotation of the wall by introducing a torsional spring at the base. Yeh (1976) has used Scott's model while allowing the wall to undergo rigid body translation and rotation. He solves the resulting system of coupled integro-partial differential equations by the Galerkin's method.

In the present paper Scott's model has been extended to incorporate the wall flexibility (bending deflections) in a direct yet simple manner. In fact the procedure can also conveniently account for rotation and translation of the wall at its base.

ANALYTICAL MODEL

Fig. 1 shows the cross-section of a cantilever retaining wall with a horizontal backfill and natural soil. The wall is to be analyzed for known input motion at the base of the wall. The proposed model (Fig. 2) treats the wall as Euler - Bernoulli beam, the soil backfill as a shear beam and the interaction between the wall and the backfill is modelled by Winkler springs. The soil backfill, being of infinite extent, is assumed to respond to base motion as a shear beam independent of the wall. Thus, the wall is vibrated by (i) the input motion acting at its base, and (ii) the shear beam which itself is being excited by the base motion.

Let $u(x,t)$ be the lateral displacement of the wall; $v(x,t)$ the lateral displacement of the shear beam; $K(x)$ the stiffness of Winkler springs per unit height and length of wall; $EI(x)$ the wall stiffness per unit length; $m(x)$ the mass of retaining wall, including added mass of a part of backfill vibrating with the wall, per unit height and length; h the height of the wall; and $\ddot{u}_g(t)$ the earthquake ground acceleration at the base of the wall, assumed not to vary spatially at the base level of wall. The equation of motion for the wall is

$$\frac{\partial^2}{\partial x^2}\left[EI(x)\frac{\partial^2 u(x,t)}{\partial x^2}\right] + m(x)\frac{\partial^2 u(x,t)}{\partial t^2} + K(x)u(x,t) = -m(x)\ddot{u}_g(t) + K(x)v(x,t) \qquad (1)$$

Solution of homogeneous part of Eq. (1) gives natural frequencies and mode shapes of the wall. Rigid body translation and rotation of the wall may be accounted for by introducing translational and rotational springs at the base. This affects only the boundary conditions and hence the natural frequencies and mode shapes; the analysis approach being otherwise the same. Forced vibration solution is then obtained by combining (i) response due to base motion, $\ddot{u}_g(t)$, with no shear beam deformation ($i.e.$, $v(x,t)=0$), and (ii) response due to vibration of the shear beam in its first few modes. The total response is then the combination of response in first few modes of the wall.

As the backfill is assumed to vibrate due to ground excitation independently of the wall, its equation of motion is

$$\frac{\partial}{\partial x}\left[G(x)\frac{\partial v(x,t)}{\partial x}\right] - \rho(x)\frac{\partial^2 v(x,t)}{\partial t^2} = \rho(x)\ddot{u}_g(t) \qquad (2)$$

where $G(x)$ and $\rho(x)$ are shear modulus and mass density of soil, respectively. Solution of this equation gives $v(x,t)$ which is needed to solve Eq. (1).

ANALYSIS FOR PROPERTIES CONSTANT WITH DEPTH

To illustrate the procedure, analysis has been carried out for wall and soil properties (EI, m, K, G, and ρ) constant with depth (x). Also, for simplicity retaining wall has been assumed as fixed at the base.

Free Vibration

Consider the homogeneous part of Eq. (1) for free vibration analysis :

$$EI\frac{\partial^4 u(x,t)}{\partial t^4} + m\frac{\partial^2 u(x,t)}{\partial t^2} + Ku(x,t) = 0 \qquad (3)$$

For the wall fixed at its base, solution of Eq. (3) gives characteristic equation as

$$\cos\ \alpha h\ \cosh\ \alpha h + 1 = 0 \qquad (4)$$

which is the same as characteristic equation for a uniform cantilever beam except that α is now defined as

$$\alpha^4 = \frac{(m\omega^2 - K)}{EI} \qquad (5)$$

where ω is natural frequency of the wall. Eq. (4) gives roots as αh = 1.875, 4.694, 7.855, etc. The mode shapes are same as those for a cantilever beam, given by

$$U(x) = A\left[\frac{\sin\ \alpha x - \sinh\ \alpha x}{\sin\ \alpha h + \sinh\ \alpha h} - \frac{\cos\ \alpha x - \cosh\ \alpha x}{\cos\ \alpha h + \cosh\ \alpha h}\right] \qquad (6)$$

where A is an arbitrary constant. If the modes are normalized to have $U(x=h)=1.0$, value of A is 1.519, -27.32, 644.5, ... in the first few modes, respectively.

