ABSTRACT

TROCKI, AARON DAVID. Designing and Examining the Effects of a Dynamic Geometry Task Analysis Framework on Teachers' Written Geometer's Sketchpad Tasks. (Under the direction of Dr. Karen Hollebrands).

This study investigates the usefulness of a Dynamic Geometry Task Analysis Framework for indicating task quality in dynamic geometry environments in general, and The Geometer's Sketchpad in particular. This research sought to first establish the validity of the framework for indicating task quality, and to second explore the effects of the framework on teachers' understanding of said quality and on teachers' task writing activity. Data were collected in the form of expert feedback on the validity and usefulness of the framework followed by revisions to the framework and task ranking methods. Student work and interviews were conducted and analyzed to further establish framework validity regarding the quality of student argumentation on tasks that ranked low, medium, and high in quality according to framework coding. The second phase of data collection was in the form of case studies that documented teacher task writing and use of the framework. The framework underwent revisions throughout the course of data collection and analysis, thus making it an evolving document in this study. Results indicate that experts came to agreement on the usefulness and validity of the framework for indicating elements of task quality. Furthermore, the quality of student argumentation generally aligned with task quality as indicated by framework codes and rankings. In the latter portion of this study, all participating teachers made significant changes to their written Geometer's Sketchpad task based on their exposure to the framework. Finally, all teachers reflected favorably on the usefulness of this Dynamic Geometry Task Analysis Framework as a guide for writing high quality tasks. Implications for mathematics education and avenues for further research are provided.

Designing and Examining the Effects of a Dynamic Geometry Task Analysis Framework on Teachers' Written Geometer's Sketchpad Tasks

by
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DEDICATION

This work is dedicated to my mother, who spent countless hours instilling in her son passion for learning and persistence for dreaming. I would also like to dedicate this work to my wife, for without her it would never have come to fruition.

BIOGRAPHY

The author comes from a modest middle class family in Chicago, Illinois. He excelled in mathematics from an early age and completed a Bachelor of Arts degree in Mathematics at Judson College. He immediately entered graduate school in a Master of Mathematics degree pursuit supported by a teaching assistantship that required teaching two lab sections of basic college algebra for students who had yet to take college level math courses. Dumbfounded by a lack of excitement, understanding, and appreciation for the discipline of mathematics often manifest by students in these courses, he decided to do something about it. It was during this pursuit that he found a calling in Mathematics Education.

Two years later he graduated from Western Illinois University with a Master of Teaching degree with a focus in Secondary Mathematics. Then, a pursuit of an Ed.D. in Curriculum and Instruction at the University of North Carolina at Chapel Hill (UNC-CH) North Carolina prompted a move to Carrboro, North Carolina in the following year. During this time he taught mathematics full time while simultaneously completing course work at UNC-CH. While much of the course work completed over a four year span proved insightful and beneficial, he ultimately discontinued the pursuit of this generalist degree and focused on his teaching craft. After three years of teaching middle school mathematics and four years teaching high school mathematics including a four year stint as department chair, he decided to pursue a Ph.D. in Mathematics Education at North Carolina State University. He flourished in this program and prepared to profess in the field.

While writing his dissertation, he was hired on a tenure track position at Elon University's Department of Mathematics and Statistics. As a new professor in the field, he has found success teaching undergraduate mathematics courses and content courses for prospective math teachers. His research agenda focuses on building on his dissertation work in which he presented a case for the usefulness of a tool he crafted for Mathematics Educators entitled, the Dynamic Geometry Task Analysis Framework.

ACKNOWLEDGMENTS

I wish to thank the numerous teachers who have believed in me over the years. Professors who have supported my growth as a researcher and teacher are also due much credit, as they have taught me how to function in the field of mathematics education. Furthermore, I would like to express deep gratitude to my committee members, for their mentorship, feedback, encouragement, and friendship. Finally, to my advisor, Dr. Karen Hollebrands, who took me under her professional wing at the time when I needed it most.

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CHAPTER 1: INTRODUCTION

"Major limitations of computer use in the coming decades are likely to be less a result of technological limitations than a result of limited human imagination and the constraints of old habits and social structures" (Kaput, 1992, p. 515). High school geometry teachers have traditionally used classroom tasks that require students to make compass and straightedge constructions while responding to associated prompts. For instance, a typical high school geometry task is one where students use a compass and straight edge to construct an angle bisector. An associated written prompt is often included that requires students to justify why the steps used in their construction generate an angle bisector. The advent of dynamic geometry software has changed the way students draw, construct, and measure by using virtual not physical tools. In the case of the student constructing an angle bisector, a dynamic geometry environment would allow the Student Two options. First, the student could simply choose the angle and use the angle bisector command to construct an angle bisector. In the second case, the teacher or task instructions could require the student to use the circle construction to virtually mimic what would be done with a physical compass and straight edge to construct an angle bisector. In both cases, the student could drag the construction and notice how the two resultant angles maintain equal measure. Incorporating such dragging and noticing in dynamic geometry tasks provides students a situation where they can create, test, and verify conjectures by using a mouse or their finger to drag objects across a screen.

Dynamic geometry software (DGS) has gained traction in mathematics education as evidenced in standards based outlets such as the Common Core State Standards for

Mathematics ([CCSSM], National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010). The CCSSM High School Geometry standards require students to "make formal geometric constructions with a variety of tools and methods" (p. 76). DGS is among the tools recommended. Hollebrands (2003) has noted that within such dynamic environments students have opportunities to consider invariant relationships through dragging as well as make corresponding conjectures and conclusions. The technological affordance of dragging goes beyond the constraints of traditional engagements in paper and pencil geometry and represents a powerful avenue for students to explore geometric relationships and develop geometric understanding. The concern for curriculum writers and high school geometry teachers becomes how to exploit such technological affordances for the betterment of student understanding. Connor, Moss and Grover (2007) explained:

> It may be that, even for relatively sophisticated students, there needs to be instruction in how to incorporate the use of software programs into developing strategies for exploring the correctness of an assertion and, in particular, designing experiments to test the validity of a statement. (pp. 62-63)

While the benefits of DGS have been widely recognized, research on what constitutes task quality for such environments and guidance for writing such tasks is lacking.

Definition of Terms

Before proceeding to delineate the purpose of this study, it befits the reader to understand the use of essential terms and definitions. The Geometer's Sketch Pad (GSP) is a software package that provides the user a platform to not only draw, measure and construct objects in a two dimensional plane, but also drag points to stretch or shrink objects. When this dragging occurs, constructed objects and their inherent attributes and relationships remain invariant. For example, if one were to construct a square and then drag one of its vertices the sides will always remain congruent and the angles will remain right angles. GSP further allows the user to type in dialogue boxes to write notes or questions. A GSP task is a combination of the geometric objects (i.e., the sketch on the screen at various points in the task completion process) and the associated directions used to accomplish a particular learning goal. The term prompt indicates a written question or direction related to a sketch that requires a verbal or written response. A technological action is a question or direction that requires a drawing, construction, or measurement within or manipulation of a sketch. A prompt may require both a written response and a technological action.

There are three basic types of sketches: 1) a pre-constructed sketch refers to a sketch in which objects are completely constructed for the student with the expectation that the students use this completed construction to respond to associated prompts; 2) a student-constructed sketch refers to a sketch in which the students constructs all of the geometric object(s) as they respond to associated prompts; and 3) a partially-constructed sketch refers to one in which the student adds to the sketch with drawings, measurements, or constructions as they respond to prompts. Although the Dynamic Geometry Task Analysis Framework can be applied to tasks involving any of the aforementioned sketch types, the focus of this study concerned tasks paired with partially-constructed sketches.

A task often requires the user to make conjectures, and this term refers to mathematical hypotheses that are suspected to be true. Geometry and geometric understanding builds through making conjectures and testing them. If this testing leads to deductive proof, then the conjecture becomes a theorem. Another avenue through which a conjecture may be justified is through recognition of an invariance under dragging that supports the conjecture. It is debatable whether such justifications count as mathematical proof, a notion that will be addressed further in the literature review.

Purpose of the Study

While a significant amount of research has been documented that assesses the quality of mathematical tasks (e.g., Smith & Stein, 1998) and some limited research has been completed on the quality of pre-constructed DGS tasks (Sinclair, 2003), little research exists that documents the quality of student-constructed and partially-constructed GSP tasks, where students respond to task prompts while making constructions. Within this context, the purpose of this study was to investigate how a Dynamic Geometry Task Analysis Framework effectively classifies GSP tasks in regard to students' mathematical work and affects teachers' understanding of what constitutes quality in GSP tasks and their writing of GSP tasks. Specifically, this study seeks to document how the framework influences and changes how teachers go about writing tasks for The GSP when they write tasks that attempt to accomplish particular learning goals. To meet this purpose, the following overarching research questions are addressed:

1. What is the nature of relationships among mathematical prompts and technological actions and arguments students provide and conclusions they reach using The Geometer's Sketchpad?
2. How do pre-service and in-service teachers use the Dynamic Geometry Task Analysis Framework?

Significance of the Study

Technology in general and dynamic geometry software in particular are increasingly used in mathematics education. This increase parallels the growing availability of computers and tablets in today's schools. Many researchers have recognized the need to harness the potential for student learning with these technological tools as these tools afford the user actions previously unavailable (Herbst, 2004; Hollebrands, 2007; Holzl, 2001; Laborde, 2000; Olivero & Robutti, 2007; Sinclair, 2004). For instance, the dynamic geometry software, the GSP affords the user the technological action of dragging consequently changing the nature of how one interacts with a geometric construction. Such technological affordances represent an opportunity and opportunity necessitates responsibility. Kaput (1992) noted this opportunity more than two decades ago, "One very important aspect of mathematical thinking is the abstraction of invariance. But, of course, to recognize invariance—to see what stays the same—one must have variation. Dynamic media inherently makes variation easier to achieve" (p. 525). Kaput articulated a call to action for harnessing the potential of technology for improving engagements afforded to students in mathematics in posing the question, "What sorts of new technological tools and supports do teachers need in order to function effectively in the context of a more ambitious, reformed mathematics curriculum?" (p. 550). This paper represents an attempt to document the effectiveness of one such support, a Dynamic Geometry Task Analysis Framework, in regard to this charge.

Overview of Methods

The first phase of work in this study was to establish the validity of the Dynamic Geometry Task Analysis Framework. A face validity check was carried out by three experts in the field followed by changes to the framework and task ranking methods. Then, 12 high school students were selected to take part in Phase One of this study. Student background data was collected proceeded by video-recorded and transcribed task-based interviews. Guidance for task-based interviews was utilized in reference to the work of Goldin (2000), a scientific perspective on structured, task-based interviews in mathematics education research. The purpose of these task-based interviews was to further assess validity in analyzing the potential match between task quality as indicated by task coding with the Dynamic Geometry Task Analysis Framework and the arguments and conclusions evident in students' mathematical activity. The arguments and conclusions produced by students were coded with the core Toulmin's (1958) Argumentation Model: Data, Claim, and Warrant.

Phase Two of the study assessed the influence of the Dynamic Geometry Task Analysis Framework use on teachers' analysis of task quality and writing of tasks. Six teachers participated in task writing sessions in which a tutorial of framework use served as the treatment. Three teachers were pre-service and the other three were practicing. Background surveys were completed along with assessments of teachers' knowledge for teaching geometry. The instrument developed by Herbst and Kosko (2013) was employed to assess this knowledge. The data gathered on these teachers including the task-writing submissions and interviews comprised case studies on each teacher. The methodology guiding the processes involved with constructing cases was guided by the recommendations of Creswell (2013). Each case was documented in the findings portion of this work.

CHAPTER 2: LITERATURE REVIEW AND CONCEPTUAL FRAMEWORK

The purpose of this study was to first assess the validity of the Dynamic Geometry Task Analysis Framework for categorizing task quality. Second, the study investigated the use of the Dynamic Geometry Task Analysis Framework in its effects on how teachers describe and write Geometer's Sketchpad tasks. With these purposes in mind, the following overarching research questions were crafted:

1. How do different mathematical prompts and technological actions affect the arguments students provide and conclusions they reach using The Geometer's Sketchpad?
2. How do pre-service and in-service teachers use the Dynamic Geometry Task Analysis Framework?

To situate these questions in the associated literature four elements and their relationships to technology were addressed: mathematical activity, student, teacher, and curriculum content. Literature on these relationships was organized regarding key constructs of Zbiek, Heid, Blume and Dick's framework entitled, "Mediation relationships among technology, student, teacher, mathematical activity, and curriculum" (2007, p. 1172). They explained that constructs for their work are, "ones that have specific applications to mathematics, that have an empirical basis, and that help one understand relationships among tool, activity, students, teachers, and curriculum content" (p. 1172). Their framework is depicted in Figure 1 below (Zbiek et al., 2007. p. 1172).

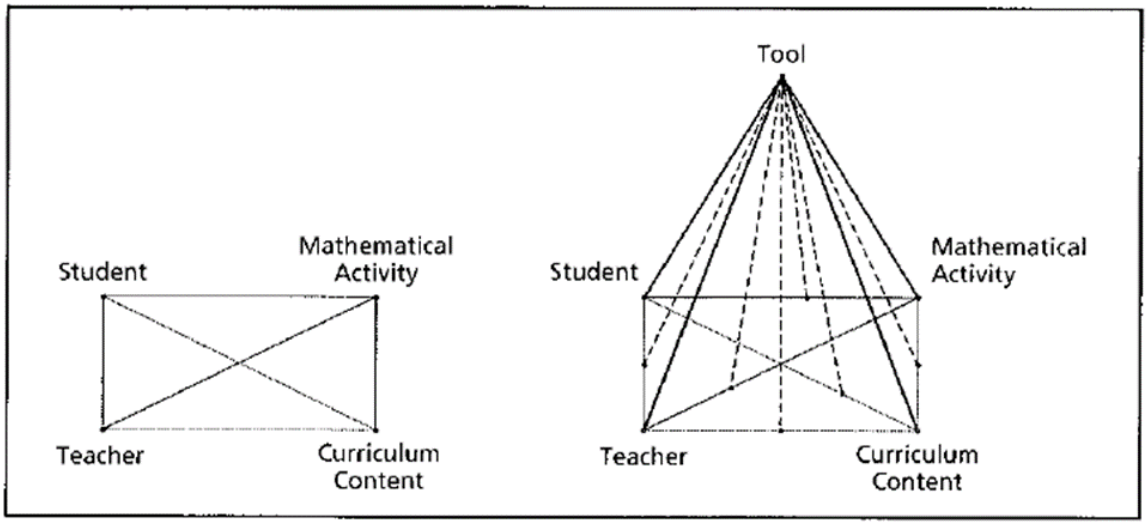


Figure 1: Mediation Relationships among Technology, Student, Teacher, Mathematical Activity, and Curriculum

The organization of this literature review reflects the objects of study found in the overarching research questions in addressing the faces of the pyramid found in Zbiek et al.'s (2007) framework. The technological tool under study is Dynamic Geometry Software in general and The Geometer's Sketchpad (GSP) in particular. The categories of the literature review are then: 1) Mathematical Activity, Tool and Student; 2) Student, Tool and Teacher; 3) Teacher, Tool and Curriculum Content; and 4) Curriculum Content, Tool and Mathematical Activity. A number of studies included in this literature review could arguably be placed in more than one category as categories share edges and vertices. As opposed to viewing such overlaps in the framework as a weakness, they are viewed as a strength as they speak to the complexity of conceptualizing the many facets of incorporating technological tools into mathematics instruction. In what follows, reviewed studies are assigned to one pyramidal face based on their dominant research question(s) and relevant findings. In some cases a study may be cited in more than one subsection. Each subsection begins with Zbiek et al.'s description of the relation (e.g., student, tool and teacher) followed by a treatment of associated literature. The author found no less than 70 studies addressing these four relationships. To appropriately narrow the breadth of the literature review, studies were excluded that were tangential to the purposes and research questions of the study.

Mathematical Activity, Tool, and Student

The design elements of a technological tool undoubtedly affect what the user of the tool can do. The "do" in this case is the mathematical activity with which one can engage. For example, one type of tool use that has received much attention in the literature on dynamic geometry software is dragging and associated mathematical activity. Zbiek et al. (2007) discuss three constructs in an effort to unpack the relationship of the user's mathematical activity to tool and provide:

> Certain characteristics of cognitive tools can serve as useful constructs for both mathematics education researchers and tool designers. The three constructs discussed here—externalized representation, mathematical fidelity and cognitive fidelity—relate to the ways cognitive tools can provide special opportunities or impediments for learning in the context of mathematical activity. (p. 1173)

In DGS, externalized representation refers to the representation on screen that may or may not link to student thinking. Zbiek et al. further explain, "Through externalized, though limited, surrogates for a student's internal mental representations displayed on the surface of the screen, externalized representations become visible phenomena that can be shared and discussed with others […]" (2007, p. 1173). Such representations serve as replication and object for users to express their mathematical thoughts and activities. In order for such mathematical activity to be productive, the representations must possess certain characteristics. Zbiek et al. explain that mathematical fidelity, or an object's "faithful[ness] to the underlying mathematical properties," is important as it influences mathematical activity (p. 1174). The authors explain, "Different [DGS] programmers make different decisions and produce ostensibly similar microworlds that act differently in selected cases because they are governed by different rules" (p. 1176). The authors cite Goldenberg, Scher and Feurzeig (2006) to provide the example of "arbitrarily placed" points behaving differently when dragged in different DGS environments. The authors further provide a caution that differences in intent of a visual display and interpretations of that display are a potential threat to mathematical fidelity.

Mathematical fidelity is closely related to the final construct, cognitive fidelity. Zbiek et al. referenced the work of Dick (2007), who expounded on Beeson's (1989) work to provide that cognitive fidelity refers, "to the degree to which the (cognitive) tool actions explicitly reflect the user's cognitive actions" (p. 1176). After conducting a case study of students using DGS, Sinclair (2004) proposed a caution, "many students do not realize or ignore the fact that the onscreen image is accurate [mathematical fidelity], and that others, who might want to use visual evidence, lack the tools [such as creating a measurement] to do so" (p. 191). While the challenges of employing DGS for the betterment of student mathematical activity have been brought to the fore, a plethora of studies have also been documented that identify the benefits of DGS for mathematical activity (Almeqdadi, 2000; Barrett, Clements, Klanderman, Pennisi, & Polaki, 2006; Choi-Koh, 1999; Jones, 2000; Pitta-Pantazi, 2008). Studies that investigate the relationship of the user's mathematical activity and tool in relation to Zbiek et al.'s constructs are treated in this section of the review. Specifically, studies that investigate technological actions when a tool is in use related to reasoning and proving in dynamic geometry software environments will be reviewed.

Reasoning and Proof

The advent of DGS has caused a stir in mathematics education communities concerning its potential to change students' mathematical activity and their corresponding opportunities to reason and prove. Geometry has historically evolved through logical deduction in which a collection of proven statements stems from an axiomatic base. Traditional geometry instruction engaged students in deducing theorems in a similar manner to that found in the evolution of Euclidean geometry. However, activity in DGS is often empirical in nature as users use measurements and dragging to form and test conjectures. Many have expressed their concern on the validity of testing in these environments and if it will replace the rigor of deduction inherent in tradition geometry work (e.g., Herbst, 2004; Hoyles & Jones, 1998). Others have worked to overcome these perceived difficulties (Holzl, 2001; Laborde, 2005). Hoyles and Jones (1998) presented a process-oriented approach to proof that was commonly used in the United Kingdom. This approach attempts to link empirical and deductive thinking by incorporating the following elements into students' mathematical activity:

1. Encouraging the students to make conjectures focusing on the relationship between geometrical objects.
2. Providing the means for the students to explain their actions and results. (p. 124)

The concern is the quality of explanations that the students are expected to provide in relationship to the rich history of deduction in the discipline. The authors studied a class of 12-year-old students who completed tasks in Cabri by utilizing the measuring and dragging features. Students distinguished among quadrilateral types by attending to invariant characteristics of each object. Hoyles and Jones concluded that the students' mathematical activity in Cabri led to significant learning gains but also noted that the unique combination of factors such as the quality of tasks, the teacher's guidance and the ability of the students may have caused such results (p. 126). The authors go on to provide a detailed warning:

> After work with this sort of software is there any need for verification in a formal sense? Can 'a way to prove' be seen as part of rather the constructive process or simply added to it as was the case in paper and pencil contexts? There is a danger that the use of this software will foster a process approach to geometry not possible before, but in so doing – at least in the UK – rather than taking a step forward we will simply replay the mistakes of the past, and limit the mathematical work of the majority to empirical argument and pattern-spotting. (p. 127)

Since Hoyles and Jones voiced this concern of mathematics educators, many have investigated and conceptualized the nature of mathematical activity in DGS and its relation to reasoning and proof (e.g., Armella & Sriraman, 2005; Furinghetti & Paola, 2003; Healy &

Hoyles, 2001; Marrades & Gutierrez, 2000; Sinclair, Healy, Osorio, & Sales, 2009). After reviewing some of that pertinent work, the literature review will address a handful of studies that concern the activities of dragging and measuring in DGS as these activities are often cited in the literature on reasoning and proof.

Stylianides (2008) studied proof and reasoning in school mathematics and provided a framework to conceptualize associated mathematical activities and constructs. While the framework is intended to analyze student reasoning and proof in general (i.e., not necessarily in a DGS environment), it directly applies to this study as it supports research on studying ways to enhance student mathematical activity related to reasoning and proof. To provide a need for her framework, Stylianides identifies a disjoint between the way students typically experience reasoning and proof and how mathematicians engage in such activities:

> In school mathematics, the development of proofs has often been treated as a formal process (primarily in high school geometry) isolated from other mathematical activities. However, this treatment of proof is problematic, because it does not afford students the same level of scaffolding that professional users of mathematics are afforded to make sense of and establish mathematical knowledge. (p. 9)

To capture the multiple activities associated with reasoning and proof, Stylianides referenced the work of Polya (1954) and others to provide the following four activities: 1) identifying patterns; 2) making conjectures; 3) providing non-proof arguments; and 4) providing proofs. It is likely that high school geometry students are most often engaged in providing proofs without engaging in activities one through three. It is further possible that student activity in

DGS environments may lend itself to activities one through three thus alleviating the problematic treatment of proof as label by Stylianides.

The framework for analyzing reasoning and proving accounts for these multiple activities and also attends to three contributing components to the mathematical activity of reasoning and proving as provided below in Table 1 (Styliandes, 2008, p. 10).

Table 1: Analytic Framework for Reasoning-and-Proving

	Reasoning-and-proving			
	Making Mathematical Generalizations		Providing Support to Mathematical Claims	
Mathematical Component	Identifying a Pattern	Making a Conjecture	Providing a Proof	Providing a Non-proof Argument
	• Plausible Pattern • Definite Pattern	• Conjecture	• Generic Example • Demonstration	• Empirical Argument • Rationale
Psychological Component	What is the solver's perception of the mathematical nature of a pattern / conjecture / proof / non-proof argument?			
Pedagogical Component	How does the mathematical nature of a pattern / conjecture / proof / non-proof argument compare with the solver's perception of this nature? How can the mathematical nature of a pattern / conjecture / proof / non-proof argument become transparent to the solver?			

The mathematical component accounts for the activities that comprise reasoning and proving and how these activities support reasoning and proving from a mathematical perspective. For instance, 'providing a non-proof argument' may involve an empirical justification of a known geometric relationship with the argument lacking a deductive path to justify the relationship. The psychological component takes into account the perception of the student regarding the nature of the mathematical object and what constitutes an appropriate conjecture and proof (p. 12). Styliandes explained, “The solver may believe that his argument is a proof, even though it may be an empirical argument according to the judgment of an observer (e.g., a researcher or teacher) who evaluates the argument using a mathematical perspective” (p. 12). The psychological component honors the subjective nature of a student’s perspective thus providing a lens by which gaps between how the student views reasoning and proving and how the field of mathematics has established reasoning and proving can come to the foreground. Styliandes provided that a teacher’s awareness of these two components and a comparison between them leads to the pedagogical component which addresses two inter-related issues. First, “Possible discrepancies emerging from this comparison can help identify potential foci for teachers’ pedagogical actions […]” and second the teacher’s, “instruction needs to actively seek ways to help students gradually refine their perceptions toward the conventional understanding” (p. 12-13). The framework offers an avenue through which the mathematical activity associated with reasoning and proving can be evaluated. Styliandes explained that the framework can be applied to both curriculum tasks and recorded episodes of mathematical activity.

Styliandes’ work also speaks to the need for the current study whose general purpose is to test the effectiveness of a framework that analyzes quality of Dynamic Geometry Tasks. Assuming that reasoning and proving are essential mathematical activities that are further decomposed with the categories: 1) identifying patterns; 2) making conjectures; 3) providing non-proof arguments; and 4) providing proofs, beckons the question of whether students are engaging in these activities. Styliandes and her research team analyzed the algebra, number theory, and geometric units in the textbook series known as the Connected Mathematics Project and analyzed 4578 tasks with their reasoning and proving framework. They found that 40% of the tasks engaged students in reasoning and proving and found, “62% were designed to offer students opportunities to give rationales; 18% to identify definite patterns; 6% to identify plausible patterns; 3% to make conjectures; 2% to provide empirical arguments; and 12% to provide demonstrations” (p. 14). This study revealed a lack of balance among mathematical activities found in this textbook series. Within this context, DGS may prove beneficial in working towards a more balanced approach to mathematical activity by giving more emphasis to conjecturing and empirical argument through the utilization of technological affordances such as dragging and measuring.

Herbst (2004) studied student mathematical activity in geometry to reveal four modes of interaction with diagrams and specified each mode’s relation to making conjectures and proving. While Herbst does not explicitly relate his findings to student work in DGS environments, it is foundational to understanding student proving in DGS environments as the diagram and perceptions of it serve a central role to activity. To establish the need for his study, he tapped one of the great thinkers on the philosophy and evolution of mathematics. Herbst paraphrased Lakatos to explain the links among making conjectures and proving and the historical growth of mathematics. Also, the activity of proving mimics the growth of mathematics thus making it, “an authentic mathematical activity” (p. 129). Herbst explained,

> Lakatos (1976) argues, the dialectic of informal proofs and refutations is key to improving a naïve conjecture and achieving a theorem. Proving is instrumental to

> conjecturing – not least because, to the extent that proofs and refutations also shape mathematical concepts (making them into abstractions), proving is the tool that best can handle them. (p. 129)

As the mathematical activity of conjecturing, refuting and proving is essential to the evolution of mathematics it is likewise essential to the individual learning of mathematics deeply. If a student can reproduce not only the conclusions (theorems) of a discipline but also the work (conjectures, refutations and proofs) that underpins the conclusions, the student is demonstrating a deep understanding of the discipline. It is possible that the technological affordances found in DGS may provide students a more meaningful experience, beyond that of textbook learning, to that of conjecturing, refuting and proving to bolster their understanding of concepts in geometry. Herbst disclosed a critique of how geometry is typically presented in textbooks,

> Lakatos's (1976) analysis of the emergence of mathematical ideas through its historical production by the mathematical community (and in contrast to the impression created by the usual a-historical, decontextualized presentation of ideas in textbooks), shows how proof is intrinsically tied to the development (and not just the justification) of knowledge. (p. 129)

After articulating the importance of conjecturing, refuting and proving for the deep learning of mathematics, Herbst proceeds to delineate modes of student interaction in DGS and their potential for deeply learning mathematics.

The first mode Herbst's work revealed is empirical, which is described as when the, "actor allows him or herself to make a variety of operations on the diagram (measuring, looking at, drawing in), only constrained by the actual features of the physical drawing and the operational constraints of the physical instruments of interaction" (p. 130). Herbst describes this mode as akin to the experiences students typically have with geometry before high school where conjectures are made in conjunction with measurements in the diagram. The empirical mode is contrasted by her second mode, representational. In this mode, "the known theoretical features of the object constrain the actor to talk in certain ways about the object and to create a diagram that is claimed to signal what is said about the object" (p. 130). Herbst explains that this mode is indicative to high school geometry instruction where, "objects of study are billed as mathematical concepts defined axiomatically and whose properties are said to be proved deductively" (p. 130). These two modes are presented as polar opposites that are likely confusing to students who are expected to make a jump between modes as they progress through mathematics classes.

Herbst proceeds with suggesting a need for students to move beyond the constraints of these two modes and posits a third mode, descriptive. In this mode, "the diagram consists of two overlapping systems: a system of referents and a system of signs" (p. 131). The diagram in this case is an actualization of the mathematical object and serves as a referent for discourse. This mode is further described as balancing between the empirical and representation modes, "some of those features of a diagram, which are available to perception along with the premises of an argument (and whatever is known), can actually feed the argument being produced" (p. 132). Working in this mode is similar to Arzarello et al.'s (2002) ascending and descending processes and Hollebrands' (2007) reactive and proactive strategies when working in DGS environments. Both studies are reviewed later in the review. The final mode of interaction that Herbst provides is generative and takes place in a way similar to that of the historical proving approach of the Greeks in which they created the geometry object while crafting their proof. Herbst provided the following thorough description of the generative mode:

> [It is] a mode of interaction between actor, object, and diagram whereby an initial, hypothetical identification between object and diagram affords conditions and constraints for the actor to anticipate operations on the diagram constrains the interpretation of the results of those operations in ascribing properties to the object, further differentiating the object from the diagram. (p. 134)

The generative mode is the most sophisticated of the four ways students can interact with a diagram to develop proof. For the purposes of this study, the focus becomes how to engage students in these hierarchical modes of interacting with a diagram in a DGS environment, which requires use of previously unavailable affordances such as dragging. The following two studies document the relationships among student tool use in DGS and their proof work.

Laborde (2005) studied how students construct meaning in geometry and particularly in DGS environments. Before investigating students work and constructing meanings, she speaks to the ambiguous nature of diagrams in student learning, "on the one hand they refer to theoretical geometrical properties, while on the other, they offer spatio-graphical properties that can give rise to a student's perceptual activity" (p. 159). Laborde speaks to the unique experience afforded to students regarding the perceived dual purposes of diagrams when students work in DGS environments, "Spatio-graphical and geometrical aspects are very much interrelated in the new kind of diagrams provided by dynamic geometry environments because their behaviour is controlled by theory" (p. 159). Furthermore, the power of this interrelated nature of work is recognized, "[DGS] environments favour a stronger link between spatio-graphical and geometrical aspects since spatial invariants in the moving diagrams almost certainly represent geometrical invariants" (p. 159).

To investigate how students link geometrical and spatio-graphical properties when working in the DGS Cabri, Laborde collected research on students working in pairs. The research transcribed student dialogue and tracked technological actions to subdivide the transcribed episodes into the following sections: "1) referring to spatio-graphical realities, SG; 2) referring to geometrical objects and relation, T; and 3) establishing a link between T and SG" (p. 167). Episode pieces where the student(s) established links between T and SG were further broken down by direction with the first being a transition from SG to T and the second being from T to SG. SG to T involves two basic kinds of moves the first of which takes place when a student recognized a spatio-graphical property in geometrical terms. For example, a student may recognize a quadrilateral as being a parallelogram by beginning with what is seen on the screen and ending with labeling it with its proper geometrical name. The second way moving from SG to T can manifest is when, "a geometrical reason is given for something observed in the behaviour of the diagram." Laborde observed this when a student expressed the theoretical reason an invariant relationship occurs after dragging an object on screen. Laborde explains, "invariance emerges from variation" (p. 171).

The other direction Laborde revealed is the move from T to SG and described it as, "a prediction based on geometrical knowledge on what should happen in the spatio-graphical domain" and provided that, "links in this category seem to be favoured by the software facilities of measuring, and especially dragging where geometric properties are preserved" (p. 172). In this category, students use their geometrical knowledge to make a prediction (conjecture) before testing it in the DGS environment with the accompanying spatio-graphical representation. Laborde provides two implications these two types of student mathematical activity in DGS environments. The first is that, "there is no place for doubt, since every spatial behaviour of the diagram may be interpreted as geometrical" and that, "experiments may become richer and based on more complex geometrical knowledge than in a paper-and-pencil environment" (p. 172). Laborde's study showed the potential for students to reason and develop geometrical understandings in linking spatio-graphical and geometrical aspects. In closing her article, Laborde explained, "learning geometry seems to involve not only learning how to use theoretical statements in deductive reasoning but also learning to recognize visually relevant spatio-graphical invariants attached to geometrical invariants" (p. 177). The allowance for students to move from spatio-graphical to theoretical representations in DGS environments marks a strength of such environments to increase the quality of student mathematical activity for reasoning and proving.

Dragging and Measuring

Holzl (2001) reported on an investigation of the case of one classroom that utilized DGS and noted the reasons the dragging feature was used. After describing the often used purpose of dragging, verifying, Holzl provided another use of dragging that emerged from his case study, searching. He articulated the research question, “Is it possible to use DGS beyond the pure confirmation of geometric facts by creating an appropriate heuristic context in such a way that the significance of the geometric situation can be recognized more easily?” (p. 68). Holzl summarized, “this mediating function [dragging] can be used in at least two principal ways: A) Drag mode as a test mode; B) Drag mode as a search mode” where with the first case, “a construction is checked as to whether it has the desired properties” and “in the second case, new properties are recognised” (p. 83). Holtz closed his article with calling for new ways, “to help students focus on invariants rather than focus on details which suppress the overall impression of a drawing in its concentration on local relationships between parts of figure” (p. 8). Since Holtz published his findings on the purposes of dragging and his call for improved ways for students to engage in DGS, other researchers have explored and further parsed out the use of dragging and related mathematical activity.

Numerous others have researched the usefulness of dragging in DGS environments (e.g., Baccaglini-Frank & Mariotti, 2010; Hadas, Hershkowitz, & Schwarz, 2000). To reveal multiple modalities of dragging, Arzarello, Olivero, Paola, and Robutti (2002) reported on student episodes in which they used dragging in DGS for a variety of purposes. The researchers used their extensive collective experience and research on student mathematical activity in DGS to define cognitive modalities of dragging and two types of student processes. The names and descriptions of each modality are provided below:

- Wandering dragging: Moving the basic points on the screen randomly, without a plan, in order to discover interesting configuration or regularities in the drawings.
- Bound dragging: Moving a semi-dragable point (it is already linked to an object).
- Guided dragging: dragging the basic points of a drawing in order to give it a particular shape.
- Dummy locus dragging: Moving a basic point so that the drawing keeps a discovered property; the point which is moved follows a path, even if the users do not realize this; the locus is not visible and does not 'speak' to the students, who do not always realize that they are dragging along a locus.
- Line dragging: Drawing new points along a line in order to keep the regularity of the figure.
- Linked dragging: Linking a point to an object and moving it onto that object.
- Dragging test: Moving dragable or semi-dragable points in order to see whether the drawing keeps the initial properties. If so, then the figure passes the test; if not, the drawing was not constructed according to the geometric properties you wanted it to have (p. 67).

The modalities of dragging were quoted at length to demonstrate how the research on DGS and the technological affordance of dragging has become more detailed and evolved.

Through their work with students completing tasks in DGS, the authors noted two types of student processes: ascending and descending. Ascending processes are, “to explore freely a situation, looking for regularities, invariants, etc.” whereas descending processes indicate when the student moves, “from theory to drawings, in order to validate or refute conjectures, to check properties, etc.” (p. 67). With these dragging modalities and processes in place, Arzarello et al. (2002, p. 69) provided the following visual to frame student work in DGS environments.

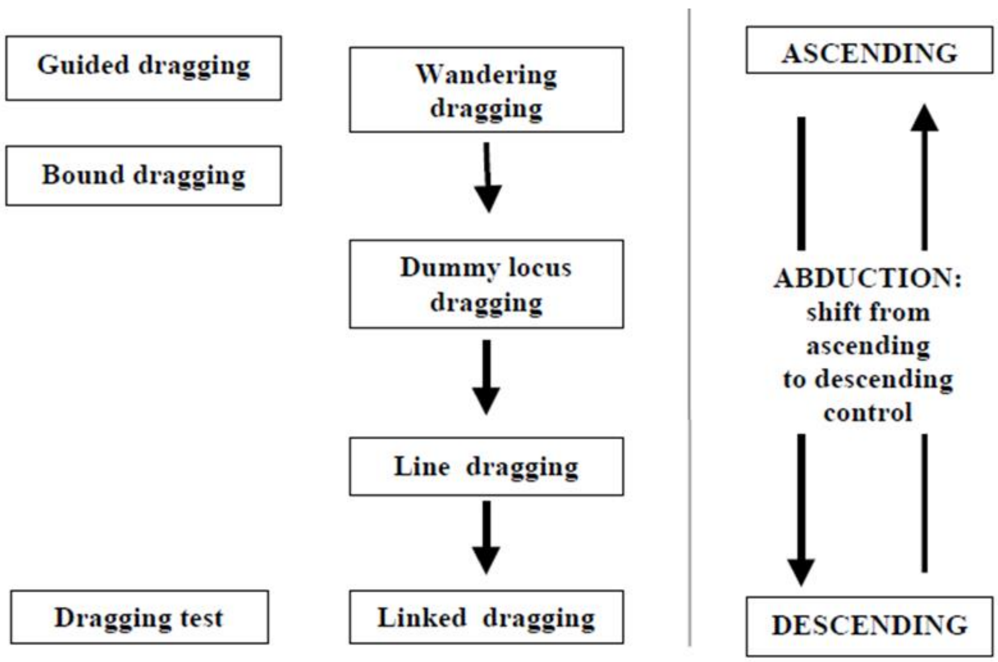


Figure 2: Dragging and Cognitive Modalities

Their framework speaks to the complexity of student mathematical activity in a DGS environment as it accounts for students moving between ascending and descending processes. The authors termed this shift abduction and site the work of Pierce (1960) who posited the term to describe a deduction statement that is based on two induction statements. In so doing, Pierce cleverly combined and legitimized inductive and deductive reasoning in mathematics reason and proof. As the multiple modalities of dragging are understood so is the mathematical potential for working in such environments. This potential may include students developing formal proofs. Arzarello et al. reference one of the fathers of mathematics education and speak to this potential:

> [S]tudents will be motivated to prove why a certain proposition is true (within a theory), after they know that it is true (within the Cabri environment). To paraphrase Polya (1954), first we need to be convinced that a proposition is true, then we can prove it. (p. 71)

This work speaks to the field's evolving understanding and documentation of powerful mathematical activity as students link their cognitive fidelity with the external representation on the computer screen to build their knowledge of geometry.

Hollebrands (2007) built on the previous work of Arzarello and others to explore purposes for dragging and measuring when students work with geometric transformations in DGS. She addressed the specific research question, "In what ways do students interpret objects created with the use of a dynamic program for geometry when they make use of the affordances of the tool as they are learning about geometric transformations?" (p. 166). Hollebrands conducted a seven week instructional segment on transformations in which students utilized the DGS The Geometer's Sketchpad. Data was collected on whole class discussions as well as in case studies on four students to reveal students technology use and thinking.

The analysis showed that students used Arzarello, Michelitti, Olivero, Robutti, Paola, and Gallino's (1998) modes of dragging: 1) to test a construction (drag test); 2) to test a conjecture (drag test); and 3) to search for invariances (lieu meut) (p. 177). Students often used the measure affordance when conducting their dragging to supplement their activity. Hollebrand's findings revealed an additional layer to the students' mathematical activity in DGS; that of their strategy. She termed two types of strategies, reactive and proactive. Reactive was described as taking place when a student makes a strategy choice on how to proceed based on what was reported on the screen. She explained, "this type of use could be described as reactive because the choice of action is in response to what the computer produced" (p. 184). In contrast, a student employing a proactive strategy has demonstrated some forethought on what to expect before an action is taken. Hollebrands explained that students, "might have certain expectations of what they want to do with the technology, determine what actions will achieve their desired results, and then perform the action and reflect on the results that appear on the screen" (p. 184). Notice parallels between Hollebrands' notion of reactive and proactive strategies and Arzarello et al.'s (2002) ascending and descending processes. Hollebrands (2007, p. 188) summarized her findings on student mathematical activity in GSP in the table below.

