Theory, Experiment and Computation of Resistance Curve for Crack Growth

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ABSTRACT

Comparison and verification are made between the theoretical J-resistance curve for crack growing in power hardening materials and results obtained by experiments and finite element numerical calculation.

INTRODUCTION

For elastic perfectly plastic materials, Rice et al (1980) have given the theoretical J-resistance curve for mode-I crack based on the crack tip field analysis. The theoretical J-resistance curve is extended to the case of power hardening materials by Dai and Hwang (1986).

The present paper aims at a unified investigation on resistance curve by experimental, computational and theoretical approaches. The experimental, computational and theoretical resistance curves are found to agree fairly well for the tested materials. This fact indicates the great potentialities of the theoretical resistance curve in the engineering structure integrity assessment.

EXPERIMENT

Compact Tension (CT) specimens of 18CrNiWA and Three-Point Bending (3PB) specimens of 38CrMoAl are subjected to displacement-controlled loading. The mechanical properties of materials are shown in Table 1. The parameter α and n are found from the least-square fitting:

$$\frac{\varepsilon}{\varepsilon_0} = \begin{cases} \dfrac{\sigma}{\sigma_0} & \sigma \le \sigma_0 \\ \dfrac{\sigma}{\sigma_0} + \alpha\left[\left(\dfrac{\sigma}{\sigma_0}\right)^n - 1\right] & \sigma > \sigma_0 \end{cases} \tag{1}$$

to the tensile curve. The crack is fatigue precracked according to the requirments suggested by Albrecht (1982). In experiments load P versus load line

Table 1 Mechanical properties of materials

Material	$\sigma_{0.2}$(kg/mm^2)	σ_b(kg/mm^2)	E (kg/mm^2)	α	n
18CrNiWA	105.2	117.6	$2.0*10^4$	0.47	22.2
38CrMoAl	86.9	100.6	$2.2*10^4$	3.59	6.3

displacement Δ are directly measured. Elastic unloading compliance method is adopted to get the varying crack length during the slow crack growth process, by use of formula by Saxena (1978) for CT specimen and formula by Albrecht(1982) for 3PB specimen. The compliance method is checked by the actural crack lengths obtained by other method, such as fatigue cycling techniques and color paints to leave marks along the current crack front. The experiments are conducted following the requirements by Albrecht (1982). Formula by Ernst et al (1981) is used for calculating J-integral for growing crack. The typical diagrams for CT specimen of 18CrNiWA and for 3PB specimen of 38CrMoAl are given in Fig.1 and Fig.2, the counterparts of J-resistance curves are shown in Fig.3 and Fig.4, respectively.

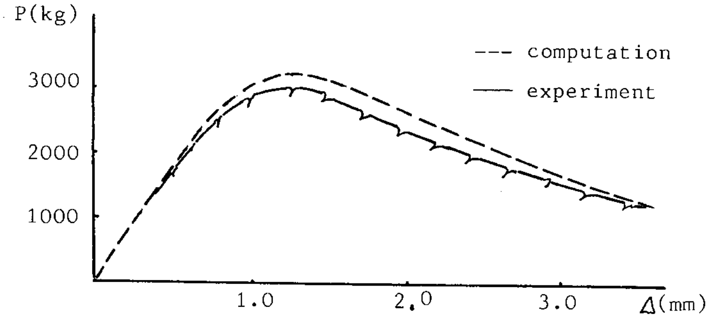

Fig.1 P versus Δ relation for 18CrNiWA (CT)

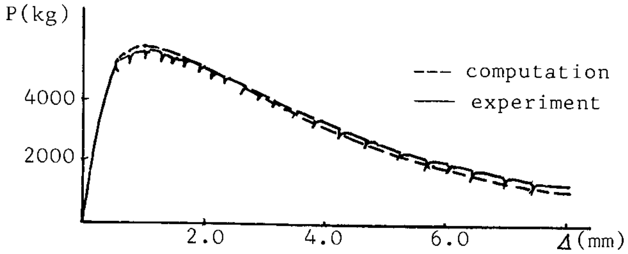

Fig.2 P versus Δ relation for 38CrMoAl (3PB)

