Disintegration of Concrete Shell Structures Under Violent Dynamic Loading Conditions

Y. Crutzen
Control Data Italia S.p.A., Palazzo Bernini, Centro Direzionale Milano 2, I-20090 Segrate, Italy

J. Reynen
Commission of the European Communities, J.R.C. Ispra Establishment, System Analysis Division, I-21020 Ispra (Varese), Italy

1. Abstract, Introduction and Conclusion

This paper addresses the highly non-linear problem of impact on reinforced concrete shell structures such as containment buildings of nuclear power plants. The general features of the computer program SLOOFSAN developed for fast-transient non-linear dynamic analysis are the following:

- Use of explicit solution strategies for time integration and thereby mass lumping. Explicit algorithms dictate small time steps but both geometrical and material non-linearities ask for small time steps anyhow.
- The geometrical non-linearities include large displacements and rotations, for which a convective coordinate formulation is applied.
- Use of the performant serendipity shell element SEMILOOF developed by Irons [1]. It is shown that techniques of explicit integration and convective coordinates can be competitively implemented for higher order elements such as SEMILOOF.
- Reinforcement is simulated by smeared bars at the upper and lower shell surfaces.
- The material non-linearities dealt with in the paper include inelastic behaviour of reinforced concrete:
  - i) cracking of concrete in tensile regime. In SLOOFSAN, an efficient model for concrete cracking is based on the concept of zero tensile strength. The crack propagation through the shell thickness can be followed very accurately.
  - ii) crushing of concrete in compressive regime. Different analytical models reproducing the ascending and descending parts of the stress-strain curve are discussed. The post-peak strain softening is one of the major factors of complete disintegration of concrete.
  - iii) plasticity of steel reinforcement encountered in tension as well as in compression.
  - iv) local steel buckling in compressive regime when concrete looses its strength for highly compressive loads (crushing) and when perfect bond with concrete is lost. SLOOFSAN has an option to delete the compressive contribution of the reinforcement to the internal forces when concrete crushing has occurred.
- Static stress distributions such as those due to dead weight and prestressing, which may influence the transient response, are defined by running SLOOFSAN in a kind of dynamic relaxation mode to obtain the steady-state starting conditions.

Numerical simulations of partial and complete damage of reinforced concrete components are shown. Applications in the field of anti-missile design of nuclear reactor buildings are discussed. The local damage of civil and military aircraft crashes onto the containment building and the complete disintegration of the concrete region under impact for the chimney will be presented.

The conclusions of this paper are:

i) The numerical methods to deal with impulsive loads on concrete structures are well established;
ii) however, the constitutive laws for the non-linear behaviour of concrete including rate effects are nearly unknown;
iii) Case studies to define the sensitivity of constitutive equations on the dynamic response have to be carried out, in particular for such remote phenomena as anti-missile design, where the time history of the load is rather unknown.

2. Equations of motion

An efficient way of formulating the equilibrium is the theorem of virtual work

$$\int \{\sigma\}^T \{\delta\epsilon\} dVol + \int \{\rho\ddot{u}\}^T \{\delta u\} dVol = \int \{f\}^T \{\delta u\} dVol \quad (1)$$

σ	stress	u	displacement	T	transposed
ϵ	strain	$\ddot{u}$	acceleration	{ }	column vector
f	external load	ρ	density	$\{ \}^T$	row vector

$\delta u, \delta\epsilon$ kinematically admissible variations of displacement and strain.

Using the concept of FEM, the discretized equations of motion result:

$$[M] \{\ddot{u}\} + [K] \{u\} = \{f\} + \{f^0\} \quad (2)$$

$[M]$	mass matrix	
$[K] = \int [B]^T [D][B] dVol$	stiffness matrix	(3)
$\{f^0\} = \int [B]^T [D] \{\epsilon^0\} dVol$	equivalent load due to initial strains ϵ^0	(4)
$\{\epsilon\} = [B] \{u\}$	total strain-displacement relation	(5)
$\{\sigma\} = [D] \{\epsilon^{e\ell}\}$	Hooke's law relating stress to elastic strain	(6)
$\{\epsilon^{e\ell}\} = \{\epsilon\} - \{\epsilon^0\}$	elastic strain	(7)

The solution of the equations of motion (2) with an implicit scheme needs at every time step - in particular for geometrical non-linearities - the assembling and inversion of a matrix, and in addition, - for material non linearities - a certain number of iterations.

For the explicit scheme, the equations are written in an alternative way:

$$[M] \{\ddot{u}\} = \{f\} - \{f^{int}\} \quad (8)$$

where the internal load is given by:

$$\{f^{int}\} = - \{f^0\} + [K] \{u\} = \int [B]^T \{\sigma\} dVol \quad (9)$$

The explicit scheme becomes attractive when the mass matrix [M] can be diagonalized (lumped), because then the equations of motion (8) are uncoupled. For simple elements, the procedure is straightforward, for the SEMILOOF element special procedures have been tested numerically [2]. The decoupled equations (8) can be integrated using the central difference scheme. The combination of the central difference scheme and mass lumping has a compensating effect on the frequency spectrum as shown by the work of Krieg and Key [3].