Table 2: Students' Strategies Related to Their Uses of Dragging and Measuring

Students' Strategies Related to Their Uses of Dragging and Measuring

	Use of a feature	
Strategy	Dragging	Measuring
Reactive	Dragging randomly	
Reactive or Proactive	Dragging to formulate a conjecture or search for invariance	Measuring to formulate a conjecture or search for invariances
	Dragging to test a construction	Measuring to test a conjecture
Proactive	Dragging to test a conjecture	Measuring to test a construction

Hollebrands' work emphasizes the importance of students using multiple strategies when working in DGS environments to utilize its full potential. As part of the concluding discussion to her reported work, she related students' strategies to Piaget's (1970) levels of abstraction. Students who acted solely on physical appearances demonstrated empirical abstractions; students who first planned their technological actions and then interpreted the subsequent visual information demonstrated quasi-empirical abstractions; while students that, "were reasoning only about the properties and theoretical objects (the figure) can be described as making reflective abstractions" (p. 189). Identifying the connections between students' strategies and levels of abstraction as they are mathematically active in DGS environments provided an important step for those concerned with designing and researching student engagements that hold up to rigorous types of abstraction traditionally associated with geometry.

Reasoning, proof, dragging and measuring have been extensively studied and documented regarding student mathematical activity in DGS environments. This research speaks to the enormous potential of DGS environments to be employed as a platform to change the way students bridge inductive and deductive approaches to building their understanding of geometry. Another player in the dynamic of students building their understanding of geometry is the teacher. The following section addresses research that recognizes the teacher as well.

Student, Tool, and Teacher

Researchers have also studied the related dynamic of student, tool and teacher. This subcategory is represented in Zbiek et al.'s (2007) left hand side of the pyramid. The literature that addresses student, tool and teacher often emphasizes a teacher's knowledge and views of technology regarding incorporation in instruction and influences on students' activity and learning. Zbiek et al. explained, "A teacher's readiness to use a particular form of technology and the nature of how a teacher's use of the technology unfolds center around how the teacher's practice and the nature of that technology align" (p. 1187). They term this match between practice and technology, pedagogical fidelity. Researchers have studied this match or lack thereof. For example, Ruthven and Hennessy (2002) studied how high school math teachers that valued classroom investigations also used technologies such as geometry programs successfully in their instruction. In another case, Farrell (1996) studied the roles precalculus teachers undertook when graphing calculators were and were not in use. Three studies are reviewed in what follows as a sample of the extensive research documented by Zbiek and Hollebrands (2008) on the relationships among student, tool (technology) and teacher.

Goos, Galbraith, Renshaw, and Geiger (2000) reported on a longitudinal study of secondary mathematics teachers' uses of technology in the classroom. Their analysis revealed four roles of technology in teacher and students learning interactions. These roles are important to the study at hand as a teacher's perception of the role of technology likely affects what the teacher values about the technology and mathematical tasks designed to utilize affordances of the technology. The four roles of technology are master, servant, partner, and extension of self. Goos et al. (2000) explain the purpose of their work to, "examine interactions between teachers and students, amongst students themselves, and between humans and technology, in order to investigate the extent to which different participation patterns provide opportunities for students to engage constructively and critically with mathematical ideas" (p. 304). The roles technology takes according to student and teacher preferences affect the extent to which the technology is utilized. In what follows, technological roles as provided by Goos et al. are reviewed and considered for the purposes of this study.

The first role of technology Goos et al.'s (2000) work revealed is technology as master, which they describe as the teacher being, "subservient to the technology and is able to employ only such features as are permitted either by limited individual knowledge or force of circumstance" (p. 307). The authors explain that this role is often manifest when teachers have experienced, "large-scale transmissive programs, where course organisation imposes the [master] relationship" (p. 307). A teacher who is functioning with this mode may use a DGS to create a student task simply to say that they used the technology. Such tasks may not utilize the full benefit of the technological affordances of the software due to the teacher's limited perspective on the role of the technology. The second role proposed is technology as servant described as when, "the user may be knowledgeable with respect to the technology but uses it only in limited ways" (p. 307). An example of a teacher functioning under this role in regard to DGS use may be one where the teacher shares a sketch in The Geometer's Sketchpad on an overhead screen to the whole class. Suppose the sketch demonstrated some geometric property, but the fact that the teacher has made the technology a servant limits the teacher to using it only in whole class discussions thus eliminating the possibility for students to discover the property on their own.

Unlike these two limited roles of technology, the third role, technology as partner, indicates a shift from the teacher exercising power over the students, to, "increase the power that students collectively exercise over their learning" (p. 307). This role is characterized by the teacher creatively using the technology and resources available. Consider a teacher who writes a task in DGS for students to explore the properties of different kinds of quadrilaterals by dragging vertices and noticing invariant relationships. The teacher may offer minimal guidance as the students work in small groups to build their understandings of what qualities quadrilaterals possess. In this hypothetical example, the students are empowered to utilize the technology as a partner and take responsibility for their learning. The fourth role is technology as extension of self and was labeled as the highest level of functioning. The authors admit that this role likely rarely occurs in practice. Goos et al. (2000) described, "Here powerful and creative use of both mathematical and communications technology forms as a natural part of a teacher's repertoire as do fundamental pedagogical and mathematical skills" (p. 308). This role may be evidenced as the teacher and students establish the mathematical norm of making written conjectures when manipulating sketches in DGS environments and sharing their work within their learning community.

Like Goos and colleagues, Drijvers, Doorman, Boon, Reed, and Gravemeijer (2010) studied how teachers use technology in their instruction at the secondary level. Drijvers et al. (2010) employed the term orchestrations to name different ways teachers incorporate technology, and, "to what extent these are related to teachers' views on mathematics education and the role of technology therein" (p. 213). Researchers video-recorded lessons from three teachers and analyzed them in regard to each teacher's technology use and profile. One aspect researchers paid attention to was instrumental genesis defined as taking place when an object or artefact is turned into an instrument (p. 214).

Their findings provided a number of categories of instrumental orchestrations. The first of six orchestration types is technical-demo where the teacher demonstrates a tool to their class. In a DGS environment, this orchestration could look like a teacher using an overhead projector to show students how to measure the length of a line segment in DGS. The second orchestration provided is explain-the-screen, and it is similar to the first in that the teacher uses the projected technology to provide an explanation of mathematical content. The following scenario is a hypothetical example of explain-the-screen orchestration. Consider the previous example but assume the teacher not only demonstrated how to measure the sides of a rectangle but also explained how these measurements provide evidence that the opposite sides of rectangles have equal measures. With the next technology orchestration, link-screen-board, the teacher, “stresses the relationship between what happens in the technological environment and how this is represented in conventional mathematics of paper, book and blackboard. This orchestration may manifest when a teacher reproduces a constructed object as seen on a blackboard and recreates it in a DGS environment. The diagram in the DGS environment can then be used to dynamically display attributes of the object through the use of measuring and dragging.

Unlike the first three orchestrations, the latter three involve student contributions for learning with the technology. In the fourth, discuss-the-screen, a whole-class discussion takes place about what is happening on the computer screen. Drijvers et al. (2010) explain, “The goal is to enhance collective instrumental genesis” (p. 219). A possible DGS engagement demonstrating this orchestration type may be a teacher dragging vertices of a constructed object on a screen and posing questions to the class about what remains invariant. In this episode, the class is encouraged to discuss what is seen on the screen to decipher properties of an object (e.g., opposite angles of a parallelogram have equal measurements). Further ownership for learning is passed to students in the fifth orchestration, spot-and-show, where, “student reasoning is brought to the fore through this identification of interesting DME [digital mathematics environment] student work during preparation of the lesson, and its deliberate use in a classroom discussion” (p. 219). In this orchestration, the teacher may notice interesting student work in a DGS environment prior to the whole class discussion and choose to display that work with the students providing explanation. In the sixth and final orchestration, sherpa-at-work, the student uses technology to present their work. This orchestration allows for the greatest student ownership of learning, and Drijvers et al. recommend, “The classroom setting should be such that the Sherpa-student can be in control of using the technology, with all students able to follow the actions of both Sherpa-student and teacher easily” (p. 220). Consider a hypothetical example in which the students are asked to show that a theorem is true using The Geometer’s Sketchpad. The students could work in small groups to construct appropriate geometric objects and then display their objects along with any desired measures and dragging to the whole class. In this case the students are orchestrating the technology as a sherpa-at-work.

Part of the data to be collected in this study involves teachers reporting how they employ DGS in their instruction. Goos et al.’s (2000) roles of technology and Drijvers et. al’s (2010) technological orchestrations serve as a lens through which teachers’ reports of their technology use may be better understood. These works are also be valuable in interpreting how teachers use technology related to how they approach writing tasks in the technological environment The Geometer’s Sketchpad.

Teacher, Tool, and Curricular Context

Zbiek et al. (2007) provided the third (front side) of their framework, “Mediation relationships among technology, student, teacher, mathematical activity, and curriculum” with vertices of Teacher, Tool and Curricular Content. The teacher who successfully incorporates technology into their mathematics instruction must be aware of the relationships between the technological tool and the curricular context. Zbiek et al. provided, “several central constructs that capture the opportunities for change in curriculum facilitated by technology: representational fluency, mathematical concordance, amplifiers, reorganizers, microprocedures, and macroprocedures" (p. 1192). While a number of research studies exist addressing these constructs (e.g., Lumb, Monaghan, & Mulligan, 2000; Ruthven & Hennessy, 2002; Zbiek, 1995), for the purposes of this study, two of these constructs are addressed, amplifiers and reorganizers. Technology as an amplifier of the curriculum is described according to Pea (1985), "certain uses of technology are amplifiers in that they accept the goals of the current curriculum and work to achieve those goals better" (p. 1199). One of the practice goals found in the CCSSM is to, "construct viable arguments and critique the reasoning of others" (2010, p. 6). The possibility of DGS environments to amplify this curricular goal needs to be further researched and realized. For instance, if DGS environments encourage high school geometry students to link empirical and deductive arguments, then the teachers should harness this potential in the tasks they choose or create for their students. Another function of technology in relation to the curriculum is reorganizer, "that change the goals of the curriculum by replacing some things, adding others, and reordering still others" (p. 1199). To revisit the CCSSM, one of the high school geometry standards reads, "experiment with transformations in the plane" (p. 75). DGS environments have changed the way experiments with transformations can take place in instruction. Hollebrands (2007) noted,

> [S]tudents learn about geometric transformations while using technology, they perform actions, afforded and mediated by the computer, which produce results.

> Students interpret these results and may compare them to what they anticipate will happen based on their understandings of the technology (knowledge about how to operate the technology and how it will behave under certain conditions) and of the geometric transformations. (p. 170)

Dynamic Geometry Software environments have added to goals of the curriculum in requiring students to know how to ‘operate the technology and how it will behave under certain conditions’ and have amplified the curriculum in allowing students to make transformations quickly as opposed to using paper and pencil. While significant research has been conducted on the relationship of technology to the curriculum, little research exists on how to assist teachers seeking to add to the curriculum and intending to write tasks in technological environments in general and DGS environments in particular.

Prestage and Perks (2007) introduced a tool they used in a graduate teaching program that assists teachers in writing non-technological mathematics tasks. The authors’ simple tool has four components: 1) notice the givens in the task and in the resources; 2) change, add, or remove a given; 3) analyze the resulting mathematics and choices for pupils and teachers; and 4) choose an appropriate task for the classroom (p. 383). The authors explained that this tool evolved from their practice and resulted from their attempt to make their practice explicit (p. 389). The authors attempted to satisfy a perceived need in the teachers to prepare them to write their own tasks for immediate and future instruction.

Laborde (2002) studied teachers’ incorporations of the DGS Cabri into their mathematics instruction over a three year period. She noted the association between tasks and the “role that the designer of a task attributes to Cabri” (p. 293). The following roles of Cabri in task design were proposed:

1. Cabri is used mainly as facilitating material aspects of the task while not changing it conceptually.
2. Cabri is supposed to facilitate the mathematical task that is considered as unchanged: this is the case where Cabri is used as a visual amplifier (Pea, 1985) in the task of identifying properties; it is assumed that it is easier to observe that three lines always intersect in one point during the deformation of the diagram by the drag mode than in a static paper-and-pencil diagram.
3. Cabri is supposed to modify the solving strategies of the task due to the use of some of its tools and to the possibility that the task might be rendered more difficult.
4. The task itself takes its meaning or its “raison d’etre” from Cabri (p. 293).

Laborde highlighted how the nature of a task is dependent on how the designer of the tasks realizes the potential of the technological environment. For example, in reference to number three, the author provided that constructing a square using paper and pencil on graph paper is qualitatively different than the task of constructing a square in Cabri, because, “The task in Cabri requires more mathematical knowledge about the properties of a square and the characteristic property of a circle which students find difficult to put into action” (p. 294). In his dissertation, Sherman (2011) studied how the use of technology related to the cognitive demand of tasks. He found that technology use does not relate to the cognitive demand of tasks, but that the way the technology is used is related to these demands (p. 292). Tasks are fundamentally different when working in a DGS environment and this fact brings to the fore a need for guidance when writing and implementing such tasks.

While some work has been documented to assist teachers in writing mathematics tasks, there is an identified need for explicit guidance in writing tasks for DGS environments. The Dynamic Geometry Task Analysis Framework proposed in this study works to meet this need. In the next section, work that offers guidance in assessing task quality for mathematics tasks in general and DGS tasks in particular is reviewed. Undoubtedly, this work could also assist researchers and teachers to write tasks, but it is reviewed in a separate section in accordance with Zbiek et al.'s "Mediation relationships among technology, student, teacher, mathematical activity, and curriculum" framework.

Curricular Context, Tool, and Mathematical Activity

Zbiek et al. (2007) provided the right face of their pyramid with the vertices curricular content, tool and mathematical activity. For the scope of this study the object of focus for this section is the mathematical task. A number of researchers have posited characteristics of quality mathematical tasks such as Smith and Stein (1998) Task Analysis Guide, which posits four categories of task quality: 1) Memorization; 2) Procedures without connections; 3) Procedures with connections; and 4) Doing mathematics. Smith and Stein based their framework on Doyle's (1988) research on characteristics of academic tasks and Resnick's (1987) work on high-level thinking skills. Doyle investigated the influence mathematics tasks related to student mathematical work and thinking. In this work, tasks were considered to be either familiar or novel with the latter described in what follows:

> Novel work consists of assignments for which students are required to assemble information and operations from several sources in ways that have not been laid out explicitly by the teacher. Novel tasks in math for example, would involve such processes as deciding which operation fits a particular problem or combining algorithms already learner into a chain of operations to solve a problem. An essential feature of novel work is that students must make decisions about what to produce and how to produce it. (p. 173)

Smith and Stein based some of the characteristics of their highest category, doing mathematics on Doyle's notion of novel work and tasks. For instance, one characteristic reads, "Require complex and non-algorithmic thinking—a predictable, well-rehearsed approach or pathway is not explicitly suggested by the task, task instructions, or a worked-out example" (Smith & Stein, 1998, p. 348). Resnick synthesized studies concerning mathematical cognition and found that, "successful learners understand the task to be one of constructing meaning, of doing interpretive work rather than routine manipulation" (p. 12). Smith and Stein employed the findings of Doyle and Resnick to develop and define their categories of mathematical task quality. In the methods chapter, Smith and Stein's framework is provided in full to support the study at hand as their work has direct bearing on the development of the author written Dynamic Geometry Task Analysis Framework.

Sinclair (2003) narrowed the focus of task analysis and offered recommendations to promote student exploration in pre-constructed dynamic geometry sketches. Her recommendations speak to the need to analyze task quality in relation the written directives required of students. In a similar way, the Dynamic Geometry Task Analysis Framework is a tool to assess task quality in looking at the quality of each prompt. As there is a lack of guidance on how to assess the quality of tasks in DGS environment, her recommendations are provided at length below:

1. When a question aims to focus student attention, the sketch must provide the visual stimulus. It must draw attention through colour, motion, and markings.
2. When a statement prompts action, such as asking students to drag, observe or deduce, the sketch must contain the necessary provisions. It must provide affordances so that the student can take the required steps.
3. Questions that invite exploration are open-ended. In order to explore uncharted territory, the student requires a sketch that allows options. Thus, when a question invites exploration, the sketch must provide alternative paths.
4. A question can surprise – which may lead to further exploration; however, the teacher is not necessarily there to correct any misinterpretation. Thus the sketch must support experimentation to unmask the confusion. It must be flexible enough to prevent frustration.
5. Questions that check understanding are important parts of any learning situation. In the study tasks, the checking involved students looking together for the answer.

> Although peer-interactions were not discussed in this article, study results showed that the sketch aided this process by providing a shared image for students to consider and discuss (p. 312).

Sinclair's work is revisited in the methods chapter as it greatly influenced the writing of the Dynamic Geometry Task Analysis Framework.

To summarize, there is limited research on DGS task quality. This gap in the literature provided the author an opportunity to develop a framework that attends to this need. In what follows, detailed research questions and sub-questions are provided to set up the study of this framework and its usefulness in revealing DGS task quality and in assisting teachers attempting to write such high quality tasks.

CHAPTER 3: METHODOLOGY

The purpose of this study was twofold. First, it assessed the validity of the Dynamic Geometry Task Analysis Framework for categorizing task quality at the prompt level. Second, the study investigates how use of the Dynamic Geometry Task Analysis Framework affects how teachers describe and write Geometer's Sketchpad tasks. For clarity, a GSP task is defined as a combination of the geometric objects (i.e., the sketch on the screen at various points in the task completion process) and the associated written directions used to accomplish a particular learning goal. Student work in a sketch can take two forms: 1) a pre-constructed sketch refers to a sketch in which objects are completely constructed for the student with the expectation that the students use this completed construction and technological actions such as measuring and drawing to respond to associated prompts, and 2) a student-constructed sketch refers to a sketch in which the student construct all or a portion of the geometric object(s) as they respond to associated prompts. All tasks and associated sketches involved are in the latter category.

Qualitative research was conducted in the form of exploratory case studies (Creswell, 2013) that utilize structured, task-based interviews (Goldin, 2000). These interviews provided the structure needed to gather meaningful data on the specific research questions in addition to allowing participants the autonomy to expound on their responses as they desired. A purposeful sample of twelve students and six teachers was gathered from surrounding school districts. With this foundational methodology in place, the following overarching research and sub-research questions were addressed:

1. What is the nature of relationships among mathematical prompts and technological actions and arguments students provide and conclusions they reach using The Geometer's Sketchpad?
2. How do pre-service and in-service teachers use the Dynamic Geometry Task Analysis Framework?
    a. What are similarities and differences between teachers' use of the framework for task writing for in-service and pre-service teachers?
    b. What are similarities and differences in task quality for tasks written by in-service and pre-service teachers?
    c. What are similarities and differences in task quality for tasks written by teachers who have different levels of knowledge for teaching geometry?

Data gathered in relation to these research questions was analyzed to revise and establish the validity of the Dynamic Geometry Task Analysis Framework and to assess its impact on teachers who write tasks in The Geometer's Sketchpad.

What follows introduces the conceptual framework for the study. The Dynamic Geometry Task Analysis Framework is composed of two parts: 1) Mathematical Depth, and 2) Technological Affordances. After delineating the creation process and components of the framework, the chapter proceeds with providing descriptions of research design, participant selection, data collection, and analysis organized under each overarching research question. Finally, the trustworthiness of the methods and findings is addressed.

Dynamic Geometry Task Analysis Framework

This study used structured, task-based interviews to refine and establish the validity of the Dynamic Geometry Task Analysis Framework, and then assessed the usability of the framework in conducting teacher case studies. With this basic study design in mind, consider the pre-revision Dynamic Geometry Task Analysis Framework in Table 3 below.

Table 3: Dynamic Geometry Task Analysis Framework

Allowance for Mathematical Depth	
Levels	**Hierarchical Levels and Descriptions**
N/A	Prompt requires a technology task with no focus on mathematics.
0	Sketch does not have mathematical fidelity required to respond to prompt.
1	Prompt requires student to recall a math fact, rule, formula, or definition.
2	Prompt requires student to report information from the construction. The student is not expected to provide an explanation.
3	Prompt requires student to consider the mathematical concepts, processes, or relationships in the current sketch.
4	Prompt requires student to explain the mathematical concepts, processes, or relationships in the current sketch.
5	Prompt requires student to go beyond the current construction and generalize mathematical concepts, processes, or relationships.
Types of Technological Action	
Affordances	**Descriptions**
N/A	Prompt requires no drawing, construction, measurement, or manipulation of current sketch.
A	Prompt requires drawing within current sketch.
B	Prompt requires measurement within current sketch.
C	Prompt requires construction within current sketch.
D	Prompt requires dragging or use of other dynamic aspects of the sketch.
E	Prompt requires creation/consideration of multiple examples from which one can generalize.
F	Prompt requires a manipulation of the sketch that allows for recognition of emergent invariant relationship(s) or pattern(s) among or within geometrical object(s).
G	Prompt requires manipulation of the sketch that may surprise one exploring the relationships represented or cause one to refine thinking based on themes within the surprise (adapted from Sinclair (2003), p. 312).

The impetus for creating this framework stemmed from the researcher's perceived need in mathematics education literature and teacher knowledge for analyzing the quality dynamic geometry software tasks and writing of such tasks. The researcher completed a research internship with a grant supported research and development professional development study. The professional development researchers and designers had the intent to develop participating teachers' knowledge for incorporating technology into the secondary mathematics classroom. Part of participating teachers work in this program required them to write tasks in The Geometer's Sketchpad. The researcher was charged with developing a way to quickly assess the quality of teacher submitted written tasks.

Throughout this work, the researcher was struck by the overall lack of quality in the submitted tasks regarding the utilization of affordance in the technological environment such as constructing, measuring, and dragging. The analyzed tasks' requirements of student thinking and potential for developing understanding were considered while developing a way to assess task quality, as was video footage of classroom task enactments with an eye to the emerging mathematical discourse. The result was the Dynamic Geometry Task Analysis Framework for assessing task quality. The researcher quickly realized the framework's potential for guiding teachers who seek to identify and write quality DGS tasks.

A brief introduction to the literature that supported framework development is included here as it necessitates an understanding of the purpose of the study. The researcher used the work of Smith and Stein (1998) and Sinclair (2003) to guide the framework development process. Smith and Stein (1998) based their framework on Doyle's (1988) research on characteristics of academic tasks and Resnick's (1987) work on high-level thinking skills. They analyzed hundreds of mathematical tasks to define four categories of cognitive demand associated with math tasks: memorization, procedures without connections, procedures with connections, and doing mathematics (p. 348). Tasks that require a low level of cognitive demand are labeled either memorization or procedures without connections, while tasks that require a higher level of cognitive demand are labeled procedures with connections or doing mathematics. Smith and Stein's (2000) Task Analysis Guide is included in Table 4 below.

Table 4: Task Analysis Guide

Lower-level demands	**Higher-level demands**
Memorization • Involve either reproducing previously learned facts, rules, formulas, or definitions or committing facts, rules, formulas, or definitions to memory • Cannot be solved by using procedures, because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure • Are not ambiguous. Such tasks involve exact reproduction of previously seen material, and what is to be reproduced is clearly and directly stated. • Have no connection to the concepts or meaning that underlies the facts, rules, formulas, or definitions being learned or reproduced	Procedures with connections • Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas • Suggest, explicitly or implicitly, pathways to follow that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts • Usually are represented in multiple ways, such as visual diagrams, manipulatives, symbols, and problem situations. Making connections among multiple representations helps develop meaning.

Table 4 Continued

Lower-level demands	Higher-level demands
	Procedures with connections • Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with conceptual ideas that underlie the procedures to complete the task successfully and that develop understanding.
Procedures without connections • Are algorithmic. Use of the procedure is either specifically called for or is evident from prior instruction, experience, or placement of the task. • Require limited cognitive demand for successful completion. Little ambiguity exists about what needs to be done or how to do it. • Have no connection to the concepts or meaning that underlies the procedure being used • Are focused on producing correct answers instead of developing mathematical understanding • Require no explanations or explanations that focus solely on describing the procedure that was used	Doing mathematics • Require complex and nonalgorithmic thinking—a predictable, well-rehearsed approach or pathway is not explicitly suggested by the task, task instructions, or a worked-out example • Require students to explore and understand the nature of mathematical concepts, processes, or relationships • Demand self-monitoring or self-regulation of one's own cognitive processes • Require students to access relevant knowledge and experiences and make appropriate use of them in working through the task • Require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions • Require considerable cognitive effort and may involve some level of anxiety for the student because of the unpredictable nature of the solution process required

Notice similarities between task characteristics included by Smith and Stein and allowances for mathematics depth in the Dynamic Geometry Task Analysis Framework. For example, in their category, Doing mathematics, they provide the description, "Require students to explore and understand the nature of mathematical relationships." The researcher attempted to assimilate the intent of this task characteristic for tasks in dynamic geometry software environments with a combination of codes such as mathematical depth of Level 4, "Prompt requires student to explain the mathematical concepts, processes, or relationships in the current sketch" and technological aspect of Affordance F, "Prompt requires a manipulation of the sketch that allows for recognition of emergent invariant relationship(s) or pattern(s) among or within geometrical object(s)." A combination of mathematical depth and technology use in this instance pushes the student towards doing mathematics. Smith and Stein's Task Analysis Guide greatly influenced the way the researcher interpreted quality of tasks for dynamic geometry software environments.

Sinclair's (2003) work also influenced aspects of the Dynamic Geometry Task Analysis Framework. Sinclair offers recommendations to promote student exploration with pre-constructed dynamic geometry sketches, one of which reads,

> A question can surprise – which may lead to further exploration; however, the teacher is not necessarily there to correct any misinterpretations. Thus, the sketch must support experimentation to unmask the confusion. It must be flexible enough to help students examine cases, yet constrained enough to prevent frustration. (p. 312)

Sinclair's recommendation influenced the technological affordances in the Dynamic Geometry Task Analysis Framework regarding Affordance G, "Prompt requires manipulation of the sketch that may surprise one exploring the relationships represented or cause one to refine thinking based on themes within the surprise." The researcher used the work of Smith and Stein, and Sinclair, to begin to unpack attributes of quality tasks by analyzing at the prompt level, with prompt being defined as a written question or direction related to a sketch that requires a verbal or written response. A technological action is a question or direction that requires a drawing, construction, or measurement within or manipulation of a sketch. A prompt may thus require both a written response and a technological action. During analysis, tasks in which technological actions were coordinated with mathematical prompts seemed to have higher potential for engaging students in mathematical reasoning, a theme thoroughly explored in this study. The coordination of mathematical depth and technological action seemed to indicate higher quality tasks.

To further understand the intended use and value of the conceptual framework, consider the following researcher written Geometer's Sketchpad Task (Trocki, 2014, p. 702).

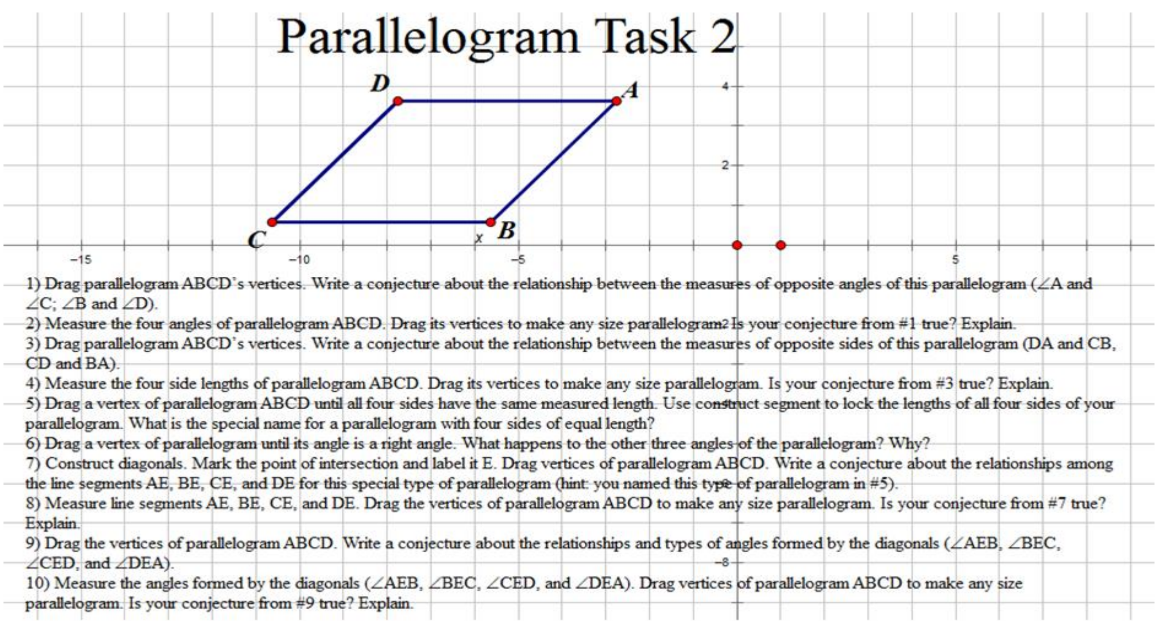


Figure 3: Parallelogram Task 2

This task has the following summary according the Dynamic Geometry Task Analysis Framework.

Table 5: Parallelogram Task Coding Summary

Prompt Number	Parallelogram Task 2 Codes
1	(1, 2, 3, D)
2	(3, 4, 5, B, D, E, F)
3	(1, 2, 3, D)
4	(3, 4, 5, B, D, E, F)
5	(1, C, D)
6	(1, 2, 3, 4, D)
7	(1, 2, 3, D)
8	(3, 4, 5, B, D, E, F)
9	(1, 2, 3, D)
10	(3, 4, 5, B, D, E, F)

Note the coding of Prompt 2, which reads, "Measure the four angles of parallelogram ABCD. Drag its vertices to make any size parallelogram. Is your conjecture from #1 true? Explain." The following is Trocki's (2014) explanation of its coding using the Dynamic Geometry Task Analysis Framework:

> It was coded accordingly as (3, 4, 5, B, D, E, F). The student is required to consider the relationship between opposite angles as part of their conjecture from Prompt 1 (coded 3) and explain why their conjecture was true or not (coded 4). The codes of D, E, and F emerged due to the prompt requiring students to drag (coded D), consider multiple examples (coded E) and recognize invariant relationships (coded F). The requirement of student explanation coupled with the technological action of dragging, considering multiple examples, and recognizing invariant relationships required the

> student to go beyond the current construction to make any desired size parallelogram and then speak to the relationship between opposite angles. This coupling of explanation along with technological actions coded D, E, and F meets the highest level of Allowance for Mathematical Depth (code 5). (p. 704)

Due to the high proportion of prompts that coordinate technological actions with mathematical depth, this task was considered to be of high quality. High, medium and low task qualities were defined in reference to the codes a task receives according to the Dynamic Geometry Task Analysis Framework. Mathematical depth is considered to be evident when a prompt ranks four and/or five in the mathematical depth coding. Tasks were ranked low, medium, or high in quality, and these ranks are based on two measures. First, the mathematical depth of the tasks was taken into account by answering the question, "Does the task possess mathematical depth related to the learning goal of the task?" Secondly, the coordination of technological actions with mathematical depth was considered. The question, "How often are prompts possessing mathematical depth coordinated with technological actions?" was answered to assess the second component of task quality.

The initial task ranking coding methodology is described below. Tasks that do not possess mathematical depth are automatically ranked low. Further, tasks that rank low in quality rarely if ever (< 30% of prompts possessing mathematical depth) coordinate technological actions with the mathematical depth in these prompts. Tasks that rank medium contain prompts possessing mathematical depth with some (> 30%, < 70%) prompts coordinating technological action with mathematical depth. A task that ranks high must contain mathematical depth with a high percentage of prompts (> 70%) that possess mathematical depth coordinated to technological actions. As found in the face validity portion of this study in Chapter 4, this initial coding methodology was refined based on expert feedback. Figure 4 below shows an image of the low-level task in GSP.

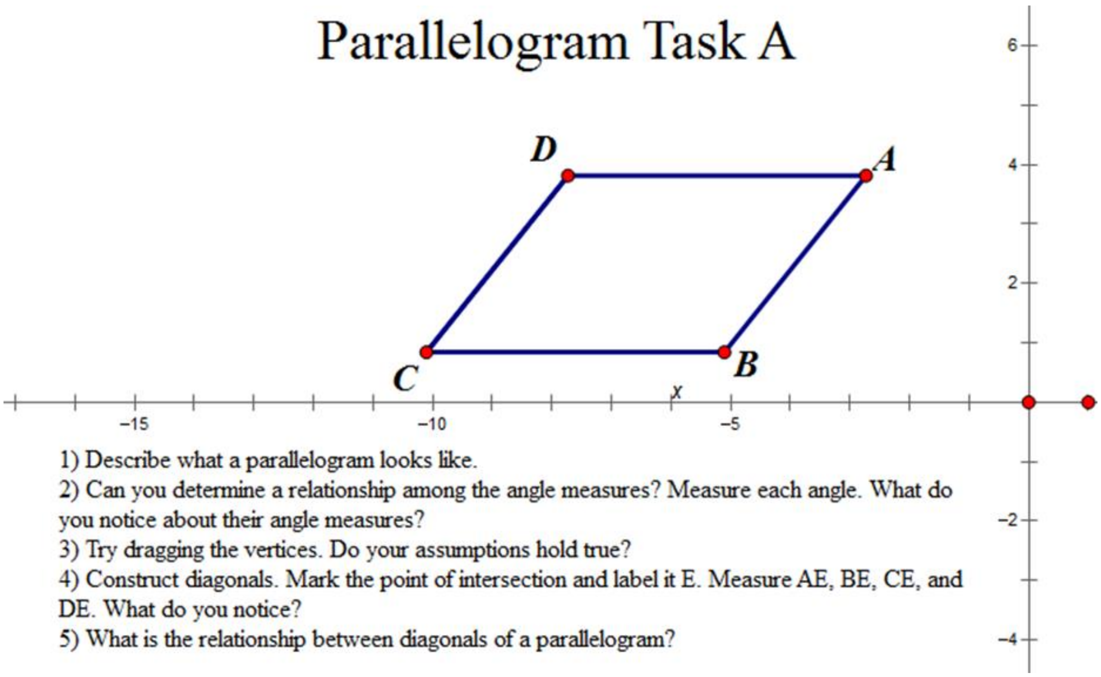


Figure 4: Low-Level GSP Task

Task A received the following codes as summarized below.

Table 6: Task A Coding Summary

Prompt Number	Parallelogram Task A Codes
1	(2, N/A)
2	(1, 2, 3, B)
3	(N/A, D)
4	(1, 2, 3, B, C)
5	(3, N/A)

These codes resulted from the nature of the associated prompts. Prompt 1 simply requires the student to report information from the sketch with not technological action. Prompts 2 through 4 require technological actions however none of these actions are associated with mathematical depth beyond three. In the last prompt the student is to consider the relationship between diagonals, but no explanation is required. Therefore, this task fits the description of low level because it does not contain prompts (i.e., mathematical depth of 4 and/or 5) that possess mathematical depth. Student work on this task was conjectured to be similar to Leung's (2011) epistemic mode of interacting with a DGS sketch known as practice (PM). Leung describes practice mode in the following two ways.

- PM1: Construct mathematical objects or manipulate pre-designed mathematical objects using tools embedded in a technology-rich environment.
- PM2: Interact with the tools in a technology-rich environment to develop (a) skill-based routines; (b) modalities of behavior; (c) modes of situated dialogue. (p. 327)

The data gathered during student task-based interviews was considered in light of Leung's epistemic modes. The following image is of the medium task in GSP.

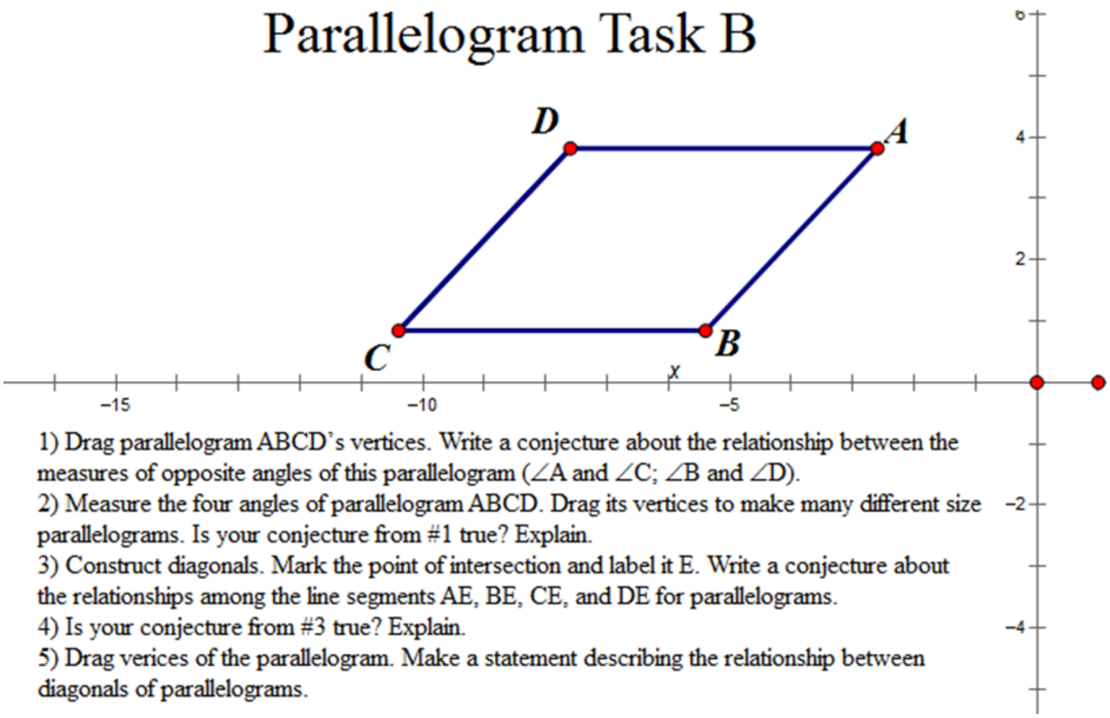


Figure 5: Medium-Level GSP Task

Task B received the following codes as summarized below.

Table 7: Task B Coding Summary

Prompt Number	Parallelogram Task B Codes
1	(1, 2, 3, D)
2	(3, 4, 5, B, D, E, F)
3	(1, 2, 3, C)
4	(3, 4)
5	(3, 5, D)

The codes for Task B resulted from the mathematical depth and technological actions found in the prompts. This task was coded as medium in quality as it contains three prompts (i.e., #s 2, 4, & 5) that possess mathematical depth. Two of these three prompts are coordinated with technological action(s), thus providing approximately 67% of prompts that possess mathematical depth also coordinating to technological action. Therefore, this task meets the stipulations of medium-level quality in the initial coding methodology.

The following image if of the high task in GSP.

