ABSTRACT

TRIPATHY, MANISH. Optimal Strategies in Platform Economies: Applications to On-Demand Ride Hailing Service, Logistics and Spectrum Sub-Leasing. (Under the direction of Hans Sebastian Heese and Michael Kay.)

With the advancement in technology and ease of access to the internet, new businesses are emerging which are disruptive in nature. These developments have led to the emergence of a relatively new and rapidly growing stream of literature, platform economics. Although initially conceived as representative of traditional two-sided platforms like credit card companies or dating websites, all the on-demand service businesses fall under the aegis of platform economics. Examples of industries where these platforms are ubiquitous include transportation (Uber, Lyft), food delivery (Uber Eats, GrubHub), and home services (Rover, TaskRabbit). Being algorithmic in nature, the decision-making process in these businesses is susceptible to exploitation through strategic actions of the agents involved. Furthermore, due to the evolving nature and recent emergence of these platform-based setups, there are ample opportunities to improve different aspects of their operational process. We address two aspects of such an on-demand ride-hailing service in the first two bodies of research in this dissertation. In the first body of work, we focus on the strategic behavior by the service providers to game the system's pricing algorithm. Subsequently, in the second body of work, we focus on the pricing mechanism adopted by an on-demand ride-hailing service platform in the logistics industry focusing on packages. Finally in the third work, we focus on firms in the telecommunications industry that lease and/or sub-lease spectrum and elicit insights into the decisions like pricing, quality, and sourcing in the presence of competition.

Many on-demand service platforms employ state-dependent pricing strategies to balance supply capacity and customer demand. In the context of ride-hailing platforms, it has been observed that drivers strategically exploit the structure of such pricing policies by coordinating with each other to deactivate some drivers in order to create an artificial shortage of supply capacity and trigger so-called *surge pricing*. We develop a simple and

high-level analytical framework to structurally characterize the drivers of such collusive behavior and the consequences for drivers and the platform. We find that collusive driver behavior is more likely in settings where customers exhibit moderate sensitivity to waiting time. For some of these cases, if customers continue to request service under driver collusion, the platform may benefit from the higher surge prices. For settings where driver collusion is harmful to the platform, we consider two possible mitigation strategies: a *bonus payment* structure to eliminate drivers' incentives to collude, which comes at a direct cost to the platform, and a *freeze* period after deactivating the app during which drivers cannot reactivate. We show that with the appropriate duration, such a freeze period can effectively eliminate driver collusion without any direct costs to the platform.

The next chapter is an exploratory analysis of an on-demand service platform for packages, where the packages bid for transportation service through various auction mechanisms, trucks offer transportation services, and Distribution Centers match demand and supply; all agents being independent and individually incentivized to participate. Using a utility-based model, we characterize the participation incentives for all the agents, implement the state-of-the-art pricing mechanisms from industry and academia, and design and implement a first-price auction-based mechanism. Using simulation and through performance indicators like throughput, profit of Distribution Center, consumer surplus, among others, we find that the package bidding mechanism significantly outperforms the status-quo. Furthermore, we extend our analysis to include uniform price auctions and Vickrey-Clarke-Groves auction. We find that the packages prefer the Vickrey-Clarke-Groves auction whereas the trucks and Distribution Centers prefer the first-price auction; although all of them prefer the bidding mechanism to the status-quo pricing mechanism.

In this next chapter, we study an oligopolistic cellular network market, wherein, firms lease and/or sub-lease spectrum and compete over prices, quality, and bandwidth usage. We focus on the impact of competition, in the form of sub-leasing spectrum to competitors, on managing the trade-off between cannibalization and revenue. Using a vertically

differentiated spatial model of product positioning, we examine parameters such as quality of service, sub-leasing wholesale price, and pricing, given that there are competitors in the market. We show the monotonic relation between the MNO's profit and bandwidth usage with the quality level of the MVNO. We characterize equilibrium under different market conditions, with firms who only lease and with firms who both lease and then sublease to other firms. We identify and characterize equilibrium conditions signifying the optimal sourcing decision of an MVNO i.e., when to source from both MNOs and when to source from the high-quality MNO. Also, we gain insights into market entry decisions and entry deterrence strategies. Finally, we explain the managerial implications of this study, draw an analogy with the current market structure and draw structural insights for the decision-makers to consider before taking sourcing, quality, and pricing decisions.

Optimal Strategies in Platform Economies: Applications to
On-Demand Ride Hailing Service, Logistics
and Spectrum Sub-Leasing

by
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DEDICATION

In memory of my Aja, Prof. Mahendra Prasad Panda, an academician ahead of his time.

BIOGRAPHY

Manish Tripathy is from Odisha, India, and graduated with a Bachelor of Technology in Manufacturing Science and Engineering from the University College of Engineering, Burla in 2011. He then joined Tata Motors Ltd., Jamshedpur as a Graduate Engineering Trainee in the Commercial Vehicle Business Unit and was eventually posted at the Capital Equipment Manufacturing Center as an Assistant Manager. Following his master's from TU Delft and Duke University, he joined Edward P. Fitts Department of Industrial and System Engineering as a Ph.D. student. His research interests include Operations Management and Economic Modeling, and upon graduation, he intends to pursue a career in academia.

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CHAPTER 1

Introduction

With the aid of technological innovations and ease of availability of the internet, emerging businesses are disrupting and cannibalized existing businesses. The most prevalent and representative examples of disruptive businesses are on-demand service platforms, for example, Uber and Lyft competing with traditional taxi services and Airbnb competing with the traditional hotel industry. The striking thing about these firms is that they operate without owning any inventory or capital; they can do this by attracting both the service providers and the consumers to their platform, resembling a classic two-sided platform. Currently, business models based on the principles of two-sided platforms are ubiquitous and can be seen in a plethora of industries. Examples include urban transportation, logistics, food delivery, and repair services. The basic premise of a two-sided platform is that the platform purports to be attractive to both the demand side and the supply side by trying to attract users to both sides and then it matches demand with supply, earning revenue by taking a fraction of what the consumers pay in lieu of facilitating the matching; credit cards and dating platforms are one of the earliest examples. However, with the rapidly evolving technology, most of the functions like matching and pricing are now done algorithmically, without any human interference. Although the basic principles of the two-sided platforms are well known and studied, the algorithmic nature of the decision-making process and the intense competition in the on-demand service sector has resulted in each firms' operating principles being highly proprietary in nature. Since these firms are relatively recent phenomena, there exists ample scope for improving the operational performance of the firms. Additionally, there have been instances of the users learning the general principle behind the decision-making algorithms and trying to game the system by strategically altering their behavior. In Chapter 2 and Chapter 3, we focus on such an on-demand service platform and analyze the strategic behavior of service providers and pricing mechanism respectively.

In Chapter 2, we focus on how the service providers strategically change their behavior to manipulate the *Surge Pricing*. Although the exact operational mechanism of on-demand ride-hailing platforms has never been verified due to its proprietary nature, the generic principles are well known, both in academia and industry. These firms attract and facilitate the transaction between demand and supply, and they do it whenever a service need arises, hence, on-demand. However, this very nature of the operation necessitates taking decisions like using independent agents in the supply side and ensuring that service is provided within a reasonable time frame. From a financial point of view, they need to ensure the prices are attractive on the demand side and yet sufficient on the supply side. A key aspect of these platforms is managing demand and supply, and paramount to maintaining a healthy revenue stream is to manage asymmetry between demand and supply. According to one on-demand ride-hailing platform, they have two sets of prices, a base price, and a surge price multiplier which is multiplied by the base price under the surge price regime. Whenever the demand is higher than supply, the surge pricing regime is affected and depending on the severity, the magnitude of the surge price changes. Given that this mechanism, at least in the broadest sense, is common knowledge, a new phenomenon is emerging amongst the service providers or drivers. In places where there is an expected high demand, like airports, concert venues, and sporting venues, some drivers are colluding to create an artificial scarcity of supply which triggers the surge price regime. They switch off the app for a certain duration, tricking the system into believing that there are no drivers in the vicinity; once the surge multiplier is high enough, all of them go back online at those prices resulting in a higher income. To analyze this phenomenon mathematically with the associated economic implications, we develop a high-level game-theoretic model mimicking an on-demand ride-hailing service. We characterize the conditions under which the drivers collude. We also derive the effect of such collusive behavior on the platform, drivers, and the customers. Furthermore, should the platform want to curb such collusive behavior, we propose two mechanisms to disincentivize the drivers from colluding.

In Chapter 3, we consider an on-demand ride-hailing service, similar to the one in Chapter 2. However, in this research work, our set-up is a logistics network with packages instead of passengers as our customers and we focus on efficient pricing and matching mechanisms. Several companies like UberFreight and Convoy are already operating on-demand transportation platforms for packages, however, they haven't been able to cannibalize the traditional businesses like FedEx and UPS. This lack of success can be partially explained owing to the basic differences between packages and passengers concerning sensitivity to waiting, different payment structures, and different capacity constraints. However, these factors do not completely explain the lack of substantial demand for these platforms. We focus attention on the pricing scheme used by the platform; they use a similar pricing scheme wherein the platform acts as a two-sided platform and using the price, and consequently, wages, influences the demand and supply. It is worth noting that under such a system the customer is a price taker and the demand is driven by the price charged by the platform. Consequently, motivated by the lack of disruption in the logistics industry, we propose an auction-based mechanism where the packages bid for services. The service providers who find the bids sufficient offer their services and the platform focuses on matching demand with supply efficiently. We do not change the structural setup; so all the agents, the platform i.e., Distribution Centre (DC), packages, and the service providers i.e., trucks, are independent of each other and only participate when sufficiently incentivized. Thus, in our proposed mechanism, demand is driven by the valuations or the willingness to pay for the packages. We use a queue-theoretic formulation to model the on-demand platform for packages. Owing to the system being dynamic and lack of analytical tractability, we use simulation to evaluate the performance of the system and compare it against the status -quo pricing scheme used by practitioners and in academia, the platform-based pricing scheme. We define several performance indicators like throughput at the DC, profit of the DC, and consumer surplus to compare the performance and also consider different auction environments like a first-price auction, uniform-price auction, and the Vickrey-Clarke-Groves (VCG) auction.

In Chapter 4, we study another emerging business model in the field of telecommunications. Recent developments in the telecommunications industry have resulted in the unfettered growth of cellular network service providers as well as consumers. However, providing cellular service is a very capital-intensive operation, requiring significant investments and infrastructure. Traditionally, the firms which participated in auctions conducted by the Federal Communications Commission (FCC), leased spectrum, invested in infrastructure, and provided cellular service, were known as Mobile Network Operators (MNO); firms like T-Mobile and Verizon are considered as MNOs. However, being enabled by technical advancements in wireless communications technology, a new class of firms has emerged recently. These firms sub-lease spectrum from the MNOs for a wholesale price and compete for the same set of consumers and are known as the Mobile Virtual Network Operators (MVNO); firms like Google Fi and Republic Wireless are considered as MVNOs. This business model ensures that the MVNO can enter a market without having to make substantial investments, and the MNOs can recoup some of the investments through the wholesale price charged to the MVNO. However, this results in several decisions and outcomes which are non-trivial i.e., pricing decisions for all the firms, setting the quality level for the MVNO by the MNOs, and sourcing decisions by the MVNO. In order to characterize the equilibrium, we develop a stylized game-theoretic model using a vertically differentiated spatial product differentiation model.

CHAPTER 2

Driver Collusion in Ride-Hailing Platforms

2.1 Introduction

On-demand service platforms which connect individual providers with customers—such as Uber and Lyft for ride-hailing, Doordash and Uber Eats for dining, and Upwork and Freelancer for talent—have grown dramatically over the past years and now are ubiquitous across the globe. In 2019, 280 companies were engaged in the on-demand services sector, spanning 16 industries, which was up from just 76 companies in 6 industries in 2014. Roughly a third of the US workforce is engaged in this sector, with revenues of 58 billion USD [Hrz19].

Ride-hailing platforms such as Uber, Lyft, and Didi represent a significant and quickly expanding segment of the on-demand services sector, whose worldwide valuation is projected to grow from 65 billion USD in 2019 to 150 billion USD in 2026 [Cur21]. US-based Uber now is available in 69 countries, with roughly 100 million active monthly customers and revenue of about 60 billion USD per year [Ube21b; Ube20]. Similarly, China's dominant ride-hailing platform, Didi, provides about 21 million rides each day and earned 32 billion USD in revenue in 2020 [The21].

A key feature of on-demand service platforms is that their capacity is composed of individual providers, and platforms need to design appropriate pricing schemes to motivate provider participation. The adoption of *surge pricing* has been a critical factor behind the success of on-demand ride-hailing platforms [Ube20a]. Given the variability in both platform capacity and customer demand, platforms typically resort to dynamic pricing to reduce imbalances between supply and demand. Specifically, in periods of limited supply, under surge pricing the platform increases both the price it charges to customers and the wage it pays to drivers, thereby reducing customer demand and attracting additional drivers to the platform [Ube20a].

The practice of surge pricing has raised concerns regarding the perception of fairness, price gouging [Dho15], and trust issues due to unpredictable and at times drastic price changes and the platforms' secrecy with regard to their pricing algorithms [Dho16]. But because surge pricing is triggered based on pricing policies which are set ahead of time and do not change frequently, drivers have found ways to reverse engineer and manipulate the algorithms used to implement these policies [Gri15]. According to the authors' conversations with researchers at Didi, the company has observed that drivers are gaming the pricing algorithms by turning off their apps in a coordinated fashion, tricking the system into believing that there are fewer drivers in the area and triggering the surge pricing regime to combat the resulting (artificial) supply shortage. Once surge pricing is triggered, drivers then return online and enjoy higher incomes. Similar strategic behavior has been observed among drivers from other ride-hailing platforms as well. For example, drivers of Uber and Lyft connect and coordinate through social forums (e.g., UberPeople, Surge Club) to identify key characteristics of the pricing policy and then actively act so as to trigger beneficial pricing regimes [McG17; Swe20; Kea20]. Such driver collusion has been widespread for a long time and occurs daily in a variety of cities, including London, New

York, and Washington, DC (see, e.g., [Ham19], [McG17] and [Smi17]). Although platforms like Uber and Lyft have threatened that such fraudulent driver behavior will lead to account deactivation [Ube20b], driver collusion is hard to detect and continues largely unfettered [Ham19].

In this paper, we develop and analyze a concise high-level game-theoretic model to structurally characterize drivers of collusion in ride-hailing platforms and the consequences of such collusion on platform profitability. We characterize the equilibrium outcomes that arise due to the interplay between price-sensitive and waiting time–sensitive customers and income-sensitive drivers, who might behave strategically.

In practice, ride-hailing platforms implement their pricing policies through algorithms, which set the desired prices and wages in reaction to observed market conditions. Since policies and hence algorithms are determined ahead of time and changed rather infrequently [Sol17], we assume the pricing policy to be exogenous and known to the drivers, and we focus on the impact of strategic driver behavior, considering the steady-state performance under a given pricing policy. In our model, customers decide whether to request the service from the platform, and drivers choose whether to offer their services through the platform. Drivers who participate can coordinate to reduce the number of drivers who are showing as active on the platform. Drivers then take turns serving customers, so earnings are evenly shared among all drivers.

We find that collusive driver behavior is more likely in settings where customers exhibit moderate sensitivity to waiting time. This is more likely in areas and at times with limited alternative options, for example, at airports or concert venues. Interestingly, we find that driver collusion may not always be harmful to the platform, which benefits from higher surge prices. For settings where driver collusion is harmful to the platform, we consider two possible mitigation strategies. The first is based on observations in practice and employs a *bonus payment* structure to eliminate the drivers' incentives to collude. While effective in combating collusion, such a bonus policy clearly comes at a direct cost to the platform.

We therefore suggest a second potential instrument, which introduces a *freeze period* after deactivating the app during which drivers are locked out. We show that with the appropriate duration, such a freeze period can effectively eliminate driver collusion without any direct costs to the platform.

To the best of our knowledge, this is the first study of strategic collusive driver behavior in the context of ride-hailing platforms. Specifically, we consider drivers that can strategically create an artificial supply shortage to trigger surge pricing, and we examine the impact of such behavior. In addition, based on the derived properties of driver collusion, we analyze two possible mitigation strategies that a platform could apply to curb harmful strategic driver behavior. Our results offer direct managerial insights for on-demand service platforms that need to identify when strategic driver behavior is detrimental to platform profits and how to adjust their policies to eliminate such collusion.

The remainder of this paper is organized as follows. We review the related literature in the following Section 2.2. Section 2.3 introduces our modeling framework. In Section 2.4, we analyze the impact of collusion, and in Section 2.5, we consider two mitigating strategies and discuss their effectiveness. Finally, we conclude with a summary of our findings in Section 2.6. Proofs of all results given in the main part of the paper are provided in Appendix A.

2.2 Literature Review

Our research paper relates to a growing literature on two-sided markets where platforms connect individual sellers and buyers. We refer the reader to [Roc03], [Arm06], [Rys09], [Par05], [Cai01], and [Cai03] for discussions of a variety of contexts and market structures. Most of the work in this stream of research focuses on settings in which customers place an order in advance, while we consider an on-demand service platform setting in which customers demand immediate service and are sensitive to possible waiting times.

The literature contains numerous studies of operational issues that arise in on-demand service platform contexts. [Gur19] use a variant of the newsvendor model to capture the service provider's trade-off between compensation and flexibility under self-scheduling, and they characterize the impact of driver self-scheduling on the platform's revenue stream. [Cac17] evaluate different possible pricing contracts that a platform can use, considering both platform profit and social welfare. [Ber20] investigate competition between two ride-hailing platforms. They find that allowing drivers to multi-home can decrease customer welfare and platform profit. Using an M/M/k model to capture the waiting time, [Tay18] examines the impact of congestion, demand uncertainty, and worker independence on platform profit and pricing decisions. [Bai19] find that a dynamic payout ratio policy that changes based on prevailing market characteristics can substantially outperform a static payout ratio policy. We refer the reader to [Nar18] for a recent overview of the related literature.

Our work is most closely related to those studies that investigate surge pricing policies. One stream of this literature investigates optimal surge pricing policies and their effectiveness compared to static pricing. For example, the results in [Ban15] suggest that dynamic pricing offers no better results than static pricing in maximizing platform profit or system welfare. [Che16] empirically analyzes the impact of surge pricing on driver participation and finds that surge pricing can significantly increase driver participation and hence platform capacity. [Cas17] show that surge pricing can resolve the "wild goose chase" problem and improve social welfare. [Cas20] empirically analyze the impact of surge pricing on the welfare of riders, drivers, and the platform; their findings suggest that it is the riders who are benefiting most from surge pricing, at the expense of both the drivers and the platform. [Gar20] consider a dynamic stochastic ride-hailing model to study which surge-pricing policy structure—additive or multiplicative—is incentive compatible and how drivers choose to either accept or reject a ride request. They find that either structure can be incentive compatible, depending on whether or not the system switches between pricing regimes.

[Gud19] find that a platform can use surge pricing to strategically price out drivers in zones with excess driver supply, so as to increase the supply of drivers in nearby regions with capacity shortage. In contrast to this stream of literature, where the optimal pricing structure is the primary focus, we take the pricing policy as given and examine the drivers' strategic behavior.

Our paper thus also relates to recent work on the strategic behavior of customers and providers. [Hu21] examine optimal surge pricing policies in a setting in which customers can strategically choose to wait for lower prices and drivers can decide whether to reposition themselves in regions with active surge pricing. [Gud19] assume that customers are myopic, whereas drivers can strategically choose where to locate to maximize their earnings. [Afe18] also consider a scenario in which drivers are forward looking in deciding whether to reposition themselves in a surge region. These papers focus on the strategic incentives and behaviors of individual drivers across time periods and locations, such as whether to move to a particular region in anticipation of a higher wage. Our study differs in that we analyze a setting in which drivers coordinate and collectively decide whether and how best to induce surge pricing to maximize their earnings. To the best of our knowledge, we are the first to consider such strategic collusion behavior among drivers in ride-hailing platforms.

2.3 Model Framework

In this section, we describe our mathematical model and assumptions regarding the customers' decisions, the platform's pricing scheme, and the drivers' objectives and behavior.

2.3.1 Customer Decision Model

Customer arrivals to the focal region are random, with rate $\bar{\lambda}$. Customers are heterogeneous in terms of the value v they attribute to the service, and we assume their types are distributed uniformly from 0 to 1.