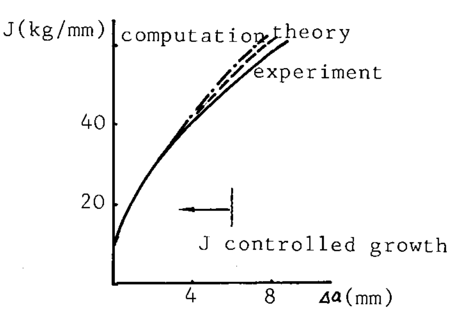

Fig.3 J-resistance curve for 18CrNiWA (CT)

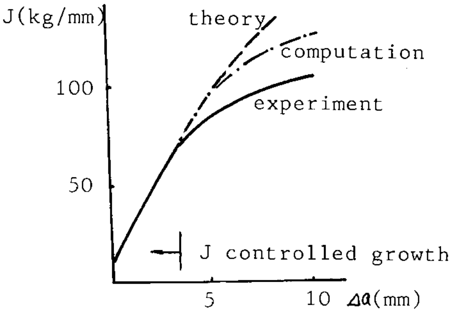

Fig.4 J-resistance curve for 38CrMOAl (3PB)

COMPUTATION

Finite element method of incremental elastic-plastic theory is adopted to simulate the crack growth. The relation between load line displacement Δ and crack growth Δa , which is obtained from experiment, is taken as the input of the numerical calculation. The crack growth is simulated by releasing the current crack tip nodal force to zero through several incremental steps. After each displacement loading, the loadind force P and J-integral are calculated. The J-integral is obtained along the contour far from the crack tip. In this way we get the P versus Δ and J versus Δa relations numerically. The results for CT specimen of 18CrNiWA are shown in Fig.1 and Fig.3, and those for 3PB specimen of 38CrMoAl in Fig.2 and Fig.4, respectively.

THEORY

On the basis of the HRR near tip field and the near tip asymptotic solution by Gao and Hwang (1981), the expression for crack tip opening displacement for mode-I crack growing in power hardening materials

$$\delta = \phi\, r \left(\frac{E\,J}{\sigma_0^2 r}\right)^{-1/(n+1)} \frac{dJ}{\sigma_0 da} + \frac{\beta\sigma_0 r}{E} \left(ln\frac{e\,\lambda EJ}{\sigma_0^2\, r} \right)^{n/(n-1)} \tag{2}$$

and values of β and λ are given by Dai and Hwang (1986), and similar expression is given by Hwang et al (1987) for mode-III crack. Here e — natural logarithmic base, ϕ , β , λ —— constants. By using the fracture criterion of critical opening displacement

$$\delta = \delta_c \qquad \text{at} \qquad r = r_c$$

Dai and Hwang (1986) obtains the theoretical J-resistance curves. Yang (1987) put eq.(2) in nondimensional form

$$T^* = \frac{dJ^*}{da^*} = \left(\frac{J^*}{J^*_{Ic}}\right)^{\frac{1}{(n+1)}} T^*_{Ic} - J^{*\,1/(n+1)}\left[(lnJ^*)^{n/(n-1)} - (lnJ^*_{Ic})^{n/(n-1)} \right] \tag{3}$$

where

$$J^* = \frac{e\,\lambda EJ}{\sigma_0^2 r_c} \qquad J^*_{Ic} = \frac{e\,\lambda}{\mu\varepsilon_0 d} \qquad d = (\alpha\varepsilon_0)^{1/n}$$

$$a^* = \frac{\beta}{\phi\; r_c}\,(e\,\lambda)^{n(n+1)}\,a \qquad r_c = \mu\delta_{Ic}$$

$$\phi = \frac{2n}{n+1}\,\frac{\tilde{u}_2(\pi,n)}{I_n}\,(\alpha I_n)^{1/(n+1)} \qquad C = 25J/\sigma_0$$

I_n , D_n and $\tilde{u}_2(\pi,n)$ are only functions of hardening exponent n and fully tabulated by Shih (1983). The values of β and λ are refined by Luo et al (1987). The J-resistance curve obtained by integrating (3) is practically insensitive to the value of μ in the range from 0.5 to 2 suggested by Rice et al (1980). Here μ is arbitrarily taken as 0.5.