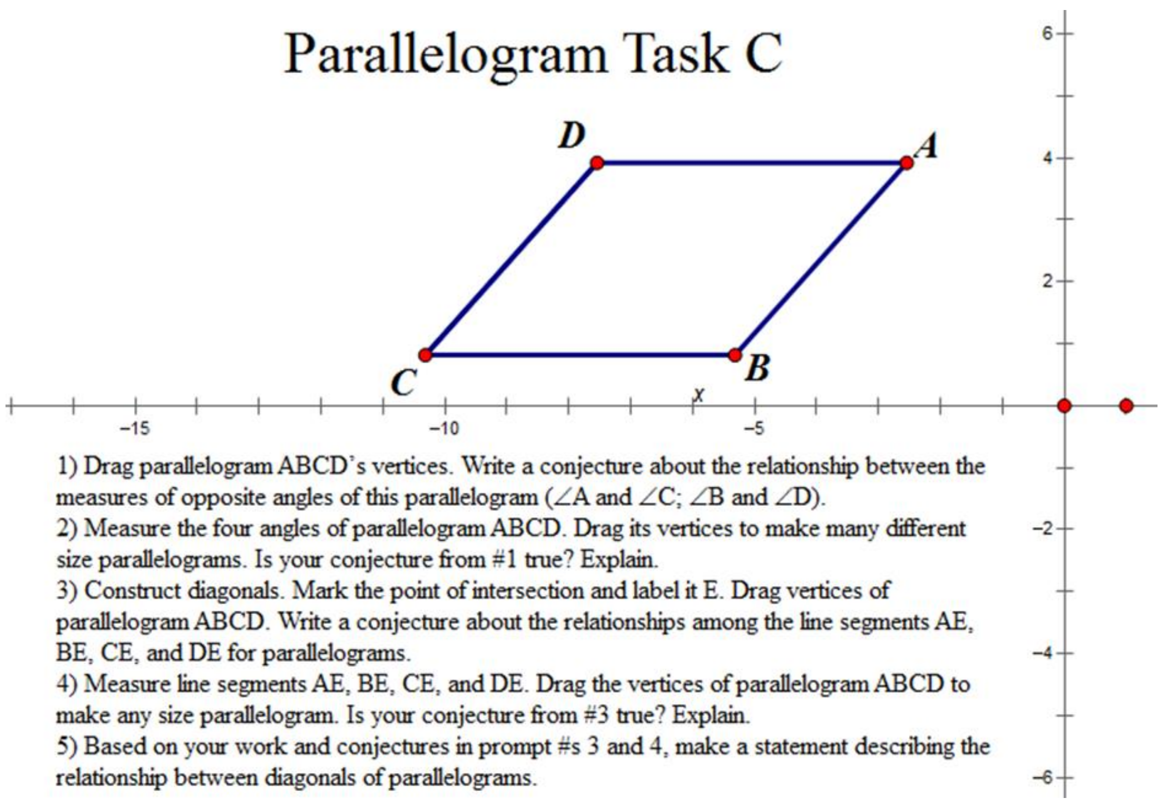


Figure 6: High-Level GSP Task

This task received the following codes summarized below

Table 8: Task C Coding Summary

Prompt Number	Parallelogram Task C Codes
1	(1, 2, 3, D)
2	(3, 4, 5, B, D, E, F)
3	(1, 2, 3, C, D)
4	(3, 4, 5, B, D, E, F)
5	(3, 4, 5, B, D, E, F)

Overall, the codes for this task are different from the low- and medium-level tasks in that all prompts that contain mathematical depth are also coordinated with technological actions. This tasks ranks high in quality according to the definition of high quality provided previously.

It was conjectured that coding of tasks with the Dynamic Geometry Task Analysis Framework as low, medium, or high would correspond to student argumentation and conclusions as labeled with the core of Toulmin's (1958) argumentation model. It was further conjectured that the framework would assist those concerned with assessing and writing quality tasks. These conjectures were researched with documented findings in in this study.

Overview and Justification of Methods

The study was composed of two phases: 1) a validity study of the Dynamic Geometry Task Analysis Framework followed by possible revisions; and 2) a study of how the framework affects teachers' perceptions of task quality and how they write and revise tasks. To address overarching Research Question One that concerns the validity of the framework, the researcher crafted three tasks for The Geometer's Sketchpad that ranked low, medium and high on the Dynamic Geometry Task Analysis Framework. The framework itself along with the three written tasks was assessed for face validity by three experts in the field of DGS in Mathematics Education. The framework and tasks were then be revised in accordance with the feedback received in the face validity process. The role of each expert in establishing face validity as described by Anastasi (1998) was invoked, "Face validity pertains to whether the test [Dynamic Geometry Task Analysis Framework] "looks valid" to the examinees who take it, the administrative personnel who decide on its use, and other technically untrained observers" and, "refers, not to what the test actually measures [task quality], but to what it appears superficially to measure" (p. 144). The face validity check concerned the Dynamic Geometry Tasks Analysis Framework's potential for describing task quality concerning the arguments and conclusions required of students. After the face validity portion of this study was completed, Research Question One was directly addressed.

Research Question One

Recall Research Question One, "What is the nature of relationships among mathematical prompts and technological actions and arguments students provide and conclusions they reach using The Geometer's Sketchpad?" The first phase of this study addressed this question in conducting with students working on the researcher written tasks. Twelve students provided written work and explained their thinking for one of three researcher written tasks. Four students completed the low-ranked task, four the medium, and four the high. Student assignment to each task ranking was randomized. Structured, task-based interviews were then carried out as described by Goldin (2000):

> Structured, task-based interviews for the study of mathematical behavior involve minimally a subject (the problem solver) and an interviewer (the clinician), interacting in relation to one or more tasks (questions, problems, or activities) introduced to the subject by the clinician in a preplanned way." (p. 519)

This preplanned way included the expectation that the researcher offer little to no guidance while the student completed the task. After the student completed their task work, the researcher asked a predetermined set of interview questions (see Appendix B). Video footage of the student working and the interview was gathered to assess the arguments and conclusions made. Goldin explained, "By analyzing verbal and nonverbal behavior or interactions, the researcher hopes to make inferences about the mathematical thinking, learning and/or problem solving of the subjects" (p. 519). The researcher's exploration of student work on each task and accompanying questioning followed the four recommendations for conducting structured, task-based interviews as provided by Goldin:

1. Posing the question (free problem solving), with sufficient time for the child to respond and only nondirective follow-up questions, such as, "Can you tell me more about that?"
2. Minimal heuristic suggestions, if the response is not spontaneous, such as, "Can you show me using some of these materials?"

3. The guided use of heuristic suggestions, again only when the requested description or anticipated behavior does not occur spontaneously, such as, "Do you see a pattern in the cards?"
4. Exploratory, metacognitive questions, such as, "Do you think you could explain how you thought about the problem?" (p. 523)

The hypothesis tested in conjunction with overarching Research Question One was that students will make qualitatively different arguments and conclusions on tasks that rank low, medium, and high on the Dynamic Geometry Task Analysis Framework. The objects of study in this phase were the tasks themselves and at one layer removed the Dynamic Geometry Task Analysis Framework. The student structured, task-based interview transcriptions were analyzed with the core of Toulmin's (1958) argumentation structure: 1) Data, 2) Claim, and 3) Warrants. The essential parts employed in this study are summarized in what follows.

- Data: The facts or evidence used to prove the argument
- Claim: The statement being argued (a thesis)
- Warrants: The general, hypothetical (and often implicit) logical statements that serve as bridges between the claim and the data (p. 98)

During the interview portion of the structured, task-based interview the students responded to questions about each prompt they addressed in their assigned task. Student written responses to task prompts provided a small enough "grain size" to code student's argumentation during the interview using Toulmin's model of argument. These findings addressed overarching

Research Question One: “How do the mathematical prompts and technological actions as identified in the Dynamic Geometry Task Analysis Framework relate to students' mathematical arguments and conclusions?” After structured, task-based data was coded, the usefulness of the framework was once again revisited. Finally, the framework underwent additional revisions as necessitated by the analysis and findings of student structured interview data. Due to pragmatic constraints, a second face validity phase of this study did not take place. It represents an avenue for further refinement of the framework.

Participant Selection

To satisfy the two phases of data collection, a purposeful sample of twelve students and six teachers was collected. Requirements for the twelve students were that they have taken a course in high school geometry or its equivalent and have had some experience using DGS environments. These students were recommended by teachers in local school districts and then asked to participate by the researcher. Recall that four students completed the low-level task, four the medium, and four the high. Descriptions of participating students are provided below.

Student Three, Student Six, Student Nine and Student Twelve completed Task A (low). Student Three was a senior in high school who had taken Geometry as a freshman, Algebra II as a sophomore, Pre-Calculus as a junior, and was currently taking Advanced Placement Statistics. She described their experience in Geometry as enjoyable because she liked working with shapes and appreciated how her teacher conducted class. She was familiar with the dynamic geometry program of Geogebra, but had not used Geometer's Sketchpad before. This student's responses are documented in the table below.

Student Six was a senior in high school who had taken Algebra I as a freshman, Geometry as a sophomore, Algebra II as a junior, and was currently taking Pre-Calculus. The student reflected that they did not particularly enjoy Geometry but rather liked, "more straightforward stuff" and preferred Algebra over Geometry. Student Six had used The Geometer's Sketchpad before in her Geometry class and was currently using Geogebra in their Pre-Calculus class.

Student Nine was a junior in high school who took Geometry as a freshman, Algebra II as a sophomore, and was currently taking Pre-Calculus. She reported that she did not enjoy Geometry but did like finding missing sides of triangles. She thought she had seen the program Geometer's Sketchpad before but could not remember when.

Student Twelve was a junior in high school and took Geometry as a freshman, Algebra II in the summer before her sophomore year, Pre-Calculus as a sophomore, and was currently taking AP Calculus AB and BC in her junior year. She reported that she really liked Geometry and did very well in the course. She had used The Geometer's Sketchpad quite often in her Pre-Calculus class.

Student One, Student Four, Student Seven, and Student Ten completed Task B (medium). Student One was a senior in high school who had taken Geometry as a freshman, Algebra II as a sophomore, Pre-Calculus as a junior and was currently taking both Advanced Placement Calculus and Statistics. He responded that he liked taking Geometry and did very well. He had never used The Geometer's Sketchpad but had seen Geogebra used in his Calculus class.

Student Four was a high school senior who had taken Algebra I as a freshman, Geometry as a sophomore, Algebra II as a junior, and was currently taking Advanced Functions and Modeling as a senior. She reflected that she liked Geometry better than Algebra, but that is was still challenging for her. She had never used The Geometer's Sketchpad before, but had used Geogebra.

Student Seven was a senior in high school who had taken Algebra I as a freshman, Geometry as a sophomore, Algebra II as a junior, and was currently taking Pre-Calculus. She reflected that Geometry was more difficult for her than other math class as she is, "more of an x equals person." She had never used The Geometer's Sketchpad but had seen Geogebra in her Pre-Calculus class.

Student Ten was a junior in high school who took Geometry and Algebra II as a freshman, Pre-Calculus as a sophomore, and was current taking Advanced Placement Calculus AB and BC. He reflected that, "I enjoyed Geometry because I enjoy math, but it was the least enjoyable of all the math I've taken." He had never used The Geometer's Sketchpad but had some minimal use with Geogebra.

Student Two, Student Five, Student Eight and Student Eleven completed Task C (high). Student Two was a senior in high school who took Geometry as a freshman, Algebra II as a sophomore, Pre-Calculus as a junior, and was currently taking Advanced Placement Calculus. He reflected that he did not pay very good attention in Geometry but did alright in the class. He had not used The Geometer's Sketchpad before but had used Geogebra in Pre-Calculus.

Student Five was a senior in high school took Geometry as a freshman, Algebra II as a sophomore, Pre-Calculus as junior and was currently taking Advanced Placement Statistics. He reflected that he did not like Geometry very much and really did not get proofs. He had used The Geometer's Sketchpad in the past and had seen Geogebra as well.

Student Eight was a senior in high school who took Geometry as a freshman, Algebra II as a sophomore, Pre-Calculus as a junior, and was currently taking both Advanced Placement Calculus and Statistics. He indicated that although he did well in Geometry, he did not feel that he learned it very well. I further articulated that he felt he was missing some parts and that this became evident while taking Calculus. He had seen The Geometer's Sketchpad before but had not used it.

Student eleven was a junior in high school who had taken Geometry in the summer before their freshman year, Algebra II as a freshman, Pre-Calculus as a sophomore, and was currently taking Advanced Placement Calculus AB and BC. Concerning their experience in Geometry they reflected that was not a great experience because they did it on their own in the format of an online course. They saw The Geometer's Sketchpad used by their teacher in Pre-Calculus.

Data Collection

For the first phase of the study, data was collected to inform the validity of the Dynamic Geometry Task Analysis Framework. The first validity test took place be in the form of face validity in which experts on dynamic geometry software in mathematics education reviewed the framework and provided written feedback on its usefulness in identifying tasks that require high quality arguments and conclusions of the student user. Student generated data was then collected on their work for each of three tasks. Data gathering appointment dates were scheduled individually for each participant and took place in the month of December, 2014. Video footage of students' task work and interviews was analyzed in relation to how they perceived each task in relation to the learning goal of each task. All on screen and written work was collected and analyzed. During the data collection process, the researcher took field notes to capture any questions participants may have had and any feedback they provided. The researcher served to assist with any technological issues.

In the first of the two-part, video-recorded student interview, prior to completion of the tasks, individuals were asked the following questions.

<u>Part 1: Student Background and Geometer's Sketchpad Use Interview Questionnaire</u>

1. What grade are you in and what math classes have you taken so far in high school?
2. Tell me about your experience in Geometry. Did you enjoy the class? How well did you do? Is there a part of the class you liked best? If so, tell me about it.
3. Have you used Geometer's Sketchpad before? How often did you use it when taking Geometry or another math class?
4. Describe how you have typically used Geometer's Sketchpad to the best of your memory.

In the second of the two-part, video-recorded student interview, after completion of each task, individuals were asked the following questions:

<u>Part 2: Student Reflection Interview Questionnaire</u>

1. Please read each prompt beginning with the first. After you read each prompt, please respond to the following questions:
    a. In your own words, what is the prompt telling you to do?
    b. What did you do in Geometer's Sketchpad and/or in writing to respond to the prompt?
    c. What have you learned from responding to this prompt?
2. Describe what you started with in the sketch, what you did, and what you accomplished or learned at the end of this task.
3. What else would you like to share about your work with this task?

Data Analysis

Data analysis for Phase One of data gathering served to test the validity of the Dynamic Geometry Task Analysis Framework. The following three interview questions were asked about each task prompt the student responds to:

a. In your own words, what is the prompt telling you to do?
b. What did you do in Geometer's Sketchpad and/or in writing to respond to the prompt?
c. What have you learned from responding to this prompt?

A coding matrix based on the work of Toulmin (1958) was used to categorize student responses concerning each task prompt.

Table 9: Sample Coding Table

Task Name:	Argument Component: Claim	Argument Component: Data	Argument Component: Warrant
Prompt 1:			
Prompt 2:			
Prompt 3:			
Etc.			

Each student interview was analyzed for the task they completed. This analysis produced a collection of code summaries for each student. Each task was then assessed in light of what problem solving goals types were exhibited when completed. Furthermore, a separate list of argument components was created for each student on each task to assess differences in task quality, thus addressing overarching Research Question One directly.

Research Question Two

Research Question Two with its sub-research questions guided the second phase of this study and reads:

How do pre-service and in-service teachers use the Dynamic Geometry Task Analysis Framework?

a. What are similarities and differences between teachers' use of the framework for task writing for in-service and pre-service teachers?

b. What are similarities and differences in task quality for tasks written by in-service and pre-service teachers?

c. What are similarities and differences in task quality for tasks written by teachers who have different levels of knowledge for teaching geometry?

The second phase of this study concerned teacher use of the Dynamic Geometry Task Analysis Framework. Case studies of six teachers (three in-service and three pre-service) were conducted. Case study methodology is preferred as Creswell (2013) explained it, “is a good approach when the inquirer has clearly identifiable cases with boundaries and seeks to provide an in-depth understanding of the case or several cases” (p. 100). The cases in this phase of the study were the teachers who provided data on their professional background, their use of dynamic geometry software in general and The Geometer’s Sketchpad in particular, and the affect the Dynamic Geometry Tasks Analysis Framework had on their perception of task quality and the written tasks they produced. The cases were therefore clearly identified and bounded with data gathering that will provide an in-depth understanding of each case.

Two groups of participants constituted the sample for this portion of the study: 1) in-service teachers; and 2) pre-service teachers. Participants in both groups experienced the same treatment in the study and the same data collection procedure. The pre-service teachers had each taken the course at North Carolina State University entitled EMS 480, Teaching Mathematics with Technology. Each student in the course was required to complete a lesson-planning project (see Appendix). These pre-service teachers were all pursuing a Bachelor of Science degree in Mathematics Education for teaching Secondary Mathematics. One participant, Participant Three, was also pursuing a Bachelor’s Degree in Mathematics. The Bachelor of Science Degree in Mathematics Education for teaching Secondary Mathematics requires the student to complete at least 11 courses in mathematics and statistics and six courses directly related to mathematics education. Because of the additional major in Mathematics, Participant Three's course work included an additional three to five courses in Mathematics. To keep the task writing requirements for both in-service and pre-service teachers parallel, participants in both groups were given the same learning goal to write their task. Furthermore, data gathered from participants in both groups focused on the task writing process and product with the same directions provided to each participant.

Before the task writing session, all participants were first provided the task writing description for the study, which asked them to write a GSP task for a potential student who is assumed to know how to use the technology (e.g., measure) but who has not achieved the provided learning goal. Furthermore, each participant was give the expectation that a student completing their task was to make at least one construction and one measurement. Each participant was then provided a researcher chosen Common Core State Standards for Mathematics Geometry goal to serve as the impetus for their task writing, "justify that the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length." Having the goal preselected circumvented the situation in which a participant chooses their favorite and likely well understood goal to write a task. After the goal was assigned, they had a high school Geometry book available to them as a reference to have on hand when they completed the task writing and data gathering portion of the study. Prior to the task writing session, teachers also completed a questionnaire regarding their background and their use of Geometer's Sketchpad (see Appendix). Participating teachers also took an online test developed by Herbst and Kosko (2014) to assess their knowledge for teaching geometry. All details concerning this data and analysis are documented in Chapter 5.

The next part of the study was video-recorded and included participant task writing and interview conduction. The task writing session was done independently with the researcher available to offer minimal technological assistance as needed. The writing session involved three major parts: 1) writing a task in The Geometer's Sketchpad based on the given learning goal (see above); 2) reading an article that introduces the Dynamic Geometry Task Analysis Framework and participating in a researcher led tutorial session on the framework; and 3) making revisions to their written task after the reading and tutorial session. In Part One, participants were allowed the use of a high school Geometry text. In Part Two, participants read the article entitled, Evaluating and Writing Dynamic Geometry Tasks, written by the researcher for practicing teachers intending to assess the quality of written tasks for The Geometer's Sketchpad. After each participant completed the reading, the researcher asked them questions about their impressions of the article and framework. The researcher then did some practice coding on Task A with the participant until the researcher felt that the participant understood how they should code with the framework. Part Three required participants to make any desired revisions to their written tasks in response to what they read in the article and the tutorial session. During this part, each participant had the article and a copy of the Dynamic Geometry Task Analysis Framework available to them. These interviews were structured in nature and closely followed the recommendations structured, task-based interviews provided by Goldin (2000). Carrying out these interviews agreed with Creswell's (2013) description of qualitative research, "conducting a qualitative research study means that researchers try to get as close as possible to the participants being studied […] subjective evidence is assembled base on individual views. This is how knowledge is known-through the subjective experience of people" (p. 20). The video footage was transcribed and analyzed in light of overarching Research Question Two and its sub-parts.

In summary, the research questions lend themselves to qualitative methods as they deal with human perceptions, interpretations, and conclusions that are not easily quantifiable. Data gathered from participating students and teachers include that of questionnaires, submitted work, video footage, and interviews. This data was analyzed to assess the usability of the Dynamic Geometry Task Analysis Framework.

Participant Selection

For Phase Two of the study, two groups of teachers made up the sample. The second group was three practicing teachers who use or have used the software The Geometer's Sketchpad in their instruction. Potential participants in this group were invited via email and provided contact information of the researcher if they desired further information before deciding to participate.

The first group was three pre-service high school math teachers who had completed the course, EMS 480, Teaching Mathematics with Technology. The course description reads, "This course prepares prospective mathematics teachers to use technology in their classrooms to assist students in formulating and solving math problems in the middle and high school mathematics curricula." It was conjectured that students who took this course would be interested in writing tasks in The Geometer's Sketchpad and willing to participate in this study. Potential participants from this group were invited in person during an early class session of EMS 480. Descriptions of participants are included below.

At the time of this study, Participant One was completing course work towards a Bachelor's Degree in Mathematics Education with a Concentration in Secondary Teaching. As a junior in college, he had no formal teaching experience but did express a passion for teaching and an eagerness to begin his teaching career. Participant One had used Geometers Sketchpad and Geogebra in the technology in mathematics education course, but reported limited experience writing tasks in dynamic geometry environments having only modified a task that involved transformations of polygons. He saw dynamic geometry environments as beneficial because they allow students to see many cases in a short amount of time through the use of dragging.

Participant Two was a junior pursuing a Bachelor's Degree in Mathematics Education with a Concentration in Second Teaching who had recently completed the same course in technology in secondary mathematics education. She considered herself a proficient user of dynamic geometry environments and expressed that she has used help functions or has gone online if she ever gets stuck using the technology. She had written a lesson plan including tasks in The Geometer's Sketchpad while in a technology in secondary mathematics course, a mathematics education course for pre-service and in-service teachers to learn about incorporating various technologies into their instruction.

Like the first two pre-service teachers, Participant Three was a junior pursuing a Bachelor’s Degree in Mathematics Education with a Concentration in Secondary Teaching who had recently completed the same course in technology in secondary mathematics education. Like the Participant Two, he had written a lesson plan in the past that utilized The Geometer’s Sketchpad. He stated a benefit of dynamic geometry environments as that of being able to see concepts in real time such as a square changing shape along with the measurements of its area and lengths.

Participant Four is the first in-service teacher to be documented in this study. She has a Bachelor of Science Degree in Mathematics with a Concentration in Secondary Teaching. At the time of this study she had been teaching high school mathematics for three and a half years in the courses of Math I, Math II, Pre-Calculus and Statistics. Math I and Math II contain the equivalent of one course in high school Geometry thus insuring that Participant Four was familiar with teaching the concepts of Geometry. She had used The Geometer’s Sketchpad in her undergraduate mathematics and mathematics education courses and had incorporated it on a limited basis in her high school teaching.

Participant Five has Bachelor’s Degrees in Mathematics and Mathematics Education along with a Master’s Degree in Education. At the time of this study she was in her eleventh year of teaching high school mathematics. She had previously taught courses ranging from Pre-Algebra through Advanced Placement Statistics and was currently teaching Math I and Pre-Calculus. She explained that Math I had a component of Geometry. She had used The Geometer’s Sketchpad in a teaching with technology course she took over 10 years ago. She had used The Geometer's Sketchpad again during her Master's work but had limited use with it in her own classroom.

At the time of this study, Participant Six was in his second year of high school mathematics teaching. He has a Bachelor's of Art degree in International Studies and a Master's of Art Degree in Teaching with an emphasis in Secondary Mathematics. He had taught courses ranging from Math II to Advanced Placement Calculus. He had used dynamic geometry software in his experience earning his Master's degree in Teaching. He reported using The Geometer's Sketchpad a few times in his own instruction.

Data Collection

Participants from both groups, pre-service and in-service, had the same task writing and data gathering experience. Data gathering appointment dates were scheduled individually for each participant and took place in the months of February and March, 2015. During the data collection process, the researcher took field notes to capture any questions participants may have and any feedback they provide. The researcher also served to assist with any technological issues. In the final video-recorded interview the following questions will be asked to individual participants:

1. In your first attempt at task writing, how did you plan to meet the learning goal for the task you wrote?
2. In your first attempt at task writing, describe in detail what the student was to do.
3. In your first attempt at task writing, how did the prompts you included push the student to achieve the learning goal?

4. In your first attempt at task writing, how did you envision student use of the technology tools to contribute to their learning?
5. After you were given the Dynamic Geometry Task Analysis Framework and reading an article about it, you were asked to make any desired changes to your task. What changes did you make and why?
6. In your second attempt at task writing, describe in detail what the student was to do.
7. In your second attempt at task writing, how did the prompts you included push the student to achieve the learning goal?
8. How did your use of the Dynamic Geometry Task Analysis Framework affect your thinking about task quality and task writing?
9. Do you see the Dynamic Geometry Task Analysis Framework as a valuable tool for teachers? Why or why not?
10. What other reflections do you have on this task writing experience?

All data collected was securely stored.

Data Analysis

Data analysis in Phase Two followed case study methodology as prescribed by Creswell (2013). Three defining features of quality case studies apply here. Creswell recommends that a, "complete findings section of a case study would then involve both a description of the case and themes or issues that the researcher has uncovered" (p. 99). Each teacher in Phase Two of the study represented a case with each being thoroughly described in terms of their professional background and use of dynamic geometry software including The

Geometer's Sketchpad. After each case was described, the organization of data analysis followed the second recommendation of reporting, "similarities and differences among cases." In so doing, the organization directly matched the focus of overarching research questions one and two. Finally, the case studies and comparisons among cases were summarized in detail satisfying Creswell's recommendation to summarize overall meanings. Final cases include a description of the teacher that includes their professional background and history with DGS. This description is followed by a reporting of each task (initial and revised) along with coding per the Dynamic Geometry Task Analysis Framework of each task. Interpretations of teachers use of the framework were supported by data obtained from transcribed interviews. These complete case documentations provided the researcher a thorough set of data to address the research questions associated with Phase Two of this study.

Description of Protections

As this research will be used to write a published dissertation, a full Institutional Review Board (IRB) approval process was undertaken in order to protect all participants in the study. The purpose of the research was disclosed to each participant along with a full description of what will be asked of them in the study. Per the IRB documentation submitted, participants were treated ethically with all identifying information changed before data analysis was conducted and findings are documented. Further, transcripts of all interviews had the interviewee's name changed to a pseudonym for research reporting. All data was stored on an external hard drive that was secured in a locked cabinet when not in use. With these protections in place, participants in the study experienced no harm and their identities are absent from all published work.

Trustworthiness

Throughout the research design, data collection and documentation every effort was made to ensure the trustworthiness of the study. Creswell sites the work of Lincoln and Guba (1985) where they use ideas such as credibility transferability to foundational elements to establish a qualitative study as trustworthy (p. 246). Traditional notions of validity and reliability are subsumed in Lincoln and Guba's descriptions of trustworthiness.

Credibility

Two important components addressed the need for credibility: expertise and reliability. The researcher has a background in teaching high school geometry and has used The Geometer's Sketchpad extensively with former students. Furthermore, the researcher has spent numerous hours analyzing teacher written tasks for The Geometer's Sketchpad and video footage of classroom episodes in which the tasks were employed. The results of this experience contributed to the development of the Dynamic Geometry Task Analysis Framework. The researcher thus possesses some level of expertise concerning the objects under study. Furthermore, this study was overseen by an expert in the field of mathematics education on dynamic geometry software use. This expert monitored all data collection and analysis regarding the fit between the data and conclusions made from the data, validity. To maintain these standards of expertise and reliability the researcher and committee chairperson as an expert in the field meet regularly and debriefed on progress and enactment of the methods.

Transferability

Concerning Phase One of this study, it is assumed that other researchers in mathematics education with expertise in dynamic geometry software would come to similar conclusions concerning the face validity of the instrument. In this sense, the processes and conclusions of establishing face validity are considered transferable. However, due to the small sample size of students attempting tasks and nature of qualitative research the results may not be as readily transferable. That is, twelve different students may provide substantially different feedback. Recall that the purpose of having students complete tasks that were coded and assessed with the Dynamic Geometry Task Analysis Framework is to test whether the quality of the tasks as indicated by the framework matches the quality of work students' experience.

With this intent of assessing the impact of the Dynamic Geometry Task Analysis Framework on teacher's writing of tasks in The Geometer's Sketchpad, a sample of six teachers was purposefully formed. Because each participant has a unique professional background and knowledge, it is expected that similar results are only guaranteed if the study were replicated with the same or teachers with similar backgrounds and knowledge. While these limits on transferability exist, the findings and implications of the study hold merit for those concerned with writing high quality tasks for The Geometer's Sketchpad in providing a lens through which teachers interpret and capitalize on the usefulness of the framework.

Summary

The methods of this study align well with the qualitative nature of assessing the usability of a framework. Answering the following research questions will provide a thorough treatment of the specific purpose of the study to investigate how the Dynamic Geometry Task Analysis Framework affects what teachers focus upon and describe as task quality in Geometer's Sketchpad (GSP) tasks and how they write GSP tasks.

1. How do the mathematical prompts and technological actions as identified in the Dynamic Geometry Task Analysis Framework relate to students' mathematical arguments and conclusions?
2. How do pre-service and in-service teachers use the Dynamic Geometry Task Analysis Framework?
    a. What are similarities and differences between teachers' use of the framework for task writing for in-service and pre-service teachers?
    b. What are similarities and differences in task quality for tasks written by in-service and pre-service teachers?
    c. What are similarities and differences in task quality for tasks written by teachers who have different levels of knowledge for teaching geometry?

A helpful summary of research questions, data collected and analysis is included below.

Table 10: Research Questions and Data Sources

Research Question	Data Collected	Analytical Methods
1. How do the mathematical prompts and technological actions as identified in the Dynamic Geometry Task Analysis Framework relate to students' mathematical arguments and conclusions?	• Twelve coded GSP tasks Video-recorded and transcribed task-based interviews	• Task coding with Dynamic Geometry Task Analysis Framework • Coded interviews with core of Toulmin's Argumentation Model
2. How do pre-service and in-service teachers use the Dynamic Geometry Task Analysis Framework?	• Twelve coded GSP tasks Video-recorded and transcribed task-based interviews	• Teachers' tasks to be coded with Dynamic Geometry Task Analysis Framework • Case studies including analysis of teachers' submitted tasks supported with transcribed interviews to profile each teacher according to framework use and perceptions
2a. What are similarities and differences between teacher's use of the framework for task writing for in-service and pre-service teachers?	• Twelve coded GSP tasks Video-recorded and transcribed task-based interviews	• Teachers' tasks to be coded with Dynamic Geometry Task Analysis Framework • Case studies including analysis of teachers' submitted tasks supported with transcribed interviews to profile each teacher according to framework use and perceptions
2b. What are similarities and differences in task quality for tasks written by in-service and pre-service teachers?	• Teacher written tasks • Coded tasks with Dynamic Geometry Tasks Analysis Framework Teacher written pre-questionnaire and video-recorded interview	• Teachers' tasks to be coded with Dynamic Geometry Task Analysis Framework • Case studies including analysis of teachers' submitted tasks supported with transcribed interviews to profile each teacher according to framework use and perceptions

Table 10 Continued

Research Question	• **Data Collected**	• **Analytical Methods**
2c. What are similarities and differences in task quality for tasks written by teachers of various levels of knowledge for teaching geometry?	• Teacher written tasks • Coded tasks with Dynamic Geometry Tasks Analysis Framework Teacher written pre-questionnaire and video-recorded interview	• Teachers' tasks to be coded with Dynamic Geometry Task Analysis Framework • Case studies including analysis of teachers' submitted tasks supported with transcribed interviews to profile each teacher according to framework use and perceptions

This chapter included a delineation of the qualitative research approach to addressing the purpose and research questions of the study. In particular it provided a justification for methods used to first establish validity of the Dynamic Geometry Task Analysis Framework and then to assess its usability by conducting case studies. This chapter is followed by chapters in which research findings are reported and then situated as implications for research and practice in Mathematics Education.

CHAPTER 4: ESTABLISHING VALIDITY

As indicated in the proposed methods of this work, the study is composed of two phases: 1) a validity study of the Dynamic Geometry Task Analysis Framework followed by possible revisions; and 2) a study of how the framework affects teachers' perceptions of task quality and how they write and revise tasks. To address overarching Research Question One that concerns the validity of the framework, the researcher crafted three tasks for The Geometer's Sketchpad that rank low, medium and high based on prompt codes generated by the Dynamic Geometry Task Analysis Framework. The framework itself along with the three written tasks were assessed for face validity by three experts in the field of dynamic geometry software in mathematics education. This chapter addresses the feedback provided by three experts, herein after referred to as expert one, expert two, and expert three, followed by corresponding revisions made to the framework, the three tasks, and the ranking methods employed. Upon completion of this documentation, Phase Two of establishing face validity was undertaken involving an assessment of student work and thinking associated with the three tasks that rank low, medium, and high according the framework codes. Before addressing the feedback provided by the three experts, a synopsis of the face validity check instructions and process as completed by each expert is provided.

Phase One

The role of each expert in establishing face validity as described by Anastasi (1998) was invoked, "Face validity pertains to whether the test [Dynamic Geometry Task Analysis Framework] "looks valid" to the examinees who take it, the administrative personnel who decide on its use, and other technically untrained observers" and, "refers, not to what the test actually measures [task quality], but to what it appears superficially to measure" (p. 144). The face validity check concerns the Dynamic Geometry Tasks Analysis Framework's potential for describing task quality regarding the arguments and conclusions required of students.

To initiate the work of each expert, identical email requests were sent by the researcher. Instructions included in the email are provided below.

The specific steps of the face validity work asked of you are the following:

- Read the framework and an associated practitioner article (attached).
- Read three Geometer's Sketchpad tasks and an explanation of their rankings per the framework (attached).
- Write a short reaction to the framework and the use of the framework to rank the three tasks as low, medium, and high in quality.

Guiding questions to assist you the face validity work are:

1. What changes if any do you recommend to the framework? Why?
2. Does the framework seem useful to those concerned with writing quality dynamic geometry tasks and assessing task quality? Why?
3. Are the framework rankings of low, medium, and high task quality appropriate in regard to the quality of student argumentation you predict will likely take place on each task (low, medium, high)? (Appendix G)

In addition to these general directives, an attached document to the email entitled, “Face Validity Assessment: Instructions” was provided that expounded on the instructions found in the email, offered the researcher written practitioner article (Trocki, 2014) as found in Appendix E, and included the portion of the methods chapter that explained the ranking of each task. The “Face Validity Assessment: Instructions” document can be found in Appendix H. Each expert was to use the article, tasks, and ranking explanations to respond to the three questions found in the email request. Each expert who responded used the instructions and materials provided to provide written feedback on each of the three questions.

Summary of Each Expert’s Feedback and Review Processes

Three experts provided feedback on the framework, task coding, and ranking methods. Expert one is a graduate student pursuing a doctoral degree in mathematics education at a state university. She has an extensive background in utilizing technology in secondary mathematics classrooms and a deep understanding and interest in utilizing dynamic geometry environments. Expert two is an associate professor in mathematics education at a state university and specializes in teaching mathematics with technology and best practices for mathematical discourse. Expert three is an assistant professor in mathematics education with self-described research interests of, “examin[ing] how tools, both technological and non-technological, mediate students understanding of mathematics,” while investigating the specific question of, “how do tools influence the content and structure of students’ arguments?” These experts provided invaluable feedback for this study.

Expert one found much potential in the utilization of the framework and tasks and initially focused on the coding of technological actions suggesting that technological action be coded before mathematical depth. Expert one analyzed the tasks as found in the practitioner article. Although this expert did not report their analysis of coding of the three tasks to potentially be used in this study, the researcher found this feedback helpful as the coding of two tasks as found in the practitioner article accurately reflects the initial coding of the three tasks employed in this study. This expert questioned the accuracy of some codes relating to technological action and mathematical depth. For instance, she reported that in Prompt 7 of Task 2 the student is asked to construct diagonals before dragging while the technological actions for that prompt only indicate code D, "Prompt requires dragging or use of other dynamic aspects of the sketch." The other major suggestion provided by expert one was that of accounting for student conjecturing in the codes for mathematical depth. This feedback is incorporated in the synthesis of additional feedback provided by experts two and three.

Expert two did not recommend any changes to the framework itself but rather focused on how the types of technological action, "helps to coordinate mathematical content." They expounded,

> I like how the mathematical depth levels are hierarchical, and the progression relates well to level of cognitive demand. I realize the technological actions are not hierarchical in nature and some overlap may occur. I would make sure to be explicit about that in your write up of the framework.

They followed this reflection with suggesting a hierarchical coding of dragging. For instance, dragging for the sake of dragging is lower in demand than dragging to consider invariant relationships. This consideration is taken into account in the *Changes to Task Ranking* portion of this chapter. Regarding question three, “Are the framework rankings of low, medium, and high task quality appropriate in regard to the quality of student argumentation you predict will likely take place on each task (low, medium, high)?” expert two inquired about the potential of code three, “Prompt requires student to consider the mathematical concepts, processes, or relationships in the current sketch,” paired with technological codes D that focuses on dragging and/or F that focuses on recognition of invariant relationships and how this pairing may influence one’s coding of a task as low, medium, or high. They suggested that such a combination of codes (depth of 3 with technological actions of D/F) may represent high coordination of depth and technological action. Finally, expert two agreed with the ranking of each tasks according to the percentage of prompts that coordinate technological actions with mathematical depth, but was unclear on how the percentages that define low, medium, and high quality come about according to the framework. They admitted, “I did need to read this several times to interpret (hopefully correctly) that the percent refers to MD [mathematical depth] not percent coordination with tech actions.” Although expert three was mistaken on how tasks are ranked as low, medium, and high according codes produced with the framework, their feedback in this regard has caused the researcher to further clarify how the assignment of task quality ranking takes place in the study. This clarification is provided in the *Changes to Task Ranking* subsection below.

Before providing feedback on the three face validity questions as found in the Face Validity Request Email (Appendix G), expert three sought written clarification on some components of the study. Pertinent questions he asked are provided below:

1. How do you define argumentation?
2. What constitutes low, medium, and high levels of argumentation?
3. How does one judge the quality of a mathematical argument?
4. [Are] [a]ll components explicit?
5. [Is] [t]he claim mathematically valid?

These questions were responded to by the researcher in writing, and then expert three provided his feedback.

Concerning item one, "What changes if any do you recommend to the framework? Why?" the expert offered five points. First, he reported that technological actions E, "Prompt requires creation/consideration of multiple examples from which one can generalize," and F, "Prompt requires manipulation of the sketch that allows for recognition of emergent invariant relationship(s) or pattern(s) among or within geometrical object(s)," seem to be the same action. He cited the recent National Council of Teachers of Mathematics publication, Developing Essential Understandings, to explain that any geometry theorem is an invariance (Ellis, Bieda, & Knuth, 2012). The second point this expert offered comes in the form of a question, "Is the appearance of the diagram on the screen an affordance of the technology?" He correspondingly suggested the technological affordance of looking at the diagram on the screen. For this purposes of this study, looking at the screen is assumed to take place in any

DGS task and is therefore not considered necessary to include as a technological affordance. Third, they drew into question the binary nature of coding for mathematical fidelity as a sketch that limits students' explorations may be considered lacking mathematical fidelity. Fourth, this expert drew into question the utility of assessing task quality at the prompt level as doing so may give a potentially faulty impression of overall task quality. This critique is similar to that offered by expert two in coding a task based on percentages of tasks that coordinate mathematical depth and technological actions. It is addressed in the *Changes to Task Ranking Methods* subsection of this chapter.