Ride-hailing platforms provide customers with information on pricing and some indication of estimated waiting times. We assume customers take such information about the market state into consideration when determining whether or not to request service. For tractability, consistent with the literature (see, e.g., [Ber20]), we use the squared system utilization as a proxy for waiting time. Specifically, observing an indicated system utilization ρ, we assume that a customer of type v associates a utility $U(v) = v - p - s\rho^2$ with purchasing the service at price p from the platform. Here the parameter s indicates customer sensitivity with respect to waiting, which often is also reflective of the competitive setting and the alternative options available to customers (public transportation, rental cars, etc.) Normalizing the value of nonparticipation to zero, a customer of type v participates if and only if $U(v) \geq 0$, so the customer service request rate λ can be expressed as

$$\begin{aligned} \lambda &= \bar{\lambda} Prob\{U(v) \geq 0\} \\ &= \bar{\lambda} Prob\{v \geq p + s\rho^2\} \\ &= \bar{\lambda}(1 - p - s\rho^2). \end{aligned} \tag{2.1}$$

2.3.2 Platform Pricing Scheme

Based on the authors' conversations with managers and providers from major ride-hailing platforms, it is the case that platforms do not frequently adjust their pricing algorithms. This observation is consistent with Uber's own description of its pricing [Ube21], suggesting that there is a rather fixed base price for any given region. We assume the platform's pricing scheme is exogenous and given, and known to customers and drivers. We determine and analyze the steady-state system performance under a given pricing policy and focus on the impact of strategic driver behavior. Consistent with industry practice and related research [Tay18; Bai19; Ber20], we assume that the platform uses time-based pricing, where prices are only adjusted after certain time intervals to allow the system to settle into the new equilibrium state. The platform's pricing policy specifies the service price it charges to customers, p, and the wage it pays to drivers for completing a service request, w; to ensure the viability of the platform, we assume $0 < w < p < 1$.

We consider a ride-hailing platform, operating in a certain focal location with frequent instances of high demand, such as airports, concert venues or stadiums. In practice, the platforms' pricing algorithms trigger surge pricing when there are more riders in a given area than available drivers [Ube21a]. We consider two pricing regimes set by the platform, depending on the asymmetry between the available number of drivers and customers requesting service: the base pricing regime (denoted by subscript n); and the surge pricing regime (denoted by subscript s). Specifically, consistent with this notion and similar to [Ban15], we assume that the platform adopts a threshold pricing policy based on the observed system utilization, that is, customer demand divided by system capacity. Specifically, the pricing policy stipulates the use of surge pricing with (p_s, w_s) whenever the system utilization ρ exceeds a given threshold $\overline{\rho}$, and base pricing (p_n, w_n) otherwise.

If switching between the base pricing regime and the surge pricing regime is always triggered based on the utilization that is actually observed (i.e., ρ_n under base pricing and ρ_s under surge pricing), it can occur that, in equilibrium, the system cycles between these two regimes, as the utilization under base pricing triggers surge pricing, which then reduces utilization sufficiently to trigger a return to base pricing (see analysis in Appendix A.1). For simplicity and analytical tractability, in the following we assume that the platform always adopts the base pricing regime in such cases, so that the algorithm switches to the surge pricing regime if and only if the utilization in the equilibrium observed after such a switch continues to satisfy the condition for surge pricing; that is, we base the definition of the pricing policy on the utilization under surge pricing, ρ_s.

$$(p, w) = \begin{cases} (p_n, w_n) & \text{if } \rho_s < \overline{\rho} \\ (p_s, w_s) & \text{if } \rho_s \geq \overline{\rho} \end{cases} \tag{2.2}$$

The rationale behind this policy and, more specifically, the use of surge pricing, is that in times with excess demand a higher surge price ($p_s > p_n$) reduces customer demand, while the higher payments to drivers ($w_s > w_n$) attract more drivers to the focal area, both effects serving the purpose of improving the balance between supply and demand.

Also, consistent with current practice at ride-hailing firms like Uber and Lyft, where the payout to drivers is defined relative to the prices charged to customers [Che20; Ube20a; Dec14; Haw16], we introduce $\delta_i = w_i/p_i$ to denote the payout ratio under pricing regime $i \in \{n, s\}$, which determines the fraction of revenues collected by the drivers (a fraction $(1-\delta_i)$ is retained by the platform). For simplicity, we drop the subscript $i = \{n, s\}$ whenever no confusion can arise. Using the effective demand rate from (2.1), the platform's profit is

$$\pi = \lambda(1-\delta)p. \tag{2.3}$$

2.3.3 Driver Decision Model

There are $\mathscr{D}$ registered drivers in the region around the focal location who can participate in the platform at any time. These drivers are heterogeneous with respect to their reservation wage per unit time r, which we assume to be distributed uniformly between 0 and 1. A driver decides to participate if and only if the expected average earnings under participation e exceed his reservation wage r. Consequently, the number of drivers who participate in the platform is $\mathscr{M} = \mathscr{D}e$.

Drivers that participate in the platform can coordinate and collude to trigger surge pricing. Our assumptions are consistent with broad evidence from practice that drivers strategically collude to affect the platform pricing algorithm, and that they coordinate to ensure an equitable sharing of the resulting benefits [McG17; Kea20; Smi17; Möh17].

In the absence of collusion, the number of drivers online is equal to the number of participating drivers, that is, $m = \mathscr{M}$. Each driver then earns $e = w\mu\rho$, where μ is the service rate of each driver (i.e., the number of customers a driver can serve per unit of time), and the system utilization is $\rho = \frac{\lambda}{\mathcal{M}\mu}$.

If the $\mathcal{M}$ drivers collude, then they collectively decide the number of active drivers who participate in the platform $m \leq \mathcal{M}$ to maximize their earnings and coordinate their activities to ensure that there are always m drivers online. Suppose that upon service completion a driver goes offline and waits for the next $\mathcal{M} - m$ service completions before returning online.

The system utilization then equals $\rho = \frac{\lambda(m)}{m\mu}$ and the total earnings of all drivers are $\lambda(m)w(m)$. As these total earnings are shared equally between the drivers through the coordination mechanism defined above, each individual driver earns

$$e(m) = w(m)\frac{\lambda(m)}{\mathcal{M}(m)}. \tag{2.4}$$

The drivers jointly determine the optimal number of drivers to be online, m^*, as the solution to

$$\max_{0 \leq m \leq \mathcal{M}(m)} e(m). \tag{2.5}$$

We will refer to driver behavior as collusion if and only if the drivers purposefully reduce the number of drivers showing online, that is, if $m^* < \mathcal{M}$. For ease of exposition, we omit references to the argument (m) in the following.

2.3.4 Game Sequence

The sequence of events of the overall game is as follows. First, based on the platform's pricing policy (and possibly in anticipation of potential driver collusion), drivers form (rational) expectations regarding their potential earnings and make themselves available at the focal location if these expected earnings exceed their reservation wage. Second, having arrived at the location, drivers decide whether or not to behave strategically, that is, whether to coordinate and collude so as to prompt the pricing algorithm to trigger surge pricing. If they decide to collude, they also determine the number of drivers to show as active online.

Third, observing the market state, the pricing algorithm generates the price per trip charged to the customers and the wage per trip paid to the drivers, consistent with the prevailing pricing policy. Fourth, customers observe the market state, that is, the posted price and system utilization (a proxy for expected waiting time), and decide whether or not to request service. We summarize our notation in Table 2.1.

Table 2.1 Summary of Notation

Parameter	Definition
$\mathcal{D}$	Number of registered drivers
$\bar{\lambda}$	Customer arrival rate
$\bar{\rho}$	Threshold on utilization that triggers surge price
p	Price per trip
w	Wage per trip
δ	Payout ratio to the drivers, $\delta = w/p$
μ	Service rate
s	Customer sensitivity to waiting
Variable	**Definition**
$\mathcal{M}$	Number of drivers that participate in the platform
m	Number of active drivers online
λ	Demand (customer request) rate
ρ	Utilization of the platform
e	Driver earnings
π	Platform profit

2.4 Analysis

In this section, we characterize the drivers' collusion behavior and its impact on driver earnings and platform profitability. First, in Section 2.4.1, we analyze the equilibrium decisions and outcomes for the benchmark case without driver collusion. In Section 2.4.2, we consider the case in which the drivers have an option to collude. We then compare these two cases to characterize the impact of collusive driver behavior in Section 2.4.3.

2.4.1 Benchmark: No Driver Collusion

We derive the equilibrium using backward induction. In the absence of an option to collude, all drivers who choose to participate show as available online, that is, we let $m = \mathcal{M}$. Then for any pricing regime (p, w), from (2.1) and since $e = w\frac{\lambda}{\mathcal{M}}$ and $\rho = \frac{\lambda}{\mathcal{M}\mu}$, the equilibrium can be derived as follows.

Lemma 1. *Suppose there is no collusion. For any given pricing regime (p, w), there exists a unique equilibrium where the number of participating and online drivers, the customer request rate, the system utilization, and the individual driver earning rate are, respectively,*

$$m = \mathcal{M} = \frac{\mu\mathcal{D}\delta\sqrt{\bar{\lambda}(1-p)}p}{\sqrt{\mu^2\mathcal{D}\delta p + \bar{\lambda}s}},$$

$$\lambda = \frac{\bar{\lambda}\mathcal{D}\mu^2\delta p(1-p)}{s\bar{\lambda} + \mathcal{D}\mu^2 p\delta},$$

$$\rho = \frac{\sqrt{\bar{\lambda}(1-p)}}{\sqrt{\mu^2\mathcal{D}\delta p + \bar{\lambda}s}}, and$$

$$e = w\mu\frac{\sqrt{\bar{\lambda}(1-p)}}{\sqrt{\mu^2\mathcal{D}\delta p + \bar{\lambda}s}}.$$

Considering the pricing policy in (2.2), the equilibrium outcomes given in Lemma 1 imply the following conditions for surge pricing.

Proposition 1. *Suppose there is no collusion and the platform uses the pricing scheme defined in* (2.2)*. Define $\hat{s} = \frac{1-p_s}{\bar{\rho}^2} - \frac{D\mu^2\delta_s p_s}{\bar{\lambda}}$. In equilibrium, the platform adopts the surge pricing regime (p_s, w_s) if $s \leq \hat{s}$ and adopts the base pricing regime (p_n, w_n) if $s > \hat{s}$.*

2.4.2 Equilibrium with Strategic Driver Behavior

We again proceed using backward induction. First, for any given pricing regime in (2.2) and number of online drivers m, from (2.1), (2.4), and the supply function $\mathcal{M} = \mathcal{D}e$, we characterize the equilibrium in the following lemma.

Lemma 2. *For a given pricing regime (p, w) and a given number of drivers online m, there exists a unique equilibrium where the customer request rate, the number of participating drivers $\mathscr{M}$, the system utilization, and the individual driver earning rate are, respectively,*

$$\lambda = \frac{m\mu(-m\mu + \sqrt{m^2\mu^2 - 4\bar{\lambda}^2(-1+p)s})}{2\bar{\lambda}s}, \tag{2.6}$$

$$\mathscr{M} = \sqrt{\frac{\delta D\mu m p\sqrt{\mu^2 m^2 + 4\bar{\lambda}^2(1-p)s} - \delta D\mu^2 m^2 p}{2\bar{\lambda}s}}, \tag{2.7}$$

$$\rho = \frac{-m\mu + \sqrt{m^2\mu^2 - 4\bar{\lambda}^2(-1+p)s}}{2\bar{\lambda}s}, and \tag{2.8}$$

$$e = \sqrt{\frac{\delta\mu m p\left(\sqrt{\mu^2 m^2 + 4\bar{\lambda}^2(1-p)s} - m\mu\right)}{2D\bar{\lambda}s}}. \tag{2.9}$$

The results in Lemma 2 are for a given number of drivers online on the platform (m). It is straightforward to confirm that substituting $m = \mathscr{M}$ into these results yields the equilibrium for the case without collusion, covered in Lemma 1. We next examine how changes in the number of drivers online affects the equilibrium characterized in Lemma 2.

Proposition 2. *Assume a given pricing regime (p, w). In equilibrium, as the number of active drivers m increases, the request rate λ, the number of drivers who participate in the platform $\mathscr{M}$, and the average earnings of a driver e increase, but the system utilization ρ decreases.*

Proposition 2 provides several key insights regarding the impact of the number of drivers that are online, m. First, an increase in drivers online leads to an increase in the service request rate, as the increased capacity reduces the anticipated waiting time and thus attracts more customers. Given a fixed wage per service completion, aggregate driver earnings then increase, and we see that the overall effect on individual driver earnings is also positive, even though the higher level of earnings attracts additional drivers to the region. A direct consequence of this result is that, if drivers collude, they keep the number of online drivers at the highest level that still ensures surge pricing. Finally, we find that the increase in system capacity induced by an increase in drivers online always dominates the corresponding increase in the customer service request rate, so increasing numbers of drivers online always reduce the system utilization.

The result that a smaller number of online drivers corresponds to a higher system utilization indicates the potential for and possible benefit of strategic driver behavior. If drivers collude to sustain a sufficiently small number of drivers online (i.e., $m \leq \bar{m}$, where $\bar{m}$ is an exogenous threshold), they can induce a sufficiently pronounced supply shortage to reach the system utilization level that triggers surge pricing (cf. (2.2)).

Lemma 3. *Assume a given pricing regime* (p, w). *In equilibrium, as the waiting-time sensitivity* s *increases, the average driver earnings* e *decrease.*

Lemma 3 also shows that individual driver earnings decrease in customers' waiting sensitivity. As a stronger waiting sensitivity reduces the customer request rate, it decreases driver utilization, resulting in reduced earnings.

Anticipating the earnings in (2.9), drivers collectively determine the optimal m^* by solving the optimization problem defined in (2.5). The next proposition summarizes our key findings.

Proposition 3. *(a) There exist thresholds* s_d^- *and* s_d^+ *with* $s_d^+ \geq s_d^-$ *such that drivers collude at*

$$m^* = \bar{m} = \frac{\bar{\lambda} - \bar{\lambda} s \bar{\rho}^2 - \bar{\lambda} p_s}{\mu \bar{\rho}}$$

to induce surge pricing if $s \in (\max\{\hat{s}, s_d^-\}, s_d^+)$, *and do not collude (i.e.,* $m^* = \mathcal{M}$*) otherwise.*

(b) When drivers collude, in equilibrium, the number of drivers in the region, customer service

request rate, and driver earnings rate are given, respectively, by

$$\mathcal{M} = \sqrt{\mathcal{D}\bar{\lambda}p_s\delta_s(1-p_s-s\bar{\rho}^2)},$$
$$\lambda = \bar{\lambda}(1-p_s-s\bar{\rho}^2), and$$
$$e = \frac{\sqrt{p_s\delta_s\bar{\lambda}\left(-s\bar{\rho}^2-p_s+1\right)}}{\sqrt{\mathcal{D}}}. \tag{2.10}$$

Proposition 3 provides conditions under which drivers collude. We know from Proposition 1 that surge pricing occurs even without collusion when customers are sufficiently insensitive to waiting ($s \le \hat{s}$), so the drivers maximize their earnings if they all are online ($m^* = \mathcal{M}$). On the other hand, for $s > \hat{s}$, the platform would always adopt the base pricing regime if there is no driver collusion. Clearly, drivers never have a reason to collude, unless $s > \hat{s}$. To better understand drivers' incentive to collude, consider three intervals for the customer waiting time sensitivity, s. For low waiting-time sensitivity ($s \in (\hat{s}, s_d^-]$), drivers do not collude and the platform adopts the base pricing regime. With a low waiting-time sensitivity, customers are relatively more sensitive to pricing. Under surge pricing, drivers earn a higher wage per trip, but the higher price charged to customers would also significantly reduce the customer service request rate and thus lower driver utilization, ultimately reducing driver earnings. However, this negative effect is attenuated as the customers' waiting-time sensitivity increases. For moderate waiting-time sensitivities ($s \in (\max\{\hat{s}, s_d^-\}, s_d^+)$), drivers benefit from colluding to induce surge pricing. Finally, since drivers' earnings decrease in customers' waiting-time sensitivity (cf. Lemma 3), the benefit of collusion to drivers, that is, their earnings at $m = \bar{m}$ minus their earnings at $m = \mathcal{M}$, shrinks as s increases, and collusion becomes unattractive as the customers' waiting-time sensitivity becomes sufficiently high ($s \ge s_d^+$). Consequently, drivers have an incentive to collude to trigger surge pricing when the customers' waiting-time sensitivity is moderate, but they choose not to collude when it is either low or high.

2.4.3 Impact of Driver Collusion

Sections 2.4.1 and 2.4.2 characterize the equilibrium outcomes without and with strategic driver behavior, respectively. In this subsection, we examine the impact of driver collusion on platform profit. Substituting the equilibrium results from Propositions 1 and 3 into the platform's profit function in (2.3), we can establish the following result.

Proposition 4. *There exist thresholds s_p^- and s_p^+ with $s_p^+ \geq s_p^-$, such that drivers collude and their collusion benefits the platform if $s \in (\max\{\hat{s}, s_d^-, s_p^-\}, s_p^+)$; and drivers collude and their collusion harms the platform if $s \in (\max\{\hat{s}, s_d^-\}, s_p^-) \cup (s_p^+, s_d^+)$. Otherwise, the platform's profit is not affected.*

According to Proposition 4, strategic driver collusion may benefit or harm the platform. The platform profit function in (2.3) is composed of two parts, the customer service request rate and the profit margin. On the one hand, driver collusion decreases the number of active drivers online m, decreasing the customer service request rate and harming the platform (cf. Proposition 2). Also, as collusion triggers surge pricing, it further reduces the customer service request rate, with a negative impact on the platform's profit. On the other hand, the platform may benefit if the profit margin under surge pricing is higher than that under the base pricing regime.

Proposition 4 shows that driver collusion harms the platform when customer waiting-time sensitivity is very low or very high, but that, interestingly, it may benefit the platform when customer waiting-time sensitivity is moderate.

A low customer sensitivity to waiting ($s \in (\max\{\hat{s}, s_d^-\}, s_p^-)$) indicates a relatively higher price sensitivity, implying a more pronounced negative impact of surge pricing on the customer service request rate. Driver collusion then is harmful to the platform. However, this negative impact is reduced for moderate customer waiting-time sensitivities ($s \in (\max\{\hat{s}, s_d^-, s_p^-\}, s_p^+)$), and the platform then may benefit from driver collusion. Moreover, as the equilibrium customer service request rate λ decreases in the waiting-time sensitivity (cf. Lemma 3), so does platform profit for any given price regime. Therefore, the benefit to the platform from collusion (i.e., platform profit with driver collusion minus platform profit without driver collusion) shrinks as customer waiting-time sensitivity grows. Consequently, the platform is more likely to be harmed by driver collusion when waiting-time sensitivity is high ($s \in (s_p^+, s_d^+)$). Finally, as shown in Proposition 2, when $s \leq \hat{s}$ or $s \geq s_d^+$, drivers do not collude, so their potential strategic behavior does not affect the platform.

Figure 2.1 illustrates the findings of Proposition 4 and shows (by example) that each of these possible outcomes could occur (i.e., that the associated parametric conditions can be met). For this illustration, we let $p_n = 0.1256, p_s = 0.9016, D = 0.9021, \mu = 0.4631, \delta_s = 0.9, \delta_n = 0.8$, and $\bar{\rho} = 0.7904$.

