

DYNAMIC PRESSURES IN ANNULUS-SHAPED PRESSURE SUPPRESSION POOLS OF BOILING WATER REACTORS GENERATED BY EARTHQUAKE GROUND MOTIONS

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Abstract

Pressure-suppression pools of annular-circular geometry are used in containments for boiling water reactor (BWR) plants of Mark II and Mark III configurations and must be designed to withstand earthquake effects.

Analytical solutions are obtained for dynamic pressures and free-surface displacements (sloshing) induced in annular-circular water pools with horizontal (flat) bottom when subjected to earthquake ground motions. Water is modeled as an inviscid, compressible fluid and free surface effects (sloshing) are accounted for. The pool boundaries are assumed rigid. Analytical solutions are obtained in the frequency domain as Fourier-Bessel or Fourier-Modified Bessel functions depending upon the magnitude of the exciting (forcing) frequency and the compressibility of water.

To obtain the effects flexible pool boundaries have on induced hydrodynamic pressures and free-surface displacements (sloshing) a two-step analysis is implemented. In the first step, the solution described above is used to obtain a set of frequency dependent hydrodynamic (added) masses.

In the second step, the structure with the previously computed hydrodynamic masses added at discrete locations on the fluid-structure interface is analyzed for an earthquake excitation using the finite element method. The solution (structural accelerations) is obtained in the frequency domain. With known pool boundary accelerations, dynamic pressures induced in the fluid are then obtained.

Analytical results are compared with results of experiments performed elsewhere.

An annulus shaped structure representing a pressure-suppression pool in a Mark III containment building was analyzed for earthquake effects. It was found that the flexibility of the structure affects the hydrodynamic pressures induced in the pool.

1. Introduction

The response of fluid containers during seismic events is of special importance in design of nuclear power plants. Approximate solutions for tanks with circular and rectangular cross-sections are presented in [1, 2, 3]. Aslam, et al, [4] obtained an analytical solution for annular circular tanks with rigid boundaries. (This geometry is representative of suppression pools in boiling water reactor plants of Mark II and Mark III configurations.) This paper presents the solution for flexible tanks of annular circular cross-sections enclosing compressible fluid.

2. Theoretical Background

2.1 Solution of Hydrodynamic Equation

Consider the annulus tank shown in Figure 1, assumed to have flexible side boundaries and rigid bottom. The fluid in the tank is assumed to be inviscid, linear and compressible, and its dynamic pressure governed by equation (1), [5]:

$$p_{,rr} + \frac{1}{r} p_{,r} + \frac{1}{r^2} p_{,\theta\theta} + p_{,zz} = \frac{1}{c^2} p_{,tt} ; \quad (1)$$

and the boundary conditions:

$$\begin{aligned} p_{,z} &= 0 & \text{at} & \quad z = 0 ; \\ p_{,r} &= -\rho \ddot{u}_{na} & \text{at} & \quad r = a ; \\ p_{,r} &= -\rho \ddot{u}_{nb} & \text{at} & \quad r = b ; \\ p_{,tt} + gp_{,z} &= 0 & \text{at} & \quad z = H ; \end{aligned} \quad (2)$$

where:

c is the sound wave velocity in fluid, \ddot{u}_{na} and \ddot{u}_{nb} are prescribed normal accelerations of the outer and inner boundary, respectively, ρ is the mass density of the fluid, g is the gravitational acceleration, and p is the dynamic pressure.