Finally, expert three considered scaffolds provided in tasks and explained, "I've become a big fan of having my students [pre-service teachers] remove the scaffolding from tasks in order to have [their future] students develop authentic conjectures (Dan Meyer's TED Talk video is fantastic)." Expert three continued with explaining how he would amend the three parallelogram tasks to accomplish such a change. Expert three concluded their fifth point, "As a teacher, I could always provide struggling students a prompt to examine particular relationships as I am monitoring their work. At this time [in providing feedback], I really don't have a good way to account for this type of task." Expert three's admission reflects the complexity of predicting the quality of student thinking and argumentation on a particular task as such predictions cannot account for potential human interactions among students and teachers. However, it is the opinion of the researcher that the prompts associated with a Geometer's Sketchpad Task may influence the quality of student thinking, argumentation, and understanding. Knowledge on this potential influence of tasks will be gained in Phase Two of the validity portion of this study where student work is analyzed. Therefore, an emerging theme in this study is that of unpacking some of the complexities of how a task influences individual student argumentation and thinking in conducting a close assessment of prompt and task quality.

Concerning question two, “Does the framework seem useful to those concerned with writing quality dynamic geometry tasks and assessing task quality?” expert three provides,

> The framework provides a means to assess the prompts associated with a dynamic sketch and how the students would potentially use the features of the dynamic environment in their investigations. It allows one to coordinate the possible students’ actions with the technology with the level of depth of mathematical reasoning. I really like it because it can show how a task is lacking.

Based, on this quote it seems the framework has potential to influence teachers’ perceptions of task quality and their writing of tasks for dynamic geometry software environments. This potential is explored in the Teachers’ *Utilization of Framework* chapter of this work.

Expert 3 responded favorably to Prompt Three, “Are the framework rankings of low, medium, and high task quality appropriate in regard to the quality of student argumentation you predict will likely take place on each task (low, medium, high)?” and made some important suggestions. Based on their extensive background with Toulmin’s (1958) model of argumentation and coding student argumentation, they suggest that the coding used on student work be limited to Data, Claim, and Warrant, the core of Toulmin’s model. Particularly, he recommends that warrants be coded as implicit or explicit according to

Toulmin's model. This expert then provided their predictions of student work on prompts found in each parallelogram task as implicit or explicit with a brief justification of each prediction. Their feedback further pointed out the difficulty in distinguishing task quality between task B and C. This feedback has prompted changes in how tasks are ranked low, medium, and high according to their associated codes. Expert three's detailed feedback was synthesized with the feedback of experts one and two as a basis for the following revisions made to the framework, task coding, and task ranking methods.

Changes to Framework

Recall that each expert responded to question one, "What changes if any do you recommend to the framework? Why?" Based on the feedback provided by three experts, one major revision is in order. A lack of distinction between technological action code, E, "Prompt requires creation/consideration of multiple examples from which one can generalize" and F, "Prompt requires a manipulation of the sketch that allows for recognition of emergent invariant relationship(s) or pattern(s) among or within geometrical object(s)" was pointed out. Therefore, code E was dropped from the framework. The revised framework is provided below.

Table 11: Dynamic Geometry Task Analysis Framework

Allowance for Mathematical Depth	
Levels	**Hierarchical Levels and Descriptions**
N/A	Prompt requires a technology task with no focus on mathematics.
0	Sketch does not have mathematical fidelity required to respond to prompt.
1	Prompt requires student to recall a math fact, rule, formula, or definition.
2	Prompt requires student to report information from the construction. The student is not expected to provide an explanation.
3	Prompt requires student to consider the mathematical concepts, processes, or relationships in the current sketch.
4	Prompt requires student to explain the mathematical concepts, processes, or relationships in the current sketch.
5	Prompt requires student to go beyond the current construction and generalize mathematical concepts, processes, or relationships.
Types of Technological Action	
Affordances	**Descriptions**
N/A	Prompt requires no drawing, construction, measurement, or manipulation of current sketch.
A	Prompt requires drawing within current sketch.
B	Prompt requires measurement within current sketch.
C	Prompt requires construction within current sketch.
D	Prompt requires dragging or use of other dynamic aspects of the sketch.
E	Prompt requires a manipulation of the sketch that allows for recognition of emergent invariant relationship(s) or pattern(s) among or within geometrical object(s).
F	Prompt requires manipulation of the sketch that may surprise one exploring the relationships represented or cause one to refine thinking based on themes within the surprise (adapted from Sinclair (2003), p. 312).

Changes to Task Coding

Concerning task coding, all experts responded to the following prompt, "Does the framework seem useful to those concerned with writing quality dynamic geometry tasks and assessing task quality? Why?" All experts responded favorably in concluding that the framework is beneficial to those concerned with task quality and all agreed that that it would assist practitioners to discriminate task quality. No significant changes were recommended to the coding at the prompt level other than expert one pointing out that a code C, "Prompt requires construction within current sketch" was missing from the coding on one task in the practitioner article. All codes in the tasks A, B, and C used in this study were double checked for completeness and accuracy. Concerns with how the prompt coding relates to overall task quality labels were provided and are addressed in the subsection below.

Changes to Task-Ranking Methods

Experts responded to the final prompt, "Are the framework rankings of low, medium, and high task quality appropriate in regard to the quality of student argumentation you predict will likely take place on each task (low, medium, high)?" Experts two and three raised a similar concern about ambiguities in ranking tasks according to codes associated with task prompts. Expert three is extensively quoted as their thoughts articulate the issue at hand:

> To me, the quality of a task isn't dependent on the percentage of prompts that ask students to explain, justify, and generalize. The quality of the task is based, in part, on whether the task requires them to do these things for all of their conjectures/claims. I

> could write the Parallelogram Task with 20 prompts and only 3 of the prompts would receive a code of 4 and 5 and none of those prompts would ask the students to coordinate their explanations with a technological action. Using this framework, the task would be rated as low-level. Yet, the task would ask students to reach the same conclusions and provide similar explanations as the tasks rated at a higher level. And, the task would still ask students to use the technology in a similar manner. Perhaps the framework would show how my task could be improved, which would be very beneficial. But I still think this is an issue that you need to address.

Like expert two, this expert confused the ranking of tasks as being solely based on the percentage of total prompts that include mathematical depth when in fact the coding is based on the percentage of prompts containing mathematical depth that also coordinate with technological action. The formula for attaining such a percentage would read [(# of prompts that coordinate mathematical depth with technological action)/(# of prompts that contain mathematical depth)]*100%. However, the general concern expressed by these two experts still holds in that one task may lump mathematical depth in one or two prompts while another task spreads mathematical depth across eight or nine prompts. In this situation, the comparison of the tasks under the proposed ranking scheme is susceptible to being unfairly labeled.

In deference to this concern, task ranking methods as based on prompt coding have been amended to ameliorate the difficulties associated with labeling task quality according to prompt percentages. The revised ranking scheme reflects the underlying subjective nature of labeling task quality. After holistically considering all prompt codes associated with a given task, ranking according to the following descriptions is to be completed:

- *Low:* Task does not contain a collection of prompts that coordinate mathematical depth and technological actions in such a way to require the student to make generalized conclusions based on emergent invariant relationships that go beyond a static sketch.
- *Medium:* Task contains a collection of prompts that coordinate mathematical depth and technological actions in such a way that may push to but does not necessitate that the student to make generalized conclusions based on emergent invariant relationships that go beyond a static sketch.
- *High:* Task contains a collection of prompts that coordinate mathematical depth and technological actions in such a way that requires the student to make generalized conclusions based on emergent invariant relationships that go beyond a static sketch.

This revised ranking scheme does away with a dependence on percentages of prompts and brings into the fore the framework user's expertise in judging the quality of a dynamic geometry task. In doing so, it reflects changes based on concerns expressed in experts' feedback.

Conclusion of Phase One

Phase one of the face validity portion of this study concluded with making one significant change to the framework in removing redundancy found in Types of Technological Action codes E and F. The revised framework reflects the change of removing the descriptor for code E and replacing it with the descriptor for code F. Doing so affects the coding of the three tasks in changing all pairs of E and F codes with the code of E. In the pre-revised framework task coding, codes E and F always appeared together. Another major change resulted in this phase with removing the percentage of prompts containing mathematical depth paired with technological action as the measure by which a task is ranked low, medium, or high. The new ranking methods are more qualitative in nature with details provided in the previous subsection of this chapter. An additional minor change to the study design is to use only the core of Toulmin's argumentation model (data, claim and warrant) rather than the full model and to track warrants as explicit or implicit with explicit reflecting stronger argumentation. With these changes in place, Phase Two of establishing face validity was undertaken.

Phase Two

The overarching research question that guided the validity portion and Phase Two is provided below.

> How do the mathematical prompts and technological actions as identified in the Dynamic Geometry Task Analysis Framework relate to students' mathematical arguments and conclusions?

Each of the three employed tasks were written to achieve two goals concerning parallelograms: 1) justify opposite angles of parallelograms are congruent; and 2) justify that diagonals of parallelograms bisect each other. It is conjectured that the quality of student argumentation relating to these goals will reflect the quality of each task as indicated by codes produced with the dynamic geometry task analysis framework. If this conjecture holds true, it will provide evidence of the framework's validity in discriminating types of arguments students produce.

Coding of student responses is based Toulmin's (1958) definitions. Data is defined as the facts or evidence used to prove the argument; claim as the statement to be argued; and warrant as the general logical statements that serve as bridges between the claim and the data. A warrant is one in which the claim is directly supported by data in a written statement. Other mathematics educations researchers have utilized Toulmin's argumentation framework. For instance, Hollebrands, Conner and Smith (2010, p.327) developed the following diagram based on Toulmins (1958) model and used it to analyze argumentation in technology environments.

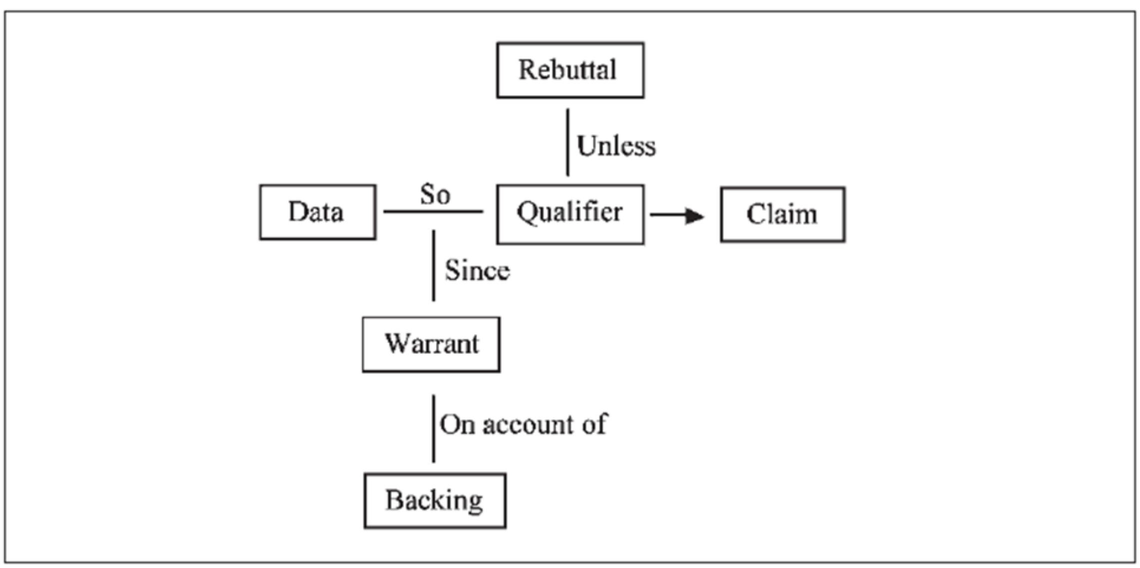


Figure 7: Toulmin's Argumentation Model Graphic

Three elements of their diagram that constituted the core of Toulmin's argumentation model were employed in this study: Claim, Data, and Warrant. Each student's written response was coded using Toulmin's core model as seen in the sample coding table below. Each response was dissected according to the argument component it contained. For example, if a student provided a claim, then the portion of their written response representing the claim was quoted under the "Claim" column for that particular prompt. If a student included data in their response, then it was coded appropriately in providing the language of the response or the referent (e.g., diagram on screen). A warrant occurred if the student provided written evidence of logically connecting their claim to the data.

Table 12: Sample Coding Table

Task Name:	**Argument Component: Claim**	**Argument Component: Data**	**Argument Component: Warrant**
Prompt 1:			
Prompt 2:			
Prompt 3:			
Etc.			

It should be noted that a potential extraneous variable in all of this work is that of students' individual work expectations and background knowledge. For example, students may have a background in mathematics that expected them to always justify their conjectures while other students may not. To ameliorate this concern, four students were randomly assigned to work through one of the tasks (low, medium, and high). Refer to the methods chapter for further details.

After this revised coding scheme was established, the researcher carried out student interviews. As found in the methods chapter, students responded to interview questions based on the following questionnaire (see Appendix B).

1. Please read each prompt beginning with the first. After you read each prompt, please respond to the following questions:
   a. In your own words, what is the prompt telling you to do?
   b. What did you do in Geometer's Sketchpad and/or in writing to respond to the prompt?
   c. What have you learned from responding to this prompt?
2. Describe what you started with in the sketch, what you did, and what you accomplished or learned at the end of the task.
3. What else would you like to share about your work with this task?

The questions found under number one were asked after the student responded in writing to each prompt. Students responded to items two and three after they finished their work on all task prompts. These interviews were transcribed and then student written responses were coded. The remainder of the chapter is organized by task and then by student work on each task. A summary coding table is provided for each student's work. Supporting evidence in summarizing the quality of student argumentation is incorporated from the transcripts as necessary. After student argumentation is presented for each of the three tasks, a subsection comparing tasks based on arguments manifest on each task is provided followed by a conclusion that revisits the related research questions.

Task A

Recall the description of a low-level task; task does not contain a collection of prompts that coordinate mathematical depth and technological actions in such a way to require the student to make generalized conclusions based on emergent invariant relationships that go beyond a static sketch. The codes for this task reflect that mathematical depth is not coordinated with technological actions. It is conjectured that student argumentation on this task reflects the low quality of the task as indicated by the codes. The following subsection summarizes each student's argumentation on Task A (low).

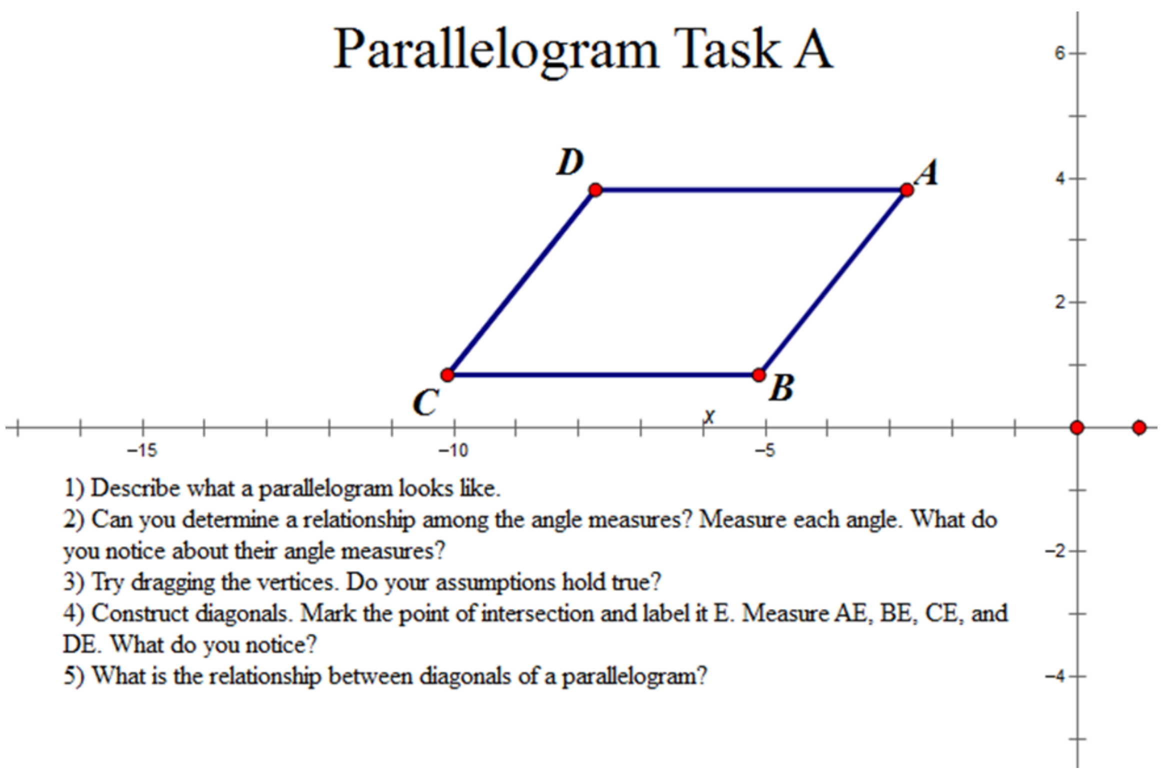


Figure 8: Low-Level GSP Task

Table 13: Task A Coding Summary

Prompt Number	Parallelogram Task A Codes
1	(2, N/A)
2	(1, 2, 3, B)
3	(2, D)
4	(1, 2, 3, B, C)
5	(3, N/A)

Student Argumentation on Task A (Low)

Student Three, Student Six, Student Nine, and Student Twelve completed Task A (low). Student Three was a senior in high school who had taken Geometry as a freshman, Algebra II as a sophomore, Pre-Calculus as a junior, and was currently taking Advanced Placement Statistics. She described their experience in Geometry as enjoyable because she liked working with shapes and appreciated how her teacher conducted class. She was familiar with the dynamic geometry program of Geogebra, but had not used Geometer's Sketchpad before. This student's responses are documented in the table below.

Table 14: Student Three Argumentation

Task Name: Task A	Argument Component: Claim	Argument Component: Data	Argument Component: Warrant
Prompt 1: Describe what a parallelogram looks like.	It's sort of like a skewed diamond or a square with two obtuse angles and two acute angles. It's a square leaning towards something.	Diagram on screen	

Table 14 Continued

Task Name: Task A	Argument Component: Claim	Argument Component: Data	Argument Component: Warrant
2: Can you determine a relationship among the angle measures? Measure each angle. What do you notice about their angle measures?	Angles ∠DCB and ∠DAB are equivalent as are angles ∠CDA and ∠ABC.	Angles	
3: Try dragging the vertices. Do your assumptions hold true?	Yes		
4: Construct diagonals. Mark the point of intersection and label it E. Measure AE, BE, CE, and DE. What do you notice?	Both lines going from obtuse angle to obtuse angle had equal distances as did both lines going from acute angle to acute angle.	Obtuse, acute, distances	
5: What is the relationship between diagonals of a parallelogram?	Angles in a parallelogram that are diagonally across from each other are equal.		

After reading Prompt 3, this student immediately asked, “Which assumptions are they talking about?” The ambiguity inherent in this prompt was further reflected in the students response to the interviewer follow up question on this prompt, “What have you learned from responding to this prompt?” as the student responded, “I guess what happens visually when you move a line of the parallelogram.” Notice that the student did not connect their response to Prompt 3 with claim they provided in response to Prompt 2, thus reflecting a weakness in this task.

Student Three responded to the overall task follow up interview prompt, “Describe what you started with in the sketch, what you did, and what you accomplished or learned at the end of the task.” She said,

> I looked at the parallelogram and then described it and then I measured all the different angles and looked at how they related to each other and how diagonals are equal. And then I put it on the vertices and started messing up the parallelogram and fixing it and made some diagonals connecting the point. Then I measured the length of the segment and made conclusions about the length of the diagonals and the angles.

The student makes a curious response in stating, "[I] started messing up the parallelogram and fixing it." Although she makes correct conclusions that opposite angles are equal in measure and that the two constructed segments of each diagonal are equal, there is no indication that the student has generalized these relationships to all parallelograms. Hence, this student did not meet the learning goals as provided above concerning parallelograms: 1) justify opposite angles of parallelograms are congruent; and 2) justify that diagonals of parallelograms bisect each other.

Student Six was a senior in high school who had taken Algebra I as a freshman, Geometry as a sophomore, Algebra II as a junior, and was currently taking Pre-Calculus. The student reflected that they did not particularly enjoy Geometry but rather liked, "more straightforward stuff" and preferred Algebra over Geometry. Student Six had used The Geometer's Sketchpad before in her Geometry class and was currently using Geogebra in their Pre-Calculus class. Their work is documented in the table below.

Table 15: Student Six Argumentation

Task Name: Task A	Argument Component: Claim	Argument Component: Data	Argument Component: Warrant
Prompt 1: Describe what a parallelogram looks like.	Has 4 equal sides like a diamond and is symmetrical.	Diagram on screen	
2: Can you determine a relationship among the angle measures? Measure each angle. What do you notice about their angle measures?	Yes. Two of the measures are the same and the others are as well. Symmetrical.	Measures	
3: Try dragging the vertices. Do your assumptions hold true?	Yes, angles A and C are the same and angles D and B are the same. The sides remain parallel.	Diagram on screen	
4: Construct diagonals. Mark the point of intersection and label it E. Measure AE, BE, CE, and DE. What do you notice?	The distances of EA and CE are the same and the distances of BE and DE are the same.	Distances	
5: What is the relationship between diagonals of a parallelogram?	The diagonals go through the center of a parallelogram and two of the sides or distances will always be equal	Diagram on screen Distances	because they will remain parallel

Student Six responded to the follow up question to Prompt Three, “What have you learned from responding to this prompt?” with saying, “the sides remain parallel.” While this student correctly concluded that angles A and C remain equal in measure and angles B and D do so as well, they did include this conclusion in response to what they have learned but rather stated that the sides remain parallel. This lack of coordination in student responses leaves one to wonder if the student truly understands that opposite angles remain equal (invariance) for any parallelogram (under dragging). At the end of the task when asked what was learned or accomplished, the student simply said that the sides always remain parallel. It can be surmised that the student focused on the definition of a parallelogram when providing their warrants without connecting their claims to invariant relationships maintained under dragging. Student Six did not meet the task learning goals as their justifications lacked quality. The student’s responses reflect the low quality of this task.

Student Nine was a junior in high school who took Geometry as a freshman, Algebra II as a sophomore, and was currently taking Pre-Calculus. She reported that she did not enjoy Geometry but did like finding missing sides of triangles. She thought she had seen the program Geometer’s Sketchpad before but could not remember when. Her responses are included below.

Table 16: Student Nine Argumentation

Task Name: Task A	Argument Component: Claim	Argument Component: Data	Argument Component: Warrant
Prompt 1: Describe what a parallelogram looks like.	A slanted rectangle	Diagram on screen	
2: Can you determine a relationship among the angle measures? Measure each angle. What do you notice about their angle measures?	The angles opposite each other are equal.	Angles	
3: Try dragging the vertices. Do your assumptions hold true?	Yes, my assumptions hold true, the angles remain equal	Angles	Even, with major shifting of the shape
4: Construct diagonals. Mark the point of intersection and label it E. Measure AE, BE, CE, and DE. What do you notice?	The point of intersection, *E*, bisects each line.	Diagram on screen	
5: What is the relationship between diagonals of a parallelogram?	Except for intersecting, the diagonals do not have a corresponding measurement, they are different lengths.	Measurement	

Student Nine provided an explicit warrant for the claim that opposite angles in parallelograms are equal. This reflected her work in looking at invariance under dragging thus providing evidence that she met one of the two goals associated with this task, use invariance under dragging to justify that opposite angles are congruent. Regarding goal two, use invariance under dragging to justify that diagonals bisect each other, Student Nine gave evidence of understanding that diagonals bisect each other in a particular instance (see response four), but did not give evidence of recognizing invariance under dragging to conclude that relationship holds for all parallelograms (see response five). Achieving only one of the two task learning goals reflects weaknesses in Task A.

Student Twelve was a junior in high school and took Geometry as a freshman, Algebra II in the summer before her sophomore year, Pre-Calculus as a sophomore, and was currently taking AP Calculus AB and BC in her junior year. She reported that she really liked Geometry and did very well in the course. She had used The Geometer's Sketchpad quite often in her Pre-Calculus class. Her responses are documented in the table below.

Table 17: Student Twelve Argumentation

Task Name: Task A	**Argument Component: Claim**	**Argument Component: Data**	**Argument Component: Warrant**
Prompt 1: Describe what a parallelogram looks like.	A quadrilateral, four sides, two obtuse angles and two acute angles, like a slanted rectangle	Diagram on screen	
2: Can you determine a relationship among the angle measures? Measure each angle. What do you notice about their angle measures?	Angle D and angle B seems to be equal, as well as angle C and A. D and B are obtuse, while A and C are acute.	Acute Obtuse Diagram on screen	I notice that I was correct with my earlier statement. D and B are equal, and A and C are equal.
3: Try dragging the vertices. Do your assumptions hold true?	Yes, my assumption that the opposite angles are equal holds true, but my statement about the types of angles changes,	Angles	when I drag the vertices

Table 17 Continued

Task Name: Task A	Argument Component: Claim	Argument Component: Data	Argument Component: Warrant
4: Construct diagonals. Mark the point of intersection and label it E. Measure AE, BE, CE, and DE. What do you notice?	I notice that AE and CE are equal, as well as BE and DE.	Diagram on screen	
5: What is the relationship between diagonals of a parallelogram?	Where the diagonals meet will always divide the distance of the diagonals equally in half. Their actual lengths have no relationship;	Distance	It just depends on where you drag it.

Student Twelve was a highly motivated student who excelled in mathematics. She provided evidence of achieving both learning goals associated with the task: 1) justify opposite angles of parallelograms are congruent; and 2) justify that diagonals of parallelograms bisect each other. It is reasonable to speculate that her extensive background using The Geometer's Sketchpad along with her accelerated track in taking high school math classes influenced the quality of the arguments she provided on this task. She was the only student of the four that completed Task A that achieved both learning goals.

Task B

Recall the description of a medium-level task; task contains a collection of prompts that coordinate mathematical depth and technological actions in such a way that may push to but does not necessitate that the student to make generalized conclusions based on emergent invariant relationships that go beyond a static sketch. The codes for this task reflect that mathematical depth is at times coordinated with technological actions. It is conjectured that student argumentation on this task reflects the medium quality of the task as indicated by the codes. The following subsection summarizes each student's argumentation on Task B (medium).

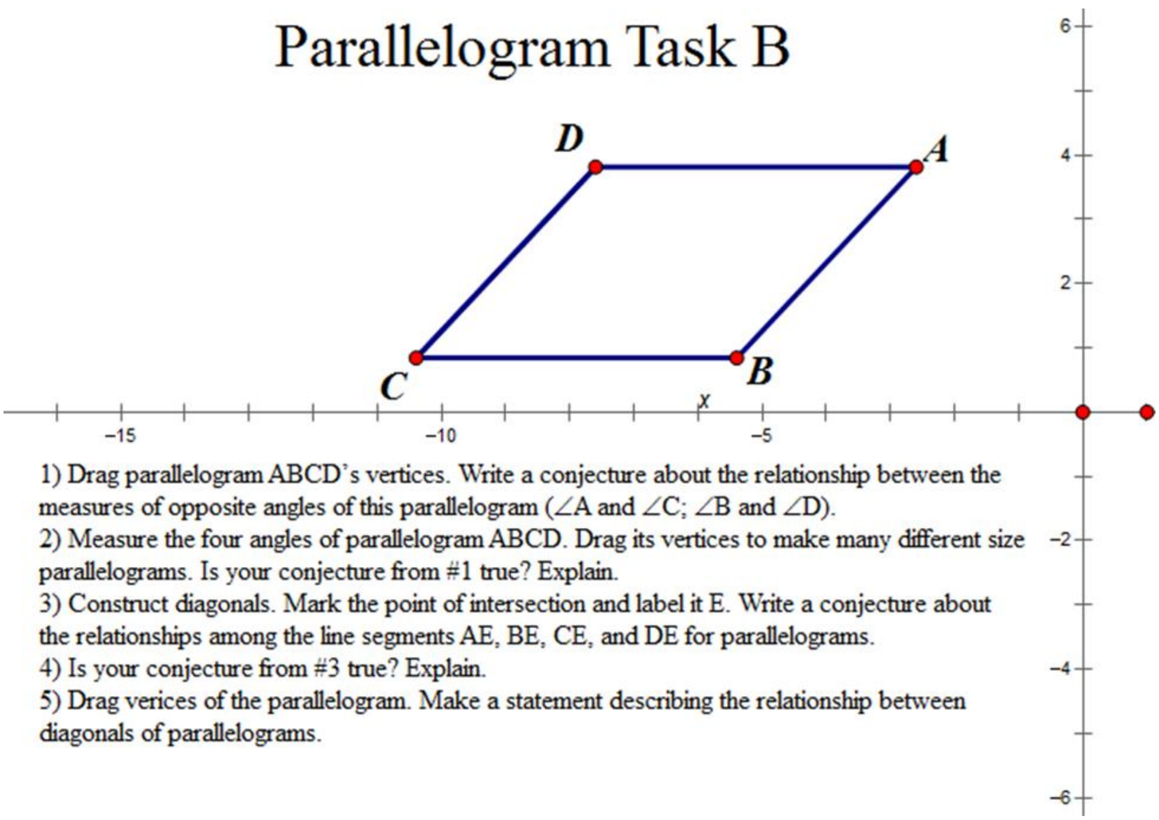


Figure 9: Medium-Level GSP Task

Task B received the following codes as summarized below.

Table 18: Task B Coding Summary

Prompt Number	Parallelogram Task B Codes
1	(1, 2, 3, D)
2	(3, 4, 5, B, D, E)
3	(1, 2, 3, C)
4	(3, 4)
5	(3, 5, D)

Student Argumentation on Task B (Medium)

Student One, Student Four, Student Seven, and Student Ten completed Task B (medium). Student One was a senior in high school who had taken Geometry as a freshman, Algebra II as a sophomore, Pre-Calculus as a junior and was currently taking both Advanced Placement Calculus and Statistics. He responded that he liked taking Geometry and did very well. He had never used The Geometer's Sketchpad but had seen Geogebra used in his Calculus class.

Table 19: Student One Argumentation

Task Name: Task B (medium)	**Argument Component: Claim**	**Argument Component: Data**	**Argument Component: Warrant**
Prompt 1: Drag parallelogram ABCD's vertices. Write a conjecture about the relationship between the measures of opposite angles of this parallelogram (∠A and ∠C; ∠B and ∠D).	Opposite angles of this parallelogram will be equal to one another.	Diagram on screen	

Table 19 Continued

Task Name: Task B (medium)	Argument Component: Claim	Argument Component: Data	Argument Component: Warrant
2. Measure the four angles of parallelogram ABCD. Drag its vertices to make many different size parallelograms. Is your conjecture from #1 true? Explain.	My answer to number one has proven true	Degrees	because angle D and angle B both have the same angle degree, 131.64 degrees.
3. Construct diagonals. Mark the point of intersection and label it E. Write a conjecture about the relationships among the line segments AE, BE, CE, and DE for parallelograms.	Line segment AE is equal to line segment CE while line segment BE is equal to line segment DE.	Diagram on screen	
4. Is your conjecture from #3 true? Explain.	The answer given for number three is true	Measured	because when measured, line segment AE does equal line segment CE and line segment BE does equal line segment DE.
5. Drag vertices of the parallelogram. Make a statement describing the relationship between diagonals of parallelograms.	The diagonals always split each other right in the middle. It is always the same distance on both side of E.	Distance	Always

Student One achieved both learning goals for this task: 1) justify opposite angles of parallelograms are congruent; and 2) justify that diagonals of parallelograms bisect each other. However, there was some explanation to be desired concerning the warrants the student provided. In response to Prompt 2, this student reported their warrant based on a static diagram, but did in fact use dragging before making their response. Thus, they provided no written evidence that invariance under dragging, but did provide verbal evidence. In response to Prompt 4, he measured without being told to do so and then dragged vertices of the parallelogram. However, like in his response to Prompt 2, he did not mention invariance under dragging in his written response. In response to an overall task follow up interviewer prompt this student concluded, “I started with a plain old parallelogram. It had no lines in it and no angles measures. What I did was put diagonals across and I found the distance between them and found the degrees for each of the angles. I learned that no matter what the opposite angles will equal each other and the line segment drawn across from one angle to another [diagonal] will be split right down the middle.” This student provided adequate evidence of achieving the two learning goals associated with the task and provided evidence of the task quality.

Student Four was a high school senior who had taken Algebra I as a freshman, Geometry as a sophomore, Algebra II as a junior, and was currently taking Advanced Functions and Modeling as a senior. She reflected that she liked Geometry better than Algebra, but that is was still challenging for her. She had never used The Geometer’s Sketchpad before, but had used Geogebra. Her written responses are coded in the table below.

Table 20: Student Four Argumentation

Task Name: Task B (medium)	Argument Component: Claim	Argument Component: Data	Argument Component: Warrant
Prompt 1: Drag parallelogram ABCD's vertices. Write a conjecture about the relationship between the measures of opposite angles of this parallelogram (∠A and ∠C; ∠B and ∠D).	m∠A =m ∠C; m∠D = m∠B	Diagram on screen	
2. Measure the four angles of parallelogram ABCD. Drag its vertices to make many different size parallelograms. Is your conjecture from #1 true? Explain.	Yes, the angle measures adjust and stay true to the above statement	Measures	if you drag the parallelogram,
3. Construct diagonals. Mark the point of intersection and label it E. Write a conjecture about the relationships among the line segments AE, BE, CE, and DE for parallelograms.	mAE=mCE; mDE= mEB.	Diagram on screen	
4. Is your conjecture from #3 true? Explain.	Yes, the distances are true to the above statement.	Distances	Tests claim using "distances" *student does not use dragging
5. Drag vertices of the parallelogram. Make a statement describing the relationship between diagonals of parallelograms.	The length of the diagonals do not always equal each other.	Length	Always

Student Four provided evidence of achieving the first learning goal; use invariance under dragging to justify that opposite angles are congruent. In their response to Prompt 2, it is apparent that they recognize the invariance of opposite angle measures under dragging. She came close to achieving learning goal two; use invariance under dragging to justify that diagonals bisect each other. She did conclude that a diagonal's lines segments on either side of the point of intersection are equal but her warrant was based on measurements of a static diagram. Therefore, learning goal two was not achieved. Student Four's argumentation reflects the medium-level quality of Task B.

Student Seven was a senior in high school who had taken Algebra I as a freshman, Geometry as a sophomore, Algebra II as a Junior, and was currently taking Pre-Calculus. She reflected that Geometry was more difficult for her than other math class as she is, "more of an x equals person." She had never used The Geometer's Sketchpad but had seen Geogebra in her Pre-Calculus class. Her work in summarized in the table below.

Table 21: Student Seven Argumentation

Task Name: Task B (medium)	**Argument Component: Claim**	**Argument Component: Data**	**Argument Component: Warrant**
Prompt 1: Drag parallelogram ABCD's vertices. Write a conjecture about the relationship between the measures of opposite angles of this parallelogram (∠A and ∠C; ∠B and ∠D).	The opposite angles are - they seem to be the same measurement.	Measurement	

Table 21 Continued

Task Name: Task B (medium)	**Argument Component: Claim**	**Argument Component: Data**	**Argument Component: Warrant**
2. Measure the four angles of parallelogram ABCD. Drag its vertices to make many different size parallelograms. Is your conjecture from #1 true? Explain.	My conjecture from number one was true,	Angles	The opposite angles do stay the same.
3. Construct diagonals. Mark the point of intersection and label it E. Write a conjecture about the relationships among the line segments AE, BE, CE, and DE for parallelograms.	mAE is equal to mCE. mBE is equal to mDE.	Diagram on screen	
4. Is your conjecture from #3 true? Explain.	My conjecture from #3 was correct.	Measurement (per field notes)	AE and CE and BE and DE are the same.
5. Drag vertices of the parallelogram. Make a statement describing the relationship between diagonals of parallelograms.	The distances stayed equal to each other.	Distances	Even with moving the vertices of the parallelogram,

Student Seven provided evidence of achieving both learning goals associated with this task: 1) justify opposite angles of parallelograms are congruent; and 2) justify that diagonals of parallelograms bisect each other. Regarding both goals she measured, dragged and linked both actions to support her two claims. Although she did not explicitly state that diagonals of parallelograms bisect each other, she did accurately conclude that the line segments on a diagonal on either side of the point of intersection always remain equal. Her responses to the prompts reflect the quality of Task B.

Student Ten was a junior in high school who took Geometry and Algebra II as a freshman, Pre-Calculus as a sophomore, and was current taking Advanced Placement Calculus AB and BC. He reflected that, "I enjoyed Geometry because I enjoy math, but it was the least enjoyable of all the math I've taken." He had never used The Geometer's Sketchpad but had some minimal use with Geogebra. His work is summarized below.

Table 22: Student Ten Argumentation

Task Name: Task B (medium)	**Argument Component: Claim**	**Argument Component: Data**	**Argument Component: Warrant**
Prompt 1: Drag parallelogram ABCD's vertices. Write a conjecture about the relationship between the measures of opposite angles of this parallelogram (∠A and ∠C; ∠B and ∠D).	As Angle A moves, Angle C moves accordingly to ensure that Angle A = Angle C and vice versa. The same is true for Angles B and D.	Diagram on screen	
2. Measure the four angles of parallelogram ABCD. Drag its vertices to make many different size parallelograms. Is your conjecture from #1 true? Explain.	My conjecture from #1 is true	Diagram on screen	because as I adjust various points along the parallelogram, the corresponding angle and the one across from it remain equal

Table 22 Continued

Task Name: Task B (medium)	Argument Component: Claim	Argument Component: Data	Argument Component: Warrant
3. Construct diagonals. Mark the point of intersection and label it E. Write a conjecture about the relationships among the line segments AE, BE, CE, and DE for parallelograms.	E acts as a midpoint for line segments AC and DB which creates segments AE, CE, DE, and BE respectively. This means that lines DE and BE are equal and lines CE and AE are equal.	Diagram on screen Midpoint and segments	
4. Is your conjecture from #3 true? Explain.	My conjecture from #3 is true	Measured	and I know this because I measured each line segment and the ones that I thought were equal were equal regardless of where and how the parallelogram was positioned.
5. Drag vertices of the parallelogram. Make a statement describing the relationship between diagonals of parallelograms.	The diagonals of the parallelogram across from each other are equal and parallel.	Diagram on screen	

Student Ten provided convincing evidence for achieving the two learning goals associated with this task: 1) justify opposite angles of parallelograms are congruent; and 2) justify that diagonals of parallelograms bisect each other. For both goals he made a claim based on dragging followed by measuring and dragging to confirm each claim. He further articulated his warrants giving evidence of achieving both goals and the quality of Task B.

Task C

Recall the description of a high-level task; task contains a collection of prompts that coordinate mathematical depth and technological actions in such a way that requires the student to make generalized conclusions based on emergent invariant relationships that go beyond a static sketch. The codes for Task C reflect that mathematical depth is often coordinated with technological actions. A conjectured significant difference between Task B and Task C is found in Prompt Four. Task B has no requirement for measuring while Task C does. This may affect the quality of student argumentation manifest on each task. It is conjectured that student argumentation on this task reflects the high quality of the task as indicated by the codes. The following subsection summarizes each student's argumentation on Task C (high).