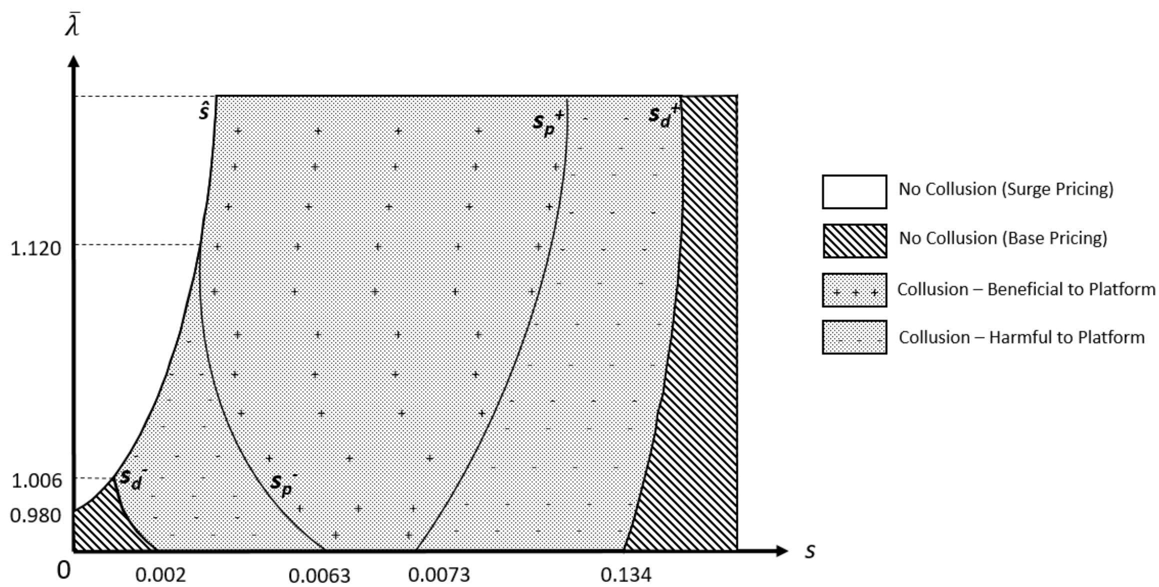


Figure 2.1 Impact of Collusion

Figure 2.1 reflects the different outcomes based on customer sensitivity to waiting s and customer arrival rate $\bar{\lambda}$, which vary along the axes. Specifically, when $\bar{\lambda}$ is high (here $\bar{\lambda} \geq 1.12$), the region in which collusion occurs and harms the platform is relatively small, and it only exists for moderate to high customer waiting-time sensitivities ($s \in [s_p^+, s_d^+]$). For higher customer arrival rates, the threat of driver collusion thus is not very significant to the platform, which may even benefit from this strategic driver behavior. A higher customer arrival rate leads directly to a higher service request rate, increasing the likelihood of surge pricing. Drivers thus do not have an incentive to collude when customers are insensitive to waiting.

When the customer arrival rate is a little lower (here $1.006 \le \bar{\lambda} < 1.12$), the platform may also be harmed by driver collusion in the case of moderately smaller customer waiting-time sensitivities ($s \in [s_d^-, s_p^-]$). In this region, under base pricing, the platform's demand is similar to its supply. If drivers then decide to collude, the decrease in supply always reduces platform profitability. Finally, when the customer arrival rate is low (here $\bar{\lambda} < 1.006$), we observe a region of lower customer waiting-time sensitivities ($s \in [0, s_d^-]$), where the drivers do not collude and the platform uses base pricing. Give the rather low customer arrival rate, the higher price under surge pricing then significantly reduces the customer service request rate, which decreases the drivers' earnings rate. Hence, drivers choose not to collude in this region.

Our results characterize when driver collusion exists, and when such collusion is beneficial or harmful to the platform. In settings where driver collusion is likely and would be detrimental to the platform's profits, the platform should consider mechanisms to prevent collusive behavior. In the next section, we consider two mitigation strategies that the platform could use to eliminate driver collusion.

2.5 Mitigation Strategies

In this section, we focus on settings where drivers have an incentive to collude and collusive driver behavior is harmful to the platform. We consider two mitigation strategies to prevent the drivers from colluding. We first consider a *bonus payment* structure to eliminate the drivers' incentives to collude (in Section 2.5.1). We then, in Section 2.5.2, suggest a second potential instrument, which introduces a *freeze period* after deactivating the app during which drivers cannot reactivate.

2.5.1 Bonus Structure

In this section, we consider a bonus payment structure, under which the platform pays an extra bonus on top of the prevailing pricing structure in (2.2). Specifically, drivers remaining online after completing a trip will receive an additional payment of b on top of the wage w for each following trip (see, e.g., [Ube20]). Ride-sharing platforms such as Uber and Lyft frequently adopt bonus-payment policies to encourage driver participation [Ube21a; Lyf21]. We focus on the question whether a bonus-type structure could be used to prevent collusive driver behavior.

Under such a structure, if drivers choose not to collude and stay online, each subsequent trip completion then would yield a total of $w+b$, whereas colluding drivers would only earn w per completed trip, as collusion necessarily requires drivers to turn off their apps after each service completion. To eliminate drivers' incentive to collude, the platform needs to offer a bonus that ensures that driver earnings without collusion are no less than driver earnings with collusion. We characterize the optimal bonus structure in the next proposition.

Proposition 5. *Suppose the platform uses the pricing structure in* (2.2) *and parameters are such that drivers collude and collusion harms the platform. The platform then benefits from adopting a bonus structure with* $b = \delta_s p_s \frac{\lambda_s}{\lambda_n} - \delta_n p_n$*, if* $\lambda_n p_n \geq \lambda_s p_s$*, where* λ_n *is given in Lemma 1 with* $(p, w) = (p_n, w_n + b)$ *and* λ_s *is given in Proposition 3; it would not adopt a bonus structure otherwise.*

The bonus payment structure described in Proposition 5 is determined to ensure that driver earnings without collusion under the bonus structure are equal to what drivers earn under collusion, thereby eliminating any incentive for drivers to collude. However, while it is intuitive that a sufficiently high bonus payment might eliminate collusion, offering such additional payments clearly comes at a cost to the platform. The platform's (operating) profit is equal to the revenues collected from customers minus the payments to drivers. By design, the total payments to the drivers are the same with and without the bonus structure, so the platform benefits from employing the bonus structure if and only if the associated revenues are higher. Proposition 5 indicates that a bonus structure can be effective and beneficial to the platform if and only if $\lambda_n p_n \geq \lambda_s p_s$. Otherwise, the platform should not adopt the bonus policy, since eliminating collusion leads to a reduction in revenues.

In conclusion, while a bonus structure can effectively be used to counteract driver collusion, the required bonus payments might be substantial, and they might often exceed the benefits that the platform derives from elimination of collusive behavior. In the next subsection, we propose another mitigating strategy that does not imply any financial transfers to drivers and thus always improves platform profit.

2.5.2 Freeze Period

In this section, we consider a *freeze period* policy for preventing driver collusion. Under such a policy, once drivers log off from their account, they are locked out of the app for a certain duration of time t_f. Under collusion, drivers coordinate to turn off their apps after each service completion and return online just in time to maintain the optimal number of drivers online (i.e., $m^* = \overline{m}$). By enforcing a sufficiently long freeze period after logoff, the platform can delay a driver's return to the group of online drivers, thereby reducing the number of drivers online and overall driver earnings under collusion (cf. Proposition 2). The platform then can eliminate collusion by enforcing any freeze period duration that ensures that driver earnings under collusion are lower than those without collusion. The following proposition characterizes the platform's optimal freeze period policy.

Proposition 6. *Suppose the platform uses the pricing structure in* (2.2) *and parameters are such that drivers collude and collusion harms the platform. There exists a threshold $\tilde{t}_f$ such* *that drivers do not collude for any* $t_f \geq \tilde{t}_f$.

The threshold $\tilde{t}_f$ is set to ensure that drivers are indifferent with respect to collusion (i.e., $e_s = e_n$), so drivers prefer not to collude for freeze periods with duration $t_f > \tilde{t}_f$. The freeze period only affects drivers after they go offline. In the absence of collusion all active drivers remain online, so this policy effectively eliminates collusion without necessitating any financial transfers. Specifically, both driver earnings and platform profits revert to the levels of the no-collusion scenario (cf. Section 2.4.1).

Unlike the bonus payment policy, the freeze period policy does not imply any direct costs to the platform, so it is clearly preferable from the platform's perspective. However, while the freeze period effectively eliminates driver collusion, too long a freeze period might also adversely interfere with the drivers' regular activities, making participation in the platform less attractive. While we abstract away from capturing such additional aspects in our model, such considerations suggest the platform should always adopt the minimum effective freeze period duration (i.e., $t_f = \tilde{t}_f$).

2.6 Discussion and Conclusion

On-demand platforms and, especially, ride-hailing platforms use surge pricing policies to counteract supply shortages and reestablish the balance between supply and demand. However, it has been widely observed among ride-hailing platforms that drivers manipulate platform pricing algorithms by artificially creating supply shortages to induce the surge pricing regime.

This paper develops a high-level game-theoretic model to examine the impact of such collusive driver behavior on the platform and to evaluate possible mitigation strategies. Unlike the extensive literature on on-demand service platforms with a focus on the structure and parameters of optimal pricing policies, we take the surge pricing algorithm as given and rather focus on the drivers and the impact of collusive provider behavior.

Our work makes several contributions to the literature. First, by characterizing the equilibrium outcomes for settings with and without consideration of strategic driver behavior, our analysis characterizes the conditions under which drivers collude to trigger surge pricing. Understanding these conditions is vital for platforms to be able to monitor and identify the existence and threat of collusive behavior.

Second, we show that collusive driver behavior may at times be beneficial to the platforms, which may explain their lack of clear and forceful counteraction. We characterize the conditions under which driver collusion benefits or harms the platform.

Finally, since there are settings in which collusive behavior is detrimental to the platform, we consider and evaluate two mitigation strategies that a platform can use to eliminate driver collusion, a *bonus payment* policy and a *freeze period* policy. We show that both of these strategies can be effective in eliminating driver collusion and improving platform profit. While bonus payment policy structures are already widely used among ride-hailing platforms (albeit for different purposes), we show that these policies cannot always be used to eliminate collusion and that their implementation is costly to the platform. The freeze period policy, however, can always be used to eliminate collusion, and it fully recovers the losses that the platform would otherwise incur due to collusive driver behavior.

In this paper, we take a very high-level view and abstract away from many operational details to derive structural insights into the drivers and impact of driver collusion in ride-hailing platforms. Our approach suggests several potential directions for future research. For example, motivated by observations from practice, where pricing policies are implemented and changed infrequently, we assumed a given and fixed pricing policy with a utilization threshold structure. It would be interesting to consider whether and how the (anticipated) presence of strategic drivers affects the optimal structure of the pricing policy. In our view, a more detailed investigation of customer behavior would also be interesting, specifically whether customers are willing to wait for nonsurge prices, and whether strategic and experienced customers realize the differential between the actually experienced waiting time and the waiting time suggested on the app, which might be artificially inflated by driver collusion.

CHAPTER 3

On-demand Logistics Service For Packages: Package Bidding Mechanism Vs Platform Pricing

3.1 Introduction

Disruptive Economics refers to the new business models which, through technological innovation, disrupt and cannibalize existing businesses [Nau14], and, these disruptions are getting more and more prevalent owing to the ubiquity of the internet. In the transportation and logistics industry, and specifically, the urban transportation industry, ride-hailing companies have revolutionized and changed the way customers travel between an origin and destination (OD) pair. For better or for worse, different aspects of urban transportation like owning personal vehicles, taxi service prices, and congestion/ traffic have been affected by the presence of ride-hailing companies [Ach20; Kam17]. However, despite these advances, some fundamental market forces have remained constant; for example, the consumers are still the price takers i.e., ride-hailing companies quote a take-it-or-leave-it price to the consumers who then decide whether or not to participate [Ube21b]. Consequently, the demand for such services is primarily driven by the price quoted by the platform. Also, the objective of these companies remains profit maximization with scant attention given to consumer surplus maximization.

In the logistics industry, trucks hauled 72.5% of all freight transported in 2019 and generated revenue of $791.7 billion in 2019 [Ame20]. One of the major challenges this industry faces is the under-utilization of truck capacity [MAI21], resulting in moderate to large transportation companies losing money due to the associated sunk costs. Some firms tried to address this issue by designing their operations similar to ride-hailing companies and offering transport between an OD pair for packages, e.g., Uber Freight, Convoy. Their operating principle resembles the existing ride-hailing companies— the person sending a package requests transportation service for a given package through a platform, (s)he is quoted a price by the platform and then the sender decides whether to accept the price or not. Finally, the sender is matched with a service provider who agrees to accept a fraction of the quoted price as wages [Ube21c]. However, these companies have not been able to cannibalize the traditional companies in the logistics industry like FedEx and/or UPS. This can be, in part, ascribed to the fact that this business model is not particularly suited for packages owing to different sensitivity to waiting, different payment structures, and different capacity constraints [For18; Dof21]; however, these factors do not completely explain the lack of success. Factors like pricing strategies and prioritizing profits also affect demand and, consequently, market penetration. It is worth noting that the sender is still a price taker and demand is driven primarily by the price quoted by the platform, as in the case with ride-hailing services.

In this exploratory paper, we focus on the logistics industry and propose an alternative system for implementing package transportation across different geographic locations. We are motivated by the lack of disruption in this traditional industry and are aware of avenues to improve efficiency as well as consumer welfare by bringing the cost to the customer down and driving demand through an auction-based bidding mechanism. Our proposal can be seen either as a policy decision by the government, for example through the state-owned carrier USPS to make this market more efficient, or as an operational model by a technology company looking to disrupt traditional businesses. Consequently, this research aims to characterize a new mechanism, inspired by the auctions, to match demand and supply in the logistics industry, and compare it with current mechanisms adopted by industry experts and academicians.

To address the research objective and gain insight into the operations model, we build a stylized model with consumers, trucks, and Distribution Centers (DC). Using a utility-based model, given problem parameters, and using an auction-based bidding process, we elicit the optimal decision for all the agents involved. Finally, using simulation we solve the problem computationally and elicit insights into different performance indicators. Furthermore, we analyze several extensions by relaxing some of the assumptions to highlight the performance of our proposed mechanism as well as ensuring robustness. The findings address the failure of the companies in disrupting traditional logistics companies as well as contribute to the development of new mechanisms to drive demand and efficiently match supply. From a practitioner's point of view, our findings show the efficacy of the proposed auction-based bidding by the packages across different performance indicators against the state-of-the-art industry practices. Furthermore, our study also paves ways for future research towards mechanisms that are more efficient and benefit all the involved agents through both theoretical and computational analysis.

The rest of the paper is structured as follows. In Section 3.2, we present an overview of the relevant literature. In Section 3.3, we present the context and explain the modeling methodology. We describe the experimental setup and results in Section 3.4. In Section 3.5, we discuss some extensions to further showcase the attractiveness and robustness of our model, and finally, in Section 3.6, we present concluding remarks, insights as well as future research directions. Proofs of all results are delegated to Appendix B for the sake of brevity.

3.2 Literature Review

This paper belongs to the growing stream of literature in two-sided markets where platforms facilitate the exchange of service between demand and supply, and we refer the readers to [Roc03], [Rys09], and [Par05] for a variety of contexts and market structures. We focus on on-demand service platforms for logistics in which customers demand service immediately and are sensitive to waiting time and prices while the drivers are sensitive to wages.

With respect to the analysis of on-demand service platforms, most of the research is focused on finding the equilibrium structure of parameters like optimal pricing structure, efficient matching, and efficient staffing ([Ban15; Cac17; Bai19; Tay18; Gur19]). Since our focus is to evaluate a pricing mechanism, we use results from papers, which focus on pricing, as a benchmark against which we compare our proposed mechanism. [Ban15] uses a queuing-theoretic economic model to compare dynamic pricing and static pricing under stochastic demand. [Tay18] uses a similar queuing formulation and examines how various sources of uncertainty, like congestion-driven delay and heterogeneity in valuations, affect the optimal price and wage. Finally, [Bai19], which uses a similar model as the two papers listed above, focuses on influencing the supply and demand through price and wages with impatient customers. With respect to context and modeling approach, these papers are closest to our research. All of these papers share a common theme, they use $M/M/k$ queue to model a ride-hailing platform and, with varying degrees of differences, elicit the optimal pricing structure. We use the results in these papers to model the base scenario in our study with some contextual changes.

The application of ride-hailing platforms in supply chain and logistics has received scant attention from both academia as well as practitioners. [Qi18] focus on the last-mile delivery and how the traditional last-mile delivery compares with a new paradigm of using ride-hailing platforms for last-mile deliveries. They find that while ride-hailing services might not be scalable as trucks, some compelling benefits warrant a serious study. Our work seamlessly complements their paper because we model the operational and pricing aspects of the logistics of the packages before the last-mile delivery i.e., between DCs and other smaller DCs in the network hierarchy. Also, it is important to point out that our work is distinct from the crowd shipping literature; crowd shipping refers to harnessing the already traveling users to transport goods ([Sim19]), but we are solely focusing on traditional modes of transportation, albeit, through a new mechanism.

We also draw upon the concepts of auction theory and mechanism design to design and implement our proposed mechanism which involves the packages bidding under a given auction mechanism. We abstract away from a rigorous economic analysis of auctions; rather we use established methodologies and tailor them contextually. We direct the interested readers to [Kri09] for a fundamental and excellent treatment on the subject. Finally, within this stream, we refer to the seminal work of [Vic61], [Cla71], and [Gro73] which provide a framework for designing auction mechanisms. For the practical applications of auction theory and mechanism design to logistics and supply chain problems, we refer the readers to [Kuy15; Qia20; Lia20; Xu17; Xu14; Laf19], and [Hua13]. Finally, simulation has long been used to analyze dynamic systems which cannot be analyzed analytically. We refer the reader to [Li20; Nik20; Bar19; Dev17; Fat15; VD07], and [Jan08] for a representative example of how simulation is used to model logistic systems and elicit operational insights.

3.3 Model And Pricing Mechanisms

We consider a logistics setup that deals with the transportation of packages between DCs and focus our attention on all the involved agents, DCs, packages that need to be transported, and the trucks that transport the packages. We assume all the agents are independent and participate only when they are sufficiently incentivized. Before describing the central aspects of the model, it is useful to provide a high-level description of the timeline of events here. When a package has to be sent between an OD pair, the sender goes to the nearest DC, observes the waiting time for service, number of packages vying for the same service, and pays a certain amount, p, while requesting transportation; we use two mechanisms to derive p, (i) Two-Sided Platform Pricing, which can be considered as status-quo, and (ii) Package Bid Pricing, which is our proposed mechanism, and describe both of them later in this section. The DC offers a fraction, δ, of this price, p, to trucks as wages for offering transportation as a service. Trucks who find this wage, $w = \delta p$, sufficient, accept the request. The DC's revenue stream, per package, stems from the fraction of the price paid by the packages in lieu of facilitating the matching of packages and trucks, $\pi = p(1-\delta)$. We now present the modeling details concerning the agents and the pricing schemes.

3.3.1 Customers/ Packages

Following [Bai19], [Tay18], and [Ban15], we use a queue theoretic framework to model the customers; we use the terms customer and package interchangeably. To be specific, we use a $M/M/k$ queue to model customer arrival and define a utility-based formulation to model customer decisions. Customers arrive at the DC randomly at a rate λ following a Poisson process. Each package has three characteristics, valuation (v), weight (M), and destination distance (D); all the packages require unit capacity. We define valuation as the maximum the customer is willing to pay to transport the package and it is drawn from a continuous distribution with a known probability distribution ($f_v(.)$), cumulative distribution ($F_v(.)$) function with a support of $[0, 1]$. The weight and destination distance are independent random variables drawn from known distributions with known expectations, $\mathbb{E}[M] = m$, $\mathbb{E}[D] = d$. Since the standard unit for freight pricing is 'weight-distance' for example 'tonne-mile' [USE21], we define the service requested by the package as a product of its weight and distance, MD. We define the utility function of a customer as $u_p = v - pMD - cW_q$, where p is the price paid by the package, c is the sensitivity of the packages to the waiting time and W_q is the expected waiting time. The package requests service only if it gains non-negative utility from the transaction, hence, the actual rate of arrival is given by the following,

$$\begin{aligned}\lambda_a &= \lambda\mathbb{P}(u_p \geq 0) \\ &= \lambda\mathbb{P}(v \geq pMD + cW_q)\end{aligned} \tag{3.1}$$

We assume that the packages do not know how many units the trucks can accommodate; in other words, the packages always bid as if the trucks can only accommodate a single package. We make this assumption here for the sake of tractability, simplicity, and eliciting insights from a simple modeling framework. We later relax this assumption in Section 3.5 and analyze the ensuing scenario.

3.3.2 Service Providers/ Trucks

From the supply side, there are K trucks registered in the system and they are heterogeneous with respect to their outside options, o. Here, outside options refer to other employment avenues like regular logistics hauling services. We assume that o is drawn from a known probability and cumulative distribution, $g_o(.)$ and $G_o(.)$ respectively, with support between $[0,1]$. Only those trucks who make at least as much as their outside option join this system and we define that fraction as β. The expected earning rate, e, for a truck in this system is given by, $e = w\mu MD\rho$, where μ is the service rate of the trucks which we assume to be exponentially distributed and ρ is the utilization of the system, which is given by $\frac{\lambda}{\mu k}$ and k is the number of active trucks in the system at that instance. The trucks, depending on their capacity, select the load bid (i.e., the set of packages they choose to carry) by selecting the packages in decreasing order of their bids. Once the truck accepts a load bid, it goes away from the system for the service duration and again becomes active or online following successful completion of the delivery.

3.3.3 Distribution Center

The DC facilitates the matching between the trucks and the packages and its revenue stream is a fraction of the amount paid by the packages who participate. The DC offers the price that a participating customer is willing to pay to the nearest active truck and continues to offer the quote to trucks in increasing order of distance until it is accepted. The profit function of the DC is given by, $\Pi_{DC} = \lambda(p - w)MD$.