It was shown [4, 6], for the case of horizontal earthquakes, that the hydrodynamic pressure can be expressed by

$$p(r, z, \theta, t) = P(r, z, t) \cos \theta . \quad (3)$$

With the boundary acceleration expressed as

$$\ddot{u}_n(z, \theta, t) = \ddot{U}_n(z, t) \cos \theta \quad (4)$$

the steady state solution of equation (1) for a frequency of excitation ω , subject to boundary conditions (2), and using relations (3) and (4), becomes:

$$p(\omega) = P(\omega) \cos \theta \exp(i\omega t) ; \quad (5)$$

where: ω is the frequency of excitation;

$$P(\omega) = \sum_{i=0} [C_1^i(\lambda_i r) Z(\alpha_i z)] + \sum_{i=0} [D_1^i(q_i r) \cos(\ell_i z)]$$

$$C_1^i(\lambda_i r) = A_{i1} J_1(\lambda_i r) + B_{i1} Y_1(\lambda_i r);$$

$$D_1^i(q_i r) = \begin{cases} A_{i2} J_1(q_i r) + B_{i2} Y_1(q_i r), & \text{for } q_i^2 > 0; \\ A_{i2} I_1(q_i r) + B_{i2} K_1(q_i r), & \text{for } q_i^2 < 0; \\ A_{i2} r^1 + B_{i2} r^{-1}, & \text{for } q_i^2 = 0.0; \end{cases}$$

$$Z(\alpha_i z) = \begin{cases} \cosh(\alpha_i z), & \text{for } \alpha_i^2 > 0; \\ \cos(\alpha_i z), & \text{for } \alpha_i^2 < 0; \end{cases}$$

$$\alpha_i^2 = \lambda_i^2 - \omega^2/c^2$$

$$q_i^2 = \omega^2/c^2 - \ell_i^2$$

$$\ell_i = i\pi/H, \quad i = 0, 1, 2, 3, \dots$$

λ_i are the roots of the equation:

$$J_1'(\lambda_i a) Y_1'(\lambda_i b) - J_1'(\lambda_i b) Y_1'(\lambda_i a) = 0;$$

A_{ij}, B_{ij} are constants which could be obtained from the specified boundary accelerations [4, 5, 6].

If sloshing effects are not important, the free surface boundary condition in equation (2) becomes [6, 7],

$$P = 0 \text{ at } z = H;$$

and

$$\ell_i = (2i - 1)\pi/2H, \quad i = 1, 2, 3, \dots$$

2.2 Hydrodynamic (Added) Mass Formulation

In the case of horizontal earthquakes, the acceleration term in the side boundary conditions of equation (2) becomes, using expression (4):

$$\ddot{U}_n(z, t) = \ddot{U}_g(t) + \ddot{U}_s(z, t); \quad (6)$$

where $\ddot{U}_g(t)$ is the input ground acceleration and $\ddot{U}_s(z, t)$ is the structural acceleration (relative to grade).

The structural acceleration is a function of the structural properties as well as the fluid motion in this coupled fluid-structure interaction problem. The structural system, modified to include a frequency dependent added

mass matrix [6, 7], representing the dynamic effects of the fluid on the structure, may be analyzed to obtain $\ddot{U}_s(z,t)$. The added mass matrix can be obtained [6, 7] from equation (5). The hydrodynamic pressure in the fluid may be obtained from equation (5), using the total structural boundary accelerations $\ddot{U}_n(z,t)$ of equation (6).

3. Case Studies

3.1 Comparison with Available Test Results

Aslam, J., et al, [4] performed tests to study the sloshing response of fluids in rigid annular-circular tanks subjected to horizontal earthquakes.

The test tank utilized is schematically represented in Figure 2. Table I shows a comparison between the sloshing natural frequencies recorded during tests, and the frequencies computed using the present analytical formulation for the same tank. The comparison is excellent. Table II shows a comparison of the maximum sloshing displacement recorded at the inner boundary during tests and the displacement predicted using the analytical formulation prescribed in this paper. The agreement is again excellent.

3.2 Response of Flexible Annular-Circular Tank

The tank schematically shown in Figure 3 was subjected to horizontal steady state ground acceleration of unit amplitude.

The resulting transfer functions for the hydrodynamic pressure at the free surface are plotted in Figure 4 for two cases: flexible and rigid outer boundary. The two curves coincide indicating that boundary flexibility does not affect the dynamic pressure at this location. This result was expected since the dynamic pressure here is governed by the low frequency sloshing response.