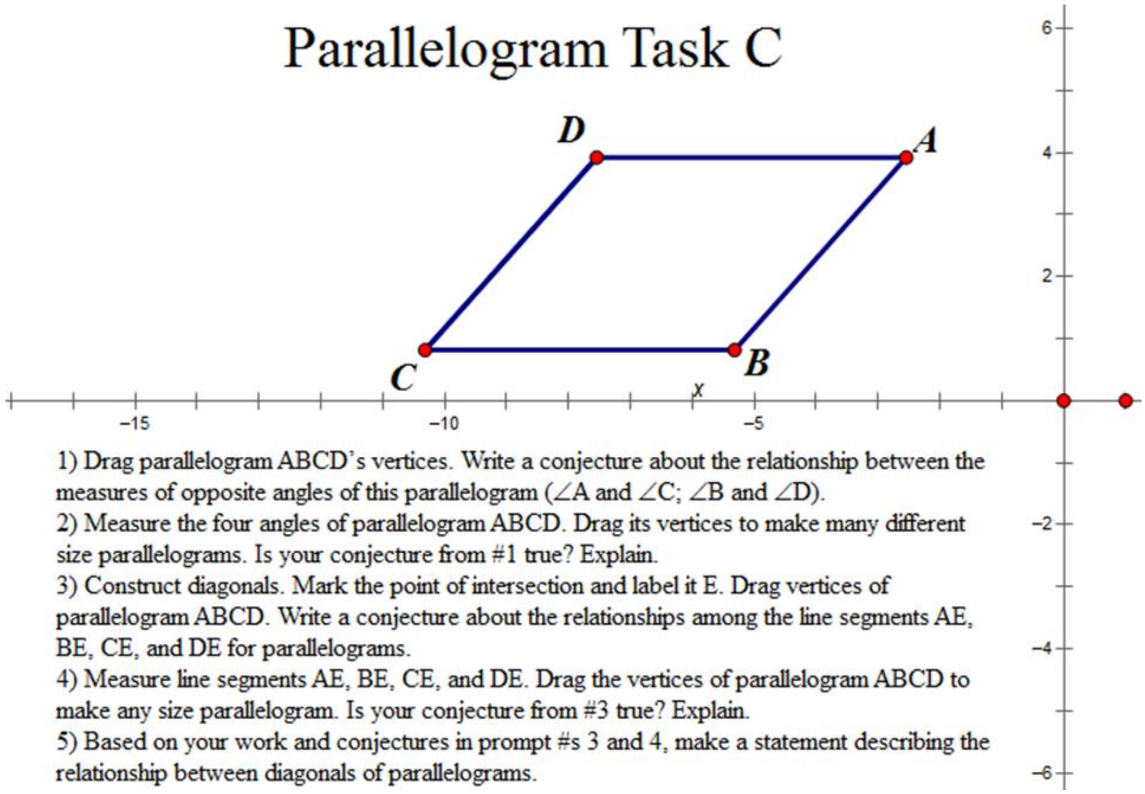


Figure 10: High-Level GSP Task

Table 23: Task C Coding Summary

Prompt Number	Parallelogram Task C Codes
1	(1, 2, 3, D)
2	(3, 4, 5, B, D, E,)
3	(1, 2, 3, C, D)
4	(3, 4, 5, B, D, E,)
5	(3, 4, 5, B, D, E,)

Student Argumentation on Task C (High)

Student Two, Student Five, Student Eight, and Student Eleven completed Task C (high). Student Two was a senior in high school who took Geometry as a freshman, Algebra

II as a sophomore, Pre-Calculus as a junior, and was currently taking Advanced Placement Calculus. He reflected that he did not pay very good attention in Geometry but did alright in the class. He had not used The Geometer's Sketchpad before but had used Geogebra in Pre-Calculus. His work is documented below.

Table 24: Student Two Argumentation

Task Name: Task C (high)	**Argument Component: Claim**	**Argument Component: Data**	**Argument Component: Warrant**
Prompt 1: Drag parallelogram ABCD's vertices. Write a conjecture about the relationship between the measures of opposite angles of this parallelogram (∠A and ∠C; ∠B and ∠D).	m∠A and m∠C are going to be the same value, and the same with m∠B and m∠D	Diagram on screen	
2: Measure the four angles of parallelogram ABCD. Drag its vertices to make many different size parallelograms. Is your conjecture from #1 true? Explain.	Yes.	Dimensions Angles	Even after changed dimensions of the parallelogram, the opposite angles stayed the same.
3: Construct diagonals. Mark the point of intersection and label it E. Drag vertices of parallelogram ABCD. Write a conjecture about the relationships among the line segments AE, BE, CE, and DE for parallelograms.	AE and EC are going to be equal value, and the same is true with DE and EB.	Diagram on screen	

Table 24 Continued

Task Name: Task C (high)	Argument Component: Claim	Argument Component: Data	Argument Component: Warrant
4: Measure line segments AE, BE, CE, and DE. Drag the vertices of parallelogram ABCD to make any size parallelogram. Is your conjecture from #3 true? Explain.	My answer from #3 is true.	Measuring	After measuring the lengths of the diagonals, it proved that it was right.
5: Based on your work and conjectures in prompt #s 3 and 4, make a statement describing the relationship between diagonals of parallelograms.	The diagonals of parallelograms, when intersected by the other diagonal, will be half the value it was before the intersection.	Diagram on screen	

Student Two provided convincing evidence for achieving the two learning goals associated with this task: 1) justify opposite angles of parallelograms are congruent; and 2) justify that diagonals of parallelograms bisect each other.1For both goals he made a claim based on dragging followed by measuring and dragging to confirm each claim. He further articulated his warrants giving evidence of achieving both goals and the quality of Task C.

Student Five was a senior in high school took Geometry as a freshman, Algebra II as a sophomore, Pre-Calculus as junior and was currently taking Advanced Placement Statistics. He reflected that he did not like Geometry very much and really did not get proofs. He had used The Geometer's Sketchpad in the past and had seen Geogebra as well. His work is documented below.

Table 25: Student Five Argumentation

Task Name: Task C (high)	Argument Component: Claim	Argument Component: Data	Argument Component: Warrant
Prompt 1: Drag parallelogram ABCD's vertices. Write a conjecture about the relationship between the measures of opposite angles of this parallelogram (∠A and ∠C; ∠B and ∠D).	When I move vertices A or D, the angle adjacent to it moves, so if I move angle A, angle D moves the same direction as angle A. Same thing happens with angle B and C.	Diagram on screen	the angle adjacent to it moves if I move angle A, angle D moves the same direction as angle A
2: Measure the four angles of parallelogram ABCD. Drag its vertices to make many different size parallelograms. Is your conjecture from #1 true? Explain.	There is a relationship between the opposite angles, for instance, angle D and angle B are very similar, And there is the same kind of relationship between the angle A and angle C.	Diagram on screen	whichever way I move the points.
3: Construct diagonals. Mark the point of intersection and label it E. Drag vertices of parallelogram ABCD. Write a conjecture about the relationships among the line segments AE, BE, CE, and DE for parallelograms.	the angles opposite each other on the point of intersection are equal. Moving the vertices causes the angles to increase or decrease.	Diagram on screen angles	As each vertex moves around,

Table 25 Continued

Task Name: Task C (high)	Argument Component: Claim	Argument Component: Data	Argument Component: Warrant
4: Measure line segments AE, BE, CE, and DE. Drag the vertices of parallelogram ABCD to make any size parallelogram. Is your conjecture from #3 true? Explain.	The distance between line AE, and CE are equal, and so are BE and DE.	Distance	This proves that my conjecture in question 3 is true.
5: Based on your work and conjectures in prompt #s 3 and 4, make a statement describing the relationship between diagonals of parallelograms.	The angles opposite each other are equal, and also the line segments are equal.	Diagram on screen Angles (previous measurement)	This is because the intersection point E is the midpoint for the line segments.

Student Five provided convincing evidence for achieving the two learning goals associated with this task: 1) justify opposite angles of parallelograms are congruent; and 2) justify that diagonals of parallelograms bisect each other. For both goals he made a claim based on dragging followed by measuring and dragging to confirm each claim. He further articulated his warrants especially in response to the final prompt giving evidence of achieving both goals and the high quality of Task C.

Student Eight was a senior in high school who took Geometry as a freshman, Algebra II as a sophomore, Pre-Calculus as a junior, and was currently taking both Advanced Placement Calculus and Statistics. He indicated that although he did well in Geometry, he did not feel that he learned it very well. I further articulated that he felt he was missing some parts and that this became evident while taking Calculus. He had seen The Geometer's Sketchpad before but had not used it. His work is summarized in the table below.

Table 26: Student Eight Argumentation

Task Name: Task C (high)	**Argument Component: Claim**	**Argument Component: Data**	**Argument Component: Warrant**
Prompt 1: Drag parallelogram ABCD's vertices. Write a conjecture about the relationship between the measures of opposite angles of this parallelogram (∠A and ∠C; ∠B and ∠D).	Opposite angles of a parallelogram have the same angles.	Diagram on screen	
2: Measure the four angles of parallelogram ABCD. Drag its vertices to make many different size parallelograms. Is your conjecture from #1 true? Explain.	The conjecture from the first question is true,	Diagram on screen Angles	because the changes are angles still have the same angle.
3: Construct diagonals. Mark the point of intersection and label it E. Drag vertices of parallelogram ABCD. Write a conjecture about the relationships among the line segments AE, BE, CE, and DE for parallelograms.	The line segments AE and CE appear to be congruent, and the segments DE and BE appear to be congruent as well.	Diagram on screen	

Table 26 Continued

Task Name: Task C (high)	Argument Component: Claim	Argument Component: Data	Argument Component: Warrant
4: Measure line segments AE, BE, CE, and DE. Drag the vertices of parallelogram ABCD to make any size parallelogram. Is your conjecture from #3 true? Explain.	Yes, the conjecture from question three is true,	Measured	once measured the segments were congruent, even when moved.
5: Based on your work and conjectures in prompt #s 3 and 4, make a statement describing the relationship between diagonals of parallelograms.	The diagonals of parallelograms define the size of the shape,	Diagram on screen (previous measurements)	and determine if the shape is truly a parallelogram.

Student Eight provided strong evidence for achieving the two learning goals associated with this task: 1) justify opposite angles of parallelograms are congruent; and 2) justify that diagonals of parallelograms bisect each other. For both goals he made a claim based on dragging followed by measuring and dragging to confirm each claim. He further articulated his recognition of invariance under dragging in his responses to prompts two and four. His quality argumentation reflects the high quality of Task C.

Student Eleven was a junior in high school who had taken Geometry in the summer before their freshman year, Algebra II as a freshman, Pre-Calculus as a sophomore, and was currently taking Advanced Placement Calculus AB and BC. Concerning their experience in

Geometry they reflected that was not a great experience because they did it on their own in the format of an online course. They saw The Geometer's Sketchpad used by their teacher in Pre-Calculus. Their work is documented below.

Table 27: Student Eleven Argumentation

Task Name: Task C (high)	**Argument Component: Claim**	**Argument Component: Data**	**Argument Component: Warrant**
Prompt 1: Drag parallelogram ABCD's vertices. Write a conjecture about the relationship between the measures of opposite angles of this parallelogram (∠A and ∠C; ∠B and ∠D).	m∠A = m∠C and m∠B = m∠D	Diagram on screen	
2: Measure the four angles of parallelogram ABCD. Drag its vertices to make many different size parallelograms. Is your conjecture from #1 true? Explain.	Yes my conjecture from #1 is true	Measure	because the measure of ∠A = the measure of ∠C and the measure of ∠D = the measure of ∠B as the parallelogram changes size.
3: Construct diagonals. Mark the point of intersection and label it E. Drag vertices of parallelogram ABCD. Write a conjecture about the relationships among the line segments AE, BE, CE, and DE for parallelograms.	Line segment AE = line segment CE and line segment BE = line segment DE.	Diagram on screen	

Table 27 Continued

Task Name: Task C (high)	Argument Component: Claim	Argument Component: Data	Argument Component: Warrant
4: Measure line segments AE, BE, CE, and DE. Drag the vertices of parallelogram ABCD to make any size parallelogram. Is your conjecture from #3 true? Explain.	Yes my conjecture from #3 is true	Diagram on screen (previous measurements)	because as the parallelogram's size changes, line segments AE and CE are equal and line segments BE and DE are equal.
5: Based on your work and conjectures in prompt #s 3 and 4, make a statement describing the relationship between diagonals of parallelograms.	The diagonals distance from opposite angles of parallelograms are equal at the intersection point of the two diagonals.	Distance	

Student Eleven provided adequate evidence for achieving the two learning goals associated with this task: 1) justify opposite angles of parallelograms are congruent; and 2) justify that diagonals of parallelograms bisect each other. Regarding both goals she made a claim based on dragging followed by measuring and dragging to confirm each claim. She further articulated his recognition of invariance under dragging in his responses to prompts two and four using the language of, “even when the parallelogram changes size.” The quality of her argumentation reflects the high quality of Task C.

Task Comparisons

It was conjectured that the quality student argumentation and learning goal achievement evident on each of the three tasks that ranked low, medium, and high in quality according to the Dynamic Geometry Task Analysis Framework would differ. To test this conjecture, the researcher assessed student work in two ways. First, the researcher tracked the achievement of task learning goals for each student on each task. Second, the researcher assessed which tasks reflected warrants in student responses. Higher instances of warrants reflect quality student argumentation.

Student argumentation was first assessed in reference to the learning goals associated with the tasks. Recall the two learning goals associated with all tasks: 1) justify opposite angles of parallelograms are congruent; and 2) justify that diagonals of parallelograms bisect each other. See the tables below for a summary of goal achievement for each student on each task. Each table can be interpreted by noticing the task label (low, medium, and high) and then the students who completed that task. Summary descriptors are then included for each student that include: 1) Years since taking Geometry; 2) Disposition to Geometry; and 3) Familiarity with The Geometer's Sketchpad. Disposition to geometry is ranked as poor, fair, or favorable and familiarity with The Geometer's Sketchpad is ranked as minimal, some, or much. Both category rankings are based on student interview data. Finally, the Learning Goals Achieved column reflects which learnings goals each students provided evidence of achieving. Summarizing student data related to goal achievement in these tables allows for a succinct summary of performance on each task.

Table 28: Student Goal Achievement

Task A (low)	Years since taking Geometry	Disposition to Geometry	Familiarity with The Geometer's Sketchpad	Learning Goals Achieved
Student 3	3	Favorable	Minimal	None
Student 6	2	Poor	Some	None
Student 9	2	Poor	Minimal	Goal 1
Student 12	2	Favorable	Much	Goals 1 & 2
Task B (medium)				
Student 1	3	Favorable	Minimal	Goals 1 & 2
Student 4	3	Fair	Minimal	Goal 1
Student 7	2	Poor	Minimal	Goals 1 & 2
Student 10	2	Fair	Minimal	Goals 1 & 2
Task C (high)				
Student 2	3	Fair	Minimal	Goals 1 & 2
Student 5	3	Poor	Much	Goals 1 & 2
Student 8	3	Fair	Minimal	Goals 1 & 2
Student 11	2	Poor	Fair	Goals 1 & 2

From the data in the table, it is evident that Task A was particularly weak regarding student achievement of learning goals. Tasks B and C reflect similar quality under this comparison technique with Task C performing slightly better. To further discriminate task quality a second comparison technique was required. Other variables such as years since taking geometry or familiarity with The Geometer's Sketchpad may have influenced each student's achievement of the learning goals, however no pattern emerged that indicates that these variables correlated to goal achievement. Larger samples are necessary to speak to such variable influences.

Student responses on each prompt for each task were assessed for warrants. The results of this assessment are documented in the following table. The maximum number of instances of tight argumentation on any prompt is four as four students completed each task.

Table 29: Student Argumentation Summary

Task A (low)	**Instances of Warrants**	**Task B (medium)**	**Instances of Warrants**	**Task C (high)**	**Instances of Warrants∠**
Prompt 1	0	Prompt 1	0	Prompt 1	1
Prompt 2	1	Prompt 2	4	Prompt 2	4
Prompt 3	2	Prompt 3	0	Prompt 3	1
Prompt 4	0	Prompt 4	4	Prompt 4	4
Prompt 5	2	Prompt 5	0	Prompt 5	0
Total	5		8		10

Under this comparison technique, it is evident that Tasks B and C far outperformed Task A, and Task C slightly outperformed Task B. Task C's high number of instances of tight argumentation further reflect its high task quality. These finding provide evidence that task quality as indicated by codes produced with the Dynamic Geometry Task Analysis Framework is valid.

Conclusion

The overarching research question and sub-questions that guided the validity portion of this work are provided below.

> How do the mathematical prompts and technological actions as identified in the Dynamic Geometry Task Analysis Framework relate to students' mathematical arguments and conclusions?

Each of the three employed tasks were written to achieve two goals: 1) justify opposite angles of parallelograms are congruent; and 2) justify that diagonals of parallelograms bisect each other. Establishing framework validity proceeded with two phases. In Phase One, the researcher sought and documented feedback from three experts in the field of mathematics education. This feedback was used to refine the Dynamic Geometry Task Analysis Framework and components of the study. A major refined component was the descriptors of low-, medium-, and high-ranking tasks. The new descriptors are more qualitative in nature and are the following:

- *Low:* Task does not contain a collection of prompts that coordinate mathematical depth and technological actions in such a way to require the student to make generalized conclusions based on emergent invariant relationships that go beyond a static sketch.
- *Medium:* Task contains a collection of prompts that coordinate mathematical depth and technological actions in such a way that may push to but does not necessitate that the student to make generalized conclusions based on emergent invariant relationships that go beyond a static sketch.
- *High:* Task contains a collection of prompts that coordinate mathematical depth and technological actions in such a way that requires the student to make generalized conclusions based on emergent invariant relationships that go beyond a static sketch.

These refinements were documented in the first section of this chapter.

For Phase Two of this study, the researcher crafted three tasks that ranked low, medium and high according to task prompt coding using the Dynamic Geometry Task Analysis Framework. Each task fit its respective qualitative description (low, medium, high) and each task was completed by four high school students. Students provided written responses to all task prompts. These written responses were coded using the core of Toulmin's (1958) argumentation model. The researcher interviewed students throughout their task completion work. These interviews were transcribed and used to clarify the coding of students' written responses. Task comparisons based on these codes revealed qualitative differences in student argumentation. These differences provided further evidence of the effectiveness of the Dynamic Geometry Task Analysis to discriminate task quality. With the face validity of the framework established, this study continued with conducting interviews with pre-service and in-service teachers concerning their utilization of the framework.

CHAPTER 5: FRAMEWORK UTILIZATION

This study was divided into two major phases: 1) a validity study of the Dynamic Geometry Task Analysis Framework followed by revisions; and 2) a study of how the framework affects teachers' perceptions of task quality and how they write and revise tasks. The research and analysis of the latter is documented in this chapter in response to the following associated research question, "How do pre-service and in-service teachers use the Dynamic Geometry Task Analysis Framework?" Three sub-research questions were also addressed:

a. What are similarities and differences between teachers' use of the framework for task writing for in-service and pre-service teachers?
b. What are similarities and differences in task quality for tasks written by in-service and pre-service teachers?
c. What are similarities and differences in task quality for tasks written by teachers who have different levels of knowledge for teaching geometry?

To address these research questions, three pre-service and three in-service teachers participated in individual task writing interview sessions with the researcher. Each participant completed the following steps:

- Took an online test that measures their knowledge for teaching geometry.
- Completed a written questionnaire concerning their background as a student and teacher and their use of The Geometer's Sketchpad.
- Completed an interview during and after the task writing process.

- Wrote a Geometer's Sketchpad task.
- Underwent a tutorial session with the researcher on the use of a dynamic geometry task analysis framework.
- Rewrote the task in reference to the tutorial session.

Information obtained on the background questionnaire and geometry test along with the transcribed interview and two teacher-written tasks compose the basis for each teacher's documented case. Each participant submitted two tasks based on the same learning goal, "justify that the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length." This learning goal was copied directly from the Common Core State Standards for High School Geometry (2010, p. 76).

Each participant had the same high school geometry textbook available to them for reference and they were asked to write a task using The Geometer's Sketchpad that would support students' achievement of the learning goal. Two other stipulations were requested of each participant: 1) that the student is to make at least on construction; and 2) that the student is to make at least one measurement. Each participant was instructed to assume that any student completing the task knows how to use the technological features of the program such as using the technology to find the slope of a line. Finally, each participant was instructed to assume that any student completing the task has the prerequisite background knowledge of the mathematical terms and ideas employed in the task directions, but not the knowledge of the learning goal itself or why it is true. With this setup, each participant wrote and submitted their first task. This activity was followed by an interaction with the Dynamic Geometry

Task Analysis Framework. See the methods chapter for details on this interaction. Each participant then wrote and submitted a potentially revised task in reaction to their exposure to the framework.

The remainder of this chapter includes the documentation of each of the six cases followed by a discussion of the documentation process and findings. In conclusion, relevant research questions were revisited in light of the reported findings. The first two sub-research questions reflect comparisons between pre-service and in-service teachers regarding framework use. Therefore, the cases are reported in two groups with the pre-service teachers first and the in-service teachers second. Within each group, the cases are organized from least affected by the framework to the most affected. Affect in this context refers to the number and substance of changes made from teacher written task one to their revised task. To assess such changes, each teacher written task was coded with the Dynamic Geometry Task Analysis Framework to provide a basis for task comparison. The framework is provided below for the reader's reference.

Table 30: Dynamic Geometry Task Analysis Framework

Allowance for Mathematical Depth	
Levels	**Descriptions**
N/A	Prompt requires a technology task with no focus on mathematics.
0	Sketch does not have mathematical fidelity required to respond to prompt.
1	Prompt requires student to recall a math fact, rule, formula, or definition.
2	Prompt requires student to report information from the sketch. The student is not expected to provide an explanation.
3	Prompt requires student to consider the mathematical concepts, processes, or relationships in the current sketch.
4	Prompt requires student to explain the mathematical concepts, processes, or relationships in the current sketch.

Table 30 Continued

Allowance for Mathematical Depth	
Levels	**Descriptions**
5	Prompt requires student to go beyond the current construction and generalize mathematical concepts, processes, or relationships.
Types of Technological Action	
Affordances	**Descriptions**
N/A	Prompt requires no drawing, construction, measurement, or manipulation of current sketch.
A	Prompt requires drawing within current sketch.
B	Prompt requires measurement within current sketch.
C	Prompt requires construction within current sketch.
D	Prompt requires dragging or use of other dynamic aspects of the sketch.
E	Prompt requires a manipulation of the sketch that allows for recognition of emergent invariant relationship(s) or pattern(s) among or within geometrical object(s).
F	Prompt requires manipulation of the sketch that may surprise one exploring the relationships represented or cause one to refine thinking based on themes within the surprise (adapted from Sinclair (2003), p. 312).

Participant reasoning concerning their task writing is revealed in the transcribed teacher interviews and referenced in each case write up. Task comparisons and participant comparisons informed by the data and accompanying analysis provides a basis for responding to the overall and sub-research questions of this chapter.

Participant Cases

The three participating pre-service teachers were invited while they were enrolled in a mathematics education course that focused on technology use in secondary education. Part of their work in this course required them to use, consider and write mathematics tasks using The Geometer's Sketchpad. Students in this course were therefore considered ideal candidates for this study.

Participant One: "Adds a Conjecture and an Explanation"

At the time of this study, Participant One was completing course work towards a Bachelor's Degree in Mathematics Education with a Concentration in Secondary Teaching. His score of $\phi = .27$ on the Mathematics Knowledge for Teaching Geometry test reflects his slightly above average knowledge of disciplinary content. As a junior in college, he had no formal teaching experience but did express a passion for teaching and an eagerness to begin his teaching career. Participant One had used The Geometer's Sketchpad and Geogebra in a technology in mathematics education course, but reported limited experience writing tasks in dynamic geometry environments, having only modified a task that involved transformations of polygons. He saw dynamic geometry environments as beneficial because they allow students to see many cases in a short amount of time through the use of dragging. He offered an example of a hypothetical student convincing themselves that diagonals of parallelogram are perpendicular by constructing a rectangle with diagonals and dragging vertices to check that diagonals remain perpendicular. Participant One further emphasized that dynamic geometry environments allow students to interact with pictures and see relationships between different shapes. Participant One articulated numerous benefits of using dynamic geometry environments for the betterment of student understanding.

Like all other participants, he wrote a task in reference to the learning goal, "justify that the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length." Participant One submitted the following task prompts:

1. Create a triangle using the line segment tool.

2. Label the vertices of the triangle A, B, and C.
3. Since we want to find the line segment joining the midpoints of two sides of a triangle, what construction should we perform next?
4. We can use the midpoints of any two sides, but I will be using sides AC and CB. Construct the midpoints of side AC and CB and create a line segment connecting them. Call the midpoint of AC D, and the midpoint of CB E.
5. Now that we have constructed the line segment joining the midpoints of the two sides of a triangle, what GSP tools can we use to justify that j is twice the length of k, and parallel?
6. For determining if k is half of j, use the measurement tool to determine if this is correct. Move point B around to determine if this is correct for multiple lengths of j and k.
7. In algebra, how did we determine if two lines were parallel? We found the slopes of the two lines and determined if they were equal. How can we determine if these lines have equal slopes?
8. Measure the slopes of lines j and k to determine if this is correct for this triangle. Drag point B to determine if this is correct for multiple slopes of j and k.

Participant One’s task was assessed with the Dynamic Geometry Task Analysis Framework by the researcher to produce the following codes.

Table 31: Participant One Task Code Summary

Prompt Number	Codes
1	1, C
2	1, C
3	1, N/A
4	1, C
5	1, N/A
6	3, 5, B, D, E
7	1, N/A
8	3, 5, B, D, E

In reflecting on his task, Participant One indicated that the student completing the task would accomplish the learning goal by making the appropriate triangle construction, measuring, and then dragging to establish the conclusions found in the learning goal. He articulated that the prompts would build on themselves to get the student to see what was accomplished at each step and what should be addressed next. He envisioned the student using the technological tools to make justifications in explaining:

> So they can use this technology to move points around and see that it's not a proof, but it does justify that this works for not just one case but for plenty of cases. And you can use this technology to see that.

Participant One showed a strong understanding of coordinating student technological activity with mathematical questions. Due to the coordination of mathematical depth and technological actions this tasks ranked medium per codes received using the Dynamic Geometry Task Analysis Framework.

Per the methods for this portion of the study, after submitting his first task and responding to interview questions, Participant One read an article about the task analysis framework (Appendix E) and then discussed the contents of the article and the use of the Revised Dynamic Geometry Task Analysis Framework in a preplanned tutorial session with the researcher. The participant then revisited the task they wrote and made any desired changes in reference to what was experienced in the tutorial session. Participant One's revised task prompts are provided below.

1. Create a triangle using the line segment tool.
2. Label the vertices of the triangle A, B, and C.
3. Since we want to find the line segment joining the midpoints of two sides of a triangle, what construction should we perform next?
4. We can use the midpoints of any two sides, but I will be using sides AC and CB. Construct the midpoints of side AC and CB and create a line segment connecting them. Call the midpoint of AC, D, and the midpoint of CB, E.
5. Now that we have constructed the line segment joining the midpoints of two sides of a triangle, construct the segment that connects the midpoints. What relationship do you expect there to be between lines j and k?
6. For determining if k is half of j, use the measurement tool to determine if this is correct. Move point B around to determine if this is correct for multiple lengths of j and k.
7. How do you expect the slopes of lines j and k to be related? Explain.

8. In algebra, how did we determine if two lines were parallel? We found the slopes of the two lines and determined if they were equal. How can we determine if these lines have equal slopes?
9. Measure the slopes of lines j and k to determine if this is correct for this triangle. Drag point B to determine if this is correct for multiple slopes of j and k.

Participant One's revised task was assessed with the Dynamic Geometry Task Analysis Framework by the researcher to produce the following codes.

Table 32: Participant One Revised Task Code Summary

Prompt Number	**Codes**
1	1, C
2	1, C
3	1, N/A
4	1, C
5	1, 3, C
6	3, 5, B, D, E
7	3, 4, 5
8	1, NA
9	3, 5, B, D, E

Participant One felt that their original task was well written and would push students to accomplish the desired learning goal. However, two significant changes to the task are reflected in the prompt codes: 1) a request for a conjecture was included in Prompt 5; and 2) a request for explanation was included in Prompt 7. The latter earned this task a code of 4, "Prompt requires student to explain the mathematical concepts, processes, or relationships in the current sketch." Hence, the revised task coordinates technological actions with greater mathematical depth compared to the original task making it a high-ranked task. This finding is further supported by the participant's reaction to their revised task:

> I made additions so after each part that was spelled out in the learning goal for finding that the segment joining the two midpoints is half the length and parallel to the third side, I forced them to go a little deeper to say what is your prediction for this or why do you think this will happen." In the participant's opinion, second task was of higher quality due to its insistence for students to consider and explain why they expect a certain relationship to hold.

In summary, Participant One made two small but important revisions to their task that reflected a major intent of the Dynamic Geometry Task Analysis Framework, to coordinate mathematical depth with technological actions. When asked how the framework affected their thinking about task quality and writing, he responded:

> I knew how to use the tool [The Geometer's Sketchpad] but not in a way in reference to a framework. I've learned about incorporating the mathematical practices with the technology but I had never really thought about it before having the framework. It really helps because there's so much you can do with it [The Geometer's Sketchpad] and so much you can build with it like constructions and transformation. So having the framework gives recommendations on what you should do and what you might not need so much of. I liked my first task but then I got more of an idea from the framework on what I could do to make it even better.

Participant One responded favorable to using the framework to revise his task. Furthermore, when asked if they thought the framework is beneficial, he indicated that it would be because it would get teachers and students to use dragging to consider multiple cases.

Participant Two: "Reorders Dragging and Measuring"

Like Participant One, Participant Two was a junior pursuing a Bachelor's Degree in Mathematics Education with a Concentration in Second Teaching who had recently completed the same course in technology in secondary mathematics education. Her score of ϕ = -.48 on the Mathematics Knowledge for Teaching Geometry test reflects her below average knowledge of disciplinary content. She considered herself a proficient user of dynamic geometry environments and expressed that she has used help functions or has gone online if she ever gets stuck using the technology. She had written a lesson plan including tasks in The Geometer's Sketchpad while in a technology in secondary mathematics course, a mathematics education course for pre-service and in-service teachers to learn about incorporating various technologies into their instruction. She expressed her belief in the potential of using The Geometer's Sketchpad in mathematics instruction, "I think that using GSP in the classroom allows the students to see exactly how geometry works, in all cases. That is something that can't be done in with just a paper and pencil." She further provided that a quality task in The Geometer's Sketchpad is one that, "involves multiple images and reference points." Based on the information she provided in the background questionnaire, it can be inferred that Participant Two recognized the power of the dynamic geometry environments for changing the way students experience the discipline in generating multiple examples for consideration.

Participant Two wrote and submitted a task based on the same learning goal that all other participants used, “justify that the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length.” Her first task submission is included below.

1. To begin construct a triangle of any sort. Label the vertices A, B and C. Then mark the midpoint on any combination of two sides, for example, the midpoints of segment AB and segment AC. After constructing the points create a line connecting them. You can call this segment DE.
2. Before you do any measurements, do you think the measure of DE is going to be half of the measure of the third side? Drag a vertex of your triangle. Do your thoughts remain true? To determine this you should measure your segment DE and the third side, for example is DE half of BC? If you need to justify if it is truly half of the length of the third side, you can use the calculate feature to divide the third side by DE. If it is half it should equate to 2.
3. Is DE parallel to the third side? If you drag a point does this fact remain true? You can determine this by using the slope measure to compare the two segments.
4. Now that we have determined these two things to be true if you were to drag a vertex of your triangle do these measurements hold true?

Participant Two’s task was assessed with the Dynamic Geometry Task Analysis Framework by the researcher to produce the following codes.

Table 33: Participant Two Task Code Summary

Prompt Number	Codes
1	1, C
2	2, 3, 5, B, D
3	2, 3, 5, B, D
4	3, 5, B, D, E

During the interview portion that followed her first task writing experience, Participant Two explained that the student would make the mid-segment construction and appropriate measurements followed by dragging to "see how things don't change." Even though her task included the technological actions of dragging and measuring, it is interesting to note that the student is asked to drag first and then to make a measurement as seen in prompts two and three. Participant Two thus assumes the student will make a proof based on a static diagram. However, in the final prompt the student is to drag a vertex and report if the measurements hold true. Participant Two showed an understanding of the importance of using dragging, but leaves room for improvement in coordinating this technological action with mathematical depth and allowing students to notice what is invariant. Her task received a medium ranking.

Per the methods for this portion of the study, after submitting his first task and responding to interview questions, Participant Two read an article about the task analysis framework (see Appendix E) and then discussed the contents of the article and the use of the Dynamic Geometry Task Analysis Framework in a preplanned tutorial session. The participant then revisited the task they wrote and made any desired changes in reference to what was experienced in the tutorial session. Participant Two's revised task prompts are provided below:

1. First construct a triangle of any sort. Label the vertices A, B and C.
2. Mark the midpoints of segments AB and AC, we will call these points D and E. Then construct a line connecting D and E. We will call this segment DE.
3. Do you notice any relationships between the lengths of segments DE and BC? Write your hypothesis below. If you were to drag vertex C of your triangle would your hypothesis remain true?
4. Using the measure tools determine the length of segments DE and BC. Do you see a correlation?
5. Use the GSP calculate function to determine the value of the measure of BC divided by the measure of DE. What do you get?
6. Is that value going to remain true no matter where you move a vertex to? What does that mean about the measure of DE in comparison to BC?
7. Looking at segments DE and BC, would you classify DE and parallel to BC? Does your classification hold true when moving a vertex?
8. Using the measure tools, calculate the slope of DE and BC. Is there a connection between them? What do these measurements say about their relationship?
9. Does the relationship stated about true when you move a vertex of the triangle?

Participant Two's revised task was assessed with the Dynamic Geometry Task Analysis Framework by the researcher to produce the following codes.

Table 34: Participant Two Revised Task Code Summary

Prompt Number	Codes
1	1, C
2	1, C
3	3, 5, D
4	3, B
5	3, B
6	3, 4, 5, D, E
7	3, 5, D, E
8	3, B
9	5, D, E

Participant Two explained that her revised task directions were further broken down to give students more concrete steps. She also provided that in the revised task the student was to write down conjectures before seeing if they are true. When the researcher asked her why she thought this was beneficial, she responded, “I think that it holds them accountable, like having them really do it because they have to write it down.” Participant Two spoke to a major change in this task, the reordering of dragging and measuring with measuring preceding dragging as she explained:

> And then they’ll use the calculate tool to divide and if it’s two; then one segment is half the other. And then they’ll drag the point to see if the number remains to be two. And then I ask them what that value means—what that value says about the relationship between the two segments?

This example of a reordering reflects another change in the revised task of requiring an explanation. Notice Prompt 6 in the revised task reads, “Is that value going to remain true no matter where you move a vertex to? What does that mean about the measure of DE in comparison to BC?" Unlike any prompt in Participant Two's first task, this prompt coordinates mathematical depth of code four, "Prompt requires student to explain the mathematical concepts, processes, or relationships in the current sketch" with measuring and dragging. The reordering of dragging and measuring along with the increased coordination of technological action with mathematical depth provided evidence of a change in understanding of task quality on behalf of Participant Two, thus earning the revised task a high ranking. When asked about the influence the framework had on her task writing she provided, she replied:

> Basically, I went back and I made the student think about the hypotheses before they did any measurements and then didn't really tell them what the measurements meant. Rather they had to determine why this is equal to what that says—why rather than what.

Participant Two correspondingly found the framework beneficial to teachers in explaining:

> I definitely think that it helps you figure out how to go from a mediocre task to a higher level task that uses all of the things that GSP is capable of-like if you were to sit down and make this triangle with a compass and measure by hand-because it allows you to move everything—it allows you make your students prove it to themselves in a more visual manner.

Based on the coding for both tasks and the participant's reaction to the framework, it is apparent that the framework influenced Participant Two's understanding of task quality in a way that she viewed as beneficial.

Participant Three: "Coordinates Mathematical Depth and Technological Actions"

Like the first two pre-service teachers, Participant Three was a junior pursuing a Bachelor's Degree in Mathematics Education with a Concentration in Secondary Teaching who had recently completed the same course in technology in secondary mathematics education. His score of ϕ = -.10 on the Mathematics Knowledge for Teaching Geometry test reflects his average knowledge of disciplinary content. Like the previous participant, he had written a lesson plan in the past that utilized The Geometer's Sketchpad. He stated a benefit of dynamic geometry environments as that of being able to see concepts in real time such as a square changing shape along with the measurements of its area and lengths. He added to this conclusion in explaining that a high quality task in The Geometer's Sketchpad, "should have the students do some type of manipulation to come to a conclusion." Participant Three demonstrated knowledge of the importance of using dynamic aspects of The Geometer's Sketchpad to aid student understanding.

Participant Three wrote and submitted a task based on the same learning goal that all other participants used, "justify that the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length." His first task submission is included below.

1. Using the measuring tool, measure the length of segment AC and the length of segment A'C'. What do you notice about the length of each segment?
2. Using the angle measuring tool or some other method, show that segment AC is parallel to segment A'C'.

3. Drag any of points A, B, or C to alter triangle ABC. Do your conclusions still hold true?
4. Next, create your own obtuse triangle. Label the triangle DEF.
5. Using the triangle you created show that the segment joining the midpoints of two sides is half the length of the third side. Then show that the segment joining the midpoints of two sides is parallel to the third side.

Participant Three's task was assessed with the Dynamic Geometry Task Analysis Framework by the researcher to produce the following codes.

Table 35: Participant Three Task Code Summary

Prompt Number	Codes
1	1, 2, B
2	1, 2, 3, B
3	5, D
4	1, C
5	1, N/A

In the interview that followed Participant Three's initial task writing, he explained that the student was to make measurements to support their understanding of the learning goal. He further explained that the student was to use dragging to ensure that the relationships found by measuring hold true. Students completing his task would know in advance what they are looking for as he explained, "I tell them [based on the directions I wrote] it should be parallel and then they kind of prove it to themselves." He did not explain what he meant by "prove it." When asked about how the student was to use the technological features available to them, he explained that, “measuring is much easier to do and it can be done for any shape […] and the measurements change in real time.” It is evident that he understands that dragging is important regarding task quality and student understanding and this task earned a medium rank.

After submitting his first task and responding to interview questions, Participant Three read an article about the task analysis framework (Appendix E) and then discussed the contents of the article and the use of the Dynamic Geometry Task Analysis Framework in a preplanned tutorial session. The participant then revisited the task they wrote and made any desired changes in reference to what was experienced in the tutorial session. Participant Three’s revised task prompts are provided below:

1. Using the measuring tool, measure the length of segment AC and the length of segment A'C'. Write a conjecture about the relationship of the lengths of segments AC and A'C'.
2. Drag any of points A, B, or C to alter triangle ABC. Does your conjecture still hold true? Explain your reasoning.
3. Using the angle measuring tool or some other method, show that segment AC is parallel to segment A'C'.
4. Drag any of points A, B, or C to alter triangle ABC. Will segment AC always be parallel to segment A'C'?
5. Next, create your own obtuse triangle. Label the triangle DEF.

6. Using the triangle you created and the measuring tool, test the conjecture you wrote in question 1. Drag any of points D, E, or F to alter triangle DEF. Does the conjecture still hold true?
7. Using the triangle you created show that the segment joining the midpoints of two sides is parallel to the third side. Drag any of points D, E, F to alter triangle DEF. Will the two segments always be parallel?
8. Using your conjecture and what you discovered about the two segments being parallel, write a theorem to extend these concepts to any generic triangle.

Participant Three's revised task prompts were assessed with the Dynamic Geometry Task Analysis Framework by the researcher to produce the following codes.