3.3.4 Two-Sided Platform Pricing

We follow [Bai19] to establish this model, with some contextual changes, as the status-quo pricing scheme. In this system, the DC plays the role of a two-sided platform, with packages demanding service and trucks offering service forming the two sides. Once packages arrive, the DC shows an expected waiting time to the package which is based on the $M/M/k$ framework, and quotes the price p. Once the packages accept this price, conditional on gaining net positive utility, the DC offers a fraction of this price to the trucks as wages, $w = \delta p$. Similarly, trucks, conditional on earning at least as much as their outside option, accept this request. It is worth noting that the DC can influence the participation of both packages and trucks by setting p and w accordingly. The following theorem details the optimal price and wage,

Theorem 1. *The optimal price charged by the platform is given by the solution of the following constrained optimization problem,*

$$\begin{aligned} &\max_{k,\lambda} \Pi_{DC} \\ &s.t.\ \rho < 1 \\ &\qquad w = \delta p \end{aligned}$$

3.3.5 Package Bid Mechanism

In our proposed mechanism, instead of DC setting the price, we allow the packages to bid based on their valuations through an auction environment. This changes the dynamics of the system drastically because now the DC cannot influence the arrivals of the packages and trucks by adjusting p. On the contrary, the packages determine their optimal bids based on the auction format. The rate of arrival of the packages is the same as above, λ, and once the package arrives, it observes the number of packages currently waiting for service and the expected waiting time. Given these values, the package determines its optimal bidding strategy, and consequently, the optimal bid. The package only requests service if it gains a non-negative utility from the service. We also allow the packages to update their bid every time there is an arrival from the system to be fair to all packages and not implicitly favor packages that come in early or late. In contrast to the DC setting prices and thus influencing demand, in this scenario, the demand or package participation is driven by their valuations and bidding strategies. We analyze a first-price auction environment in this section; in Section 3.5, we analyze uniform price auction as well as Vickrey-Clarke-Groves (VCG) auction. The following theorem details the optimal bidding strategy adopted by the packages,

Theorem 2. *The optimal bid, b_n, of an incoming package, when there are $n-1$ packages in the system, is given by the following,*

$$b_n = \frac{n-1}{n}(v - c\,W_q)$$

3.4 Experimental Setup And Results

In this section, we describe our simulation set-up, the performance indicators, and provide a representative analysis of the simulation results. Owing to the dynamic nature of the setup and lack of analytical tractability, we implement a discrete event simulation to model both mechanisms. This enables us to implement the dynamics of different mechanisms, analyze the output and derive insights. We first implement the platform pricing mechanism, then we implement the package bid mechanism, and finally compare the output through some performance indicators, described in the subsequent section.

3.4.1 Initialization

We generate values for the input parameters in order to initialize our system; we have two types of input parameters, stochastic and deterministic. Consistent with the literature, stochastic parameters are generated randomly using predefined distributions, and deterministic parameters remain constant throughout. We use the following numerical distributions for the input parameters to present an illustrative example and analysis in the subsequent sections, $\lambda \sim PP(10)$, $\mu \sim exp(5)$, $v \sim \mathscr{U}[25,50]$, $M \sim \mathscr{U}[25,50]$, $D \sim exp(12)$, $o \sim \mathscr{U}[80,120]$, $c = 5$, $K = 100$, and, $\delta = 0.8$. We then implement the two mechanisms using results from Section 3.3. It is worth noting that since we use the same set of input parameters to simulate the two different mechanisms and extensions, the results serve to illustrate the ordinal performance of the mechanisms rather than absolute performance which would require rigorous normalization and parametrization. Thus, although the choice of parameters does impact the absolute performance, from a pure comparison point of view, any set of feasible input parameters, which remain the same for all mechanisms, are sufficient to illustrate the relative performance of different mechanisms. The use of different ranges and underlying distributions for the stochastic problem parameters does not affect our results.

3.4.2 Performance Indicators

To evaluate the performance of the package bid mechanism and ensure that all the agents have the incentive to participate in such a mechanism, we focus on the following performance indicators.

3.4.2.1 Cumulative Throughput at the DC

Within our framework, the DC plays an important role under both mechanisms. The DC influences the demand by setting the price and wages under the two-sided platform pricing mechanism whereas, in the package bid mechanism, the DC matches demand and supply. Since the revenue stream of the DC depends on a fraction of what the participating customers pay, the DC has an implicit incentive to join a mechanism that maximizes its throughput. Hence, we compare the throughput at DC under different mechanisms to evaluate which mechanism leads to higher throughput.

3.4.2.2 Profit of DC

Higher throughput does not automatically translate to higher profit for the DC. This is because, in the pricing mechanism, the DC might end up with high earning from a low throughput because of the high price charged per customer. In contrast, if the bids are sufficiently low in the package bid mechanism, then even with high throughput, the total earning might be less. Hence, we want to evaluate, given the same set of valuations and service requests, which mechanism leads to higher profits for the DC, thus incentivizing participation.

3.4.2.3 Price paid by the Customer

The price paid by the customers determines the demand in either of the mechanisms, however, it is not trivial which of the mechanism is beneficial to the packages because of variability in a multitude of parameters like service units requested and valuation or willingness to pay. While a package with a high valuation might find the platform pricing mechanism acceptable because of a possibly low quoted price, a package with a low valuation will certainly prefer the package bidding mechanism owing to the fact that a smaller bid has a possibility of getting accepted. We compare the actual price paid by the customers under the two mechanisms with respect to its valuation.

3.4.2.4 Trucks' Earnings and Package Wait Times

The actual time spent by the package waiting to get picked by a truck is an important indicator of the efficiency of service. Also, the actual amount that the trucks earn from a mechanism is indicative of the attractiveness of the system which influences the number of active trucks offering their services. Thus, we elicit these metrics from our simulation to gain insight into the attractiveness of a mechanism.

3.4.3 Sequence of Events

Following initialization, the DC offers the bids or prices of participating packages to the nearest truck. Depending on the truck's outside option and capacity, the truck selects a load bid and leaves the system for the duration of the service period i.e., time taken to deliver the package and re-enters after delivery is complete. However, if the nearest truck rejects the load bid, the DC then offers the set of bids or prices to the next nearest truck and continues the process. To maintain computational tractability, we limit our time duration to 1000 time units. Following the completion of the predefined time horizon, we extract the performance indicators described in the previous section. Finally, to gain an understanding of the long-run behavior of the system we conduct a Monte Carlo simulation 10,000 times and analyze the expected or long-run average values of the performance indicators. The sequence of events is illustrated in Figure 3.1.

3.4.4 Numerical Analysis

Our results indicate that the package bidding mechanism outperforms the platform pricing mechanism by a significant margin. From the point of view of throughput at DC, since demand is now driven by the bids, more packages find it worthwhile to participate. Consequent to more packages using the system, the profit at the DC also increases as compared to the platform pricing. Finally, since packages indulge in shade bidding i.e., bidding less

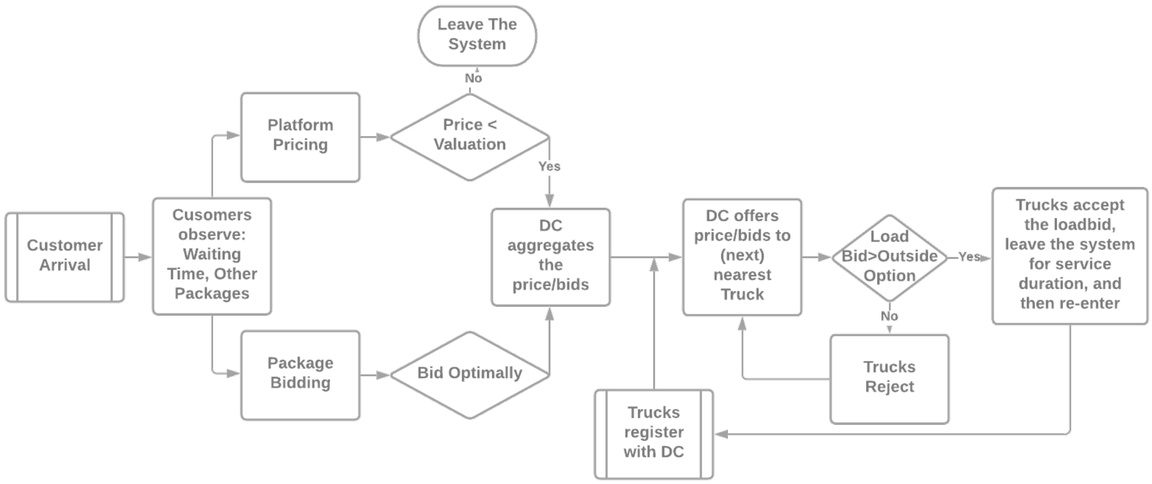
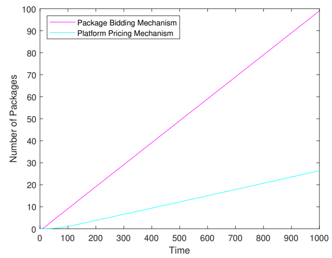
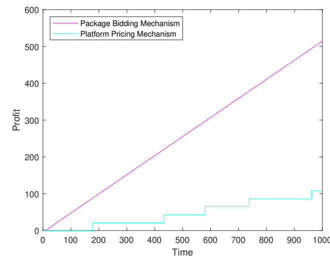


Figure 3.1 Sequence of Events

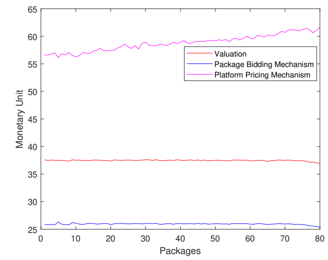
than their valuation, the amount they eventually pay is also less than their valuation and significantly less than the amount charged under the platform pricing mechanism. Table 3.1 and Figure 3.2 detail the numerical output of our simulation model. From Table 3.1, we see that the number of active trucks almost increases by 300% under the package bidding mechanism with an increase of almost 7% in earnings. From the point of view of the packages, under the package bid mechanism, their waiting time decreases by almost 90%. Furthermore, the packages pay around 32% less than their valuation and 52% less than the amount charged under the platform pricing mechanism. Finally, from the DC's point of view, the throughput and the profit increase by 400% and 500% respectively under the package bid mechanism. One of the reasons the package bid mechanism comprehensively outperforms the platform pricing mechanism is that the demand is driven by the package bids. The platform, DC, in this case, plays a passive role of matching demand and supply rather than affecting demand and supply through setting price and wages. Thus, even though the packages end up paying lesser than under platform pricing, the volume of throughput ensures a higher profit.



(a) Cumulative Throughput at DC



(b) Cumulative Profit of DC



(c) Amount paid by the Customer

Figure 3.2 Package Bid Mechanism vs Platform Pricing

Table 3.1 Package Waiting Times and Trucks' Earnings

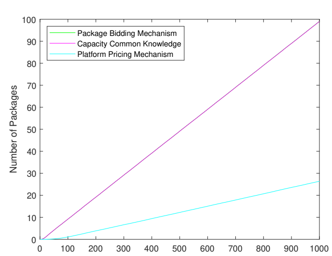
Mechanism	Average Number of Active Trucks	Average Earnings	Average Waiting Time of Packages
Platform Pricing	5.34	87.16	233 (± 119.46)
Package Bidding	22.143	93.0157	18.619 (± 18.977)

3.5 Extensions

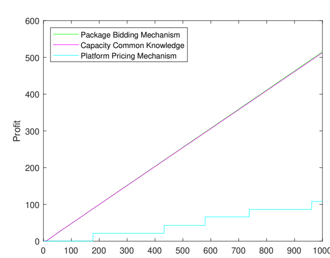
In this section, we relax some assumptions from Section 3.3 and analyze other widely used auction mechanisms. In the first extension, we relax the assumption that the packages do not know the capacity of the trucks, and hence, bid for unit capacity. We relax this assumption and assume that the truck's capacity is common knowledge. Consequently, this takes the form of a uniform price auction. Subsequently, we analyze another form of auction, the VCG auction. We analyze the findings of each of the extensions concerning all the performance indicators in the subsequent sections and present the details of trucks' earnings and packages waiting times in the end.

3.5.1 Capacity of Trucks is Common Knowledge

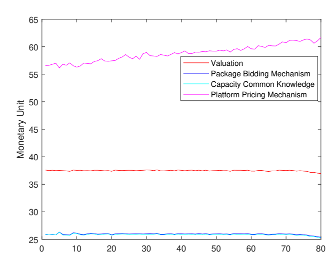
In this section, we assume that the trucks indicate their capacity to the DC and the DC makes this knowledge public. So, when the packages arrive, in addition to observing the number of other packages and waiting time, they also observe the capacity of the trucks. Since the bids are offered in the increasing order of distance from the DC, they initially bid according to the capacity of the nearest truck and subsequently bid according to the capacity of the truck to which the bid is being offered. This essentially takes the form of a uniform price auction and since each package requires unit space, the optimal bid is to bid truthfully [Kri09]. The truck accepts the packages based on their capacity and the packages have to pay the market-clearing price. The market-clearing price here refers to the lowest of the bids selected by the truck in the load bid [Kri09]. The only difference between this extension and our main analysis is that there is more credible information available to the customers, hence, we can interpret this extension as evaluating the value of information. We can see from Figure 3.3 that, in expectation, the first-price auction with no information on capacity coincides with the uniform price multi-unit auction with capacity being common knowledge and outperforms the platform pricing mechanism. This is an interesting result because these two auction formats are different and first-price auctions are not always efficient whereas the uniform price auction for unit demand is always efficient [Kri09]. However, the reason they produce almost similar results is because of the high number of incoming and participating packages. We know that the optimal bidding strategy for the first-price auction is to shade bid, $\frac{n-1}{n}v$, where n is the number of other packages and v is the valuation, and for uniform price auction with unit demand is to bid truthfully, v; thus, as n increases, the optimal bids start converging. Since the truck's decision process remains unaffected, the results of both these formats coincide in expectation.



(a) Cumulative Throughput at DC



(b) Cumulative Profit of DC

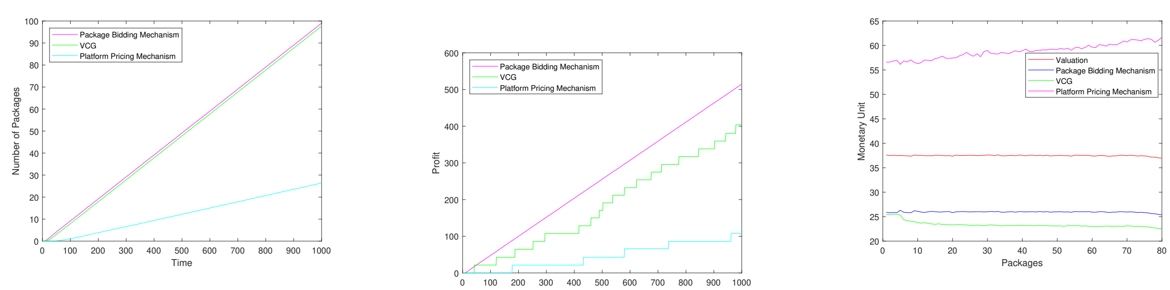


(c) Amount paid by the Customer

Figure 3.3 Capacity Common Knowledge

3.5.2 VCG Auction

In this extension, we keep assuming that the capacity of the trucks is common knowledge and use an extension of the second-price auction, the VCG auction. Owing to the established results that the VCG auction is efficient, produces socially optimal results, and induces truthful bidding [Vic61; Cla71; Gro73] and for the sake of completeness, we analyze this auction environment. We can see from Figure 3.4 that the throughput at the DC is significantly higher than that of the platform pricing mechanism but slightly lower than the package bidding mechanism. The profit of the DC, under VCG, is lower than the profit under the package bid mechanism and higher than the profit under the platform pricing mechanism. Finally, the actual amount paid by the packages is the lowest of all mechanisms and significantly lower than their valuations. Table 3.2 enumerates and quantifies these results. The results can be ascribed to the algorithmic design of VCG auctions. Since the VCG auction mechanism inherently recalculates and redistributes the bids, eventually charging individual bidders the harm they cause to other bidders [Cra06], akin to Shapley value but with opposite intent [Sha53], the packages end up paying less than their valuations thereby reducing profits at the DC. From Table 3.2, we see that both the trucks and the DC prefer the package bid mechanism to the VCG mechanism, however, the customers prefer the VCG mechanism over the package bid mechanism; all three agents prefer VCG to platform pricing mechanism.



(a) Cumulative Throughput at DC

(b) Cumulative Profit of DC

(c) Amount paid by the Customer

Figure 3.4 VCG Auction Environment

Table 3.2 Performance of VCG Auction Environment

Performance Indicator	VCG Auction	vs Package Bidding	vs Platform Pricing
Cumulative Throughput at DC	100 packages	↓ 1 %	↑ 400 %
Cumulative Profit of the DC	402 monetary units	↓ 34 %	↑ 300 %
Amount paid by the Customer	23 monetary units	↓ 11 %	↓ 60 %
Number of Active Trucks	21.0773	↓ 5%	↑ 300 %
Earnings of Trucks	87.45 monetary units	↓ 6%	↓ 0.3%
Package Wait Times	31.603 (± 22.386) time units	↑ 90 %	↓ 86%

3.6 Conclusion And Discussion

Motivated by the lack of success of on-demand service firms in the logistics sphere, we propose an alternative mechanism that serves to address the potential shortcomings of the status-quo pricing mechanisms i.e., platform pricing. In the platform pricing mechanism, the platform manipulates the demand and supply by setting appropriate prices and wages, for both sides respectively, while maximizing its profit. Since the customers are price takers, their participation is constrained by the price set by the platform, which means that packages that do not derive a positive utility from the system, choose not to participate. However, we propose a mechanism, motivated by auction theory, where the packages bid their willingness to pay to get service. In this case, instead of being constrained, all the packages participate because there exists a positive probability that they will get service for the amount they are willing to pay. Although this mechanism serves to focus primarily on the packages, we also analyze the impact of the said mechanism on the trucks as well as the DC. From the point of view of trucks, this mechanism is a means to complement their existing operations by allowing them to utilize their capacity more efficiently while enhancing their earnings. We find that the package bidding mechanism outperforms the platform pricing mechanism significantly across all performance metrics, and proves to be beneficial for all three agents. Having established the better performance of the package bidding mechanism, we extend our analysis to two different types of auctions, Uniform

Price Auctions, and VCG Auctions. We find that, in the long run, the uniform price auction is indistinguishable from the first-price auction; also, the packages prefer the VCG auction format while the DC and the trucks prefer the first-price auction.

In this paper, we analyze a stylized and simple model to gain insights into the efficacy of our proposed mechanism. However, this results in abstracting away from many operational details. For example, we analyze a single DC, whereas, in real life, a DC is almost always a part of a larger network. Thus, it would be interesting to conduct a similar analysis for a network of DCs, evaluate how our mechanism performs, and especially pay attention to whether or not the agents try to game the system, e.g., by colluding. Also, in this paper, we focus on the ordinal performance of our mechanism, it would be more rigorous if we normalize and scale the parameters to reflect real-life values and then analyze the ensuing outcomes. For example, we could normalize and scale the valuation, weight, and distance to a single parameter, $\frac{v}{MD}$, which signifies the ratio of valuation and requested service units. This parameter warrants further analysis because, intuitively, a high valuation package with low service units demand might prefer the platform pricing mechanism because otherwise, it would have to bid, and potentially, pay a high amount owing to its high valuation. Also, it would be interesting to analyze the impact of holding cost at the DC because that would implicitly place an upper bound on the time a package can spend at a DC, waiting for service.

CHAPTER 4

Competitive Implications of Spectrum Sub-Leasing on Optimal Price, Quality and Sourcing Decisions

4.1 Introduction

Technological advancement in the field of telecommunications has seen tremendous growth in both the number of consumers as well as newer technologies and strategies being implemented by the cellular network operators. The number of cell phone users is, at this moment, higher than ever before and steadily growing. Similarly, the number of firms offering cellphone network services is increasing at a steady rate. However, offering cellular services is a very capital-intensive operation and that, coupled with technological innovation, has led to a very interesting business model. Traditionally, cellular services were only offered by firms, known as Mobile Network Operators (MNO), who participated in the Federal Communications Commission (FCC) auction and leased bandwidth from FCC. But, recently, a different class of firms, known as Mobile Virtual Network Operators

(MVNO), have emerged, who do not participate in the spectrum auction by FCC, but sub-lease bandwidth from the traditional firms or the leaseholders. The idea is that, due to the capital-intensive nature of the business, the leaseholders get to recoup some of their investment through sub-leasing, and firms who sub-lease, get market entry without having to make substantial investments. For example, firms like T-Mobile and AT&T participate in the FCC auctions and also invest heavily in the infrastructure and are considered as MNOs, and firms like Republic Wireless, Cricket Wireless, and Google Fi, who sub-lease bandwidth from MNOs, are considered as MVNOs. Owing to this shift in the business model, we observe some interesting non-trivial dynamics in the market structure like cannibalization of consumers, market share shift, the interdependence of quality of service & bandwidth, and market segmentation.