Transfer functions for the hydrodynamic pressures at the mid-height of the fluid are shown in Figure 5. As the curves indicate the sloshing frequencies are unimportant; however, the structural flexibility has a significant effect on the hydrodynamic pressure at this location. Since the response of the structure is governed by the higher amplitude dynamic pressures which develop at deeper submergence locations (the mid-depth being a representative location) and not by the lower amplitude pressure near the pool surface, it is important to include the structural flexibility in the analysis.

4. Conclusions

An analytical method for the analysis of flexible tanks with annular-circular cross-section, subjected to horizontal earthquake motions, is presented. The method is based on the solutions of equation (1) governing the dynamic pressure of a compressible fluid, using Fourier-Bessel series, then

computing an added mass matrix to represent fluid effects on the structural boundaries. Comparison with available test results for rigid containers shows excellent agreement. It is shown that the tank flexibility has an important effect on the state of the hydrodynamic pressures in the fluid. It is concluded that one must account for the coupling between the fluid and the tank in this class of problems.

5. Acknowledgment

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6. References

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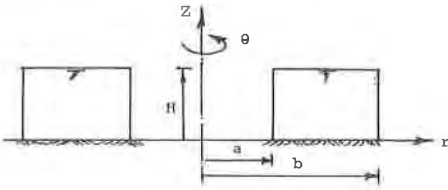


Fig. 1
Coordinate System

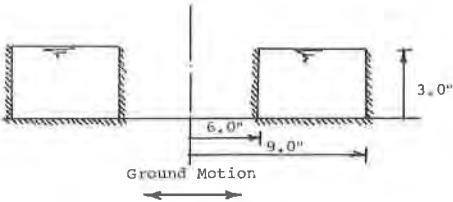


Fig. 2
Dimensions of the Test Tank

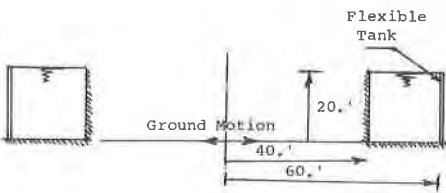


Fig. 3
Dimensions of the Flexible Tank

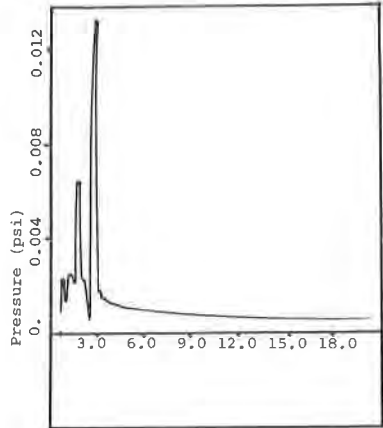


Fig. 4
Transfer Function of Surface Pressures

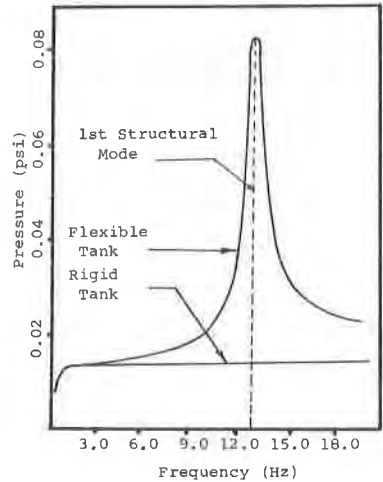


Fig. 5
Transfer Function of Pressures at
Mid-Height of the Tank

Table I - Test and Analytical Frequencies

Mode Number	Sloshing Frequencies (Hz)	
	Test 1	Theory
1	0.70	0.70
2	3.20	3.19
3	4.50	4.49
4	5.50	5.47

Table II - Steady State Sloshing Response of Water

Frequency (Hz)	Ground Acc. (g)	Max. Disp. at Inner Wall (IN)	
		Test 1	Theory
0.6	0.0059	0.167	0.159
1.2	0.0312	0.184	0.180
2.0	0.0530	0.168	0.170
3.7	0.0742	0.271	0.265