Table 36: Participant Three Revised Task Code Summary

Prompt Number	Codes
1	1, 3, B
2	3, 4, 5, D, E
3	1, 2, 3, B
4	5, D, E
5	1, C
6	3, 5, B, D, E
7	3, 5, D, E
8	1, 3, 5, N/A

In the interview following their revised task writing, Participant Three explained what changes he made and why, "I tried to change the wording and included the word conjecture and asked the student to explain their thinking and I broke it up into steps." Furthermore, he wanted, "to get them thinking that this applies to more than just one triangle and to every triangle." Participant Three expressed that he wanted students to discover the relationships expressed in the learning goal. This effort is reflected in codes for prompts two and six where the student is to manipulate the sketch in order to recognize the relationships present without being told the relationship is to be beforehand. Additionally, the revised task contains prompts that have mathematical depth of four and five per the framework whereas the initial task contained no depth beyond three. Their revised task earned a rank of high. Participant Three made substantial changes to their task as evidenced by the framework codes and his interview reflections. Regarding his use of the framework, Participant Three explained:

> It gave me something to reference in regard to quality. In the second task I wrote I went through categories N/A through 4 and 5 and made sure I hit those levels. I really wanted to include a question that was level 4 or 5.

As seen in the codes on the revised task, he did include these levels of mathematical depth and coordinated them with technological actions of measuring and dragging. Furthermore, he responded that, "As a junior in mathematics education it always feels like you are to create a task but you have no way to tell how good it is. It's nice to have something to tell you if you did what you need to do—it's like a check list." To summarize, Participant Three saw the framework as beneficial and he made major revisions to his task based on his understanding of it.

Participant Four: "Incorporates Explanations"

Participant Four is the first in-service teacher to be documented in this study. She has a Bachelor of Science Degree in Mathematics with a Concentration in Secondary Teaching.

At the time of this study, she had been teaching high school mathematics for three and a half years in the courses of Math I, Math II, Pre-Calculus, and Statistics. Math I and Math II contain the equivalent of one course in high school Geometry thus insuring that Participant Four was familiar with teaching the concepts of Geometry. Her score of $\phi = 1.18$ on the Mathematics Knowledge for Teaching Geometry test reflects her strong knowledge of disciplinary content. She had used The Geometer's Sketchpad in her undergraduate mathematics and mathematics education courses and had incorporated it on a limited basis in her high school teaching. She emphasized that The Geometer's Sketchpad tasks she has used with her students are, "self-directed as opposed to teacher-centered." Participant Four reflected on the benefits of using dynamic geometry software:

> One benefit of using Geometer's Sketchpad is that students can explore situations (such as trigonometry graphs or properties of quadrilaterals) at their own pace and make conjectures about them. By personally discovering concepts, students will understand and retain the information better than in a traditional lecture-style lesson.

She also provided her opinion of what constitutes a high quality Geometer's Sketchpad task as one that is, "designed well enough to guide them through a learning experience that allows them freedom to discover while at the same time achieving the desired learning objectives." Participant Four emphasized discovery in dynamic geometry environments and evidenced an understanding of quality student engagements with tasks in such dynamic environments.

Participant Four wrote and submitted a task based on the same learning goal that all other participants used, "justify that the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length." Her first task submission prompts are included below.

1. Making sure the arrow is highlighted on the left-hand toolbar, click in the blank space so that nothing is highlighted. Then click once on line segment AB. Then go to the "Construct" tab at the top and click "Midpoint." Then right click on the midpoint and label it D.
2. Repeat step 1 with line segment BC and label its midpoint E.
3. Select points D and E and construct a segment connecting them.
4. Q1: Given that segment connects the midpoints of two sides of the triangle, what do you think an appropriate name would be for line segment DE.
5. Q2: If you were to pick any point on line segment DE, what do you think would be true about that point and its distance from A and B? What about its distance from B and C?
6. Q3: What do you notice about the slopes of DE and CA?
7. After clicking on the blank space, select segment DE and measure its slope using the "Construct" tab. Then do the same for CA.
8. Q4: Is your thought from Q3 confirmed? Why do you think this is?
9. After clicking in the blank space, click C and drag it around the screen. Try to make the triangle isosceles, scalene, equilateral, acute, right, and obtuse.
10. Q5: What happens to the slopes of DE and CA as you move C around?
11. Q6: What general statement can you make about slopes of DE and CA?

12. After clicking on the blank space, select DE and measure its length using the "Measure" tab. Now do the same for CA.
13. Q7: What do you notice about their lengths?
14. After clicking on the blank space, create a ratio of their side lengths by going to "Number," click "Calculate," click your measurement of DE, click the division symbol, click the measurement of CA, and click OK.
15. Q8: Does this ratio confirm your thought from Q7?
16. After clicking on the blank space, move C around the grid and watch what happens to the measurements of DE and CA and their ratios. Try to make the triangle isosceles, scalene, equilateral, acute, right, and obtuse.
17. Q9: What general statement can you make about the measurements of DE and CA?

Participant Four's task was assessed with the Dynamic Geometry Task Analysis Framework by the researcher to produce the following codes.

Table 37: Participant Four Task Code Summary

Prompt Number	**Codes**
1	2, C
2	2, C
3	2, C
4	2, N/A
5	3, N/A
6	1, 2, N/A
7	1, 2, B
8	1, 2, 3, 4
9	1, D
10	2, 3, 5, D, E
11	2, 3, 5, N/A
12	1, B

Table 37 Continued

Prompt Number	Codes
13	2, N/A
14	1, B
15	2, N/A
16	1, D, E
17	2, 3, 5, NA

Participant Four explained that the student was to begin with constructing the mid-segment followed by making appropriate measurements, conjectures, and then dragging to test their conjecture. Regarding the two parts of the learning goal about the relationships between the mid-segment and the third side, she explained:

> First I had them just look at the numbers to see if they could figure it [the relationships] out and then I had them look at the ratios. Then they would make a statement about the mid-segment and the side opposite and then the slopes of both. They would move it around to see if the conjecture about the slopes and lengths is right.

Participant Four wrote explicit task directions for the process of measuring, conjecturing, and dragging to test. Notice that while her task reflects her strong understanding of utilizing the technological actions afforded in The Geometer's Sketchpad the prompts do not contain many requests for the student to explain why the relationships they find are true. Her task therefore received a medium rank according to codes generated with the Dynamic Geometry Task Analysis Framework.

After submitting her first task and responding to interview questions, Participant Four read an article about the task analysis framework (Appendix E) and then discussed the contents of the article and the use of the Dynamic Geometry Task Analysis Framework in a preplanned tutorial session. The participant then revisited the task she wrote and made any desired changes in reference to what was experienced in the tutorial session. Participant Four's revised task prompts are provided below.

1. Making sure the arrow is highlighted on the left-hand toolbar, click in the blank space so that nothing is highlighted. Then click once on AB Then go to the "Construct" tab at the top and click "Midpoint." Then right click on the midpoint and label it D.
2. Repeat step 1 with BC and label its midpoint E.
3. Q1: Given that D is the midpoint of AB, what statement can you make about the distance of D from A and B?
4. Select points D and E and construct a segment connecting them.
5. Q2: Given that segment connects the midpoints of two sides of the triangle, what do you think an appropriate name would be for DE?
6. Q3: If you were to pick any point on DE, what do you think would be true about that point and its distance from A and B? What about its distance from B and C? Why do you think this?
7. Q4: What do you notice about the slopes of DE and CA?
8. After clicking in the blank space, select segment DE and measure its slope using the "Construct" tab. Then do the same for CA.

9. Q5: Is your thought from Q4 confirmed? Why do you think this is?
10. After clicking in the blank space, click C and drag it around the screen. Try to make the triangle isosceles, scalene, equilateral, acute, right, and obtuse.
11. Q6: What happens to the slopes of DE and CA as you move C around?
12. Q7: What general statement can you make about the slopes of DE and CA? Why do you think this is?
13. After clicking on the blank space, select DE and measure its length using the "Measure" tab. Now do the same for CA.
14. Q8: What do you notice about their lengths?
15. After clicking on the blank space, create a ratio of their sides lengths by going to "Number," click "Calculate," click your measurement of DE, click the division symbol, click the measurement of CA, and click OK.
16. Q9: What does this ratio tell you? Does this ratio confirm your thought from Q8?
17. After clicking on the blank space, move C around the grid and watch what happens to the measurements of DE and CA and their ratios. Try to make the triangle isosceles, scalene, equilateral, acute, right, and obtuse.
18. Q10: What general statement can you make about the measurements of DE and CA? Why do you think this is?

Participant Four's revised task was assessed with the Dynamic Geometry Task Analysis Framework by the researcher to produce the following codes:

Table 38: Participant Four Revised Task Code Summary

Prompt Number	Codes
1	2, C
2	2, C
3	2, N/A
4	2, C
5	2, N/A
6	3, N/A
7	1, 2, N/A
8	1, 2, B
9	3, 4
10	1, D
11	2, 3, 5, D, E
12	3, 4, 5, N/A
13	1, B
14	2, N/A
15	1, B
16	2, 3, N/A
17	1, D, E
18	3, 4, 5, N/A

In the interview following their revised task writing, Participant Four explained what changes she made and why. She explained that the big changes she made, "were in the later questions by asking them to explain their thinking like why do you think this is, because I didn't have a lot of those [requests for explanations]." As a follow up question, the research asked, "So what might you predict will be different in student responses because you made these changes? What are you looking for based on these changes?" Participant Four responded, "I would like them to be able to verbalize their justification, so it's not just what, but why it is." Her task changes are reflected in the codes. For example, Prompt 11 requires the student to move a vertex and notice what happens to the slopes of both the mid-segment and third side. This technological action is capitalized on with Prompt 12 asking the student to explain why their statement about slopes is true. The second task then not only represents greater mathematical depth but also further coordination of technological actions and depth thus earning it a high ranking.

Participant Four explained that her use of the Dynamic Geometry Task Analysis Framework had affected her thinking about task quality and writing:

> It requires the teacher to be more thoughtful about it [task writing] and not just get the point across. I know why something is the way it is, but I want my students to see that too. This makes it explicit so I think the framework is helpful with that.

She further expressed that the framework would help teachers choose dynamic geometry tasks online and they look to incorporate them in their instruction.

Participant Five: "Coordinates Dragging with Explaining"

Participant Five has Bachelor's Degrees in Mathematics and Mathematics Education along with a Master's Degree in Education. At the time of this study she was in her eleventh year of teaching high school mathematics. She had previously taught courses ranging from Pre-Algebra through Advanced Placement Statistics and was currently teaching Math I and Pre-Calculus. Her score of $\phi = 1.33$ on the Mathematics Knowledge for Teaching Geometry test reflects her strong knowledge of disciplinary content. She explained that Math I had a component of Geometry. She had used The Geometer's Sketchpad in a teaching with technology course she took over 10 years ago. She had used The Geometer's Sketchpad again during her Master's work but had limited use with it in her own classroom. Participant Five expressed her opinion on why dynamic geometry environments are beneficial for students in providing, “The clicking and dragging nature of Sketchpad allows them to physically experience many of the theorems they study and thus they’re probably more likely to remember them.” The final question of the background survey asked the participant to describe a high quality The Geometer’s Sketchpad Task, and she responded:

> I suppose a high quality task would involve having the students experiment with a file where they kind of play with it at their own pace. Maybe there would be some guiding questions that would draw their attention to some tools that might be useful. Then they would be asked to draw some conclusions about what they’re experiencing that would ultimately lead into some fundamental theorem in Geometry.

Participant Five recognized the importance of using the technological affordance of dragging in a task where students were to experiment before, “drawing conclusions.”

Participant Five wrote and submitted a task based on the same learning goal that all other participants used, “justify that the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length.” Her first task submission prompts are included below:

1. Construct triangle ACE
2. Construct the midpoint of segment AC and call it B
3. Construct the midpoint of segment CE and call it D
4. Construct segment BD
5. Measure the length and the slope of segment BD
6. Measure the length and the slope of segment AE

7. Select point A and drag it around so you change the shape of triangle ACE. Observe what happens to the slope of DB and AE as you're doing this. What can you conclude about the slope of these two segments?
8. Select side CE and drag it around so you again change the shape of triangle ACE. This time observe what happens to the length of DB and AE as you're doing this. What can you conclude about the lengths of these two segments?

Participant Five's task was assessed with the Dynamic Geometry Task Analysis Framework by the researcher to produce the following codes:

Table 39: Participant Five Task Code Summary

Prompt Number	**Codes**
1	1, C
2	1, C
3	1, C
4	1, C
5	1, B
6	1, B
7	3, 5, D
8	3, 5, D

Participant Five wrote and submitted a very straightforward task in which the student was to make constructions and measurements associated with the learning goal. The student was then to using dragging to establish the two parts of the learning goal. She explained: "It [the task] tells them to measure as well to observe the relationships that are there, and it tells them to drag around so they can see relationships are there all the time and not just for one instance." Participant Five recognized the importance of using dragging in The Geometer's

Sketchpad, but did not coordinate this technological action with requiring student explanations. Her task received a rank of medium.

After submitting her first task and responding to interview questions, Participant Five read an article about the task analysis framework (see Appendix E) and then discussed the contents of the article and the use of the Dynamic Geometry Task Analysis Framework in a preplanned tutorial session. The participant then revisited the task she wrote and made any desired changes in reference to what was experienced in the tutorial session. Participant Five's revised task prompts are provided below.

1. Construct triangle ACE.
2. Construct the midpoint of segment AC and call it B.
3. Construct the midpoint of segment CE and call it D.
4. Construct segment BD.
5. Drag triangle ACE's vertices. Make a conjecture about the relationship between the segment joining the midpoints of two sides of a triangle (BD) and the third side (AE).
6. Measure the length and the slope of segment BD. Measure the length and the slope of segment AE. Drag triangle ACE's vertices again to make a triangle of any size. Is your conjecture from question 5 true? Explain.
7. What can you conclude about the slope of the segment joining the midpoints of two sides of a triangle (BD) and the third side (AE)? What can you conclude about the lengths of these two segments?

Participant Five's revised task was assessed with the Dynamic Geometry Task Analysis Framework by the researcher to produce the following codes:

Table 40: Participant Five Revised Task Code Summary

Prompt Number	Codes
1	1, C
2	1, C
3	1, C
4	1, C
5	3, 5, D
6	3, 4, 5, B, D, E
7	3, 5, N/A

The two major differences between the first task and the revised task is the addition of a conjecture and the request for an explanation. Participant Five explained, "I had them make the conjecture first and then had them drag to see if their conjecture holds true. So that thought process came out more after I read the framework." She also indicated that she required explanation in the revised task and felt that the second task had prompts that were better coordinated to the learning goal. She added:

> Before, I was really just focusing their attention on the two segments and not necessarily the goal as a whole, so when I rewrote I focused them on the relationship between the segment that joins the midpoints and the third side whereas previously I didn't really go beyond the two segments in the best way. So they may not realize this for all triangles and combining both relationships at once.

Participant Five made small but significant changes in her task that led to increased coordination between technological actions and mathematical depth by requiring students to report what they see and provide explanation. The revised task received a high rank.

Furthermore, Participant Five saw the framework as beneficial for teachers, “I see the measuring, explaining, and testing as important and definitely the dragging. Dragging is the big one! This framework is definitely valuable for a beginning teacher and without instruction from this framework many teachers would not use dragging.” She also indicated that the framework, “reminded me of some of the language I wanted to use as far as requiring them to explain. That was lacking in the first. I want them to go beyond the construction to see this for all triangles.” The task coding and interview revealed that the framework affected this veteran teacher’s perceptions of task quality and task writing.

Participant Six: “Adds Dragging and Explaining”

At the time of this study, Participant Six was in his second year of high school mathematics teaching. He has a Bachelor’s of Art degree in International Studies and a Master’s of Art Degree in Teaching with an emphasis in Secondary Mathematics. He had taught courses ranging from Math II to Advanced Placement Calculus. His score of $\phi = .77$ on the Mathematics Knowledge for Teaching Geometry test reflects his above average knowledge of disciplinary content. He had used dynamic geometry software in his experience earning his Master’s degree in Teaching. He reported using The Geometer’s Sketchpad a few times in his own instruction. Participant Six expressed that dynamic geometry environments are beneficial in allowing students to, “think deeply about a particular aspect of Geometry." He did not mention the technological affordance of dragging in any response to the background questionnaire.

Participant Six wrote and submitted a task based on the same learning goal that all other participants used, "justify that the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length." His first task submission is included below.

1. Given triangle ABC.
2. Construct the midpoint of segment AB (label it J) and the midpoint of segment BC (label it K).
3. Construct segment JK.
4. Measure angle BJK and angle BKJ.
5. Measure angle A and angle C.
6. How would you classify angle BJK and angle A? Angle BKJ and Angle C?
7. Based on your answers in questions 5 and 6, what does this imply about the two segments, AC and JK?
8. You should have two triangles in front of you: triangle ABC and triangle BJK. Which angle do they share?
9. Based on what you've done so far and answered in question 8, what can you conclude about these triangles? Justify your answer.
10. Measure segment JB and segment KB.
11. Measure segment AB and segment BC.
12. What is the ratio of segment JB to segment AB? Segment KB to segment BC?

13. By now you should notice that triangle BJK is similar to triangle BAC. And, you should have the ratio of their side lengths. Measure segment AC. Then, measure segment JK. Is the ratio the same as it was for the segments in question 12?
14. What can you conclude about a segment joining midpoints of two sides of a triangle?

Participant Six's task was assessed with the Dynamic Geometry Task Analysis Framework by the researcher to produce the following codes:

Table 41: Participant Six Task Code Summary

Prompt Number	Codes
1	1, N/A
2	1, C
3	1, C
4	1, B
5	1, B
6	1, 2, N/A
7	1, 2, N/A
8	1, 2, N/A
9	1, 2, N/A
10	1, B
11	1, B
12	1, 2, B
13	1, 2, B
14	2, 3, N/A

This task contained no request for student dragging and possess mathematical depth that only once went beyond code 2, "Prompt requires student to report information from the construction. The student is not expected to provide an explanation." Participant Six explained that he wanted students to, "go step by step and not just give them the answer, so they could go independently to reach the goal." He further explained that he wanted students to see similar triangles based on measurements in order to meet the learning goal. He saw the students as using The Geometer's Sketchpad to quickly and easily make measurements and saw the program as beneficial for students that may not have the dexterity to make measurements on paper. Because of the lack of coordination between mathematical depth and technological action, this task received a low quality ranking.

After submitting his first task and responding to interview questions, Participant Six read an article about the task analysis framework (Appendix E) and then discussed the contents of the article and the use of the Dynamic Geometry Task Analysis Framework in a preplanned tutorial session. The participant then revisited the task she wrote and made any desired changes in reference to what was experienced in the tutorial session. Participant Six's revised task prompts are provided below:

1. Given triangle ABC.
2. Construct the midpoint of segment AB (label it J) and the midpoint of segment BC (label it K).
3. Construct segment JK.
4. Measure angle BJK and angle BKJ.
5. Measure angle A and angle C.
6. How would you classify angles BJK and angle A? Angle BKJ and Angle C?
7. Click and drag point A. Does anything change about the angles in question 6? Make a conjecture about what would happen if you dragged the other vertices.

8. Test your conjecture in question 7 by repeating question 7, only this time drag points B and C. Does anything change about the angles in question 6?
9. Based on your answers in questions 5, 6, 7, and 8, what does this imply about the two segments, AC and JK?
10. You should have two triangles in front of you: triangle ABC and triangle BJK. Which angle do they share?
11. Based on what you've done so far and answered in question 8, what can you conclude about these triangles? Justify your answer.
12. Measure segment JB and segment KB.
13. Measure segment AB and segment BC.
14. What is the ratio of segment JB to segment AB? Segment KB to segment BC?
15. If you dragged a vertex and moved it around the screen, predict what would happen to the ratio of JB to AB.
16. Drag the vertices to make an equilateral triangle; note the ratio of JB to AB.
17. Repeat question 14, only this time make a scalene triangle and an isosceles triangle. Does your prediction from question 13 hold true? Why would it hold true? What can you determine about the two triangles?
18. By now you should notice that triangle BJK is similar to triangle BAC. And, you should have the ratio of their side lengths. Measure segment AC. Then, measure segment JK. Predict what the ratio of these sides will be.
19. Is the ratio the same as it was for the segments in question 12?

20. What can you conclude about a segment joining midpoints of two sides of a triangle?

Participant Six's revised task was assessed with the Dynamic Geometry Task Analysis Framework by the researcher to produce the following codes.

Table 42: Participant Six Revised Task Code Summary

Prompt Number	Codes
1	1, N/A
2	1, C
3	1, C
4	1, B
5	1, B
6	1, 2, N/A
7	2, 3, 5, D, E
8	2, 3, 5, D, E, F
9	2, 3, N/A
10	1, B
11	1, B
12	1, 2, B
13	3, 5, N/A
14	3, 5, D, E
15	3, 4, 5, D, E, F
16	2, 3, B
17	2, 3, N/A
18	3, 5, N/A

Participant Six made majors changes to his first task. Notice that in the revised task no less than four requests to use dragging are made whereas in the first task there were none. Furthermore, in the revised task there are requests for explanations after conjectures have been made thus earning codes of four for mathematical depth and E and F representing recognition of invariant relationships in a way that may surprise the student. For instance, in prompt fifteen, the student is to use dragging to test a conjecture in a way that guarantees that they pay attention to emergent invariant relationships. Because the student is not told what to expect beforehand, this prompt may contain an element of surprise as well. Participant Six reflected:

> I changed mine to have students make conjectures or answer why a relationship happened. I think that in having them make more predictions instead of just rote measurements as I originally had it makes them think about it before hand and think about why this happens and makes them connect what's happening to their predictions.

He made changes to his task that reflected his emerging understanding of the importance of dragging along with explanations. His second task coordinates technological actions and mathematical depth to a much greater degree than in the first, earning it a high ranking.

Participant Six indicated that the Dynamic Geometry Task Analysis Framework helped him realize the importance of dragging and explaining. He also saw it as valuable for teachers when he shared, "I think it would be really awesome to have a district math coach to work with teacher s to make this beneficial. As a new teacher I think it would be difficult to find the time to use the framework and program effectively." With this he expressed the difficulty of teachers to incorporate technology in their classrooms without outside guidance.

Subtleties of Coding

Documentation of summary results for the Chapter 5: Framework Utilization is preceded by a section on the task coding process. After this section, findings are summarized according to themes and patters that emerged in the data analysis. The summary section is followed by a conclusion in which the relevant research questions are revisited regarding findings.

Because the results of working with pre-service and in-service teachers are largely based on coding with the Dynamic Geometry Task Analysis Framework, it behooves the researcher to provide a treatment on the subtleties of coding at emerged in analyzing teacher submitted tasks. The first subtlety that emerged regards the nature of mathematical depth codes. This portion of the framework has been entitled, "Allowance for Mathematical Depth" with the subheading "Hierarchical Levels and Descriptions." Notice that coding for Participant Four's task one reflects prompts that have a code of 3, "Prompt requires student to consider the mathematical concepts, processes, or relationships in the current sketch," and a code of 5, "Prompt requires student to go beyond the current construction and generalize mathematical concepts, processes, or relationships," but that do not reflect a code 4, "Prompt requires student to explain the mathematical concepts, processes, or relationships in the current sketch."

Prompt 5 of Participant Four's first task is an example of how this coding can happen, as the prompt reads, "What happens to the slopes of DE and CA as you move C around?" A student responding to this prompt is required to consider mathematical relationships that go beyond the current construction due to the dragging request. However, the student is not required to explain why the relationship holds true. Coding this prompt brings to the fore the subtle difference between explanation and justification in dynamic geometry environments for the purposes of this study. A justification emerged when a student was to notice an invariance under dragging with a hypothetical response of, “the slopes remain the same no matter what triangle I make.” However, an explanation goes beyond that of only noticing of invariance with a request to consider and articulate why the invariance is manifest in the objects on the screen. Overlap between justifying and explaining undoubtedly exists. Because of the potential for this coding situation, the word “hierarchical” was removed from the framework as sometimes a level may be skipped when coding a prompt.

The second subtlety to be addressed is that of coding an E, “Prompt requires manipulation of the sketch that allows for recognition of emergent invariant relationship(s) or pattern(s) among or within geometrical object(s)” and coding an F, “Prompt requires manipulation of the sketch that may surprise one exploring the relationships represented or cause one to refine thinking based on themes within the surprise (adapted from Sinclair, 2003, p. 312).” The researcher considered a code E to likely be occurring if the student is to measure and then drag to see if the measurement changes to indicate an invariant relationship. For example, the following prompt from Participant Three’s first task received a code E, “Using the triangle you created show that the segment joining the midpoints of two sides is parallel to the third side. Drag any of points D, E, F to alter triangle DEF. Will the two segments always be parallel?” because it requires manipulation and directs the student on what to notice. However, another prompt from another participant’s task reads, “Test your conjecture in question 7 by repeating question 7, only this time drag points B and C. Does anything change about the angles in question 6?” and was coded as both E and F because it is based on a student conjecture and not on a preconceived conclusion on behalf of the task writer.

The subtleties reported above emerged for the researcher when unpacking written tasks with the Dynamic Geometry Task Analysis Framework. It should be noted that any attempt at coding with this framework will have some degree subjectivity involved. Regardless, the attempt does seem fruitful based on the summary findings reported in what follows. In this light, the framework has a purpose to bring some objectivity to the understanding of task quality and writing.

Summary of Findings and Research Questions

Coding of participants' tasks on the Revised Dynamic Geometry Task Analysis Framework revealed differences participants' initial and revised tasks and between pre-service and in-service teachers' task quality. Pre-service teachers' initial tasks all ranked medium per prompt coding and were characterized by combinations of constructing, measuring, and dragging with requests to notice what is invariant. The first two in-service teachers, Participants Four and Five, both submitted initial tasks that ranked medium according to the framework coding, while Participant Six's initial task ranked low, due it is lack of utilization of dragging connected to the learning goal. Each participant's initial task coding is summarized in Table 43 below:

Table 43: Comparison of Pre-Service and In-Service Teachers' Initial Task Submissions

PRE-SERVICE TEACHERS		IN-SERVICE TEACHERS	
Prompt	**Codes**	**Prompt**	**Codes**
PARTICIPANT 1		**PARTICIPANT 4**	
1	1, C	1	2, C
2	1, C	2	2, C
3	1, N/A	3	2, C
4	1, C	4	2, N/A
5	1, N/A	5	3, N/A
6	3, 5, B, D, E	6	1, 2, N/A
7	1, N/A	7	1, 2, B
8	3, 5, B, D, E	8	1, 2, 3, 4
		9	1, D
		10	2, 3, 5, D, E
		11	2, 3, 5, N/A
		12	1, B
		13	2, N/A
		14	1, B
		15	2, N/A
		16	1, D, E
		17	2, 3, 5, NA
PARTICIPANT TWO		**PARTICIPANT FIVE**	
1	1, C	1	1, C
2	2, 3, 5, B, D	2	1, C
3	2, 3, 5, B, D	3	1, C
4	3, 5, B, D, E	4	1, C
		5	1, B
		6	1, B
		7	3, 5, D
		8	3, 5, D
PARTICIPANT THREE		**PARTICIPANT SIX**	
1	1, 3, B	1	1, N/A
2	3, 4, 5, D, E	2	1, C
3	1, 2, 3, B	3	1, C
4	5, D, E	4	1, B
5	1, C	5	1, B
6	3, 5, B, D, E	6	1, 2, N/A
7	3, 5, D, E	7	1, 2, N/A
8	1, 3, 5, N/A	8	1, 2, N/A
		9	1, 2, N/A
		10	1, B
		11	1, B
		12	1, 2, B
		13	1, 2, B
		14	2, 3, N/A

This display of prompt coding for initially submitted teacher tasks provides details that support the discussion of findings below. Notice the differences in number and type of prompts for each participant and each group.

The impact of The Revised Dynamic Geometry Task Analysis Framework on both the pre-service and the in-service groups can be readily seen in the codes on each participant's revised task.

Table 44: Comparison of Pre-Service & In-Service Teachers' Revised Task Submissions

PRE-SERVICE TEACHERS		IN-SERVICE TEACHERS	
Prompt	**Codes**	**Prompt**	**Codes**
PARTICIPANT 1		**PARTICIPANT 4**	
1	1, C	1	2, C
2	1, C	2	2, C
3	1, N/A	3	2, N/A
4	1, C	4	2, C
5	1, 3, C	5	2, N/A
6	3, 5, B, D, E	6	3, N/A
7	3, 4, 5	7	1, 2, N/A
8	1, NA	8	1, 2, B
9	3, 5, B, D, E	9	3, 4
		10	1, D
		11	2, 3, 5, D, E
		12	3, 4, 5, N/A
		13	1, B
		14	2, N/A
		15	1, B
		16	2, 3, N/A
		17	1, D, E
		18	3, 4, 5, N/A
PARTICIPANT TWO		**PARTICIPANT FIVE**	
1	1, C	1	1, C
2	1, C	2	1, C
3	3, 5, D	3	1, C
4	3, B	4	1, C
5	3, B	5	3, 5, D
6	3, 4, 5, D, E	6	3, 4, 5, B, D, E

Table 44 Continued

PRE-SERVICE TEACHERS		IN-SERVICE TEACHERS	
Prompt	**Codes**	**Prompt**	**Codes**
PARTICIPANT TWO		**PARTICIPANT FIVE**	
7	3, 5, D, E	7	3, 5, N/A
8	3, B		
9	5, D, E		
PARTICIPANT THREE		**PARTICIPANT SIX**	
1	1, 3, B	1	1, N/A
2	3, 4, 5, D, E	2	1, C
3	1, 2, 3, B	3	1, C
4	5, D, E	4	1, B

Associated research questions are addressed in what follows regarding this summary coding information.

All participants made changes to their first written task based on their exposure to the Dynamic Geometry Task Analysis Framework in the tutorial session. Some participants made minor changes to the wording of their prompts while other completely changed the number and wording of their prompts. These changes were documented for each participant in the reported case studies and were summarized in the tables above. A concise summary of these findings as related to the relevant research questions for this chapter is provided in the table below. Then, additional differences between pre-service and in-service teachers' approaches to and work in task writing are addressed. Recall the descriptions of task quality as based on the framework:

- *Low:* Task does not contain a collection of prompts that coordinate mathematical depth and technological actions in such a way to require the student to make generalized conclusions based on emergent invariant relationships that go beyond a static sketch.

- *Medium:* Task contains a collection of prompts that coordinate mathematical depth and technological actions in such a way that may push to but does not necessitate that the student to make generalized conclusions based on emergent invariant relationships that go beyond a static sketch.
- *High:* Task contains a collection of prompts that coordinate mathematical depth and technological actions in such a way that requires the student to make generalized conclusions based on emergent invariant relationships that go beyond a static sketch.

Based on the extensive coding work completed and a major emergent theme of adding explaining to their revised directions, the researcher decided to further refine these qualitative descriptions. This small refinement only affects the description of high quality and is reported in italics for the readers benefit.

- *High:* Task contains a collection of prompts that coordinate mathematical depth and technological actions in such a way that requires the student to make and explain generalized conclusions based on emergent invariant relationships that go beyond a static sketch.

Table 45: Participant Task Findings Summary

Participant Number	Mathematics Knowledge for Teaching Geometry Score	Number of Prompts for Task and Revised Task	Summary Statement of Task Changes	Task One Quality	Task Two Quality
One		8 9	Adds a conjecture and an explanation	Medium	High
Two		4 9	Reorders Dragging and Measuring	Medium	High
Three		5 8	Coordinates Mathematical Depth with Technological Actions	Medium	High
Four		17 18	Incorporates Explaining	Medium	High
Five		8 7	Coordinates Dragging with Explaining	Medium	High
Six		14 18	Adds Dragging and Explaining	Low	High

All participants made changes to their original task and all increased their task ranking according to the Revised Dynamic Geometry Task Analysis Framework. Recall that Participants One through Three were pre-service teachers while Four through Six were in-service. Five of the six participants submitted revised tasks that had a greater number of prompts. This increase in prompts may not necessarily reflect an increase in task quality but may rather be due to revising an existing task as opposed to creating a new task. Results on number of prompts may have been different if participants were asked to create a completely new task based on a different learning goal. Regardless, all participants submitted initial tasks that ranked medium in quality according to the framework except Participant Six. Patterns emerged when comparing both groups of participants.

As seen in Table 44 above, in-service teachers submitted tasks that typically had more prompts than the tasks that pre-service teachers submitted. One possible explanation may be due to the in-service teachers' experience with students and the in-service teachers' perceived need for students to have detailed, step-by-step guidance. Participant Four's task, for example reflects a high degree of guidance concerning both the mathematical investigation and the technological aspects of The Geometer's Sketchpad. All but one participant increased the number of prompts they included from their first to their revised task. It may be inferred that the framework influences teachers to increase the number of prompts they write for Geometer's Sketchpad tasks, especially for pre-service teachers. A possible area for further investigation concerns how the framework may influence pre-service teachers to write tasks that look more similar to in-service teacher tasks concerning the breadth guidance provided to students.

Another difference that emerged was that of how pre-service teachers approached task writing compared to in-service teachers. In the first task submitted, all participants wrote tasks that focused students on some combination of dragging and measuring, but pre-service teachers more often wrote task prompts that told students what to look for thus eliminating any chance for discovery or surprise. This pattern remained true in the revised tasks as well. For example, unlike any pre-service teachers' revised tasks, Participant Six's revised task includes prompts coded F, "Prompt requires manipulation of the sketch that may surprise one exploring the relationships represented or cause one to refine thinking based on themes within the surprise." An exploration of potential causes of this difference is needed.

The research and analysis of data regarding the following research question was addressed in this chapter, “How do pre-service and in-service teachers use the Dynamic Geometry Task Analysis Framework?” Three sub-research questions were also addressed:

a. What are similarities and differences between teachers’ use of the framework for task writing for in-service and pre-service teachers?
b. What are similarities and differences in task quality for tasks written by in-service and pre-service teachers?
c. What are similarities and differences in task quality for tasks written by teachers who have different levels of knowledge for teaching geometry?

In what follows, the sub-research questions are first addressed followed by the overall research question that motivated the work documented in this chapter.

The similarities between pre-service and in-service teachers’ use of the framework outnumber the differences. Both groups recognized the framework as a useful guide for writing and assessing quality Geometer’s Sketchpad Tasks, with the former expressing their reliance more strongly. Participant Two, a pre-service teacher, explained, “I definitely think that it helps you figure out how to go from a mediocre task to a higher level task that uses all of the things that GSP is capable of.” The in-service teachers typically used less strong language as evidenced when Participant Five provided, “This framework is definitely valuable for a beginning teacher and without instruction from this framework many teachers would not use dragging.”

Furthermore, all participants viewed the framework as beneficial to revising their initial task. They each verbally responded to the interview question, “How did your use of the Dynamic Geometry Task Analysis Framework affect your thinking about task quality and task writing?” Participants are quoted at length to provide the reader an accurate impression of participant framework use and its impact. These results are summarized below.

Table 46: Participants’ Perceptions of Framework Use

Participant	**Response**
One	I knew how to use the tool but not in a way in reference to a framework. I’ve learned about incorporating the mathematical practices with this technology, but I had never really thought about it before having the framework. It really helps because there’s so much you can do with it and so much you can build with it like constructions and transformations. So having the framework gives recommendations on what you should do and what you might not need so much of. I liked my first task, but then I got more of an idea [from the framework] on what I could do to make it even better.
Two	Basically, I went back and I made the student think about the hypotheses before they did any measurements, and then didn’t really tell them what the measurements meant. Rather, they had to determine why this is equal to what that says; why rather than what. I wanted them to think outside just the numbers. Researcher: Can I paraphrase you and tell me if you agree or not and if you need to adjust my statement. It almost seems like instead of just telling them that this is a relationship that happens, it was more that they had to name the relationship based on what you made them focus on. In this case, it is the ratio between the longer side to the mid-segment being two regardless of dragging. Participant: Yes! Right!

Table 46 Continued

Participant	Response
Three	It gave me something to reference in regard to quality. In the second task I wrote, I went through categories N/A through 4 and 5 [on the framework], and made sure I hit those levels. I really wanted to include a question that was level 4 or 5. Researcher: How do you think a question that hits 4 or 5 helps student understanding? I think of course it does. So like in math education we talk about A implying B and B implying C so A implies C you know, and does it apply to the rest of them. So any time a student sees that something applies across the board is a good thing and a primary goal of math. Researcher: So when you say across the board, what do you mean by that? Participant: I mean that this theorem right here applies to all triangles.
Four	Right, so the first thing it made me think of is if I have my students explain enough—not just what they notice but why it is. And, another is to get them to verbalize their thought process and rationale. Then also I wanted to make sure I didn't have bad codes on the framework.
Five	It reminded me of some of the language I wanted to use as far as requiring them to explain. That was lacking in the first. I want them to go beyond the construction to see this for all triangles. Researcher: In this revised task, do you think the prompts do a better job to link the technological action with mathematical depth? Participant: I think that probably a little bit better and there's fewer steps now to make it easier for the kids to really see what's happening. It's still not a ton of links as there's still a lot of construct, construct, construct at the beginning. I moved the measure prompts in the second task to help student s make and test conjectures.
Six	The use of dragging is obviously really important in terms of making a generalization that it works for any triangle. This also reminded me of Bloom's taxonomy as you go from something simple or rote to something complex like making generalizations. I wanted them to go from something very simple like parallel lines and similar triangles to something very complex like what the learning goal is asking of me.

Based on participant responses, the framework had a marked impact on their perceptions of task quality. Recall that participants are ordered from least to greatest in each group (pre-service and in-service) according to how affected their revised tasks were by participant understanding of the framework. Thus, Participants One through Three were pre-service teachers with Participant Three being the most affected by the framework, while Participants Four through Six were in-service teachers with Participant Six being most affected. This trend of affects is likewise reflected in participant responses documented above regarding framework use. Participant One spoke to the small task changes he made while Participant Three explained, “In the second task I wrote, I went through categories N/A through 4 and 5 [on the framework], and made sure I hit those levels.” For the in-service teacher group, Participant Six spoke to the dramatic affect the framework had on his revised task writing, “The use of dragging is obviously really important in terms of making a generalization that it works for any triangle.” Participant Six had not incorporated dragging in his initial task. His comment reflects the change in quality from his initial task to his revised task in going from low to high.

Another similarity between both groups is participant use of conjecturing in their revised tasks. Although the Revised Dynamic Geometry Task Analysis Framework does not explicitly address conjecturing, many participants increased their use of requiring conjectures in their revised tasks. This may be due to the language of the framework that puts value on students’, “recognition of emergent invariant relationships,” and the possibility of “surpris[ing] one exploring the relationships.” Such recognitions and surprises may more often occur when the student has to refine what the previously thought as reported in a written conjecture. Another likely cause for the increased incorporation of conjecturing may be due to the article participants read in which tasks that included conjecturing were coded with the framework.

The third sub-research question addressed in this chapter concerns potential relationships between participants' knowledge for teaching Geometry and their task writing. It reads, "What are similarities and differences in task quality for tasks written by teachers who have different levels of knowledge for teaching geometry?" Data on this question was generated with an online test of teacher's Knowledge for Teaching Geometry. The creators of the assessment indicated that theta scores are the most appropriate measure of teacher knowledge as opposed to percentages correct due to the formers accounting of question difficulty based on item response theory. They also indicated that theta represents the number of standard deviations above or below the mean (z scores). We can therefore conclude that all participants scored above the mean of those who have taken this test before the conduction of this study. See Table 47 below for summary results.