The MNOs lease spectrum from the FCC through an auction process. They offer cellular service to the consumers at a pre-announced quality of service and charge a certain price for the service. The quality of service can be interpreted as the voice clarity, network strength, or the number of dropped calls and it is constrained by the bandwidth held by the firm. We define the bandwidth, in the broadest sense abstracting away from the telecommunications-related technical definition, as the capability to serve a population at a given quality of service. This definition implies that to maintain a high quality of service, a firm needs sufficiently high bandwidth. We abstract away from the mechanics of the auction process used to secure bandwidth, and rather focus on scenarios where the MNOs have limited bandwidth. In addition to being expensive, having an excessive surplus of bandwidth is considered hoarding and has anti-trust implications. Then, the consumers decide if they want to subscribe to the announced service or not. The MVNOs, when they enter the market, sub-lease spectrum from the MNOs under a certain revenue-sharing agreement, which can be one-time wholesale price, two-part tariff, or per unit demand price. The MVNOs offer similar services to the same market, announce their quality of service as well as the price, and customers decide whether to subscribe to M(V)NOs or none of the service providers.

It is important to note that, usually, the quality offered by the MVNO is always less than the quality of service of its parent MNO; this is because when the MNO sub-leases bandwidth, it prioritizes its quality, and the MVNO's quality is reduced to compensate for the limited bandwidth. The profit function for the MNOs is dependent on the number of subscribing customers, presence of MVNOs, and if present, the revenue sharing agreement with them; hence, the MNOs decisions are to decide whether to sub-lease to another firm or not and how to price its offering keeping in view the presence of other, potentially, competing firms. Similarly, MVNO decides which firm to lease from and adjust prices keeping in view the reduced quality offered by the MNO. These strategies differ in different types of markets i.e., monopoly or duopoly. Hence, we need to understand how these markets operate under different market conditions.

In this paper, we study the market equilibrium outcomes for monopolistic and duopolistic cellular service provides, motivated by examples like AT&T and Google Fi. Specifically, we focus on the sub-leasing decision faced by the firms and the subsequent effects on the market structure. We focus on the competitive implications of the sub-leasing spectrum between MNO and MVNO on the price and quality. To gain insights into the business model, we analyze two competing cellular service providers, MNOs, with different quality of service levels, with and without the presence of MVNO. We develop and analyze a stylized high-level game-theoretic model to characterize the equilibrium pricing, quality, and sourcing decisions of the MNOs and MVNOs.

We characterize the relationship between the quality of the MVNO with the bandwidth of the MNO. We identify equilibrium conditions for two sourcing strategies of the MVNO, when it is optimal for the MVNO to source from both the MNOs and when it is optimal to source from only the higher quality MNO. Furthermore, we show that the equilibrium market structures differ when the MNOs set their prices anticipating the entry of MVNOs as compared to when they do not anticipate the entry of MVNOs. In the former, the high-quality MNO can eliminate the demand for the low-quality MNO by leveraging the entry of the MVNO whereas, in the latter, all the three firms co-exist in the market.

The rest of the paper is structured as follows. In Section 4.2, we present an overview of the extant literature. In Section 4.3, we present the model formulation. In Section 4.4, we analyze different market structures and characterize the market equilibrium, and finally, in Section 4.5, we present an overview of the findings, managerial insights, and concluding remarks. Proofs of all results are delegated to Appendix C.

4.2 Literature Review

Our work is related to a stream of literature, albeit not directly related and with contextual changes, which can be considered as analogous, Omni-Channel retail. Typically, in this setting, a manufacturer sells to the end consumers in two different ways, directly through its channel and indirectly through an independent retailer. In our setting the MNOs and MVNOs are equivalent to manufacturers and retailers respectively. However, there exist subtle differences like the presence of bandwidth constraints, the product is an information good and the sequence of events is different. We refer the readers to the following paper and references therein for a thorough treatment of this stream, [Ary18], [Chi03], [Gao17], [Har19] and [Zha10].

Our work is closely related to a growing literature on the telecommunications market. Since the emergence of MVNOs is a fairly recent phenomenon, there have been very few studies that analyze the business model of spectrum sub-leasing; Most of the papers related to MVNOs focus on the technological aspects and mechanism design of spectrum auctions. Since our focus is to analyze the competitive implication of the sub-leasing spectrum to MVNO, we abstract away from those specific aspects. The papers which are closest to our in terms of context, modeling approach, and solution techniques are, [Li19], [Dua11], [Gui11], [Lot17], [Zhu19], [Kha18], and [Deb12]; all of them analyze equilibrium pricing strategy of MNOs and MVNOs. However, none of them consider, (i) the impact of competition i.e.

presence of more than one MNO and an MVNO, (ii) the impact of the bandwidth held by an MNO on the leasing, pricing, and sub-leased quality decisions, and (iii) sourcing decision of the MVNO where the MVNO is capable of sourcing from either or both the MNOs. To the best of our knowledge, we are the first to consider the competitive implication of sub-leasing of the spectrum between MNOs and MVNOs while taking into account the effect of limited or constrained bandwidth, quality levels of both MNO and MVNO, and sourcing decision of the MVNO.

From a modeling point of view, our work draws inspiration from the Product Differentiation and Quality Competition literature. In their seminal work, [Mus78], characterize the solution to the pricing problem faced by a monopolist with a quality-differentiated product line up. This work, more than others, kick-started the research on product differentiation and price & quality competition. Many marketing and economics papers, for example, [Mil86],[Sha82] and [Moo88] look at pricing problem and product differentiation under different settings like signaling, information asymmetry and seller reputation.

The next streams of literature to develop from the above papers were based on price & quality competition and vertical product positioning under the effect of competition. Gabszewicz and Thisse in a series of papers (e.g., [d'A79; Gab79; Gab82; Gab86a; Gab92; Gab86b; d'A83]), look at different scenarios involving location models, vertical/horizontal differentiation, market segmentation, quality choice, and competition and characterize the equilibrium solution for each scenario. A couple of papers, notably, [Lan90], [Kar97] and [Ban98] study the relationship between quality and competition; they obtain equilibrium pricing results for different market conditions like monopoly and oligopoly. [Mat07] studies a scenario with price and quality competition and elucidate strategies to mitigate competition. They also show results concerning consumer welfare and the impact of differentiation and competition on welfare. Finally, we refer to [Tir88] and [Shu80] for a review of theories of market structure, market equilibrium, industrial organization.

4.3 Model

In this section, we detail the modeling preliminaries and assumptions with respect to the consumers, MNOs, and MVNOs.

4.3.1 Consumer Decision

For consumers, we use a vertically differentiated spatial model to represent their utility function, with the vertical dimension representing the preference for quality, θ. Without loss of generality, we assume that consumers are distributed uniformly along a line of unity length. A customer associates the following utility with purchasing cellular connection from a firm, MNO or MVNO,

$$u_i = \theta q_i - p_i \tag{4.1}$$

where q_i is the quality of service, $\theta > 0$ is the customer's type evaluating quality, p_i is the subscription price charged by the firm and i indexes the firm. Some details are implicitly embedded in the model due to the choice of the utility model; everything else remaining the same, a consumer prefers a product of higher quality; this is a result of θ being positive. We can derive the market demand using the above defined utility function by deriving the indifferent customers (see analysis in Appendix C.1) i.e., indifferent between either subscribing to any firm or not subscribing. We define the indifferent customers, θ_{jk}, as

$$\arg_{\theta \in [0,1]} u_j = \{\theta_{jk} \in [0,1] : u_j = u_k \ \forall \theta \in [0,1]\}$$

with j, k indexing the firms.

4.3.2 MNO

The MNOs acquire spectrum with the associated bandwidth from the FCC through auctions and based on the acquired bandwidth, Q_i, they decide the quality of service, q_i. We assume that q_i and Q_i are exogenous to our model; apart from the benefit of tractability, this allows us to focus on the pricing, quality of MVNO, and sourcing decisions without losing any relevant insights. We consider two MNOs M_1 and M_2 offering products with (exogenous) quality levels q_1 and q_2 at retail prices p_1 and p_2 respectively to the market. Without loss of generality, we normalize q_1 to 1 and q_2 to α, where $\alpha \in (0,1)$; this ensures that we have two quality differentiated MNOs. Since we are dealing with information goods, we assume that there is no operating cost for the platform and normalize all the investments or sunk cost to zero. We assume an MNO sub-leases spectrum to an MVNO for a per-unit demand wholesale price. Without the presence of the MVNO, the MNOs earn revenue through the subscription price charged to the consumers. When MVNO is present in the market, the MNOs also earn revenue through a per-unit demand wholesale price charged to the MVNO. The profit of the MNOs, with and without MVNO, then is given by,

$$\pi_i = \begin{cases} p_i d_i & \text{without MVNO} \\ p_i d_i + w_i d_v & \text{with MVNO} \end{cases} \tag{4.2}$$

where w_i is the wholesale price charged by the MNO, d_i is the demand from the consumers for the MNO and d_v is the demand for the MVNO. Additionally, we define the bandwidth usage as the amount of customers served at a given quality level, $q_i d_i$. Consequently, the MNOs operate under the following constraint, $q_i d_i + q_v d_v \leq Q_i$, the expression $q_v d_v$ being applicable when subleasing to MVNO. Also, we assume that the MNOs have limited bandwidth, the implications of which we present in the next sub-section.

4.3.3 MVNO

The MVNO acquires spectrum from either of the MNOs by paying a per-unit demand wholesale price, w_i. The quality of the MVNO is determined by the multiplier δ_i; thus, it is given by $q_i\delta_i$. Depending on whether the MVNO is sourcing from M_1 or M_2, its quality is δ or $\delta\alpha$ respectively. The implication of the limited bandwidth constraint is relevant when the MVNO is sourcing from the higher quality MNO, M_1. In such a scenario, we assume that $\delta<\alpha$ i.e., the quality of the MVNO is always lower than the quality of M_2. Finally, the MVNO earns revenue from the subscription price, p_v, it charges to the consumers and incurs w_i as a cost. Profit for the MVNO then is,

$$\pi_v = (p_v - w_i)d_v \tag{4.3}$$

4.3.4 Game Sequence

The overall game sequence is as follows. After the MNOs have acquired spectrum and bandwidth and decided on the quality of service exogenously, they set the prices they charge the consumers. They can set the prices in two different ways; they can either take into account the potential entry of an MVNO and set prices accordingly or they can set the prices according to a duopoly market structure and ignore the potential entry of an MVNO. This distinction is analogous to the scenario where the MNOs are already established in a market with the MVNO entering later and when all three firms enter a new market i.e, in the former, the prices of the MNO are set at the duopoly level and do not chane, while, in the latter, the MNOs set their prices knowing the MVNO is present and going to set its price based on the MNOs prices. Then, the MNOs set the quality multiplier for the MVNO followed by setting the per-unit demand wholesale prices for the sub-leased spectrum at the set quality level. At this point, the MVNO sets the price it charges the consumers, should it enter the market. Finally, the demand is realized for all the firms through the customers' utilities.

4.4 Analysis

In this section, we characterize the optimal prices, quality levels of MVNO, and the optimal sourcing decision of the MVNO under different market structures. First, in Section 4.4.1, we analyze the base case as a duopoly competition between the two MNOs without the presence of the MVNO. For the sake of completeness, we also completely characterize the equilibrium for a monopolistic MNO with and without MVNO (see analysis in Appendix C.1). We then introduce an MVNO and first characterize the equilibrium when the MVNO sources from a single MNO and then when the MVNO has the option to either from the high-quality MNO or both the MNOs in Section 4.4.2 and Section 4.4.4 respectively. In each of the sections, we analyze the equilibrium when the MNOs do not anticipate in the presence of the MVNO and when they anticipate the presence of the MVNO.

4.4.1 Base Case: No MVNO

Following the game sequence, we derive the equilibrium using backward induction. In the absence of MVNO, the MNOs compete in a duopoly and solve the following constrained optimization problem,

$$\max_{p_i} \pi_i$$
$$s.t.\ q_i d_i \leq Q_i$$

The equilibrium is detailed in the following Proposition,

Proposition 7. *In a duopoly, the optimal prices and profits of the MNOs, given different values of the problem parameters, is given in Table 4.1.*

Table 4.1 Equilibrium: Duopoly

	$\frac{2}{4-\alpha} \le Q_1$, $\frac{\alpha}{4-\alpha} \le Q_2$	$\frac{2}{4-\alpha} > Q_1$, $\frac{\alpha}{4-\alpha} > Q_2$	$\frac{2}{4-\alpha} \le Q_1$, $\frac{\alpha}{4-\alpha} > Q_2$	$\frac{2}{4-\alpha} > Q_1$, $\frac{\alpha}{4-\alpha} \le Q_2$
p_1^*	$\frac{2(1-\alpha)}{4-\alpha}$	$1-Q_1-Q_2$	$\frac{(1-Q_2)(1-\alpha)}{2-\alpha}$	$\frac{2(1-\alpha)(1-Q_1)}{2-\alpha}$
p_2^*	$\frac{\alpha(1-\alpha)}{4-\alpha}$	$\alpha(1-Q_1)-Q_2$	$\frac{(\alpha-2Q_2)(1-\alpha)}{2-\alpha}$	$\frac{\alpha(1-Q_1)(1-\alpha)}{2-\alpha}$
π_1^*	$\frac{4(1-\alpha)}{(\alpha-4)^2}$	$Q_1(1-Q_1-Q_2)$	$\frac{(1-\alpha)(1-Q_2)^2}{(2-\alpha)^2}$	$\frac{2Q_1(1-Q_1)(1-\alpha)}{2-\alpha}$
π_2^*	$\frac{(1-\alpha)\alpha}{(\alpha-4)^2}$	$\frac{Q_2((1-Q_1)\alpha-Q_2)}{\alpha}$	$\frac{Q_2(1-\alpha)(\alpha-2Q_2)}{\alpha(2-\alpha)}$	$\frac{\alpha(1-\alpha)(1-Q_1)^2}{(2-\alpha)^2}$

We focus on the values of Q_i such that the bandwidth constraint is not binding; otherwise, the MNOs will not be left with any bandwidth to be able to sub-lease spectrum to any entering MVNO.

4.4.2 MVNO sourcing from a single MNO

In this section, we introduce an MVNO in the market where it can source from either of the two MNOs, M_1 or M_2. We analyze two scenarios as evidenced by observations from the business operations of cellular service providers. First, when the MNOs set their prices in the first step, they do not take into account the potential entry of the MVNO into the market i.e., We assume that the MNOs set their prices and do not frequently change the prices by reacting to any external presence i.e., the MVNO. This is a reasonable assumption and follows the current business practices from the telecommunications market i.e., many cellular providers have annual plans wherein the prices charged to the consumer are fixed for a year. This is akin to the scenario where the MNOs are established in the market, credibly committed to the duopoly prices, and an MVNO subsequently enters the market. Second, we analyze the scenario wherein the MNOs take into account the potential presence of the MVNO and set their prices accordingly. This is akin to when all three firms are entering a new market and the MNOs can change prices frequently, without being tied to any commitment. We again follow backward induction to characterize the equilibrium; the following Proposition details the result when the MVNO sources from M_2,

Proposition 8. *In equilibrium, the MVNO will never source from M_2.*

Proposition 8 states that when the MVNO sources from M_2, it essentially cannibalizes the market share of M_2 without affecting the market share of M_1. Thus, intuitively, M_2 sets the price and the wholesale price such that the demand for the MVNO is reduced to 0. When the MVNO sources from M_1, following the assumption of limited bandwidth, the quality of the MVNO is the lowest of all the offerings in the market i.e., $\delta < \alpha$.

Lemma 4. *The profit as well as the bandwidth usage of M_1 is a monotonic increasing function of δ_1.*

Lemma 4 provides a key insight into how the MNO determines the quality of the MVNO. Combined with the fact that M_1 has limited bandwidth and the Lemma 4, M_1 will try to set as high δ_1 as possible which will lead to an increased bandwidth usage as well, leading to all bandwidth getting exhausted. At this point, δ_1 will be determined automatically as the bandwidth constraint becomes binding. The ensuing equilibrium is detailed in the following Proposition,

Proposition 9. *When the MVNO sources from M_1, and the MNOs do not react to the presence of MVNO, all three firms co-exist in the market. However, under similar circumstances, when the MNOs set prices anticipating or reacting to the presence of the MVNO, M_1 can reduce the demand of M_2 to 0, thereby eliminating competition.*

Proposition 9 provides insight into how all three firms can co-exist in the market with an MVNO if the MNOs do not set their prices anticipating the potential entry of MVNO. Intuitively, this lack of anticipation prevents M_1 to fully leverage the presence of the MVNO to cannibalize and reduce the demand of M_2 to 0, thereby eliminating competition. However, when the MNOs set prices reacting to the MVNO's presence, M_1 sets the prices such that the MVNO's demand completely cannibalizes the market share of M_2.

4.4.3 MVNO sourcing from both MNOs

When the MVNO sources from both MNOs, we define the quality of the MVNO as, $q_v = \gamma(\delta_1 + \alpha\delta_2)$, where, γ is the parameter that determines the eventual quality of the MVNO. Depending on the value of γ, the quality of the MVNO can be best, worst, and in between the quality of all the offerings. For example, given a sufficiently high γ i.e., $\gamma > \frac{1}{\delta_1+\alpha\delta_2}$, the MVNO's quality of service is higher than that of both the MNOs. Again, this is evidenced from the real-life business operations. In this scenario, higher quality of service may refer to having a near-constant network coverage, owing to sourcing from both networks. For example, Google Fi, which sources from T-Mobile, Sprint, and US Cellular, enjoys a better network coverage than all the three firms it sources from. However, in other instances, sometimes sourcing from more than one MNO might lead to network congestion and dropped calls thus reducing the overall quality of the MVNO.

Proposition 10. *An MVNO will only source from both the MNOs iff its offering's quality is highest of all the three offerings.*

4.4.4 MVNO with option of sourcing from both MNOs

Here, we focus on cases where the MVNO has the option to source from both MNOs or from only the high quality MNO; we consider the MVNO to have the highest quality of service of all the offerings in the former and lowest quality in the latter. The following Proposition details the equilibrium in this scenario,

Proposition 11. *When the MVNO has the option to source from both the MNOs and the MNOs set their prices without anticipating the presence of MVNO, depending on the problem parameters, in equilibrium, the MVNO might source from both MNOs or only source from the high-quality MNO. Conversely, when the MNOs set their prices anticipating the MVNO, in equilibrium, the MVNO always sources from both the MNOs. In both scenarios, all three firms co-exist.*

4.5 Discussion and Conclusion

With the advent of rapid technological innovations, new firms, known as MVNOs, have emerged wherein they sub-lease spectrum from original leaseholders, known as MNOs, and compete for the same set of consumers. However, the pricing decisions, quality level of MVNO, and sourcing decisions of MVNO are non-trivial and lead to different realizations of market shares through cannibalization. In order to gain insights into the business model and characterize the equilibrium strategies, we develop and solve a stylized game-theoretic model. Unlike the extant literature focusing on MVNOs, which mostly focus on the technological aspects of MVNO and mechanism design of the spectrum auction, we focus on the competitive and economic implications of the spectrum sub-leasing on the pricing, quality, and sourcing decisions.

We make several contributions to literature. First, we characterize the relationship between the quality level of the MVNO and the bandwidth held by the MNO. This aspect is critical in formulating a constrained optimization problem, which mimics the real-world operational constraints of MNOs. Second, we identify conditions under which it is optimal for the MVNO to source from either both MNOs or from only the high-quality MNO. For an MVNO, these conditions are extremely important to understand because they are critical in identifying when to enter a market and where to source from, which in turn affect the profitability of the MVNO. Finally, we analyze the equilibrium market conditions when the MNOs commit to their prices and do not anticipate or react to the entry of MVNO and when the MNOs set their prices while anticipating the presence of the MVNO. These findings provide managerial insights into whether or not the MNO should commit to their prices in the presence of the MVNO.