Forced Vibration due to Base Motion

The equation of motion for forced vibration due to base motion $\ddot{u}_g(t)$

$$EI\frac{\partial^4 u(x,t)}{\partial x^4} + m\frac{\partial^2 u(x,t)}{\partial t^2} + Ku(x,t) = -m\ddot{u}_g(t) \tag{7}$$

may be solved by modal superposition. Let

$$u(x,t) = \sum_{i=1}^{\infty} U_i(x)T_i(t) \tag{8}$$

Substitution of Eq. (8) into Eq. (7) and application of the orthogonality condition leads to uncoupled equations of motion. Introducing damping, which will include contributions from material damping as well as radiation damping, the modal equations are

$$\ddot{T}_i + 2\omega_i\zeta_i\dot{T}_i + \omega_i^2 T_i = -P_i\ddot{u}_g(t) \qquad i = 1,2,3,... \tag{9}$$

where ζ_i is modal damping; and P_i is participation factor for the i-th mode given by

$$P_i = \frac{\int_0^L U_i dx}{\int_0^L U_i^2 dx} \tag{10}$$

For modes normalized such that $U_i(x = h) = 1.0$, value of P_i for first few modes is 1.566, -0.868, 0.509, ...

Solution of Eq. (9) is straight forward. Generally, base motion is specified by its response spectrum from which the peak value of $T_i(t)$ is directly obtained. The maximum response of wall may then be obtained by SRSS (Square Root of Sum of Square) of maximum response in the first few modes.

Forced Vibration due to Backfill

With G and ρ constant with depth, solution of homogeneous part of Eq. (2) gives natural frequencies ($\overline{\omega}_k$), mode shapes (V_k), and participation factors (β_k) for the uniform shear beam as :

$$\overline{\omega}_k = \frac{(2k-1)\pi}{2h}\sqrt{\frac{G}{\rho}} \tag{11}$$

$$V_k(x) = \sin\frac{(2k-1)\pi x}{2h} \tag{12}$$

$$\beta_k = \frac{4}{(2k-1)\pi} \tag{13}$$

Thus, maximum shear beam response in its k-th mode may be obtained as

$$v_k(x,t)_{\max\ t} = \frac{4}{(2k-1)\pi}\frac{S_a(\overline{\omega}_k,\zeta_k)}{\overline{\omega}_k^2}\sin\frac{(2k-1)\pi x}{2h} = A_k\sin\frac{(2k-1)\pi x}{2h} \tag{14}$$

where $\bar{\xi}_k$ is the damping of shear beam in its k-th mode, and $S_a(\bar{\omega}_k, \bar{\xi}_k)$ is spectral acceleration corresponding to frequency $\bar{\omega}_k$ and damping $\bar{\xi}_k$.

Shear beam motion near peak response in k-th mode may be approximated as sinusoidal with its k-th natural frequency. Hence, equation of motion of the wall due to excitation by k-th mode of the shear beam may be written as

$$EI\frac{\partial^4 u(x,t)}{\partial x^4} + m\frac{\partial^2 u(x,t)}{\partial t^2} + Ku(x,t) = KA_k \sin\frac{(2k-1)\pi x}{2h} \cdot \sin\bar{\omega}_k t \tag{15}$$