Table 47: Summary of Participants' Knowledge for Teaching Geometry Scores

Participant Group	**Percent Correct out of 42**	**Theta Score**
Pre-Service Group	47.6	-.10
Participant One	54.8	.27
Participant Two	40.5	-.48
Participant Three	47.6	-.10
In-Service Group	69.8	1.093
Participant Four	71.4	1.18
Participant Five	73.8	1.33
Participant Six	64.3	.77

The pre-service teacher group, Participant One, Participant Two, and Participant Three scored $\phi = .27$, $\phi = -.48$, and $\phi = -.10$ respectively. In the in-service group, Participant Four, Participant Five, and Participant Six scored $\phi = 1.18$, $\phi = 1.33$, and $\phi = .77$ respectively. On average, the pre-service teacher group scored ϕ = -.10 while the in-service teacher group scored ϕ = 1.093. A two sample t-test was carried out to reveal that the likelihood of this difference occurring by chance is much less than .01, indicating a significant difference between group performance at an ϕ = .01 significance level.

Participants' performance on sub-content categories were also provided but without corresponding theta (standard deviation) scores. Of particular interest to this study was the Specialized Content Knowledge category. Herbst and Kosko (2014) explain that such knowledge is, "used particularly in doing tasks of teaching, such as, for example, the knowledge teachers need to use in writing the problems they will assign to students" (p. 24). In a sense, the study reported herein involves teachers' Specialized Content Knowledge within the general domain of dynamic geometry environments and in particular The Geometer's Sketchpad. The Participants' scores on Specialized Content Knowledge are summarized below:

Table 48: Summary of Participant Specialized Content Knowledge

Participant Group	Number Correct out of 20	Percent Correct (%)
Pre-Service Group	Average: 11.3	Average: 56.7
Participant One	14	70.0
Participant Two	9	45.0
Participant Three	11	55.0
In-Service Group	Average: 14.7	Average: 73.3
Participant Four	14	70.0
Participant Five	15	75
Participant Six	15	75

The in-service teacher group scored significantly higher than the pre-service group as may be expected due to the former's experience evaluating and writing tasks on a weekly bases for their students. This difference in Specialized Content Knowledge did not manifest in differences in task quality between both groups as labeled with coding from the Dynamic Geometry Task Analysis Framework. However, this difference did match other differences in submitted tasks such as number of included prompts and level of technical guidance.

Participant scores were also reported on the sub-category of Knowledge of Content and Teaching, which Herbst and Kosko (2014) defined as, "a combination of knowledge of teaching and knowledge of mathematics and includes the knowledge needed to decide on the best examples and representations to use for given instructional objectives" (p. 24). This domain of knowledge directly related to this study as participants considered what representations to include in their written tasks.

A summary of participant scores on Knowledge of Content and Teaching is provided below:

Table 49: Summary of Participants' Mathematics Knowledge of Content and Teaching

Participant Group	Number Correct out of 7	Percent Correct (%)
Pre-Service Group	Average: 1	Average: 14.3
Participant One	1	14.3
Participant Two	1	14.3
Participant Three	1	14.3
In-Service Group	Average: 4.7	Average: 66.7
Participant Four	6	85.7
Participant Five	5	71.4
Participant Six	3	43.0

These results coincided with the results on participants' Specialized Content Knowledge where the in-service group far outscored the pre-service group. Furthermore, within both groups, participants were labeled according to how affected their revised task was in regard to the framework. This ordering reflected the levels of Knowledge of Content and Teaching within the in-service group. For example, the last in-service teacher who submitted the most revised task evidenced the lowest score in this domain.

Recall the research question that prompted gathering this data, "What are similarities and differences in task quality for tasks written by teachers who have different levels of knowledge for teaching geometry?" From the information provided in Table 45, no specific pattern emerged concerning the comparison Mathematical Knowledge for Teaching Geometry scores and the quality of initially submitted tasks. All pre-service teachers initially submitted tasks that ranked medium in quality according to coding with the framework and revised tasks that ranked high. In the other group, two of the initially revised tasks ranked medium and the third ranked low. The ranking of low came as a result of this task lacking any call for the student to use dragging to confirm the relationships found in the learning goal.

Participants were numbered within each group from least affected by the framework to the most affected by the framework per task coding on each participant's initial and revised task. For Participants Three and Six, whose task writing was perceived by the researcher to be most affected by their framework use, it can be concluded that the lower a teachers' Mathematical Knowledge for Teaching corresponds to a greater reliance on the Dynamic Geometry Task Analysis Framework for improving task quality.

The overall research question for this chapter reads, "How do pre-service and in-service teachers use the Dynamic Geometry Task Analysis Framework?" One of the most important findings is that of increased coordination of mathematical depth with technological actions as found on all of the participants revised tasks. Recall all initially submitted tasks ranked below a label of high per the coding on the framework, while all revised tasks ranked high according to framework codes. Whether this change was reflected in a participant reordering dragging and measuring or including explanations or simply incorporating the technological actions of dragging, all participants submitted revised tasks that further coordinated mathematical depth with technological actions in comparison to their initially submitted tasks.

Another prominent trend in comparing participants' initial tasks with their revised tasks is that of incorporating explanations. The Allowance for Mathematical Depth code of four, "Prompt requires student to explain the mathematical concepts, processes, or relationships in the current sketch" only manifested once for all initial tasks, whereas it emerged no less than eight times in the revised tasks. Participant Four summarized this change well, "The first thing it [the framework] made me think of is if I have my students explain enough—not just what they notice but why it is." This reported effect of the framework speaks to its promise in helping teachers consider how to assess and write tasks for dynamic geometry environments that require students to utilize technological affordances build their understanding of why relationships within and among geometrical objects are true.

Conclusion

In conclusion, the effects of exposure to the Revised Dynamic Geometry Task Analysis Framework were explored for pre-service and in-service teachers. The findings suggest that this framework is beneficial in helping teachers recognize task quality in general and specifically according to task prompts. Participants wrote revised tasks that further coordinated student technological actions with mathematical depth. Furthermore, teachers articulated how and why the framework influenced their task writing approaches along these lines. This Revised Dynamic Geometry Task Analysis Framework has thus proven useful to mathematics teachers in prompting them to more deeply consider the tasks they afford their students in dynamic geometry environments such as The Geometer's Sketchpad.

CHAPTER 6: DISCUSSION AND IMPLICATIONS

This study was motivated by the perceived need for teachers to have guidance when evaluating and writing dynamic geometry tasks in general, and for The Geometer's Sketchpad in particular. This need is echoed in mathematics education literature as well. Connor, Moss, and Grover (2007) explained:

> It may be that, even for relatively sophisticated students, there needs to be instruction in how to incorporate the use of software programs into developing strategies for exploring the correctness of an assertion and, in particular, designing experiments to test the validity of a statement. (p. 62-63)

As teachers are responsible to provide such instruction for students who engage in technological environments, it behooves the mathematics education community to take the charge of developing research based practices to aid teachers seeking to utilize such environments. To that end, the researcher crafted the Dynamic Geometry Task Analysis framework as an aid for teachers concerned with task quality in dynamic geometry software environments.

This perceived need on behalf of the researcher was further motivated as part of work in a larger professional development program. Part of the research and design work of conducting this ongoing professional development involved assessing the quality of teacher submitted Geometer's Sketchpad tasks. The attempt to conduct this assessment led to the creation of the Dynamic Geometry Task Analysis Framework. This framework was used to assess the quality of teacher submitted tasks based on coding the prompts teachers wrote for their particular tasks. The framework proved immediately useful in providing a way assess and think about task quality in dynamic geometry environments. The researcher conjectured that this framework could likewise prove immediately useful for teachers who employee tasks for these environments into their instruction. To test this conjecture, a number of specific research question were developed to motivate and guide this study:

> How do the mathematical prompts and technological actions as identified in the Dynamic Geometry Task Analysis Framework relate to students' mathematical arguments and conclusions?

The purpose of addressing this question is to test the validity of the framework regarding how close the quality of student work matched the predicted quality of tasks as indicated by the framework. Before conducting research involving students, tasks and the framework, a face validity study was undertaken. In this portion of the study, three experts on the use of dynamic geometry software environments in mathematics education provided detailed written feedback on the potential of the framework for revealing task quality. All experts indicated that the framework would be of benefit to those concerned with task quality in dynamic geometry environments. This expert feedback was used to revise the framework and refine how the framework should be used best to rank tasks.

After the face validity was established, three tasks were crafted that ranked low, medium, and high on the framework. Recall the descriptions of low, medium, and high quality per the framework are the following:

- *Low:* Task does not contain a collection of prompts that coordinate mathematical depth and technological actions in such a way to require the student to make generalized conclusions based on emergent invariant relationships that go beyond a static sketch.
- *Medium:* Task contains a collection of prompts that coordinate mathematical depth and technological actions in such a way that may push to but does not necessitate that the student to make generalized conclusions based on emergent invariant relationships that go beyond a static sketch.
- *High:* Task contains a collection of prompts that coordinate mathematical depth and technological actions in such a way that requires the student to make and explain generalized conclusions based on emergent invariant relationships that go beyond a static sketch.

The Dynamic Geometry Task Analysis Framework underwent revisions based on expert feedback (Chapter 4) and use for coding participant submitted tasks (Chapter 5). As this study constitutes the dissertation work of the researcher, an additional amendment to the framework is based on a committee member's feedback. This change is to the language of technological action F, which not ends in the phrase, "that may be based on testing extreme cases." For the reader's benefit, the post revisions Dynamic Geometry Task Analysis Framework is included below. In previous portions of this reported study, this framework was at times referred to the Revised Dynamic Geometry Tasks Analysis Framework. Due to the conclusion of revisions of this work, the framework is hereinafter referred to as the Dynamic Geometry Task Analysis Framework.

Table 50: Dynamic Geometry Task Analysis Framework

Allowance for Mathematical Depth	
Levels	**Descriptions**
N/A	Prompt requires a technology task with no focus on mathematics.
0	Sketch does not have mathematical fidelity required to respond to prompt.
1	Prompt requires student to recall a math fact, rule, formula, or definition.
2	Prompt requires student to report information from the sketch. The student is not expected to provide an explanation.
3	Prompt requires student to consider the mathematical concepts, processes, or relationships in the current sketch.
4	Prompt requires student to explain the mathematical concepts, processes, or relationships in the current sketch.
5	Prompt requires student to go beyond the current construction and generalize mathematical concepts, processes, or relationships.
Types of Technological Action	
Affordances	**Descriptions**
N/A	Prompt requires no drawing, construction, measurement, or manipulation of current sketch.
A	Prompt requires drawing within current sketch.
B	Prompt requires measurement within current sketch.
C	Prompt requires construction within current sketch.
D	Prompt requires dragging or use of other dynamic aspects of the sketch.
E	Prompt requires a manipulation of the sketch that allows for recognition of emergent invariant relationship(s) or pattern(s) among or within geometrical object(s).
F	Prompt requires manipulation of the sketch that may surprise one exploring the relationships represented or cause one to refine thinking based on themes within the surprise that may be based on testing extreme cases. (adapted from Sinclair (2003), p. 312).

After the framework was revised, it was used to design tasks (low, medium, high) that were used by students to determine if there was a relationship between the responses of students and the quality of the tasks. Twelve students participated in the study with four students randomly assigned to one of three tasks (low, medium, high). The core of Toulmin's (1958) argumentation model was used to assess the quality of student argumentation on each task. Results showed that students that completed a low-ranked task had fewer instances of quality argumentation components as defined by Toulmin. Students who completed the medium ranked task had a greater number of quality argumentation components while students who completed the high-ranked task had the highest instances of quality argumentation components. For example, Toulmin defines one argumentation component, warrant, as “the general logical statements that serve as bridges between the claim and the data” (p. 98). Results showed that five warrants, eight warrants, and 10 warrants were evident on student work for the low-, medium-, and high-ranked tasks, respectively. The overall results for this portion of the study served to establish the validity of the framework for labeling task quality. After the first research question and sub-questions were addressed, the study continued with engaging teachers with the framework.

The heart of this study involves the exploration of teachers’ use and perceptions of the framework. A purposeful sample of six was arranged, with three participants being pre-service teachers and three being in-service. A comparison of these two groups provided a convenient way to assess the effectiveness of the framework with the professionals it is purported to help, teachers. With this assessment endeavor in mind, the following overarching and sub-research questions were crafted:

How do pre-service and in-service teachers use the Dynamic Geometry Task Analysis Framework?

a. What are similarities and differences between teachers’ use of the framework for task writing for in-service and pre-service teachers?

b. What are similarities and differences in task quality for tasks written by in-service and pre-service teachers?

c. What are similarities and differences in task quality for tasks written by teachers who have different levels of knowledge for teaching geometry?

These research questions guided the methodology of engaging participants with the framework in a way to assess the effects of the framework on teachers' knowledge for writing quality dynamic geometry tasks in the program The Geometer's Sketchpad. Each participant met with the researcher and wrote a task in The Geometer's Sketchpad based on a particular learning goal. This activity was followed with the participant reading a practitioner article that the researcher wrote about the framework. The researcher then engaged the participant in a conversation about the framework and how to code task prompts with the framework. Sample tasks were visited in this conversation and partial coding carried out. After this engagement with the framework, participants revised their initially submitted task and asked to make any desired changes in regard to what was experience in the framework tutorial session.

Data produced in these teacher engagement sessions revealed that all participants made changes to their initially submitted task. The six initially submitted tasks and six revised tasks were coded by the researcher with the framework. The codes revealed that all tasks increased in quality per the descriptors low, medium, and high. Furthermore, all participants reflected positively on the usefulness of the framework for recognizing and writing quality tasks. While many similarities emerged between the two groups and their respective use of the framework, a number of differences also emerged related to two of the sub-questions addressed in this portion of the study.

The first sub-research question reads, "What are similarities and differences between teachers' use of the framework for task writing for in-service and pre-service teachers?" In general, pre-service teachers relied more heavily on the framework for guidance when revising their tasks. For example, the pre-service teacher, Participant Three explained:

> It gave me something to reference in regard to quality. In the second task I wrote, I went through categories N/A through 4 and 5 [on the framework], and made sure I hit those levels. I really wanted to include a question that was level 4 or 5.

Another difference that emerged regards participants' impressions on the usefulness of the framework for teachers as a whole. In-service teachers were slightly less enthusiastic regarding this query. Recall, all participants responded to the interview question, "Do you see the Dynamic Geometry Task Analysis Framework as a valuable tool for teachers? Why or why not?" An in-service teacher, Participant Five reflected:

> I see the measuring, explaining, and testing as important and definitely the dragging. Dragging is the big one! This framework is definitely valuable for a beginning teacher and without instruction from this framework many teachers would not use dragging.

Like the difference between the degree to which pre-service and in-service teachers relied on the framework to make task revisions, varying perspectives on its broader usefulness for mathematics teachers were evident between the pre-service and in-service groups.

The third sub-research question involves an exploration of participants' knowledge for teaching Geometry related to the quality of tasks submitted. This research question reads, "What are similarities and differences in task quality for tasks written by teachers who have different levels of knowledge for teaching geometry?" Based on the data analysis documented in Chapter 5, no specific pattern emerged concerning the comparison Mathematical Knowledge for Teaching Geometry scores and the quality of initially submitted tasks Results showed that across the board pre-service teachers evidenced lower Mathematics Knowledge for Teaching Geometry than their in-service counterparts. All pre-service teachers initially submitted tasks that ranked medium in quality according to coding with the framework and revised tasks that ranked high. In the other group, two of the initially revised tasks ranked medium and the third ranked low. The ranking of low came as a result of this task lacking any call for the student to use dragging to confirm the relationships found in the learning goal. While this task ranked low according to the framework, it is important to note that it represented a novel approach to achieving the learning relative to the other five initially submitted tasks. While the quality of initially submitted tasks did not appear to relate to participants' Mathematics Knowledge for Teaching Geometry scores, other patterns emerged as reported in Chapter 5 such as higher scores relating to providing more technological directions.

After the engagement with the framework, all participants submitted tasks that ranked high according to framework coding. At this level of analysis, it seems that both participant groups quality of revised task submissions were unrelated to participants' Knowledge for

Teaching Geometry. A significant difference emerged between pre-service and in-service teachers' average scores on the test with in-service far outperforming the pre-service group. In-service teachers' higher performance seemed to correlate to differences in task structure and other nuances that were not captured in the task ranking per the Dynamic Geometry Task Analysis Framework.

A summary of the research questions addressed and findings is provided below:

Table 51: Summary of Research Questions and Findings

Research Question	**Summary of Findings**
1. How do the mathematical prompts and technological actions as identified in the Dynamic Geometry Task Analysis Framework relate to students' mathematical arguments and conclusions?	Task quality low, medium, and high as labeled by framework coding generally matches the differences in the quality of arguments students provide and the conclusions they reach. An emergent pattern was that of few instances of warrants in student work on tasks that ranked low according to framework coding.
2. How do pre-service and in-service teachers use the Dynamic Geometry Task Analysis Framework?	All participants submitted revised tasks that ranked at least one quality level higher according to coding results on the Revised Dynamic Geometry Task Analysis Framework.
2a. What are similarities and differences between teacher's use of the framework for task writing for in-service and pre-service teachers?	All participants utilized the framework to improve the task quality of their initially submitted task, however pre-service teachers expressed a greater reliance on the framework to do so.
2b. What are similarities and differences in task quality for tasks written by in-service and pre-service teachers?	The three participating pre-service teachers initially submitted tasks that ranked medium according to framework coding. The three in-service teachers initially submitted two medium ranking tasks and on low ranking task.
2c. What are similarities and differences in task quality for tasks written by teachers of various levels of knowledge for teaching geometry?	In-service teacher participants had lower scores on the Mathematics Knowledge for Teaching Geometry test however their initial task rankings were higher than the in-service teacher group. All participants submitted revised tasks that ranked high according to framework coding.

Utilization of Dynamic Geometry Software Environments in Context

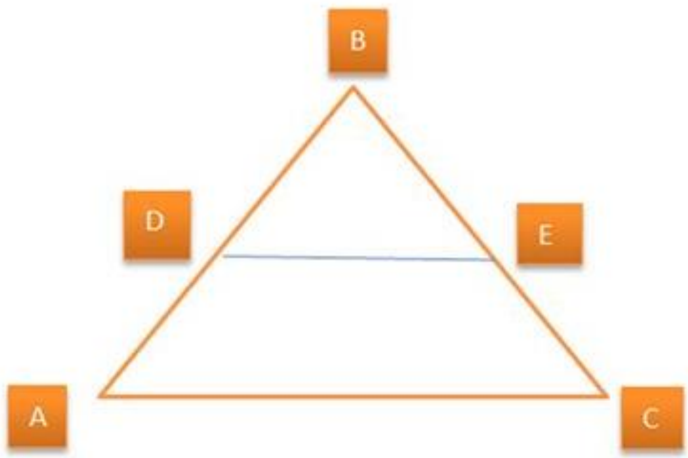
In regard to the research questions addressed and associated findings, a discussion of mathematical work within the medium, dynamic geometry environments is in order. Numerous researchers have explored how dynamic geometry environments have changed the way students interact with geometrical objects and learn Geometry. This change represents an opportunity to exploit the dynamic affordances of such environments. More than two decades ago, Kaput (1992) spelled out this opportunity, “One very important aspect of mathematical thinking is the abstraction of invariance. But, of course, to recognize invariance—to see what stays the same—one must have variation. Dynamic media inherently makes variation easier to achieve” (p. 525). The challenge is then how to guide student work in which dynamic affordances are used to recognize and understand invariant relationships or theorems.

Before the advent of dynamic geometry environments, secondary mathematics curricula in general and Geometry curricula in particular often included scaffolds to student proof activity in the form of two-column formats. These formats provided students a structure within which to document a deductive argument. These formats typically began with a place for the student to record given information and a place to document a ‘prove’ statement. This statement served as a goal that students would reach in the last row of their two-column proof, thus taking exploration out of student work. For the novice attempting to write such proofs, curriculum writers included partially completed rows in this two-column format. See Figure 11 below for a typical example. Potentially removed portions for a novice student proving task are indicated parenthetically in bold.

Instructions: Complete given two-column proof.

Prove: The mid-segment of a triangle is half of the third side.

Given: Triangle ABC with mid-segment DE and congruent angles BDE and BAD



Statement	Reason(s)
1) **(DE is mid-segment of Triangle ABC)**	1) Given
2) AD=DB; CE=EB	2) **(Definition of Mid-segment)**
3) AB=AB; CB=CB	3) **(Reflexive Property of Equality)**
4) AD+DB=AB; CE+EB=CB	4) Segment Addition Postulate
5) **(DB +DB=AB; CE+CE=CB)**	5) Substitution
6) 2DB=AB; 2EB=CB	6) Algebra: Combine Like Terms
7) Angle BDE = BAD	7) **(Congruence implies equal measures)**
8) Triangle ABC is Similar to Triangle DBE	8) **(Side-Angle-Side for Triangle Congruence)**
9) (AB=2DE)	9) Theorem: Corresponding sides of similar triangles are congruent.
10) **(The mid-segment of a triangle is half the third side.)**	10) QED

Figure 11: Sample Two-Column Proof Task

As students progressed with their ability to document proofs, the scaffold of providing pieces of the two-column proof (i.e., argument) would be incrementally removed. In some high school Geometry classes students would eventually be expected to document two-column proofs with no scaffolds. This accomplishment typically represented the height of student proving work. In all proof work, the student was to relate their statements and reasons to a static diagram. Depending on college career and major, students may have visited proof again. Proof work at this juncture typically would not necessitate a two-column format.

Recall the purpose of the Dynamic Geometry Task Analysis Framework is to offer teachers guidance for choosing and writing tasks for dynamic geometry environments. In a similar fashion to the intent of curriculum writers from years past to scaffold Student Two-column proof work, the researcher's intent was to implore teachers to consider the scaffolds they provide students' work in dynamic geometry environments. The major difference between the scaffolds that the Dynamic Geometry Task Analysis Framework encourages and those from curricular past is that the former encourages the utilization of affordances unique to dynamic environments. Such affordances have changed the way students interact with geometry objects.

The learning goal that participants used in this study read, "justify that the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length." The submitted tasks that ranked higher per coding with the Dynamic Geometry Task Analysis Framework typically had a higher number of prompts and refined requests for student noticing and explanations. Furthermore, these prompts typically required students to notice invariant relationships under dragging. It can be surmised that these higher ranking tasks had more and better scaffolds that correspond to higher likelihoods that hypothetical students completing the task attain the associated task learning goal using technological affordances. The Dynamic Geometry Task Analysis Framework bases task rankings on the prompts or scaffolds provided to the student and not the statement to be justified or proved in the task. Higher task ranking then corresponds to more highly scaffolded tasks.

A question that may arise in the reader's mind is one concerning task quality. The researcher has considered this question during the course of conducting and documenting this study, and has likewise considered if a task that contains more prompts and more refined requests for student explanations while utilizing technological affordances is actually of higher quality compared to a task that does not. The researcher has come to an evolving conclusion that a task's quality depends not only on its prompts, but also on the user it purports to educate. In light of this posited notion, it can be concluded that a task that ranks high for one group of students may rank low for other groups of students. In an ideal situation, a teacher could give a student a learning goal in a dynamic geometry environment such as, "justify that the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length," and then have the student independently complete all necessary steps to document their argument in a dynamic geometry environment. But this ideal will not likely be realized without prerequisite intermediate steps. Perhaps the Dynamic Geometry Task Analysis Framework will encourage teachers to provide task scaffolds that lead students to independently justifying conclusions of learning goals in such environments at some point in that student's mathematical future.

Study Limitations

Any study has its limitations and the study at hand is no exception. At the beginning of this study, experts in the field of mathematics were contacted with the request to provide feedback concerning the face validity of the Dynamic Geometry Task Analysis Framework and its use to rank tasks. Each expert provided detailed written feedback that the researcher used to make amendments to the framework and its applications for task ranking. Two study limitations are associated with the expert feedback work. First, only three experts provided feedback. A greater number of experts may have caused the researcher to make more nuanced changes to the framework and it applications for task coding. For instance, the researcher made a change to the framework based on his coding of teacher written tasks. It is possible that this change would not have come so late in the study if additional experts provided feedback that would have necessitated this change earlier in study's progression.

The second limitation is the lack of revisiting experts for their feedback on researcher choices to amend the framework and how tasks are ranked accordingly. A second round of expert feedback may have resulted in a stronger study. For instance, the change to task ranking methodology of further distinguishing between medium- and high-level tasks in having the latter require student explanations may have strengthened this study. However, due to pragmatic constraints of study design, data gathering, analysis, and documentation inherent to the field of mathematics education disallowed the researcher from gathering a second round of feedback. Another request for and documentation of expert feedback concerning this documented study as a whole may be in order to inform subsequent research steps. Such work may be carried out as a path for future research.

After expert feedback was gathered to inform changes to the Dynamic Geometry Task Analysis Framework and coding, the researcher engaged twelve students with tasks that were ranked according to framework coding. Four students individually completed a task that ranked low; four completed a task that ranked medium, and four completed a task that ranked high. High school students who volunteered for the study were randomly assigned to each of the three aforementioned groups. All student participants were either Juniors or Seniors with varying backgrounds in Mathematics. Because student mathematical background information was not gathered until the time task interviews were conducted, the study did not control for the potential confounding variable of student mathematical knowledge as evidence by their background in mathematical coursework. It can be seen in student background information that a fairly even spread of student background knowledge was represented for each student group (e.g., students who completed the task that ranked high). However, a study design that further controlled for student background knowledge would have been stronger than the one reported herein. Such an expanded effort may be possible in the future with increased funding for work in this area.

A limitation of this study that deems addressing is the limited ability to generalize based on a small sample size and qualitative data analysis. Only three pre-service teachers and three in-service teachers participated. This limited sample size occurred as a result of two main limiting factors. First, the researcher found it quite difficult to find undergraduate students who had taken the desired technology course in mathematics education and who were willing to participate in this study. Secondly, due the constraint of having only one researcher a limited number of task writing interviews could pragmatically take place. Like the other limitations delineated above, this limitation could easily be overcome with funded research team. However, the inability to make generalizations is not as easily overcome.

The final limitation to mention to is that of the subjective nature of qualitative data analysis. Twelve student task completion interviews and six teacher task writing interviews were conducted and transcribed. These transcriptions were referenced to support conclusions based on student or teacher submitted work. The choice of what transcript portions to reference was left to the discretion of the researcher. Study limitations due to the subjective nature human perception and qualitative research can be ameliorated with funded research teams who are, for example, able to cross check coding to establish reliability. When checks of this nature are in place the reliability of the studies will increase.

Implications for Mathematics Education

Numerous implications emerged for the field of mathematics education as a result of this study. Based on limited study results, it can be concluded that the Dynamic Geometry Task Analysis Framework proves beneficial for teachers seeking to find and write quality tasks for dynamic geometry task environments. The development of this framework was influenced by another task framework. This framework was developed by Smith and Stein (1998) can be used to assess the quality of mathematical tasks. Other work has been done to indicate qualities of pre-constructed dynamic geometry tasks (Sinclair, 2003). The framework that served as the focus of this study differentiates itself from products of the work of Smith and Stein or that of Sinclair as it considers task quality based on its collection of prompts and they degree to which they coordinate mathematical depth with technological actions related to quality student argumentation. The focus on coordination puts the Dynamic Task Analysis Framework in a unique position in mathematics education as it is the first framework to address this coordination. Implications of this framework are pertinent to both practitioners and researchers. Due to the potential overlapping nature of practitioners and researchers work in the field, the following delineated implications may likely be of interest to both constituents.

Practitioners

Based on the work from Chapter 5 of this documented research, the Dynamic Geometry Task Analysis Framework seems to aid practitioners seeking to recognize aspects of task quality based on a task's prompts. The framework then has implications for the field of mathematics education regarding practitioner use. The following implications emerged based on this study's findings.

Participating pre-service and in-service teachers evidenced two dominant patterns in their use of the Dynamic Geometry Task Analysis Framework. First, revised tasks were characterized by a higher degree of prompts that coordinate mathematical depth with technological action. Recall that all revised ranked high according to task coding with the framework. The description of a high task is included below:

> Task contains a collection of prompts that coordinate mathematical depth and technological actions in such a way that requires the student to make and explain generalized conclusions based on emergent invariant relationships that go beyond a static sketch.

As found in the description of a high task and the documented findings in Chapter 5, revised tasks also included requests for students to explain their thinking often in regard to not only the 'what' and 'how' of a conclusion but also the 'why.' Such detailed requests for explanations can likewise be found in participating teachers' reflections on their framework use. These findings suggest a high degree of framework usability amongst practitioners.

Based on such findings, the following list of questions was created as a guideline for practitioners to use in reference to the Dynamic Geometry Task Analysis Framework when finding and creating high quality tasks for dynamic geometry environments in general and the Geometer's Sketchpad in particular.

- Does the task contain prompts that utilize the dynamic affordance of the software such as dragging?
- Do the task's prompts relate and/or coordinate technological actions with mathematical explanations?
- Do explanations focus on why mathematical conclusions are valid based on requested student work in the task?

Tasks that reflect answers of 'yes' to these questions will likely rank higher according to prompt coding with the Dynamic Geometry Task Analysis Framework. These questions are meant to be used by practitioners to quickly assess task quality without having to perform a complete coding of task prompts.

Researchers

The purpose of this study was to investigate how a Dynamic Geometry Task Analysis Framework effectively classifies Geometer's Sketchpad tasks in regard to students' mathematical work and affects teachers' understanding of what constitutes quality in The Geometer's Sketchpad tasks and their writing of such tasks. The reported findings represent a beginning of investigating the usefulness of this framework. Throughout human history tools have been continually refined and have influenced the creation of other tools. This framework likewise represents a tool to be refined by mathematics education researchers and a tool to influence the creation of other tools. Therefore, an implication of this study on the field of mathematics education is that of further investigating the framework's usefulness in broader regards. One broader regard is the investigation of the framework's usability for tasks written for other dynamic geometry software such as Geogebra. It is the speculation of the researcher that task qualities revealed by the Dynamic Geometry Framework coding would likewise be revealed for tasks in these similar dynamic geometry software platforms as such platforms also offer the unique affordance of dragging to test conjectures. However, the speculation needs to be tested and represents an avenue for further research.

To investigate other broader regards, the methodology of having pre-service and in-service teachers create dynamic geometry tasks without knowledge of the Dynamic Geometry Task Analysis Framework, read and discuss an article that introduces the framework (treatment), and then revise their initially submitted tasks is recommended to be employed for interested parties. Based on the quality of findings using this process, it is accordingly recommended that the process be used to enhance mathematics teacher education programs and in-service teacher professional development. This methodology is hence viewed as a contribution to the field of mathematics education.

One potential path for investigation involves the effectiveness of the framework for changing teachers' knowledge of task quality for larger groups of teachers. Comparison studies could be carried out that have, as a control group, teachers who frequently use dynamic geometry environments, and, as an experimental group, teachers who frequently use dynamic geometry environments and utilize the Dynamic Geometry Task Analysis Framework for choosing and writing tasks. Many comparisons could be analyzed for each group such as student satisfaction and learning outcomes.

Another path for investigation involves how teacher use of the framework may influence what mathematical norms evolve for student work. Goos, Soury-Lavergne, Assude, Brown, Kong, Glover, Grugeon, Laborde, Lavicza, Miller, and Sinclair (2010) analyzed and compared, "various theoretical frameworks that illuminate the teacher's role in technology-integrated learning environments and the inter-relationship between factors influencing teachers' use of digital technologies" (p. 311). Goos et al. ended their chapter with a call for further research including the development of, "theories of practice to inform pre-service and in-service teacher education, in order to strengthen recommendations about what counts as "progress" or "success" in technology integration" (p. 327). A possible research question aligned with this call could read, “What mathematical norms may develop in student work as students complete tasks that rank low, medium, and high in the Dynamic Geometry Task Analysis Framework?” Evolving answers to this question are of particular importance as it addresses what task scaffolds need to be in place for novice, growing, and expert teacher and student users of dynamic geometry environments. Instructional and curricular recommendations would then be developed in an effort to more fully realize the potential dynamic geometry environments for the betterment of student understanding.

Conclusion

Based on the limited data gathered and analyzed for this study, it can be concluded that the Dynamic Geometry Task Analysis Framework shows promise for helping teachers recognize and write quality tasks in dynamic geometry environments in general and Geometer’s Sketchpad in particular. The development of this framework spawned from an effort to analyze the quality of teacher submitted Geometer’s Sketchpad tasks during a professional development program. A product of this effort was a framework that was predicted to help teachers seeking to utilize dynamic geometry environments in their instruction. In this study, the framework underwent revisions and was put to a test concerning its usefulness for teachers. The results of this test do not represent an end in and of themselves but rather a foundation upon which to further refine this tool and influence the creation of other tools that concern those interested with the utilization of emerging technologies for the enhancement of enacted curricula.

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APPENDICES

Appendix A: Student Background and Geometer's Sketchpad Use Questionnaire

1. What grade are you in and what math classes have you taken so far in high school?
2. Tell me about your experience in Geometry. Did you enjoy the class? How well did you do? Is there a part of the class you liked best? If so, tell me about it.
3. Have you used Geometer's Sketchpad before? How often did you use it when taking Geometry or another math class?
4. Describe how you typically used Geometer's Sketchpad to the best of your memory.

Appendix B: Structured, Task-Based Interview Student Interview Questionnaire

Please read each prompt beginning with the first. After you read each prompt, please respond to the following questions:

1. In your own words, what is the prompt telling you to do?
2. What did you do in Geometer's Sketchpad and/or in writing to respond to the prompt?
3. What have you learned from responding to this prompt?
4. Describe what you started with in the sketch, what you did, and what you accomplished or learned at the end of the task.
5. What else would you like to share about your work with this task?

Appendix C: Low-, Medium-, and High-Quality Geometer's Sketchpad Tasks

Low Task

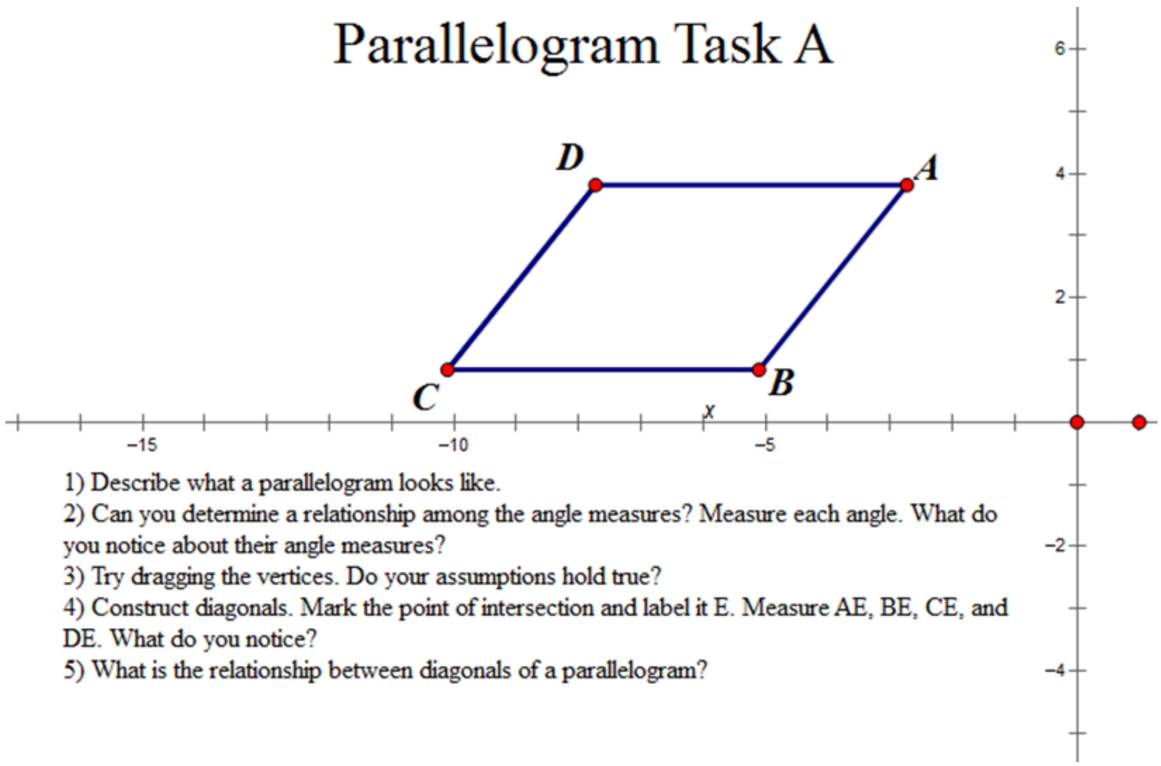

Figure 4: Low-Level GSP Task

Medium Task

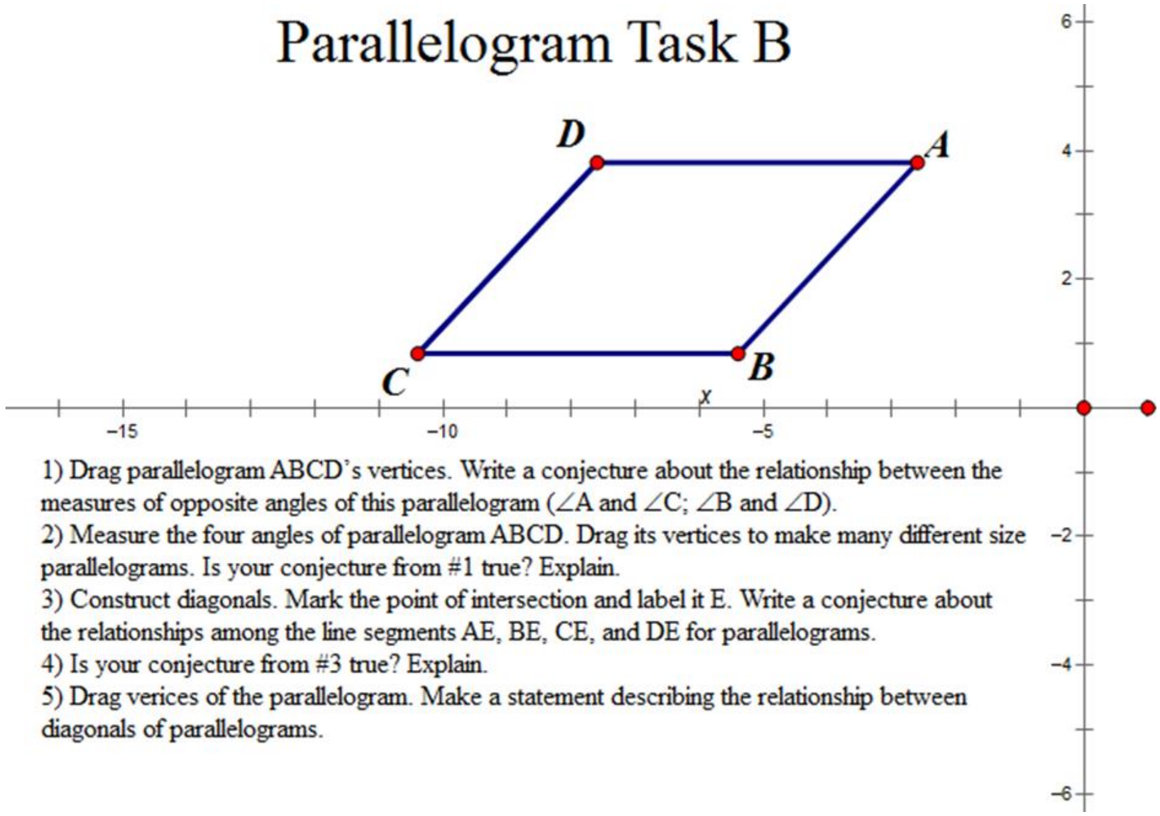

Figure 5: Medium-Level GSP Task

High Task

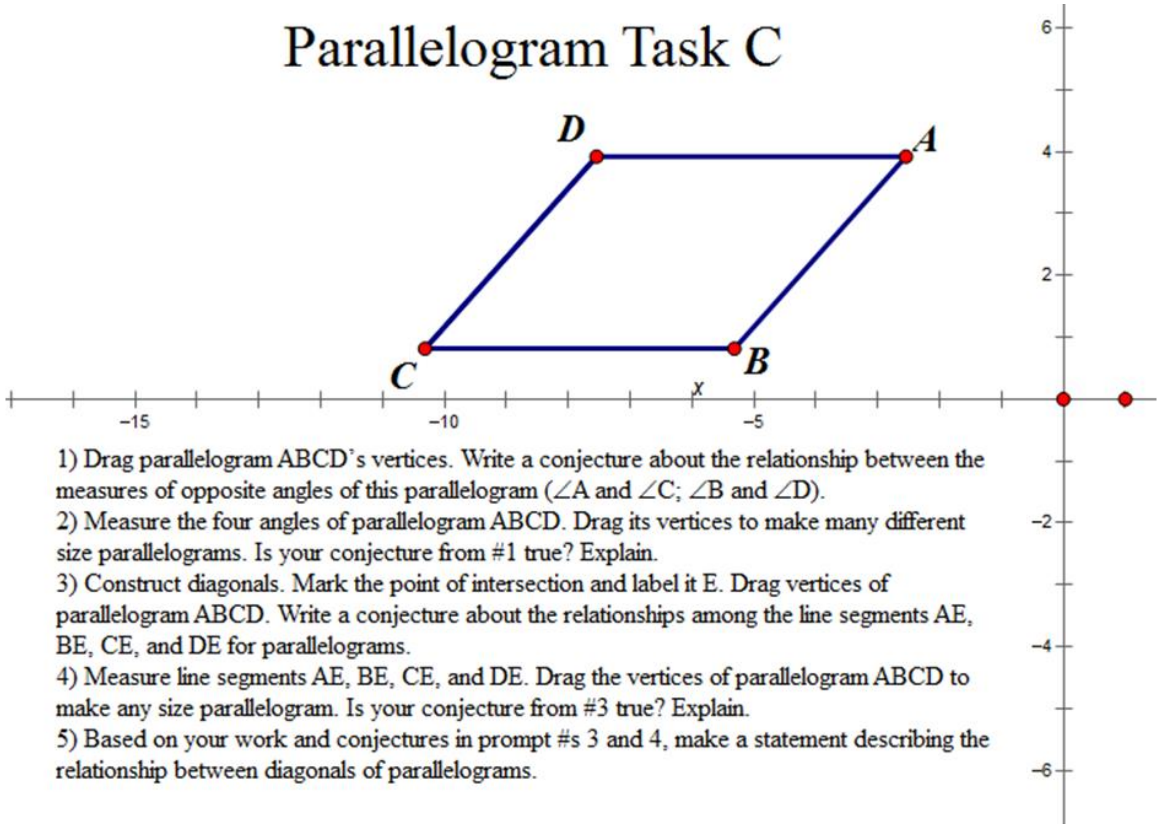

Figure 6: High-Level GSP Task

Appendix D: Teacher Background and Dynamic Geometry Use Questionnaire

Instructions: Please respond to each prompt as fully and accurately as possible. The information you provide will greatly enhance the quality of this study.