Owing to the highly stylized nature of our model, we have abstracted away from many details which present many directions for future research. For example, it would be interesting to endogenize the quality decisions into our model. Furthermore, it would also be interesting to incorporate the spectrum acquisition decision vis-à-vis bandwidth held by MNOs into our modeling framework. It is our opinion that incorporating the above two aspects would make the model even more rigorous and robust.

CHAPTER 5

Conclusion and Future Work

In this dissertation, we focus on emerging business models fuelled by technological advances in the field of transportation, logistics, and telecommunications. Specifically, we look at a two-sided on-demand service platform and focus on the strategic behavior of the service providers to game the platform's pricing algorithm to enhance their earnings. In another context, we propose, implement and analyze a new pricing mechanism based on the auction environment and evaluate its efficiency against the two-sided platform pricing mechanism using simulation. Finally, within the context of the telecommunications industry, we focus on the implications of sub-leasing spectrum on the pricing, quality, and sourcing decisions in the presence of competing firms. To gain meaningful insights, we developed stylized game-theoretic models mimicking the emerging business operations with the associated economic implications and solved them using techniques from the theory of constrained optimization.

In Chapter 2, we characterize how and under what conditions the drivers collude. Then, we elucidate the effect of this collusive behavior on the platform, drivers, and the customers.

We find that, as opposed to the customers who are always harmed by the collusion, the platform is not always hurt by this collusive behavior. Finally, we prescribe two mitigating strategies for the platform to implement, when it is negatively affected by the collusion, to dissuade the drivers from colluding. In Chapter 3, we focus on an on-demand transportation service for packages and propose a new pricing mechanism based on auctions. Using simulation we find that the new mechanism outperforms the two-sided platform pricing mechanism across all performance indicators. Finally, in Chapter 4, we characterize the equilibrium pricing, quality, and sourcing decisions of two competing MNOs and an MVNO. We show the realizations of the market structure with respect to the co-existence of all firms or elimination of a competing firm under different sourcing conditions.

There are many directions for future research in all three bodies of works. For example, in Chapter 2, we assume that the prices and the pricing structure are exogenous to our model; it would be interesting to endogenize them and find out the optimal pricing structure and optimal prices in the presence of colluding drivers. Similarly, in Chapter 3, we only consider a single DC and its local network. It would make the analysis more robust if we can extend our analysis to a network of DCs and their associated networks. Finally, in Chapter 4, it would be interesting to endogenize the quality and the bandwidth decisions in our model to see the structure of the optimal quality and bandwidth for the MNOs.

REFERENCES

[Ach20] Acheampong, R. A. et al. "Mobility-on-demand: An empirical study of internet-based ride-hailing adoption factors, travel characteristics and mode substitution effects". *Transportation Research Part C: Emerging Technologies* **115** (2020), p. 102638.

[Afe18] Afeche, P et al. *A tale of timescales: Surge pricing and dynamic matching for hotspot demand shock in ridehailing networks*. INFORMS Revenue Management and Pricing Section Conference. 2018.

[Ame20] American Trucking Association. *News and Insights*. `https://www.trucking.org/news-insights`, accessed March 28, 2021. 2020.

[Arm06] Armstrong, M. "Competition in two-sided markets". *The RAND Journal of Economics* **37**.3 (2006), pp. 668–691.

[Ary18] Arya, A. & Mittendorf, B. "Bricks-and-mortar entry by online retailers in the presence of consumer sales taxes". *Management Science* **64**.11 (2018), pp. 5220–5233.

[Bai19] Bai, J. et al. "Coordinating supply and demand on an on-demand service platform with impatient customers". *Manufacturing & Service Operations Management* **21**.3 (2019), pp. 556–570.

[Ban15] Banerjee, S. et al. "Pricing in ride-sharing platforms: A queueing-theoretic approach". *Proceedings of the Sixteenth ACM Conference on Economics and Computation* (2015), p. 639.

[Ban98] Banker, R. D. et al. "Quality and competition". *Management science* **44**.9 (1998), pp. 1179–1192.

[Bar19] Barenji, A. V. et al. "Intelligent E-commerce logistics platform using hybrid agent based approach". *Transportation Research Part E: Logistics and Transportation Review* **126** (2019), pp. 15–31.

[Ber20] Bernstein, F. et al. "Competition between two-sided platforms under demand and supply congestion effects". *Manufacturing & Service Operations Management* (2020), forthcoming.

[Cac17] Cachon, G. P. et al. "The role of surge pricing on a service platform with self-scheduling capacity". *Manufacturing & Service Operations Management* **19**.3 (2017), pp. 368–384.

[Cai01] Caillaud, B. & Jullien, B. "Competing cybermediaries". *European Economic Review* **45**.4-6 (2001), pp. 797–808.

[Cai03] Caillaud, B. & Jullien, B. “Chicken & egg: Competition among intermediation service providers”. *RAND Journal of Economics* **34**.2 (2003), pp. 309–328.

[Cas20] Castillo, J. C. *Who benefits from surge pricing?* available at `https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3245533`. 2020.

[Cas17] Castillo, J. C. et al. “Surge pricing solves the wild goose chase”. *Proceedings of the ACM Conference on Economics and Computation* (2017), pp. 241–242.

[Che16] Chen, M. K. “Dynamic pricing in a labor market: Surge pricing and flexible work on the Uber platform”. *Proceedings of the ACM Conference on Economics and Computation* (2016), p. 455.

[Che20] Cheng, M. *Uber’s new California commission caps only underscore the pricing opacity for drivers*. accessed January 15, 2021, available at `https://qz.com/1797224/uber-capped-california-service-fees-but-added-a-marketplace-fee/`. 2020.

[Chi03] Chiang, W.-y. K. et al. “Direct marketing, indirect profits: A strategic analysis of dual-channel supply-chain design”. *Management science* **49**.1 (2003), pp. 1–20.

[Cla71] Clarke, E. H. “Multipart pricing of public goods”. *Public Choice* **11**.1 (1971), pp. 17–33.

[Cra06] Cramton, P. et al. *Combinatorial Auctions*. Cambridge, MA, USA: MIT press, 2006.

[Cur21] Curry, D. *Ride hailing taxi app revenue and usage statistics*. accessed February 28, 2021, available at `https://www.businessofapps.com/data/ride-hailing-app-market/#7`. 2021.

[d’A83] d’Aspremont, C. et al. “Product differences and prices”. *Economics Letters* **11**.1-2 (1983), pp. 19–23.

[d’A79] d’Aspremont, C. et al. “On hotelling’s stability in competition”. *Econometrica: Journal of the Econometric Society* (1979), pp. 1145–1150.

[Deb12] Debbah, M. et al. “Market share analysis between MNO and MVNO under brand appeal based segmentation”. *6th International Conference on Network Games, Control and Optimization (NetGCooP)*. IEEE. 2012, pp. 9–16.

[Dec14] Decker, S & Saitto, S. *Uber seeks to patent pricing surges that critics call gouging*. accessed March 13, 2020, available at `https://www.bloomberg.com/news/articles/2014-12-18/uber-seeks-to-patent-pricing-surges-that-critics-call-gouging`. 2014.

[Dev17] Devari, A. et al. "Crowdsourcing the last mile delivery of online orders by exploiting the social networks of retail store customers". *Transportation Research Part E: Logistics and Transportation Review* **105** (2017), pp. 105–122.

[Dho15] Dholakia, U. M. *Everyone hates Uber's surge pricing*. accessed January 15, 2021, available at `https://hbr.org/2015/12/everyone-hates-ubers-surge-pricing-heres-how-to-fix-it`. 2015.

[Dho16] Dholakia, U. M. *Uber's surge pricing: 4 reasons why everyone hates it*. accessed January 15, 2021, available at `https://kinder.rice.edu/2016/01/27/uberhatesurgepricing/#.Vqkf5_krLct`. 2016.

[Dof21] Doft. *Uber freight disrupts trucking industry by guaranteeing 7-Day payments*. `https://doft.com/blog/uber-freight-disrupts-trucking-industry-guaranteeing-7-day-payments`, accessed April 10, 2021. 2021.

[Dua11] Duan, L. et al. "Duopoly competition in dynamic spectrum leasing and pricing". *IEEE Transactions on Mobile Computing* **11**.11 (2011), pp. 1706–1719.

[Fat15] Fatnassi, E. et al. "Planning and operating a shared goods and passengers on-demand rapid transit system for sustainable city-logistics". *Transportation Research Part B: Methodological* **81** (2015), pp. 440–460.

[For18] Forbes. *Is Uber freight on the verge of going out of business?* `https://www.forbes.com/sites/stevebanker/2018/04/10/is-uber-freight-on-the-verge-of-going-out-of-business/?sh=12d8e264461f/`, accessed April 10, 2021. 2018.

[Gab79] Gabszewicz, J. J. & Thisse, J.-F. "Price competition, quality and income disparities". *Journal of economic theory* **20**.3 (1979), pp. 340–359.

[Gab82] Gabszewicz, J. J. & Thisse, J.-F. "Entry (and exit) in a differentiated industry". *Noncooperative Approaches to the Theory of Perfect Competition*. Elsevier, 1982, pp. 213–224.

[Gab92] Gabszewicz, J. J. & Thisse, J.-F. "Location". *Handbook of game theory with economic applications* **1** (1992), pp. 281–304.

[Gab86a] Gabszewicz, J. J. & Thisse, J.-F. "Spatial competition and the location of firms". *Fundamentals of pure and applied economics* **5** (1986), pp. 1–71.

[Gab86b] Gabszewicz, J. J. et al. "Segmenting the market: The monopolist's optimal product mix". *Journal of economic theory* **39**.2 (1986), pp. 273–289.

[Gao17] Gao, F. & Su, X. "Online and offline information for omnichannel retailing". *Manufacturing & Service Operations Management* **19**.1 (2017), pp. 84–98.

[Gar20] Garg, N. & Nazerzadeh, H. "Driver surge pricing". *Proceedings of the 21st ACM Conference on Economics and Computation* (2020), p. 501.

[Gri15] Griffin, A. *Uber surge pricing: Researchers reverse-engineer app and find how to cheat controversial charging system*. accessed January 15, 2021, available at `https://www.independent.co.uk/life-style/gadgets-and-tech/news/uber-surge-pricing-researchers-reverse-engineer-app-and-find-how-cheat-controversial-charging-system-a6715266.html`. 2015.

[Gro73] Groves, T. "Incentives in teams". *Econometrica: Journal of the Econometric Society* **41** (1973), pp. 617–631.

[Gud19] Guda, H. & Subramanian, U. "Your Uber is arriving: Managing on-demand workers through surge pricing, forecast communication, and worker incentives". *Management Science* **65**.5 (2019), pp. 1995–2014.

[Gui11] Guijarro, L. et al. "Competition and bargaining in wireless networks with spectrum leasing". *IEEE Global Telecommunications Conference-GLOBECOM 2011*. IEEE. 2011, pp. 1–6.

[Gur19] Gurvich, I. et al. "Operations in the on-demand economy: Staffing services with self-scheduling capacity". *Sharing Economy: Making Supply Meet Demand* **6** (2019), pp. 249–278.

[Ham19] Hamilton, I. A. *Uber drivers are reportedly colluding to trigger 'surge' prices because they say the company is not paying them enough*. accessed January 15, 2021, available at `https://markets.businessinsider.com/news/stocks/uber-drivers-artificially-triggering-surge-prices-reports-abc7-2019-6-1028280277`. 2019.

[Har19] Harsha, P. et al. "Dynamic pricing of omnichannel inventories: honorable mention—2017 M&SOM Practice-Based Research Competition". *Manufacturing & Service Operations Management* **21**.1 (2019), pp. 47–65.

[Haw16] Hawkins, A. *Uber is trying to make you forget that surge pricing exists*. accessed March 13, 2020, available at `https://www.theverge.com/2016/6/23/12017002/uber-surge-pricing-upfront-fare-app-update-announcement`. 2016.

[Hrz19] Hrzic, J. *What will 2019 bring to the on-demand services economy?* accessed February 28, 2021, available at `https://medium.com/@jhrzic/what-will-2019-bring-to-the-on-demand-services-economy-c99de54bc6a7`. 2019.

[Hu21] Hu, B. et al. "Surge pricing and two-sided temporal responses in ride hailing". *Manufacturing & Service Operations Management* (2021), forthcoming.

[Hua13] Huang, G. Q. & Xu, S. X. “Truthful multi-unit transportation procurement auctions for logistics e-marketplaces”. *Transportation Research Part B: Methodological* **47** (2013), pp. 127–148.

[Jan08] Janssen, M. & Verbraeck, A. “Comparing strengths and weaknesses of Internet-based matching mechanisms for the transport market”. *Transportation Research Part E: Logistics and Transportation Review* **44**.3 (2008), pp. 475–490.

[Kam17] Kamargianni, M. & Matyas, M. “Business Ecosystem of Mobility-as-a-Service”. *Transportation Research Board*. Vol. 96. 2017.

[Kar97] Karmarkar, U. S. & Pitbladdo, R. C. “Quality, class, and competition”. *Management Science* **43**.1 (1997), pp. 27–39.

[Kea20] Keall, C. *Uber drivers colluding to trigger surge pricing: Study*. accessed April 27, 2020, available at `https://www.nbr.co.nz/article/uber-drivers-colluding-trigger-surge-pricing-study-ck-206034`. 2020.

[Kha18] Khalifa, N. B. et al. “Profit and strategic analysis for MNO-MVNO partnership”. *IFIP Networking Conference (IFIP Networking) and Workshops*. IEEE. 2018, pp. 325–333.

[Kri09] Krishna, V. *Auction Theory*. Cambridge, MA, USA: Academic Press, 2009.

[Kuy15] Kuyzu, G. et al. “Bid price optimization for truckload carriers in simultaneous transportation procurement auctions”. *Transportation Research Part B: Methodological* **73** (2015), pp. 34–58.

[Laf19] Lafkihi, M. et al. “Freight transportation service procurement: A literature review and future research opportunities in omnichannel E-commerce”. *Transportation Research Part E: Logistics and Transportation Review* **125** (2019), pp. 348–365.

[Lan90] Lancaster, K. “The economics of product variety: A survey”. *Marketing science* **9**.3 (1990), pp. 189–206.

[Li19] Li, C. et al. “Pricing game with complete or incomplete information about spectrum inventories for mobile virtual network operators”. *IEEE Transactions on Vehicular Technology* **68**.11 (2019), pp. 11118–11131.

[Li20] Li, F. et al. “Enhancing supply chain decisions with consumers’ behavioral factors: An illustration of decoy effect”. *Transportation Research Part E: Logistics and Transportation Review* **144** (2020), p. 102154.

[Lia20] Liang, R. et al. “Truthful auctions for e-market logistics services procurement with quantity discounts”. *Transportation Research Part B: Methodological* **133** (2020), pp. 165–180.

[Lot17] Lotfi, M. H. & Sarkar, S. “The economics of competition and cooperation between MNOs and MVNOs”. *51st Annual Conference on Information Sciences and Systems (CISS)*. IEEE. 2017, pp. 1–6.

[Lyf21] Lyft. *Bonuses and incentives*. accessed February 15, 2021, available at `https://help.lyft.com/hc/en-us/sections/115003494568-Bonuses-and-Incentives`. 2021.

[MAI21] MAI. *Digitized trucking set to change the logistics game*. accessed April 10, 2021, available at `https://www.mktalt.com/Blogs/digitized-trucking-set-to-change-the-logistics-game.html`. 2021.

[Mat07] Matsubayashi, N. “Price and quality competition: The effect of differentiation and vertical integration”. *European Journal of Operational Research* **180**.2 (2007), pp. 907–921.

[McG17] McGoogan, C. *Uber drivers gang up to cause surge pricing, research says*. accessed January 15, 2021, available at `https://telegraph.co.uk/technology/2017/08/02/uber-drivers-gang-cause-surge-pricing-research-says`. 2017.

[Mil86] Milgrom, P. & Roberts, J. “Price and advertising signals of product quality”. *Journal of political economy* **94**.4 (1986), pp. 796–821.

[Möh17] Möhlmann, M. & Zalmanson, L. “Hands on the wheel: Navigating algorithmic management and Uber drivers’ autonomy”. *in proceedings of the International Conference on Information Systems (ICIS), Seoul, South Korea* (2017), pp. 10–13.

[Moo88] Moorthy, K. S. “Product and price competition in a duopoly”. *Marketing science* **7**.2 (1988), pp. 141–168.

[Mus78] Mussa, M. & Rosen, S. “Monopoly and product quality”. *Journal of Economic theory* **18**.2 (1978), pp. 301–317.

[Nar18] Narasimhan, C. et al. “Sharing economy: Review of current research and future directions”. *Customer Needs and Solutions* **5**.1 (2018), pp. 93–106.

[Nau14] Naughton, J. *From Gutenberg to Zuckerberg: Disruptive innovation in the age of the Internet*. London: Quercus, 2014.

[Nik20] Nikzamir, M. & Baradaran, V. “A healthcare logistic network considering stochastic emission of contamination: Bi-objective model and solution algorithm”. *Transportation Research Part E: Logistics and Transportation Review* **142** (2020), p. 102060.

[Par05] Parker, G. G. & Van Alstyne, M. W. "Two-sided network effects: A theory of information product design". *Management Science* **51**.10 (2005), pp. 1494–1504.

[Qi18] Qi, W. et al. "Shared mobility for last-mile delivery: Design, operational prescriptions, and environmental impact". *Manufacturing & Service Operations Management* **20**.4 (2018), pp. 737–751.

[Qia20] Qiao, B. et al. "Revenue optimization for less-than-truckload carriers in physical Internet: Dynamic pricing and request selection". *Computers & Industrial Engineering* **139** (2020), p. 105563.

[Roc03] Rochet, J.-C. & Tirole, J. "Platform competition in two-sided markets". *Journal of the European Economic Association* **1**.4 (2003), pp. 990–1029.

[Rys09] Rysman, M. "The economics of two-sided markets". *Journal of Economic Perspectives* **23**.3 (2009), pp. 125–143.

[Sha82] Shaked, A. & Sutton, J. "Relaxing price competition through product differentiation". *The review of economic studies* (1982), pp. 3–13.

[Sha53] Shapley, L. S. "A value for n-person games". *Contributions to the Theory of Games* **2**.28 (1953), pp. 307–317.

[Shu80] Shubik, M. & Levitan, R. *Market structure and behavior*. Harvard Univ Pr, 1980.

[Sim19] Simoni, M. D. et al. "Potential last-mile impacts of crowdshipping services: a simulation-based evaluation". *Transportation* **47**.4 (2019), pp. 1–22.

[Smi17] Smith, O. *Uber drivers are 'gaming the system' to charge you more – really*. accessed January 15, 2021, available at `https://www.forbes.com/sites/oliversmith/2017/08/02/uber-drivers-are-gaming-the-system-to-charge-you-more-really/?sh=ff132a616c8a`. 2017.

[Sol17] Solman, P. *How Uber drivers game the app and force surge pricing*. accessed January 15, 2021, available at `https://www.pbs.org/newshour/economy/uber-drivers-game-app-force-surge-pricing`. 2017.

[Swe20] Sweeney, S. *Uber, Lyft drivers manipulate fares at Reagan National causing artificial price surges*. accessed April 27, 2020, available at `https://wjla.com/news/local/uber-and-lyft-drivers-fares-at-reagan-national`. 2020.

[Tay18] Taylor, T. A. "On-demand service platforms". *Manufacturing & Service Operations Management* **20**.4 (2018), pp. 704–720.

[The21] The Economist. *Why China's Didi can succeed where Uber has struggled*. accessed February 28, 2021, available at `https://www.economist.com/busi` ness/2021/02/15/why-chinas-didi-can-succeed-where-uber-has-struggled. 2021.

[Tir88] Tirole, J. *The theory of industrial organization*. MIT press, 1988.

[Ube20] Uber. *Financials*. accessed February 28, 2021, available at https://investor.uber.com/financials/default.aspx. 2020.

[Ube20] Uber. *How and why do drivers earn extra?* accessed January 31, 2021, available at https://marketplace.uber.com/pricing/driver-promotions. 2020.

[Ube20a] Uber. *How surge pricing works*. accessed March 13, 2020, available at https://www.uber.com/us/en/drive/driver-app/how-surge-works/. 2020.

[Ube20b] Uber. *Uber's community guidelines*. accessed December 7, 2020, available at https://www.uber.com/us/en/drive/basics/uber-community-guidelines. 2020.

[Ube21a] Uber. *Driver promotions*. accessed February 15, 2021, available at https://www.uber.com/us/en/drive/promotions/. 2021.

[Ube21b] Uber. *Financials*. accessed February 28, 2021, available at https://investor.uber.com/financials/default.aspx. 2021.