Eq. (15) also can be solved by modal superposition which gives

$$\ddot{T}_{ik} + 2\omega_i\xi_i\dot{T}_{ik} + \omega_i^2 T_{ik} = \frac{K}{m}A_k\bar{P}_{ik}\sin\bar{\omega}_k t \tag{16}$$

where $\bar{P}_{ik}$ is participation factor for i-th wall mode due to excitation by k-th shear beam mode and is given by

$$\bar{P}_{ik} = \frac{\int_0^L U_i(x)V_k(x)dx}{\int_0^L U_i^2(x)dx} \tag{17}$$

For the normalization used earlier, $\bar{P}_{11}$=1.356, $\bar{P}_{12}$=-0.392, $\bar{P}_{13}$=0.060, $\bar{P}_{21}$=-0.387, $\bar{P}_{22}$=-1.224, $\bar{P}_{23}$=0.564, $\bar{P}_{31}$=0.082, $\bar{P}_{32}$=0.561, $\bar{P}_{33}$=1.127, ...

Eq. (16) is same as equation of motion of a damped single degree of freedom system excited by a harmonic force and has a standard solution. Response in i-th wall mode may be obtained by combining response due to a few shear beam modes.

EXAMPLE

As an example, a reinforced concrete cantilever retaining wall has been analyzed by the proposed method. As contribution of higher modes of the wall and the shear beam is negligible, only the first modes have been considered. Design spectrum recommended by the U.S. Atomic Energy Commission (1973) for 10 % damping, scaled to have peak ground acceleration as 0.1 g, has been used as the horizontal ground motion input at base level of the wall. Wall and soil properties have been assumed to be constant with depth. The following properties of the wall and the soil were used in the analysis. For the wall: height (h) = 6.0 m; stiffness (EI) = 9.0 x 10^8 N-m^2/m; and mass, including added mass of soil (m) = 4500 kg/m^2. For the soil: shear modulus (G) = 3.6 x 10^7 N/m^2; Poisson's ratio (ν) = 0.4; and mass density (ρ) = 1600 kg/m^3.

Stiffness of Winkler springs (K) has been calculated by the expression (Scott, 1973)

$$K = \frac{8G\ (1-\nu)}{L\ (1-2\nu)} \tag{18}$$

where ν is the Poisson's ratio for backfill and L is the length of backfill which may be taken as $10h$ for backfills of infinite extent. This yields K = 14.4 x 10^6 N/m^3. The fundamental frequencies of the wall and the shear beam are obtained as 71.5 rad/sec and 39.3 rad/sec, respectively. Response of wall due to base excitation gives maximum base shear as 10.3 kN and maximum base moment as 44.9 kN-m. The maximum base shear and base moment in the wall due to shear beam excitation are 39.3 kN and 171.3 kN-m, respectively. Combining the two responses by SRSS, one gets design base shear and base moment as 40.6 kN and 177.0 kN-m, respectively.

If the same wall is analyzed treating it as rigid (Scott, 1973), maximum base shear and base moment are obtained as 73.7 kN and 281.5 kN-m, respectively. On the other hand, Mononobe - Okabe method, with $\alpha_h = 0.1$, $\alpha_v = 0$, $\phi = 35^{o}$, $i = 0$, $\delta = 0$, $\beta = 0$ (with usual definition of these notations), gives total dynamic active pressure as 16.0 kN.

DISCUSSION

A simple linear analytical model has been proposed for seismic analysis of flexible cantilever retaining walls. The method gives forces and moments which are substantially lower than those obtained by treating the wall as rigid. However, these are still higher than those given by the Mononobe - Okabe method. It was seen that (i) as wall stiffness is increased, forces increase; (ii) as the shear modulus of soil backfill is increased, the forces decrease; and (iii) as the mass of wall, including added mass of soil, is decreased the forces also decrease. As the variation in response due to these parameters may be substantial, proper evaluation of shear modulus and added mass of soil is quite important. In this context forced vibration tests on full - scale prototype walls may prove very useful. The response being linear at low - level vibrations that are generated by eccentric mass shakers, such tests will be of considerable value in calibrating the parameters involved in the proposed method of analysis.

REFERENCES

Hadjian, A. H., (1978). Panel on Earthquake Induced Earth Pressures: Opening Statement -- "State of the Art". Proc. Specialty Conference on Earthquake Engineering and Soil Dynamics, Pasadena, U.S.A., Vol 3, pp 1429-1435.