1. Describe your educational background including degree(s) completed along with major and/or minor courses of study.
2. Describe what drew you to teaching and if applicable an account of your teaching experience including grades and subjects taught and leadership experiences.
3. Describe when, where and to what extent you have learned about and how to use Dynamic Geometry Software (DGS). Name any relevant DGS programs and particularly speak to when, where and to what extent you have learned about and how to use Geometer's Sketchpad.
4. As applicable, describe how you have incorporated Geometer's Sketchpad in your teaching experience.
5. Have you ever written a student task within Geometer's Sketchpad? If so, describe one or two examples of your writing to the best of your ability. For any tasks you describe, please include your impression of the quality of student engagement and learning.
6. What are potential benefits of using Geometer's Sketchpad (or another similar Dynamic Geometry Software) in your teaching (include future teaching as applicable)? Please be as specific as possible.
7. Describe characteristics of a high quality task in Geometer's Sketchpad.

Appendix E: Article: Evaluating and Writing Dynamic Geometry Tasks

High school geometry teachers have traditionally used classroom tasks that require students to make compass and straightedge constructions and respond to associated prompts. For instance, a typical high school geometry task is one where students use a compass and straight edge to construct a line perpendicular to a given line. An associated written prompt is often included that requires students to justify why the steps used in their construction generate a perpendicular line. The advent of dynamic geometry software has changed the way students draw, construct, and measure by using virtual not physical tools. Students create, test, and verify conjectures by using a mouse to drag objects across a computer screen or the viewing window of calculator or other mobile device.

Technology use in general and dynamic geometry in particular has gained traction in mathematics education as evidenced in the Common Core State Standards for Mathematics ([CCSSM], National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010) The CCSSM High School Geometry standards require students to “make formal geometric constructions with a variety of tools and methods” (p. 76). Dynamic geometry software is among the tools listed. Further, Hollebrands (2003) has noted that within such dynamic environments students have opportunities to consider invariant relationships through dragging as well as make corresponding conjectures and conclusions. Dynamic geometry software is a recommended tool for students to experiment and build their mathematical understanding.

While research has shown the potential to use dynamic geometry tasks as tool for mathematical reasoning, little guidance is provided to teachers for evaluating the quality of dynamic geometry tasks much less for writing their own. The purpose of this article is to introduce a framework for analyzing and writing dynamic geometry tasks that are designed to engage students in mathematical reasoning. We begin by asking the reader to compare two sample tasks, each of which is designed to engage students in developing and testing conjectures about parallelograms. We then introduce the framework and illustrate how it can be used to evaluate the potential of each task in accomplishing the desired result.

The Parallelogram Tasks

Consider Parallelogram Tasks 1 and 2 provided in Figure 1. For each, students are provided with a pre-prepared sketch in the dynamic geometry environment. In this case, the given sketch is the quadrilateral ABCD that is constructed to be a parallelogram. That is, no matter what students do to quadrilateral ABCD by dragging edges or vertices, it will always be a parallelogram. Each task therefore possesses mathematical fidelity, defined for a sketch as possessing necessary mathematical characteristics to guarantee the objects in the sketch are what they are claimed to be. When analyzing dynamic geometry tasks, checking for mathematical fidelity is crucial since without this the remainder of the task may become meaningless. Read each task and consider: Which task contains prompts that push the student to experiment and build their mathematical understanding?

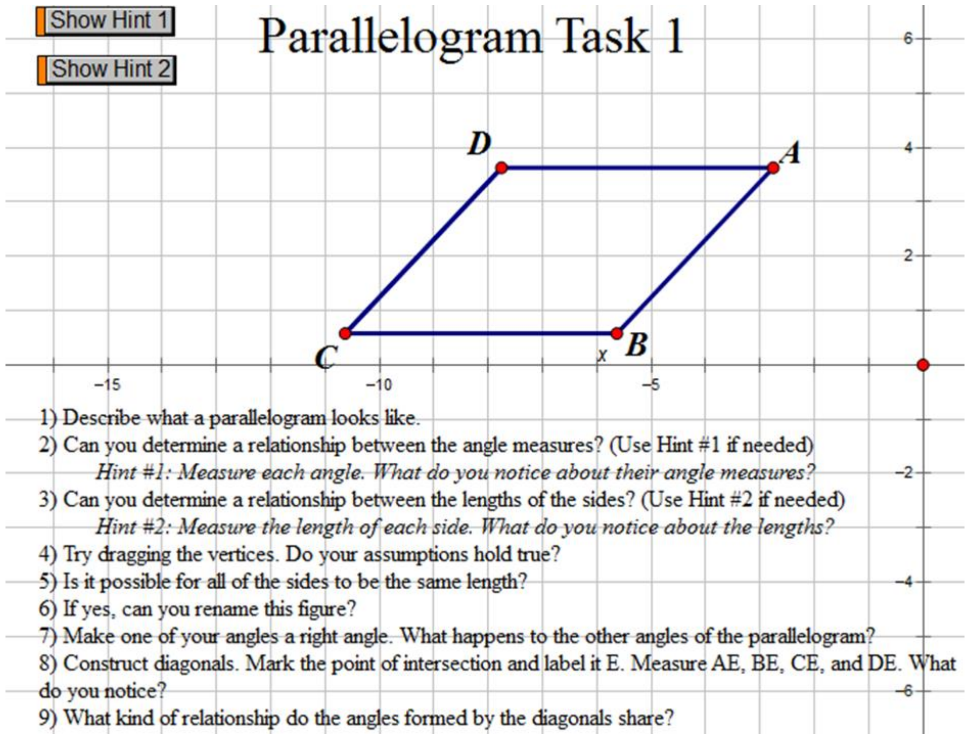

Figure 12: Parallelogram Task 1

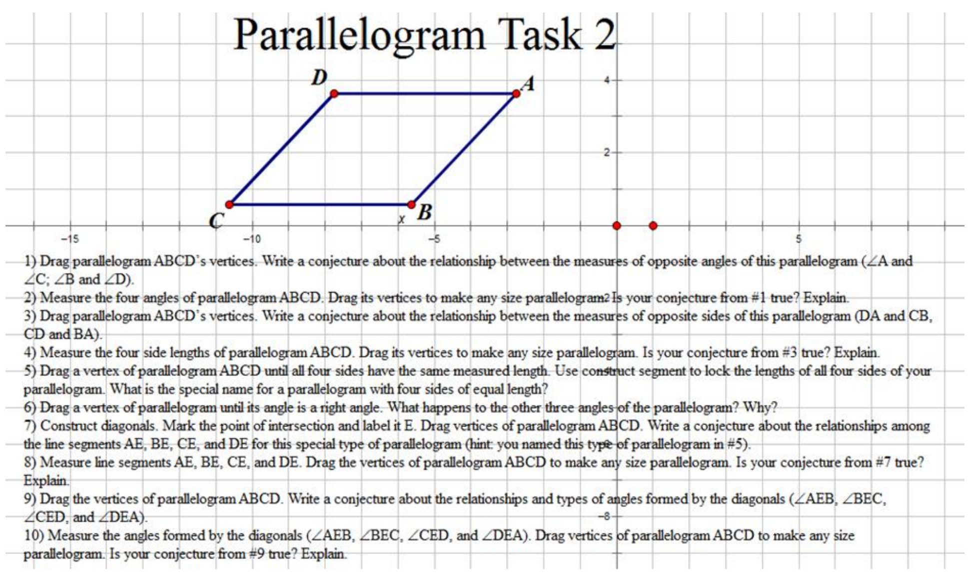

Figure 3: Parallelogram Task 2

Now that you've read through each task revisit the question: Which task contains prompts that encourage the student to experiment and build their mathematical understanding? In your evaluation you likely noticed differences in the demands of the student in each task. Consider how you would explain these differences to a colleague and then decide which task you would consider using in the classroom.

Context and Need

The impetus for this work originated in a professional development project in which the following research question was addressed: How does the design and use of tasks that incorporate dynamic geometry software support teacher-facilitated mathematical discussions? The professional development project included 24 high school geometry teachers and had as an overarching goal to increase student success in high school geometry so that they would be able to pursue Science Technology Engineering and Math (STEM) related college majors and careers. Part of the professional learning involved teachers writing tasks for the software Geometer's Sketchpad. Once submitted, the research team analyzed video footage of classroom enactments of these tasks and attended to task characteristics and the emergent discourse.

Much like traditional geometry construction tasks, many of these tasks required students to draw, measure, and construct in dynamic environments and respond to associated prompts while completing the task. Unlike traditional geometry construction tasks, some of these dynamic geometry tasks required the use of the technological action of dragging. We considered a high quality dynamic geometry task as one in which both the mathematical prompts and technological actions combine to focus students on understanding concepts, processes, and relationships as the sketch evolves.

Description of Framework

The research base for the framework is composed of the work of Smith and Stein (1998) and Sinclair (2003). In the mid-1990's Smith and Stein (1998) used Doyle's (1988) research on characteristics of academic tasks and Resnick's (1987) work on high-level thinking skills to analyze hundreds of mathematical tasks. This led them to define four categories of cognitive demand associated with classroom tasks: memorization, procedures without connections, procedures with connections, and doing mathematics (p. 348). Tasks that require a low level of cognitive demand are labeled either memorization or procedures without connections while tasks that require a higher level of cognitive demand are labeled procedures with connections or doing mathematics. Note that the level of cognitive demand depends heavily upon the students' prior experiences, not just on how the task is phrased. What might be a routine task for one student could lead to an in-depth mathematical exploration for another. Also, Sinclair (2003) offers recommendations to promote student exploration with pre-constructed dynamic geometry sketches. However, as we found in our analysis, tasks in which the technological actions were coordinated with the mathematical prompts seemed to have higher potential for engaging students in mathematical reasoning. There is an identified need in research and teaching to evaluate the potential of such dynamic geometry tasks. To this end, we developed a framework that identifies characteristics of prompts associated with how dynamic geometry tasks are written.

Table 1 presents the Framework for Analyzing Dynamic Geometry Tasks. The framework is designed to identify qualities of dynamic geometry tasks that are best suited to support mathematical learning goals outlined in the CCSSM. The framework is divided into two components: 1) allowance for mathematical depth; and 2) types of technological actions. These two divisions emerged after analyzing the teacher-written tasks. The levels of allowance for mathematical depth are based on the work of Smith and Stein (1998), whereas Sinclair's work (2003) provided a basis for the types of technological prompts.

We defined a mathematical prompt to be a written question or direction related to a sketch that requires a verbal or written response. Likewise, we defined a technological action to be a question or direction that requires a drawing, construction, or measurement within or manipulation of a sketch. Many times a prompt is both mathematical and technological. For example, Prompt 7 from the first task in Figure 1 is, "Make one of your angles a right angle. What happens to the other angles of the parallelogram?" This prompt is both mathematical and technological and so was coded accordingly as (1, 2, 3, D). Notice the levels of allowance for mathematical depth appear first in the code (the numbers) and the types of technological actions second (the letters).

Table 3: Dynamic Geometry Task Analysis Framework

Allowance for Mathematical Depth	
Levels	**Hierarchical Levels and Descriptions**
N/A	Prompt requires a technology task with no focus on mathematics.
0	Sketch does not have mathematical fidelity required to respond to prompt.
1	Prompt requires student to recall a math fact, rule, formula, or definition.
2	Prompt requires student to report information from the construction. The student is not expected to provide an explanation.

Table 3 Continued

Allowance for Mathematical Depth	
Levels	**Hierarchical Levels and Descriptions**
3	Prompt requires student to consider the mathematical concepts, processes, or relationships in the current sketch.
4	Prompt requires student to explain the mathematical concepts, processes, or relationships in the current sketch.
5	Prompt requires student to go beyond the current construction and generalize mathematical concepts, processes, or relationships.
Types of Technological Action	
Affordances	**Descriptions**
N/A	Prompt requires no drawing, construction, measurement, or manipulation of current sketch.
A	Prompt requires drawing within current sketch.
B	Prompt requires measurement within current sketch.
C	Prompt requires construction within current sketch.
D	Prompt requires dragging or use of other dynamic aspects of the sketch.
E	Prompt requires creation/consideration of multiple examples from which one can generalize.
F	Prompt requires a manipulation of the sketch that allows for recognition of emergent invariant relationship(s) or pattern(s) among or within geometrical object(s).
G	Prompt requires manipulation of the sketch that may surprise one exploring the relationships represented or cause one to refine thinking based on themes within the surprise (adapted from Sinclair (2003), p. 312).

Coding with the Framework

Here we explain how the research team used the framework to code the first five prompts of each task from Figure 1. Task 1 was written by a teacher beginning to use dynamic geometry software in their classroom, and Task 2 was written by the researcher developing the framework. Before reading further, we suggest taking some time to consider how you would use the framework to code these prompts. Then think about what the coding implies about the potential each task has to engage students in developing mathematical understanding. Each task begins with the same parallelogram and follows with similar but different prompts. Is one more likely to engage students in the mathematical reasoning desired? Our hope is that by comparing your analysis to ours that our methods for applying the framework will be clearer. You will then be prepared to use the framework to analyze the quality of other dynamic geometry tasks and to write your own high quality tasks.

In Parallelogram Task #1, Prompt 1 reads, "Describe what a parallelogram looks like." The prompt was coded as (2, N/A [not applicable]) because it simply requires the student to report information from the sketch with no need to perform a technological action. Prompt 2 reads "Can you determine a relationship between the angle measures? (Use Hint #1 if needed)" Prompts 2 and 3 were each coded as (1, 2, 3, B) as they require the student to recall some definitions (e.g., angle measure) (coded 1), to report the measures in the sketch (coded 2), to consider the relationships in the sketch (coded 3), and to carry out the technological action of measuring (coded B). The coding on neither prompt went beyond 3 for allowance for mathematical depth because the student was not asked to explain why certain relationships such as opposite sides being congruent were inherent characteristics of the parallelogram. Furthermore, the technological action of measuring without dragging does not push the student to consider invariant relationships that may be present in the parallelogram.

Prompt 4, "Try dragging the vertices. Do your assumptions hold true?" received a code of (N/A, D). It received a code of N/A for mathematical depth because it in no way indicates if the student's assumptions are about a parallelogram or even mathematical. It received the code of D because of its requirement of dragging. Prompt 5 reads, “Is it possible for all sides to be the same length?” and received a code of (1, N/A). The code of 1 for mathematical depth is a result of the prompt requiring the student to recall the definition of length to understand and respond to the question. To respond to this prompt, some students may try dragging the vertices of the parallelogram, but the prompt does not require them to do so.

In Parallelogram Task #2, Prompt 1 reads, “Drag parallelogram ABCD’s vertices. Write a conjecture about the relationship between the measures of opposite angles of this parallelogram (A and C; B and D).” and was coded as (1, 2, 3, D). Prompt 2 built on Prompt 1 by directing the student, “Measure the four angles of parallelogram ABCD. Drag its vertices to make any size parallelogram. Is your conjecture from #1 true? Explain.” It was coded accordingly as (3, 4, 5, B, D, E, F). The student is required to consider the relationship between opposite angles as part of their conjecture from Prompt 1 (coded 3) and explain why their conjecture was true or not (coded 4). The codes of D, E, and F emerged due to the prompt requiring students to drag (coded D), consider multiple examples (coded E) and recognize invariant relationships (coded F). The requirement of student explanation coupled with the technological action of dragging, considering multiple examples, and recognizing invariant relationships required the student to go beyond the current construction to make any desired size parallelogram and then speak to the relationship between opposite angles. This coupling of explanation along with technological actions coded D, E, and F meets the highest level of Allowance for Mathematical Depth (coded 5). Prompts 3 and 4 from Parallelogram

Task #2 were parallel in nature to Prompts 1 and 2 and received the same codes of (1, 2, 3, D) and (3, 4, 5, B, D, E, F) respectively.

Prompt 5 reads, “Drag a vertex of parallelogram ABCD until all four sides have the same measured length. Use construct segment to lock the lengths of all four sides of your parallelogram. What is the special name for a parallelogram with four sides of equal length?” Prompt 5 was coded as (1, C, D) due to it requiring the student to recall a math fact, namely that a parallelogram with equal side lengths is called a rhombus. It additionally requires the student to drag a vertex of the parallelogram (coded D) and construct the line segments of the rhombus (coded C). Prompt 5 did not go beyond a code of 1 because the student was not required to report information from the sketch or explain why certain relationships were guaranteed.

Table 2 provides a summary of prompts and codes for each Parallelogram Task. Organizing the codes in this way provides a summary of the degree to which each task is written to promote mathematical reasoning. Notice that for Parallelogram Task #1, five of the nine prompts are missing either mathematical or technological components. Additionally, the codes for allowance for mathematical depth never go beyond three. For Parallelogram Task #2 the prompts consistently include both a mathematical and technological component, and at times the coding for allowance for mathematical depth reached level 5. We concluded that these qualities caused Parallelogram Task #2 to be of higher potential than Parallelogram Task #1. This summary table works as a lens by which teachers and researchers can carefully consider the potential of any dynamic geometry task and how they might write similar tasks.

Table 52: Summary of Codes for Parallelogram Tasks 1 and 2

Prompt Number	Parallelogram Task 1 Codes	Parallelogram Task 2 Codes
1	(2, N/A)	(1, 2, 3, D)
2	(1, 2, 3, B)	(3, 4, 5, B, D, E, F)
3	(1, 2, 3, B)	(1, 2, 3, D)
4	(N/A, D)	(3, 4, 5, B, D, E, F)
5	(1, N/A)	(1, C, D)
6	(1, N/A)	(1, 2, 3, 4, D)
7	(1, 2, 3, D)	(1, 2, 3, D)
8	(1, 2, 3, B, C)	(3, 4, 5, B, D, E, F)
9	(3, N/A)	(1, 2, 3, D)
10		(3, 4, 5, B, D, E, F)

High-Quality Dynamic Geometry Tasks

The Dynamic Geometry Task Analysis Framework serves as a guide for analyzing and writing tasks for dynamic geometry environments. High quality tasks must contain prompts that account for the level 5 indicator, "Prompt requires student to go beyond the current construction and generalize mathematical concepts, processes, or relationships." Writing high caliber prompts also requires the utilization of technological affordances unique to a dynamic geometry environment such as dragging. We found that certain combinations of codes for Allowance for Mathematical Depth and Types of Technological Actions prompted students to be "doing mathematics" as Smith and Stein described (1998). For example, in Parallelogram Task #2, a code of (3, 4, 5, B, D, E, F) emerged for a prompt that required the student to measure and then drag to consider relationships that go beyond one particular size parallelogram. In responding to this prompt the student is "doing mathematics" by exploring and understanding the nature of mathematical relationships. Prompts of this nature will encourage the student to make drawings, measurements or constructions, explore relationships and form deeper mathematical understandings.

Because the framework was based on tasks submitted by teachers who were beginning to utilize GSP in their geometry classrooms, a potential limiting feature is that it may best apply when evaluating tasks intended for novice users of dynamic geometry environments. As students continue to master technological actions as they engage in cognitively demanding tasks, they may no longer need detailed directions that dictate technological actions. Performing such actions may become a normalized expectation over time. Tasks that rank high on the framework may help to establish these expectations. A fruitful area for further research may lie in exploring what expectations emerge as students continually engage in tasks that rank high on the Dynamic Geometry Task Analysis Framework.

Conclusion

Implications for analyzing dynamic geometry tasks and writing high quality tasks for dynamic geometry environments became apparent to us after examining many of these tasks in light of the discourse and understanding that emerged around their use. To organize our thinking and systematize our categorization of task quality, we assessed tasks at the prompt level. In so doing, we crafted a framework that worked to quickly assess task quality based on its prompts. In an effort to connect research to practice, we inferred that the framework would also prove beneficial to teachers seeking to analyze dynamic geometry tasks and write high quality tasks for their students. To this end, we offer the Dynamic Geometry Task

Analysis Framework as a guide for teachers desiring to harness the full potential of dynamic geometry environments for the betterment of student conceptual understanding.

Acknowledgements

Data for the development of this framework originated in a professional development project under the direction of Karen Hollebrands at North Carolina State University. This project was partially supported by the National Science Foundation (DRL-0929543). Any opinions, findings, and conclusions or recommendations expressed herein are those of the author and do not necessarily reflect the views of the National Science Foundation. The author thanks Karen Hollebrands for her invaluable feedback throughout the development of this work.

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Doyle, W. (1988). "Work in Mathematics Classes: The Context of Students' Thinking during Instruction." Educational Psychologist 23 (2): 167-180.

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Appendix F: Task-Writing Activity Interview Questionnaire

1. In your first attempt at task writing, how did you plan to meet the learning goal of each task you wrote?
2. In your first attempt at task writing, describe in detail what the student was to do.
3. In your first attempt at task writing, how did the prompts you included push the student to achieve the learning goal?
4. In your first attempt at task writing, how did you envision student use of the technology tools to contribute to their learning?
5. After you were given the Dynamic Geometry Task Analysis Framework and reading an article about it, you were asked to make any desired changes to your task. What changes did you make and why?
6. In your second attempt at task writing, describe in detail what the student was to do.
7. In your second attempt at task writing, how did the prompts you included push the student to achieve the learning goal?
8. How did your use of the Dynamic Geometry Task Analysis Framework affect your thinking about tasks quality and task writing?
9. Do you see the Dynamic Geometry Task Analysis Framework as a valuable tool for teachers? Why or why not?
10. What other reflections do you have on this task writing experience?

Appendix G: Face Validity Request Email

[Expert's Name Here],

The purpose of this email is to request your expertise in evaluating the face validity of a dynamic geometry task analysis framework. The quality of my dissertation study entitled, Designing and Examining the Effects of a Dynamic Geometric Task Analysis Framework on Teachers' Written Geometer's Sketchpad Tasks, relies on the use of a valid framework to assess the quality of tasks in the dynamic geometry environment Geometer's Sketchpad. The face validity check concerns this dynamic geometry task analysis framework's potential for describing task quality concerning the arguments and conclusions required of students. After I have received feedback on this face validity check, the framework will be revised as necessary.

The specific steps of the face validity work asked of you are the following:

- Read the framework and an associated practitioner article (attached)
- Read three Geometer's Sketchpad tasks and an explanation of their rankings per the framework (attached)
- Write a short reaction to the framework and the use of the framework to rank the three tasks as low, medium, and high in quality.

Guiding questions to assist you the face validity work are:

1. What changes if any do recommend to the framework? Why?
2. Does the framework seem useful to those concerned with writing quality dynamic geometry tasks and assessing task quality? Why?

3. Are the framework rankings of low, medium, and high task quality appropriate in regard to the quality of student argumentation you predict will likely take place on each task (low, medium, high)?

I would greatly appreciate if your feedback takes place in the next two weeks. Let me know if you are willing to do this work in this timeframe. Do not hesitate to contact me with any questions or clarifications.

Best,

Aaron Trocki

(919) 923.3798

Appendix H: Face Validity Request Letter

The study is composed of two phases: 1) a validity study of the Dynamic Geometry Task Analysis Framework followed by possible revisions; and 2) a study of how the framework affects teachers' perceptions of task quality and how they write and revise tasks. To address overarching Research Question One that concerns the validity of the framework, the researcher will craft three tasks for The Geometer's Sketchpad that rank low, medium and high on the Dynamic Geometry Task Analysis Framework. The framework itself along with the three written tasks will be assessed for face validity by two to three experts in the field of dynamic geometry software in mathematics education. The framework and tasks will then be revised regarding the feedback received in the face validity process. The role of the expert in establishing face validity as described by Anastasi (1998) is invoked, "Face validity pertains to whether the test [Dynamic Geometry Task Analysis Framework] "looks valid" to the examinees who take it, the administrative personnel who decide on its use, and other technically untrained observers" and, "refers, not to what the test actually measures [task quality], but to what it appears superficially to measure" (p. 144). The face validity check concerns the Dynamic Geometry Tasks Analysis Framework's potential for describing task quality concerning the arguments and conclusions required of students.

Coding of Low-, Medium-, and High-Ranked Tasks (Pre-revisions)

Low-, Medium-, and High-Ranked Geometer's Sketchpad Tasks

High, medium, and low task qualities are defined in reference to the codes a task receives according to the Dynamic Geometry Task Analysis Framework. Mathematical depth is considered to be evident when a prompt ranks four and/or five in the mathematical depth coding. Tasks are ranked low, medium, or high in quality, and these ranks are based on two measures. First, the mathematical depth of the tasks is taken into account by answering the question, "Does the task possess mathematical depth related to the learning goal of the task?" Secondly, the coordination of technological actions with mathematical depth is considered. The question, "How often are prompts possessing mathematical depth coordinated with technological actions?" will be answered to assess the second component of task quality. Tasks that do not possess mathematical depth are automatically ranked low. Further, tasks that rank low in quality rarely if ever (< 30% of prompts possessing mathematical depth) coordinate technological actions with the mathematical depth in these prompts. Tasks that rank medium contain prompts possessing mathematical depth with some (> 30%, < 70%) prompts coordinating technological action with mathematical depth. A task that ranks high must contain mathematical depth with a high percentage of prompts (> 70%) that possess mathematical depth coordinated to technological actions.

The following image is of the low-level task in GSP.

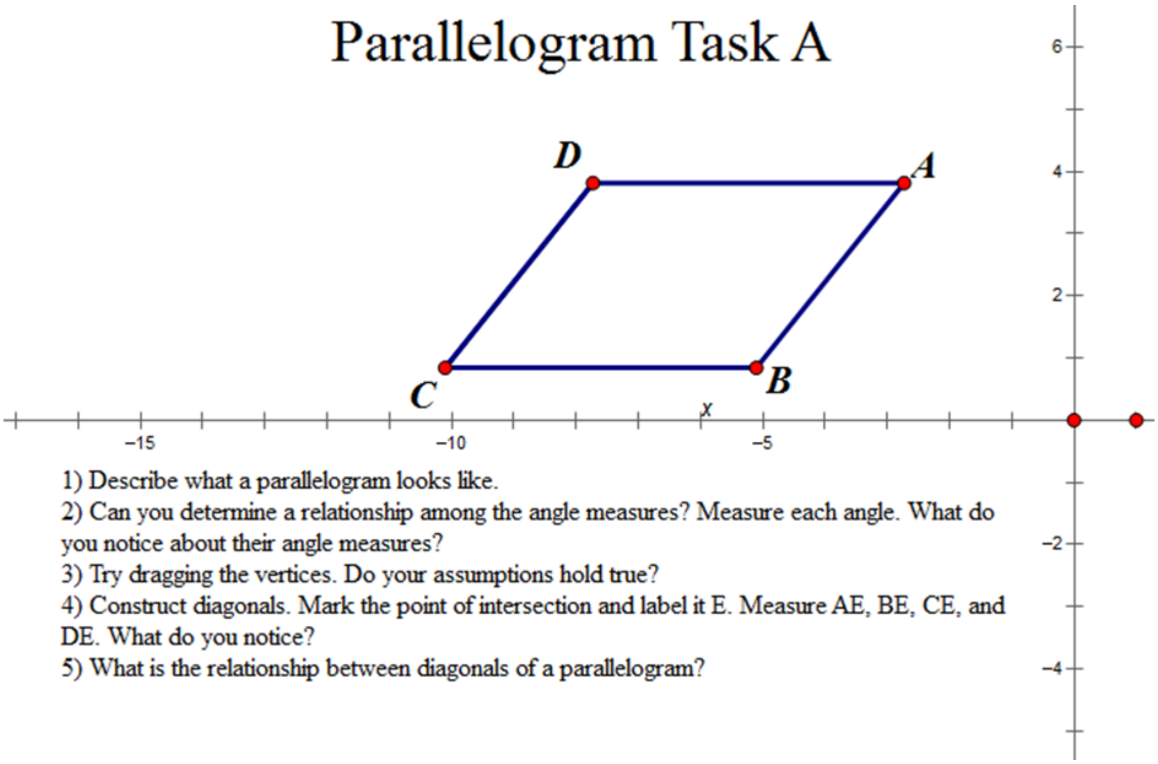

Figure 4: Low-Level GSP Task

Task A received the following codes as summarized in the table below.

Table 6: Task A Coding Summary

Prompt Number	Parallelogram Task A Codes
1	(2, N/A)
2	(1, 2, 3, B)
3	(N/A, D)
4	(1, 2, 3, B, C)
5	(3, N/A)

These codes resulted from the nature of the associated prompts. Prompt 1 simply requires the student to report information from the sketch with not technological action. Prompts 2 through 4 require technological actions however none of these actions are associated with mathematical depth beyond three. In the last prompt the student is to consider the relationship between diagonals, but no explanation is required. Therefore, this tasks fits the description of low level because it does not contain prompts (i.e., mathematical depth of 4 and/or 5) that possess mathematical depth. Student work on this tasks is conjectured to be similar to Leung's (2011) epistemic mode of interacting with a DGS sketch known as practice (PM). Leung describes practice mode in the following two ways:

- PM1: Construct mathematical objects or manipulate pre-designed mathematical objects using tools embedded in a technology-rich environment.
- PM2: Interact with the tools in a technology-rich environment to develop (a) skill-based routines; (b) modalities of behavior; (c) modes of situated dialogue. (p. 327)

The data gathered during student task-based interviews will be analyzed in reference to Leung's epistemic modes.

The following image is of the medium task in GSP.

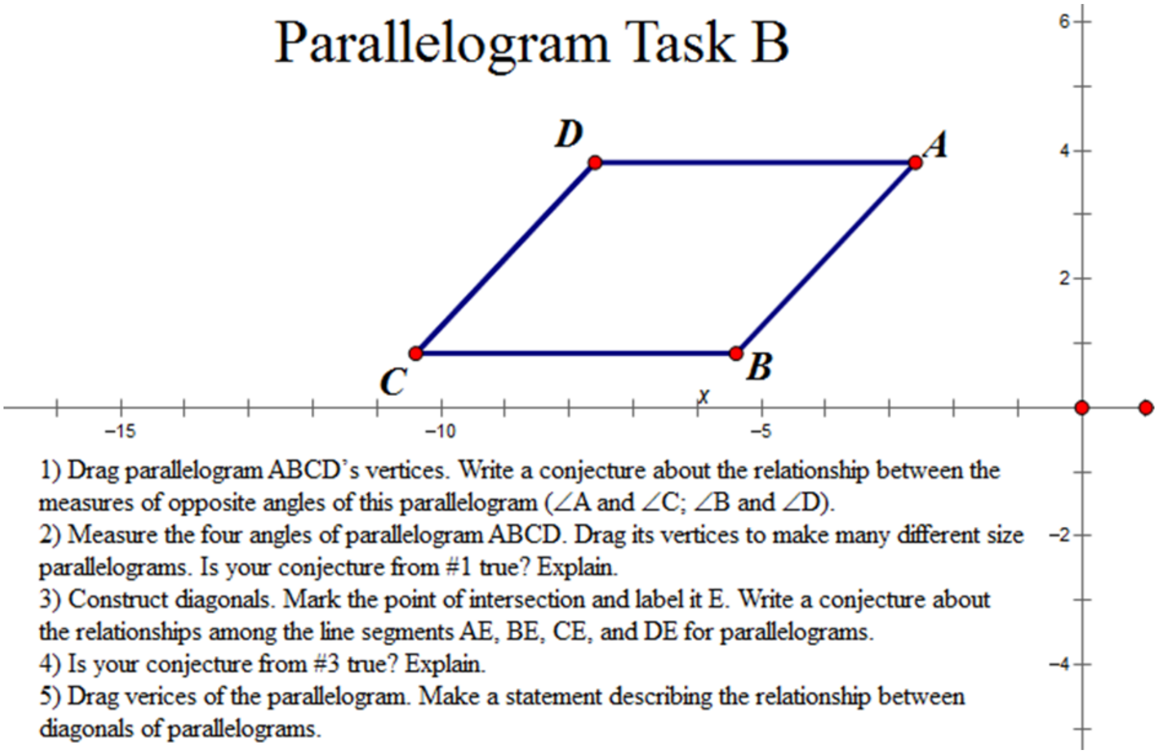

Figure 5: Medium-Level GSP Task

Task B received the following codes as summarized in the table below.

Table 7: Task B Coding Summary

Prompt Number	Parallelogram Task B Codes
1	(1, 2, 3, D)
2	(3, 4, 5, B, D, E, F)
3	(1, 2, 3, C)
4	(3, 4)
5	(3, 5, D)

The codes for Task B resulted from the mathematical depth and technological actions found in the prompts. This tasks was coded as medium in quality as it contains three prompts (i.e., #s 2, 4, & 5) that possess mathematical depth. Two of these three prompts are coordinated with technological action(s), thus providing approximately 67% of prompts that possess mathematical depth also coordinating to technological action. Therefore, this tasks meets the stipulations of medium-level quality. The following image if of the high task in GSP:

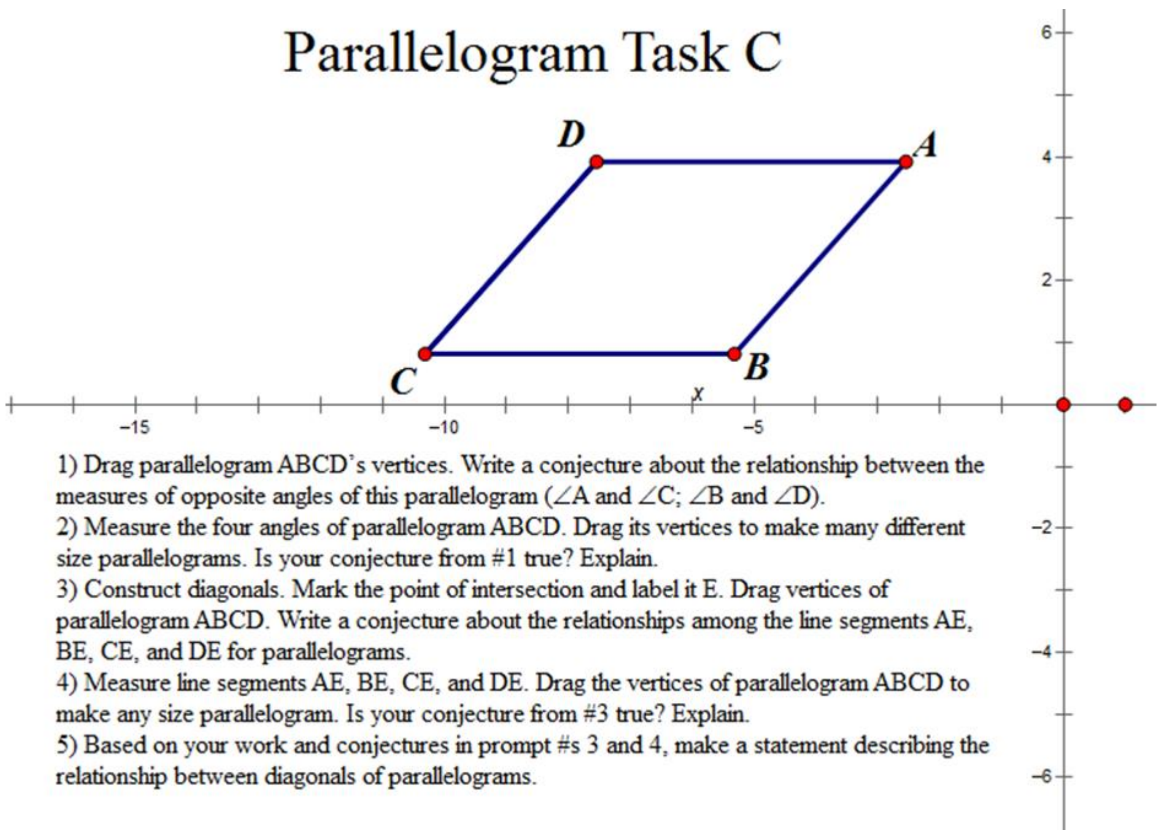

Figure 6: High-Level GSP Task

Table 8: Task C Coding Summary

Prompt Number	Parallelogram Task C Codes
1	(1, 2, 3, D)
2	(3, 4, 5, B, D, E, F)
3	(1, 2, 3, C, D)
4	(3, 4, 5, B, D, E, F)
5	(3, 4, 5, B, D, E, F)

Overall, the codes for this task are different from the low- and medium-level tasks in that all prompts that contain mathematical depth are also coordinated with technological actions. This tasks ranks high in quality according to the definition of high quality provided previously.

It is conjectured that coding of tasks with the Dynamic Geometry Task Analysis Framework as low, medium, or high will correspond to student argumentation and conclusions as labeled with Toulmin's (1958) argumentation model. It is further conjectured that the framework will assist those concerned with assessing and writing quality tasks. These conjectures will be explored and researched in this study.