[Ube21] Uber. *How much does a ride with the Uber app cost?* accessed March 2, 2021, available at https://www.uber.com/us/en/price-estimate/. 2021.

[Ube21a] Uber. *Uber marketplace*. accessed January 15, 2021, available at https://marketplace.uber.com/pricing/surge-pricing. 2021.

[Ube21b] Uber. *Uber's price estimator*. https://www.uber.com/us/en/price-estimate/, accessed April 10, 2021. 2021.

[Ube21c] UberFreight. *Moving the world, together*. https://www.uber.com/us/en/freight/, accessed April 10, 2021. 2021.

[USE21] USEIA. *Coal*. https://www.eia.gov/coal/index.php, accessed March 28, 2021. 2021.

[VD07] Van Duin, J. et al. “Real time simulation of auctioning and re-scheduling processes in hybrid freight markets”. *Transportation Research Part B: Methodological* **41**.9 (2007), pp. 1050–1066.

[Vic61] Vickrey, W. “Counterspeculation, auctions, and competitive sealed tenders”. *The Journal of Finance* **16**.1 (1961), pp. 8–37.

[Xu14] Xu, S. X. & Huang, G. Q. “Efficient auctions for distributed transportation procurement”. *Transportation Research Part B: Methodological* **65** (2014), pp. 47–64.

[Xu17] Xu, S. X. & Huang, G. Q. “Efficient multi-attribute multi-unit auctions for B2B e-Commerce logistics”. *Production and Operations Management* **26**.2 (2017), pp. 292–304.

[Zha10] Zhang, J. et al. “Crafting integrated multichannel retailing strategies”. *Journal of Interactive Marketing* **24**.2 (2010), pp. 168–180.

[Zhu19] Zhu, Y. et al. “The Cooperation and competition between an added Value MVNO and an MNO allowing secondary access”. *International Symposium on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks (WiOPT)*. IEEE. 2019, pp. 1–8.

APPENDICES

APPENDIX A

Driver Collusion in Ride-Hailing Platforms

A.1 Cycling Equilibrium (Proposition 12)

Assume a pricing policy based on the actually observed utilizations under the two pricing regimes.

$$(p, w) = \begin{cases} (p_n, w_n) & \text{if } \rho < \overline{\rho} \\ \\ (p_s, w_s) & \text{if } \rho \geq \overline{\rho} \end{cases} \tag{A.1}$$

Comparing the equilibrium system utilization in Lemma 1 with the threshold $\bar{\rho}$ in (A.1), we can characterize the equilibrium pricing regime in the following proposition.

Proposition 12. *Suppose there is no collusion and the platform uses the pricing scheme defined in* (A.1)*. Define* $\hat{s}_i = \frac{1-p_i}{\bar{\rho}^2} - \frac{D\mu^2\delta_i p_i}{\bar{\lambda}}$ *for* $i = \{n, s\}$*, and note that* $\hat{s}_s > \hat{s}_n$*. In equilibrium, the platform adopts the base pricing regime* (p_n, w_n) *if* $s > \hat{s}_n$*, the surge pricing regime* (p_s, w_s) *if* $s \leq \hat{s}_s$*, and it alternates between surge pricing regime and base pricing regime if* $\hat{s}_s < s \leq \hat{s}_n$*.*

Proof of Proposition 12: Let ρ_i denote the utilization under regime $i = \{n, s\}$. According to the pricing scheme in (A.1), the surge pricing regime remains in effect if and only if $\rho_s \geq \bar{\rho}$. Substituting the results from Lemma 1 into $\rho_s \geq \bar{\rho}$ and rearranging terms, we have $s \leq \hat{s}_s = \frac{1-p_s}{\bar{\rho}^2} - \frac{D\mu^2\delta_s p_s}{\bar{\lambda}}$. Similarly, from (A.1), the base pricing regime remains in effect if and only if $\rho_n < \bar{\rho}$. Substituting the results from Lemma 1 into $\rho_n < \bar{\rho}$ and rearranging terms, we have $s > \hat{s}_n = \frac{1-p_n}{\bar{\rho}^2} - \frac{D\mu^2\delta_n p_n}{\bar{\lambda}}$. Since $p_n < p_s$ and $w_n = \delta_n p_n < w_s = \delta_s p_s$, we have $\hat{s}_n > \hat{s}_s$. According to the above analysis, when $\hat{s}_s < s \leq \hat{s}_n$, we always have $\rho_s < \bar{\rho} \leq \rho_n$. From (A.1), the platform then alternates between base pricing and surge pricing. □

Proposition 12 shows that without collusion, surge pricing will be in effect whenever customers are sufficiently insensitive to waiting (i.e., $s \leq \hat{s}_s$). As they are more willing to wait for service, the increase in the utility they derive from the service leads to a higher service request rate and, thus, a higher system utilization. Consistent with expectation and the purpose and rationale of surge pricing policies, the platform then switches to surge pricing to re-balance the system.

Note that, under the pricing scheme in (A.1), there is the possibility of a cycling equilibrium when the customer's sensitivity to waiting is moderate (i.e., $\hat{s}_s < s \leq \hat{s}_n$). In this case, the system utilization under base pricing ρ_n is slightly higher than the threshold $\bar{\rho}$, which triggers the surge-pricing regime. Once the surge pricing regime is in effect, the higher price reduces the customer service request rate and the higher wage attracts additional drivers, overall resulting in a lower system utilization, as intended. For certain intermediate values of s, the resulting utilization ρ_s then turns out to be lower than $\bar{\rho}$, so the pricing algorithm switches back to base pricing. This cyclic behavior will repeat in equilibrium.

A.2 Lemmas 5 and 6

Lemma 5. *e_i decreases in s for $i = \{n, s\}$, and $\Delta e = e_s - e_n$ is concave in s.*

Proof of Lemma 5: Taking the derivatives of e_s from (2.10) and e_n (from Lemma 1) with respect to s, we have

$$\frac{\partial e_s}{\partial s} = -\frac{\bar{\rho}^2 \bar{\lambda} p_s \delta_s}{2\sqrt{\mathscr{D}}\sqrt{-p_s \delta_s \bar{\lambda}\left(s\bar{\rho}^2 + p_s - 1\right)}} < 0$$

and

$$\frac{\partial e_n}{\partial s} = -\frac{\bar{\lambda}\mu\delta_n\sqrt{\bar{\lambda}(1-p_n)}p_n}{2\left(\mathscr{D}\mu^2\delta_n p_n + \bar{\lambda}s\right)^{3/2}} < 0,$$

so both e_s and e_n decrease in s. Taking the second derivative of $e_s - e_n$ with respect to s, we have

$$\frac{\partial^2(e_s - e_n)}{\partial s^2} = -\frac{\bar{\rho}^4 p_s^2 \delta_s^2 \bar{\lambda}^2}{4\sqrt{\mathscr{D}}\left(p_s\delta_s\bar{\lambda}\left(-s\bar{\rho}^2 - p_s + 1\right)\right)^{3/2}} - \frac{3\bar{\lambda}^2\mu\delta_n\sqrt{\bar{\lambda}(1-p_n)}p_n}{4\left(\mathscr{D}\mu^2\delta_n p_n + \bar{\lambda}s\right)^{5/2}} < 0,$$

so Δe is concave in s. □

Lemma 6. *π_i decreases in s for $i = \{n, s\}$, and $\Delta\pi = \pi_s - \pi_n$ is concave in s.*

Proof of Lemma 6: Substituting the results from Lemma 1 and (2.6) into the platform's profit function (2.3), we can express π_n and π_s as

$$\pi_s = p_s\bar{\lambda}(1-\delta_s)(1-p_s - s\overline{\rho}^2), \tag{A.2}$$

$$\pi_n = \frac{\mathscr{D}\bar{\lambda}(1-p_n)p_n^2(1-\delta_n)\delta_n\mu^2}{s\bar{\lambda} + \mathscr{D}p_n\delta_n\mu^2}. \tag{A.3}$$

Taking the derivatives of π_s and π_n with respect to s, we get

$$\frac{\partial \pi_s}{\partial s} = -p_s\bar{\lambda}(1-\delta_s)\overline{\rho}^2 < 0$$

and

$$\frac{\partial \pi_n}{\partial s} = -\frac{\mathscr{D}\bar{\lambda}^2\mu^2(1-\delta_n)\delta_n\left(1-p_n\right)p_n^2}{\left(\mathscr{D}\mu^2\delta_n p_n + \bar{\lambda}s\right)^2} < 0,$$

so platform profit decreases in s under both pricing regimes. Since

$$\frac{\partial^2(\pi_s-\pi_n)}{\partial s^2}=-\frac{2\mathscr{D}\bar{\lambda}^3\mu^2(1-\delta_n)\delta_n\left(1-p_n\right)p_n^2}{\left(\mathscr{D}\mu^2\delta_n p_n+\bar{\lambda}s\right)^3}<0,$$

$\Delta\pi$ is concave in s. □

A.3 Proofs of the Results Given in the Main Text

Proof of Lemma 1: With $\mathscr{M}=m$, $\mathscr{M}=\mathscr{D}e$, (2.1), $e=w\frac{\lambda}{\mathscr{M}}$, and $\rho=\frac{\lambda}{\mathscr{M}\mu}$, solving for m, λ, ρ, and e yields the expressions provided in the result. □

Proof of Proposition 1: According to the pricing scheme in (2.2), surge pricing is in effect if and only if $\rho_s\geq\bar{\rho}$. Substituting the results from Lemma 1 into $\rho_s\geq\bar{\rho}$ and rearranging terms, we have surge pricing if and only if $s\leq\hat{s}=\frac{1-p_s}{\bar{\rho}^2}-\frac{D\mu^2\delta_s p_s}{\bar{\lambda}}$. Otherwise (i.e., if $\rho_s<\bar{\rho}$ or $s>\hat{s}$), the platform adopts base pricing. □

Proof of Lemma 2: With $\mathscr{M}=\mathscr{D}e$, (2.1), (2.4), and $\rho=\frac{\lambda}{m\mu}$, solving for $\mathscr{M}$, λ, ρ, and e yields the expressions provided in the result. □

Proof of Proposition 2: Taking the partial derivatives of λ, $\mathscr{M}$, ρ and e in (2.6)-(2.9) with respect to m, we have

$$\frac{\partial\lambda}{\partial m}=\frac{\mu\left(\sqrt{-4\bar{\lambda}^2sp+\mu^2m^2+4\bar{\lambda}^2s}-\mu m\right)^2}{2\bar{\lambda}s\sqrt{-4\bar{\lambda}^2sp+\mu^2m^2+4\bar{\lambda}^2s}}>0,$$

$$\frac{\partial\mathscr{M}}{\partial m}=\frac{\bar{\lambda}s\left\{\sqrt{\frac{\delta\mu mp\left(\sqrt{\mu^2m^2+4\bar{\lambda}^2(1-p)s}-\mu m\right)}{\bar{\lambda}s}}\right\}^{\frac{3}{2}}}{2\sqrt{D}m^2\mu\sqrt{2m^2\mu^2+8\bar{\lambda}^2(1-p)s}\,w}>0,$$

$$\frac{\partial\rho}{\partial m}=\frac{\mu^2m-\mu\sqrt{4\bar{\lambda}^2s\left(1-p\right)+\mu^2m^2}}{2\bar{\lambda}s\sqrt{4\bar{\lambda}^2s\left(1-p\right)+\mu^2m^2}}<0,$$

$$\frac{\partial e}{\partial m}=\frac{\bar{\lambda}s\left\{\sqrt{\frac{\delta\mu mp\sqrt{\mu^2m^2+4\bar{\lambda}^2(1-p)s}-\delta\mu^2m^2p}{\bar{\lambda}s}}\right\}^{\frac{3}{2}}}{2\sqrt{D}m^2\mu\sqrt{2m^2\mu^2+8\bar{\lambda}^2(1-p)s}\,w}>0,$$

so λ, $\mathscr{M}$, and e increase in m, but ρ decreases in m. □

Proof of Lemma 3: Taking the partial derivative of e in (2.9) with respect to s, we have

$$\frac{\partial e}{\partial s}=\frac{m\mu(-m^2\mu^2-2\bar{\lambda}^2(1-p)s+m\mu\sqrt{m^2\mu^2+4\bar{\lambda}^2(1-p)s})w}{2\sqrt{\bar{\lambda}}s^{\frac{3}{2}}\sqrt{2m^2\mu^2-8\bar{\lambda}^2(-1+p)s}\sqrt{Dm\mu(-m\mu+\sqrt{m^2\mu^2+4\bar{\lambda}^2(1-p)s})w}}<0,$$

so the driver earnings decrease in s. □

Proof of Proposition 3: The system utilization under base pricing decreases in m (Proposition 2). We first derive the condition under which the surge pricing regime is in effect. With $p=p_n$ and $w=w_n$, using the utilization in (2.8), the condition $\rho_s\geq\bar{\rho}$ can be rearranged to $m\leq\bar{m}=\frac{\bar{\lambda}(1-s\bar{\rho}^2-p_n)}{\mu\bar{\rho}}$. As individual driver earnings e increase in m (Proposition 2), if drivers choose to collude to trigger surge pricing, they will collude at the largest m that sustains surge pricing, that is, $m^*=\bar{m}$. Substituting $m=\bar{m}$ into (2.6), (2.7), and (2.9), and using $p=p_s$ and $\delta=\delta_s$, we obtain the expressions provided in the result. We now proceed to derive the conditions for collusion. We use e_s and e_n to denote the driver earning rates under surge pricing with collusion and under the base pricing regime, respectively. Substituting e_s from (2.10) and e_n from Lemma 1 into $e_n<e_s$ and using the result from Lemma 5 (in Appendix A.2) that e_s-e_n is concave in s, we can simplify the condition to $s_d^-<s<s_d^+$, where

$$s_d^-=\frac{\bar{\lambda}\delta_s p_s-\bar{\lambda}p_s\delta_s p_s-D\mu^2\bar{\rho}^2\delta_n p_n\delta_s p_s-\sqrt{A}}{2\bar{\lambda}\bar{\rho}^2\delta_s p_s},$$

$$s_d^+=\frac{\bar{\lambda}\delta_s p_s-\bar{\lambda}p_s\delta_s p_s-D\mu^2\bar{\rho}^2\delta_n p_n\delta_s p_s+\sqrt{A}}{2\bar{\lambda}\bar{\rho}^2\delta_s p_s},$$

$$A=\delta_s p_s\left\{(\bar{\lambda}(-1+p_s)+D\mu^2\bar{\rho}^2\delta_n p_n)^2\delta_s p_s+4\bar{\lambda}D\mu^2\bar{\rho}^2\delta_n p_n((-1+p_n)\delta_n p_n+\delta_s p_s-p_s\delta_s p_s)\right\}.$$

Here s_d^- and $s_d^+\geq s_d^-$ are the roots of the equation $e_n=e_s$. Drivers only consider collusion to trigger surge pricing when the base pricing regime would otherwise be in effect (i.e, $s\geq\hat{s}$), so the collusion region is the intersection of $(\hat{s},\infty)$ and (s_d^-,s_d^+), or equivalently $(\max\{\hat{s},s_d^-\},s_d^+)$. □

Proof of Proposition 4: Let π_s and π_n denote platform profits under the surge with driver collusion and base pricing regimes, respectively. Substituting π_s from (A.2) and π_n from (A.3) into $\pi_n < \pi_s$ and using the result from Lemma 6 (see Appendix A.2) that $\pi_s - \pi_n$ is concave in s, we can simplify the condition to $s_p^- < s < s_p^+$, where

$$\begin{aligned}
s_p^- &= \frac{(1-\delta_s)p_s C - \sqrt{B}}{2\bar{\lambda}(1-\delta_s)p_s\bar{\rho}^2},\\
s_p^+ &= \frac{(1-\delta_s)p_s C + \sqrt{B}}{2\bar{\lambda}(1-\delta_s)p_s\bar{\rho}^2},\\
B &= (1-\delta_s)p_s(4\bar{\lambda}D\mu^2(-(1-\delta_n)p_n + (1-\delta_n)p_n^2\\
&\quad + (1-\delta_s)(1-p_s)p_s)\bar{\rho}^2\delta_n p_n + (1-\delta_s)p_s C^2,\\
C &= \bar{\lambda}(1-p_s) - D\mu^2\delta_n p_n\bar{\rho}^2.
\end{aligned}$$

Here s_p^- and $s_p^+ \geq s_p^-$ are the roots of the equation $\pi_n = \pi_s$. It is straightforward to show that $s_d^- \leq s_p^- \leq s_p^+ \leq s_d^+$. Since drivers would only consider collusion when $s \geq \hat{s}$, the region where collusion benefits the platform is $(\max\{\hat{s}, s_d^-, s_p^-\}, s_p^+)$. □

Proof of Proposition 5: Suppose the drivers get a per-ride bonus of b if they remain continuously online after completing their first service. Denote the earnings with bonus as $e_i = (\delta_n p_n + b)\frac{\lambda_n}{\mathscr{M}_n}$ and the earnings without bonus as $e_{ii} = \delta_s p_s \frac{\lambda_s}{\mathscr{M}_s}$. The platform will choose b such that drivers are indifferent between colluding and not colluding, that is, $e_i = e_{ii}$. From $\mathscr{M} = \mathscr{D}e$, the number of drivers in the region then is the same for both cases, that is, $\mathscr{M}_s = \mathscr{M}_n$. Using $e_i = (\delta_n p_n + b)\frac{\lambda_n}{\mathscr{M}_n}$, $e_{ii} = \delta_s p_s \frac{\lambda_s}{\mathscr{M}_s}$, $e_i = e_{ii}$, and $\mathscr{M}_s = \mathscr{M}_n$, we have

$$b = \delta_s p_s \frac{\lambda_s}{\lambda_n} - \delta_n p_n. \tag{A.4}$$

The platform's profits with and without the bonus in (A.4) are $((1-\delta_n)p_n - b)\lambda_n$ and $(1-\delta_s)p_s\lambda_s$, respectively. The platform thus benefits from the bonus payment structure if and only if

$$((1-\delta_n)p_n - b)\lambda_n > (1-\delta_s)p_s\lambda_s.$$

Substituting b from (A.4), we get

$$\lambda_n p_n > \lambda_s p_s,$$

where λ_n is given in Lemma 1 with $(p, w) = (p_n, w_n + b)$ and λ_s is given in Proposition 3. $\square$

Proof of Proposition 6: Suppose there are $\mathcal{M}$ drivers in the region of interest and m_f go online. Given the customer arrival rate λ, the average inter-arrival time between two consecutive service completions is $\frac{1}{\lambda}$. According to the assumed coordination mechanism between drivers (cf. Section 3.3), after a service completion, a driver goes offline and remains offline until another $\mathcal{M} - m_f$ drivers have completed their services. Therefore, the expected time between two consecutive activations of a driver is $t_{off} = (\mathcal{M} - m_f)\frac{1}{\lambda}$.