The relevent material data for 18CrNiWA are given by Hwang et al(1987). The theoretical J-resistance curve is given in Fig.3. The material data for 38CrMoAl are as follows:

$E = 2.2 * 10^4$ kg/mm^2 $\qquad$ $\sigma_0 = 86.9$ kg/mm^2

$\alpha = 3.59$ $\qquad$ $n = 6.33$

and

$I_n = 4.84$ $\qquad$ $\tilde{u}_2(\pi, n) = 2.234$

$\beta = 2.16$ $\qquad$ $\lambda = 0.499$

Integrating (2) with the experimental initial values

$J_{1c} = 9.5$ kg/mm $\qquad$ $T_{1c} = 53$ $\qquad$ $\delta_{1c} = 0.04$ mm

gives the theoretical J-resistance curves shown in Fig.4. In Fig.3 and Fig.4 are indicated the limit of J-controlled growth $c = 25\ J/\sigma_o$, suggested by Shih (1981) for bend type specimen.

CONCLUSION

Fairly good coincidence is obtained between the experimental, computational, and theoretical resistance curves for CT specimen of 18CrNiWA and also for 3PB specimen of 38CrMoAl within the limit of J-controlled crack growth. Due to its simplicity, the theoretical J-resistance curve for power hardening materials suggested by Dai and Hwang (1986) is promising as a basis for structure integrity assessment.

The authors express their gratitude for the support from the National Science Foundation of China.

REFERENCES

Albrecht, P. et al (1982). Tentative Test Procedure for Determining the Plane Strain J-R Curve. Journal of Testing and Evaluation, Vol.10. 6, pp.245-251.

Dai, Y. and Hwang, K.C. (1986). A Numerical Investigation of Unsteady Crack Growth in Power Hardening Materials. Proc. of Int. Conf. on Computational Mechanics, Tokyo, pp. V-305-V-311.

Ernst, H. A., Paris, P. C. and Landes, J. D. (1981). Estimates on J-Integral and Tearing Modulus T from a Single Specimen Test Record. Fracture Mechanics ASTM STP 743.

Gao, Y. C. and Hwang, K.C. (1981). Elastic-Plastic Fields in Steady Crack Growth in Strain Hardening Materials. ICF5, Cannes, pp.669-682.

Hermann, L. and Rice, J. R. (1980). Comparison of Theory and Experiment for Elastic-Plastic Plane Strain Crack growth. Metal Science, Aug.-Sept., pp.285-291.

Hwang, K. C. et al (1987). Theoretical J-Resistance Curve for Mode-III and Mode-I Plane Strain Cracks in Power-Hardening Materials and Comparison with Experiments. Proc. Int. Conf. Fract. and Fract. Mech., Shanghai, pp.20-26.

Luo, X. F., Yang, W. and Hwang, K. C. (1987). Theoretical J Resistance Curve and Its Applications in Structural Integrity Assessment, Proc. Int. Conf. Fract. and Fract. Mech., Shanghai, pp.277-281.

Rice, J. R., Drugan, W. J. and Sham, T. L. (1980). Elastic-Plastic Analysis of Growing Cracks. ASTM STP 700, pp. 189-201.

Saxena, A. and Hudak, S. T. (1978). Reviews and Extensions of Compliance Information for Common Crack Growth Specimen. Int. J. Fracture, Vol.14, 5, pp. 453-468.

Shih, C. F. (1981). Requirements for One Parameter Characterization of Crack Tip Fields by the HRR Singularity. Int. J. Fracture, Vol.17, pp. 27-43.

Shih, C. F. (1983). Tables of Hutchinson-Rice-Resengren Singular Field Quantities, Brown Report, MRL E-147.

Yang, W. (1987). Defect Assessment Procedure Based on Ductile Tearing Process. Acta Mechanica Sinica, Vol. 3, 4, pp. 344-353.