Matuo, M. and Ohara, S., (1960). Lateral Earth Pressure and Stability of Quay Walls During Earthquakes. Proc. 2nd World Conference on Earthquake Engineering, Tokyo-Kyoto, Japan, Vol. 1, pp 165 - 181.

Nazarian, H. N. and Hadjian, A. H., (1979). Earthquake Induced Lateral Soil Pressures on Structures, Journal of the Geotechnical Engineering Division (ASCE), Vol. 105, GT9, pp 1049-1066.

Scott, R. F., (1973). Earthquake - Induced Earth Pressures on Retaining Walls. Proc. 5th World Conference on Earthquake Engineering, Rome, Italy, Vol. 2, pp 1611 - 1620.

Tajimi, H. (1973). Dynamic Earth Pressures on Basement Walls. Proc. 5th World Conference on Earthquake Engineering, Rome, Italy, Vol. 2, pp 1560-1569.

U. S. Atomic Energy Commission (1973). Design Response Spectra for Seismic Design of Nuclear Power Plants. Regulatory Guide No. 1.60.

Wood, J. H. (1973). Earthquake - Induced Soil Pressures on Structures. Ph .D. Thesis, California Institute of Technology, EERL 73-05, Pasadena, U.S.A.

Yeh, C.-S. (1976). Dynamic Response of Retaining Walls During Earthquake. Proc. International Symposium on Earthquake Structural Engineering, St. Louis, Mo., U.S.A., pp 387 - 392.

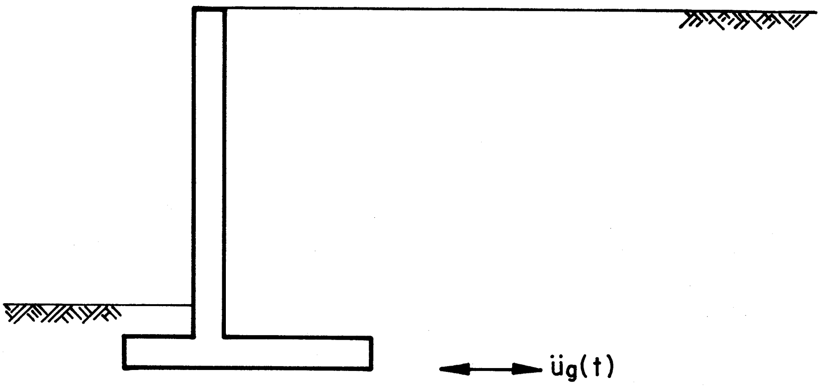

FIG. 1 CANTILEVER RETAINING WALL

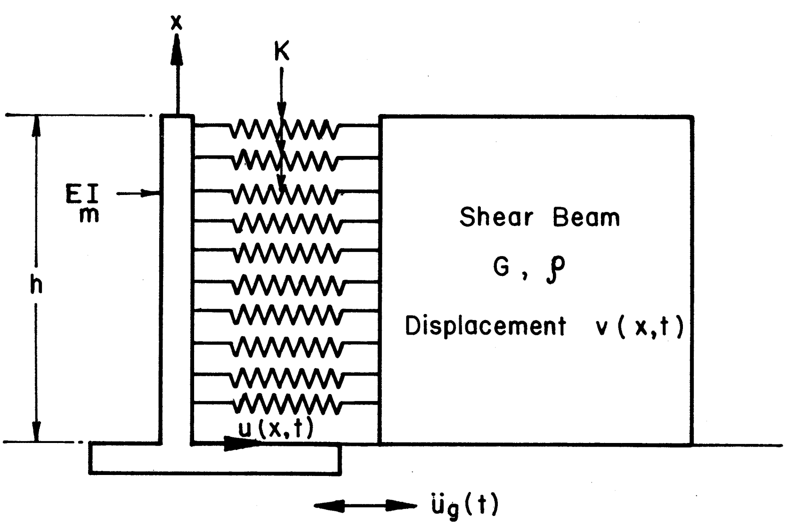

FIG. 2 ANALYTICAL MODEL