Without a freeze period, drivers will collude at $m_f = \bar{m}$ and $t_{off} = (\mathcal{M} - \bar{m})\frac{1}{\lambda}$, so implementing a freeze period with $t_f \le (\mathcal{M} - \bar{m})\frac{1}{\lambda}$ has no effect on driver behavior. If $t_f > (\mathcal{M} - \bar{m})\frac{1}{\lambda}$, drivers either choose not to collude, or they choose to collude at some $m_f < \bar{m}$. In the latter case, substituting λ from (2.6) and $\mathcal{M}$ from (2.7) into $t_{off} = (\mathcal{M} - m_f)\frac{1}{\lambda}$, we get

$$t_{off} = \frac{\sqrt{2\bar{\lambda} s \delta D \mu m_f p \sqrt{\mu^2 m_f^2 + 4\bar{\lambda}^2(1-p)s} - 2\bar{\lambda} s \delta D \mu^2 m_f^2 p} - 2 m_f \bar{\lambda} s}{m_f \mu(-m_f \mu + \sqrt{m_f^2 \mu^2 - 4\bar{\lambda}^2(-1+p)s})} \tag{A.5}$$

It is straightforward to show that t_{off} decreases in m_f. Since the driver earning rate e increases in m_f (see Proposition 2), drivers always choose the highest $m = m_f$, implying the lowest t_{off}. Because $t_{off} \ge t_f$, we have $t_{off} = t_f$, so the drivers' earning rate decreases in t_f. Let $\tilde{t}_f$ denote the value of t_f where driver earnings under collusion equal those without collusion; then drivers do not collude if and only if $t_f \ge \tilde{t}_f$. The value of $_f$ satisfies $\delta_s p_s \frac{\lambda_f}{\mathcal{M}_f} = \delta_n p_n \frac{\lambda_n}{\mathcal{M}_n}$, where λ_f and $\mathcal{M}_f$ are given in (2.6) and (2.7) with $(p, w) = (p_s, w_s)$, $\tilde{t}_f = t_{off}$ from (A.5), and $m = m_f$. $\square$

APPENDIX B

On-demand Logistics Service For Packages: Package Bidding Mechanism Vs Platform Pricing

B.1 Proofs of the Results Given in the Main Text

Proof of Theorem 1: The price, p, charged by the DC, as a function of λ is given by,

$$p = \frac{1}{md} F_v^{-1}\Big(1 - \frac{\lambda_a}{\lambda}\Big) - \frac{c}{md} W_q$$

Since trucks, who make at least as much as their outside option, will join this system and we define that fraction as $\beta = G_o(\frac{wd\lambda_a}{k})$. We then have that $k = \beta K$. We can now write the wage of a truck in this system, $w = G_o^{-1}(\frac{k}{K})\frac{k}{md\lambda_a}$. Finally, the profit function of the DC is given by,

$$\Pi_{DC} = \lambda_a\Big[\frac{1}{md} F_v^{-1}\Big(1 - \frac{\lambda_a}{\lambda}\Big) - \frac{c}{md} W_q - G_o^{-1}\Big(\frac{k}{K}\Big)\frac{k}{md\lambda_a}\Big]md$$

We have an implicit constraint here which ensures the stability of the queue, $\rho < 1$. Also, we have the fixed pay out ratio of price and wages i.e., $w = \delta p$, which adds an equality constraint to our model. Putting together the objective function and the constraints gives us the required constrained optimization problem whose solution indicates the optimal price. □

Proof of Theorem 2: An incoming package, with a bid of b_n, wants to maximize its utility, which is defined as, $u_p = v - c W_q - b_n$. However, it will only gain a positive utility if and only if it is selected by the truck. Thus, the actual utility function which a package seeks to maximize is given by,

$$(v - c W_q - b_n)\mathbb{P}(b_1 < b_n, b_2 < b_n, \ldots, b_{n-1} < b_n)$$

The package thus solves the following optimization problem,

$$\max_{b_n}(v - c W_q - b_n)\mathbb{P}(b_1 < b_n)\mathbb{P}(b_2 < b_n)\ldots\mathbb{P}(b_{n-1} < b_n) \tag{B.1}$$

Solving (B.1) by taking the first order derivative with respect to b_n and setting it to 0, we get the desired results. □

APPENDIX C

Competitive Implications of Spectrum Sub-Leasing on Optimal Price, Quality and Sourcing Decisions

C.1 Monopoly Results: Proposition 13

Proposition 13. *An MVNO can never be profitable while leasing from a monopolist MNO, and hence, will never enter such a market. When only a single MNO is present, the optimal price and profit is given by the following,*

Table C.1 Monopoly Results

	$\frac{q_1}{2} \leq Q_1$	$\frac{q_1}{2} > Q_1$
p_1^*	$\frac{q_1}{2}$	$q_1 - Q_1$
π_1^*	$\frac{q_1}{4}$	$\frac{(q_1-Q_1)Q_1}{q_1}$

Proof of Proposition 13: Using our definition of the indifferent customer from Section 4.3.1, we consider the following location of the indifferent customer, $0 < \theta_{10} < 1$, where $\theta_{10} = \frac{p_1}{q_1}$.

The demand for the MNO is given by $d_1 = 1-\theta_{10}$ and the profit is given by $\pi_1 = p_1 d_1$ along with the bandwidth constraint, $q_1 d_1 \leq Q_1$. Thus, the monopolist MNO solves the following constrained optimization problem,

$$\max_{p_1} \pi_1$$
$$s.t.\ q_1 d_1 \leq Q_1$$

We confirm the concavity of the profit function by checking the second-order derivative of the profit function with respect to p_1; $\frac{\partial^2 \pi_1}{\partial p_1^2} = -\frac{2}{q_1}$, which is always negative. The Lagrangian of the constrained optimization problem can be written as, $\mathscr{L}(p_1, \lambda) = p_1 d_1 - \lambda(q_1 d_1 - Q_1)$. Taking the first order derivative of the Lagrangian and solving using the complementary slackness conditions, we get the results in Table C.1. $\square$

C.2 Proof of Results in the Main Text

Proof of Proposition 7: We use the following ordering of the indifferent customers owing to the presence of quality differentiated MNOs, $0 < \theta_{20} < \theta_{10} < \theta_{12} < 1$. The demand for both the firms is given by,

$$\begin{aligned} d_1 &= 1-\theta_{12} \\ &= 1-\frac{p_1 - p_2}{1-\alpha} \\ d_2 &= \theta_{12} - \theta_{20} \\ &= \frac{p_1 - p_2}{1-\alpha} - \frac{p_2}{\alpha} \end{aligned}$$

Both the firms solve the following constrained optimization problem simultaneously to find the optimal prices,

$$\max_{p_i} \pi_i$$
$$s.t. \; q_i d_i \leq Q_i$$

We confirm the concavity of the profit by checking the second-order derivative of the profit function of each of the MNOs, M_1 and M_2, with respect to p_1 and p_2 respectively; $\frac{\partial^2 \pi_1}{\partial p_1^2} = \frac{-2}{1-\alpha}$ and $\frac{\partial^2 \pi_1}{\partial p_1^2} = \frac{-2}{\alpha(1-\alpha)}$, both of which are always negative. Formulating the Lagrangian for each of the MNO, $\mathscr{L}(p_i, \lambda_i) = p_i d_i - \lambda(q_i d_i - Q_i)$, taking the first derivative, and solving simultaneously using complementary slackness conditions, we get the results in Table 4.1.

□

Proof of Proposition 8: Under these circumstance, the quality of the MVNO will always be lowest of all the three offerings. We consider the following ordering of the indifferent customers as the only ordering where the equilibrium outcomes are not trivial and need to be derived,

$$0 < \theta_{v0} < \theta_{20} < \theta_{2v} < \theta_{10} < \theta_{12} < 1$$

The demand for the three firms is given by,

$$\begin{aligned}
d_1 &= 1 - \theta_{12} \\
&= 1 - \frac{p_1 - p_2}{1-\alpha} \\
d_2 &= \theta_{12} - \theta_{2v} \\
&= \frac{p_1 - p_2}{1-\alpha} - \frac{p_2 - p_v}{\alpha - \delta} \\
d_v &= \theta_{2v} - \theta_{v0} \\
&= \frac{p_2 - p_v}{\alpha - \delta} - \frac{p_v}{\delta}
\end{aligned}$$

The profit functions for the firms are,

$$\pi_1 = p_1 d_1$$
$$\pi_2 = p_2 d_2 + w d_v$$
$$\pi_v = (p_v - w) d_v$$

Solving for p_v and w, we get,

$$p_v = \frac{p_2 \delta}{\alpha}$$
$$w = \frac{p_2 \delta}{\alpha}$$

We observe that this leads to 0 demand for the MVNO. $\square$

Proof of Lemma 4 and Proposition 9: We consider the following ordering, which implies that the quality of the MVNO is lower than that of M_2,

$$0 < \theta_{v0} < \theta_{20} < \theta_{2v} < \theta_{10} < \theta_{12} < 1$$

The demand for each of the firm is given by,

$$\begin{aligned} d_1 &= 1 - \theta_{12} \\ &= 1 - \frac{p_1 - p_2}{1 - \alpha} \\ d_2 &= \theta_{12} - \theta_{2v} \\ &= \frac{p_1 - p_2}{1 - \alpha} - \frac{p_2 - p_v}{\alpha - \delta} \\ d_v &= \theta_{2v} - \theta_{v0} \\ &= \frac{p_2 - p_v}{\alpha - \delta} - \frac{p_v}{\delta} \end{aligned}$$

The profit functions for the firms are,

$$\pi_1 = p_1 d_1 + w_1 d_v$$
$$\pi_2 = p_2 d_2$$
$$\pi_v = (p_v - w_1) d_v$$

Through backward induction, we solve for p_v and w_1. Plugging these values in the profit function of M_1, we observe that the bandwidth usage as well as the profit increases in δ. That implies that MNO 1 will set δ_1 as high as possible till all the bandwidth is exhausted or the quality of the MVNO matches that of MNO 2. In order to solve for δ_1, we solve $q_1 d_1 + q_v d_v - Q_1 = 0$, which is the binding bandwidth constraint, and then solve for the prices, p_1 and p_2,

$$p_1 = 1 - Q_1$$
$$p_2 = \frac{2\alpha(1+\alpha-2Q_1)}{1+2\alpha}$$
$$\delta_1 = \frac{2\alpha((2\alpha-3)Q_1+1)}{\alpha^2+2(\alpha-2)Q_1+1}$$

This configuration leads to 0 demand for the low quality MNO, M_2. However, when the MNOs are committed to prices and do not react to the presence of the MVNO, instead of solving for prices, we use the duopoly prices and note that all three firms co-exist with positive demand. □

Proof of Proposition 10: When the MVNO has the highest quality of service, we consider the following ordering of the indifferent customers,

$$0 < \theta_{20} < \theta_{10} < \theta_{12} < \theta_{v0} < \theta_{v1} < 1$$

The demand functions of the firms are,

$$\begin{aligned}
d_v &= 1-\theta_{v1} \\
&= 1-\frac{p_v-p_1}{\gamma(\delta_1+\alpha\delta_2)-1} \\
d_1 &= \theta_{v1}-\theta_{12} \\
&= \frac{p_v-p_1}{\gamma(\delta_1+\alpha\delta_2)-1}-\frac{p_1-p_2}{1-\alpha} \\
d_2 &= \theta_{12}-\theta_{20} \\
&= \frac{p_1-p_2}{1-\alpha}-\frac{p_2}{\alpha}
\end{aligned}$$

The profit functions for the firms are,

$$\pi_1 = p_1 d_1 + w_1 d_v$$

$$\pi_2 = p_2 d_2 + w_2 d_v$$

$$\pi_v = (p_v - w_1 - w_2) d_v$$

Solving for p_v, w_1 & w_2, δ_1 & δ_2 and p_1 & p_2, we have,

$$\begin{aligned}
p_1 &= \frac{8(1-\alpha)}{3(4-\alpha)} \\
p_2 &= \frac{4\alpha(1-\alpha)}{3(4-\alpha)} \\
p_v &= 5Q_1-\frac{4-5Q_2(4-\alpha)+11\alpha}{4-\alpha} \\
\delta_1 &= \frac{3\alpha+6(\alpha-4)Q_1+4}{(\alpha-4)\gamma} \\
\delta_2 &= \frac{2(3Q_2(4-\alpha)-4\alpha)}{\alpha^2(4-\alpha)} \\
d_1 &= \frac{4+3\alpha}{24-6\alpha} \\
d_2 &= \frac{4}{12-3\alpha} \\
d_v &= \frac{1}{6}
\end{aligned}$$

When the MVNO is of intermediate quality, we consider the following ordering of the indifferent customers,

$$0 < \theta_{20} < \theta_{v0} < \theta_{v2} < \theta_{10} < \theta_{1v} < 1$$

The demand functions for the firms are,

$$\begin{aligned}
d_1 &= 1 - \theta_{1v} \\
&= 1 - \frac{p_1 - p_v}{1 - \gamma(\delta_1 + \alpha\delta_2)} \\
d_v &= \theta_{1v} - \theta_{v2} \\
&= \frac{p_1 - p_v}{1 - \gamma(\delta_1 + \alpha\delta_2)} - \frac{p_v - p_2}{\gamma(\delta_1 + \alpha\delta_2) - \alpha} \\
d_2 &= \theta_{v2} - \theta_{20} \\
&= \frac{p_v - p_2}{\gamma(\delta_1 + \alpha\delta_2) - \alpha} - \frac{p_2}{\alpha}
\end{aligned}$$

The profit functions for the firms are,

$$\pi_1 = p_1 d_1 + w_1 d_v$$

$$\pi_2 = p_2 d_2 + w_2 d_v$$

$$\pi_v = (p_v - w_1 - w_2) d_v$$

Solving for p_v and w_1 & w_2, we observe that the demand for MVNO is 0.

When the MVNO is of lowest quality, we consider the following ordering of the indifferent customers,

$$0 < \theta_{v0} < \theta_{20} < \theta_{2v} < \theta_{10} < \theta_{12} < 1$$

The demand functions for the firms are,

$$\begin{aligned}
d_1 &= 1 - \theta_{12} \\
&= 1 - \frac{p_1 - p_2}{1 - \alpha}
\end{aligned}$$

$$\begin{aligned} d_2 &= \theta_{12} - \theta_{2v} \\ &= \frac{p_1 - p_2}{1-\alpha} - \frac{p_2 - p_v}{\alpha - \gamma(\delta_1 + \alpha\delta_2)} \\ d_v &= \theta_{2v} - \theta_{v0} \\ &= \frac{p_2 - p_v}{\alpha - \gamma(\delta_1 + \alpha\delta_2)} - \frac{p_v}{\gamma(\delta_1 + \alpha\delta_2)} \end{aligned}$$

The profit functions for the firms are,

$$\begin{aligned} \pi_1 &= p_1 d_1 + w_1 d_v \\ \pi_2 &= p_2 d_2 + w_2 d_v \\ \pi_v &= (p_v - w_1 - w_2) d_v \end{aligned}$$

Solving for p_v and w_1 & w_2, we observe that the demand for MVNO is 0. $\square$

Proof of Proposition 11: The demand for all the firms is given by,

$$d_v = \begin{cases} \frac{p_2 - p_v}{\alpha - \delta_1} - \frac{p_v}{\delta_1} & \text{Source from one MNO} \\ 1 - \frac{p_v - p_1}{\gamma(\delta_1 + \alpha\delta_2) - 1} & \text{Source from Both} \end{cases}$$

$$d_1 = \begin{cases} 1 - \frac{p_1 - p_2}{1-\alpha} & \text{Source from one MNO} \\ \frac{p_v - p_1}{\gamma(\delta_1 + \alpha\delta_2) - 1} - \frac{p_1 - p_2}{1-\alpha} & \text{Source from Both} \end{cases}$$

$$d_2 = \begin{cases} \frac{p_1 - p_2}{1-\alpha} - \frac{p_2 - p_v}{\alpha - \delta_1} & \text{Source from one MNO} \\ \frac{p_1 - p_2}{1-\alpha} - \frac{p_2}{\alpha} & \text{Source from Both} \end{cases}$$

The profit function for the three firms is given by,

$$\pi_v = \begin{cases} (p_v - w_1) d_v & \text{Source from one MNO} \\ (p_v - w_1 - w_2) d_v & \text{Source from Both} \end{cases}$$

$$\pi_1 = \begin{cases} p_1 d_1 + w_1 d_v & \text{Source from one MNO} \\ p_1 d_1 + w_1 d_v & \text{Source from Both} \end{cases}$$

$$\pi_2 = \begin{cases} p_2 d_2 & \text{Source from one MNO} \\ p_2 d_2 + w_2 d_v & \text{Source from Both} \end{cases}$$

Based on our game sequence, we first solve for p_v,

$$p_v = \begin{cases} \frac{w_1\alpha + p_2\delta_1}{2\alpha} & \text{Source from one MNO} \\ \frac{-1+p_1+w_1+w_2+\gamma\delta_1+\alpha\gamma\delta_2}{2} & \text{Source from Both} \end{cases}$$

We calculate the profit of MVNO at this stage for a given set of prices, p_i, MVNO quality indicators, δ_i, and wholesale prices, w_i. We define $A = \sqrt{\alpha\delta_1(\alpha-\delta_1)}$ and $B = \sqrt{\gamma(\delta_1+\alpha\delta_2)-1}$. The profit function of MVNO is given by the following,

$$\pi_v = \begin{cases} (\frac{p_2\delta_1 - w_1\alpha}{A})^2 & \text{Source from one MNO} \\ (\frac{B^2+p_1-w_1-w_2}{B})^2 & \text{Source from Both} \end{cases}$$

Let the profits of the MVNO when sourcing from both MNOs and when sourcing from a single MNO be defined as π_{vb} and π_{vs} respectively. The MVNO will source from both MNOs if $\pi_{vb} \geq \pi_{vs}$ and $\pi_{vb} \geq 0$. Solving the above two conditions leads to the the feasible region being the convex polytope, $\mathscr{C}w \leq \mathscr{D}$, where, $\mathscr{C} = \begin{pmatrix} A+B\alpha & A \\ 1 & 1 \\ A-B\alpha & A \end{pmatrix}$ and $\mathscr{D} = \begin{pmatrix} AB^2 + Ap_1 + Bp_2\delta_1 \\ B^2 + p_1 \\ AB^2 + Ap_1 - Bp_2\delta_1 \end{pmatrix}$. Similarly, the MVNO will source from a single MNO if $\pi_{vs} > \pi_{vb}$ and $\pi_{vs} \geq 0$ and solving them leads to the following feasible region, the convex polytope,

$$\mathscr{E}w \leq \mathscr{F}, \mathscr{E} = \begin{pmatrix} A+B\alpha & A \\ 1 & 0 \\ B\alpha - A & -A \end{pmatrix} \text{ and } \mathscr{F} = \begin{pmatrix} AB^2 + Ap_1 + Bp_2\delta_1 \\ \frac{p_2\delta_1}{\alpha} \\ -AB^2 - Ap_1 + Bp_2\delta_1 \end{pmatrix}.$$

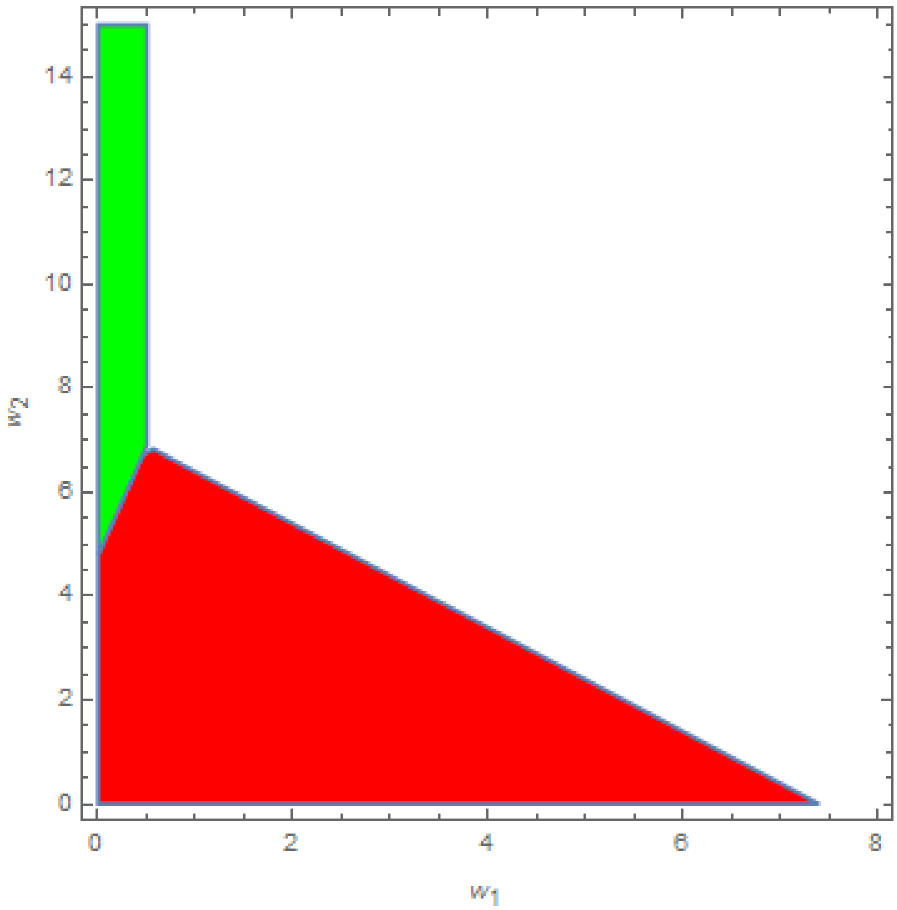


Figure C.1 Numerical representation of MVNO's sourcing decision; Red → Source from Both, Green → Source from one MNO; $\alpha = 0.8, \delta_1 = 0.4, \delta_2 = 0.3, \gamma = 10, p_1 = 2, p_2 = 1$

Now, both MNOs will set the wholesale prices simultaneously. In order to obtain the optimal wholesale prices, given a set of prices and MVNO quality indicators, we solve the response function of w_1 for a given w_2 and vice versa. Solving the response functions on a case-by-case basis, depending on the realizations of the problem parameters, we get the desired result. $\square$