The advantages of the explicit time integration scheme are manifold:

. ease of programming: no assemblage of matrices, less computer memory, no inversion of matrices, no problems of bandwidth and node and/or element numbering;

. straightforward implementation of all kinds of constitutive equations;

. straightforward implementation of geometrical non-linearities due to large rotations, using convective coordinates.

The drawback of the explicit scheme is the limitation of the time step dictated by the travel time of acoustic waves through the smallest dimension of any element and for thin structures the shell formulation is to be preferred rather than a degenerated solid-element formulation. Anyhow, also in the implicit scheme, the non-linearities limit the time step, not from the point of view of stability but rather from the point of view of accuracy, or alternatively iterations have to be applied.

Equations (2) - the implicit scheme - allow for a static solution by inverting matrix [K], the assemblage of which is foreseen anyhow in implicit schemes. For the explicit formulation (8), the static solution ($\{f\} = \{f^{int}\}$) is more difficult. In SLOOFSAN different approaches are implemented by simply solving (8) for the static load as a step load at time = 0 (steady state starting condition) by

i) adding a damping $\{f^{damp}\} = [C]\{\dot{u}\}$, where [C] is a matrix proportion to the mass matrix, or

ii) whenever the acceleration of a degree of freedom changes sign ($f_i = f_i^{int}$), the velocity of that degree of freedom is put zero ($\dot{u}_i = 0$);

iii) like (ii) but putting the velocity equal to half the average velocity of the element to which the degree of freedom belongs.

Moreover, in order to speed up the solution for static loads, the frequency response for transverse movements is artificially changed. This is not carried out by introducing an apparent density, but by multiplying in equation (8) the component of $\{f\} - \{f^{int}\}$ in the local transverse direction and for the rotations by a factor α representing the ratio of the critical time step in transverse direction to the membrane one. This means that the steady state problem is solved with an apparent time step equal to the critical time step of membrane behaviour, and that the response of the bending behaviour is made faster such that its apparent critical time step equals the one for membrane behaviour. Stress distributions due to dead weight and prestressing can then be defined correctly.

3. Reinforcement and prestressing in SEMILOOF

An efficient finite element available for the solution of thin shells is the SEMILOOF element developed by Irons [1]. This rather complex thin shell model incorporates the following improvements of the shell formulation:

. the element adopts the isoparametric concepts for geometry based on the 8-node quadratic element;
. it is a non-conforming displacement model: satisfying C^0 continuity between elements, only some measure of C^1 continuity is provided at discrete points on the element perimeter, the LOOF nodes, placed in the best way to minimize the work lost in conformity;
. according to the Patch Test (PT), it converges practically for a patch of plane quadrilateral elements when states of constant membrane or bending stress are reproduced;
. the use of a reduced integration rule permits to achieve an improved flexiblity of the thin element and remarkably good results are obtained with very few elements; the strains (membrane and flexion) are computed at 4 (2x2) Gauss-points on the mid-surface for the thin quadrilateral;
. having both membrane and bending stiffnesses and linear varying strains through the thickness, this curved three-dimensional element incorporates discrete Kirchhoff's assumptions by means of constraints on transversal shear behaviour;
. the quadrilateral element has 32 master degrees of freedom (DOF) divided into 24 corner-midside displacement components in the global coordinate system (x,y,z) and 8 local rotations normal to the element edge at LOOF nodes. The triangle element has 24 master DOF divided into 18 displacement components and 6 local rotations.

The SEMILOOF dynamic model has nodal masses and inertia moments (mass lumping process successfully tested [2]) and the lumped-mass explicit technique permits to compute directly the nodal accelerations, velocities and displacements from the basic equations of motion (8).

3.1 Reinforcement

The modelling of reinforcements consists of adding into the internal forces the stresses due to steel membrane (smeared bars) with Poisson's ratio $\nu = 0$ (Fig. 1). The membranes undergo the same total strains as the concrete (no bond slip!). The thickness of the membranes follows from the design specifications of percentage of reinforcement.

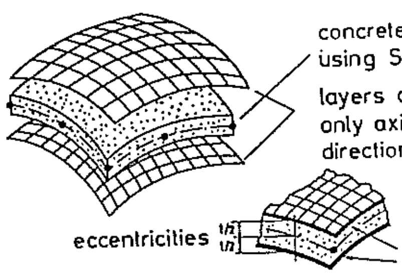

Fig. 1 - SEMILOOF model for reinforced concrete.

3.2 Prestressing

When the reinforcement membrane described in the previous paragraph is given a (negative) prestrain ϵ_0, the contribution of the internal force (9) of the reinforcement becomes tensile, and the concrete will be precompressed. In SLOOFSAN, the amount of precompressing of the concrete is an input parameter. Running the program in the static mode, by one of the methods discussed in chapter 2, the stress distribution in the concrete and reinforcement is established automatically, including eventually the effect of dead weight, being a natural way of precompressing. The RESTART option allows to introduce the impulsive load in a subsequent run.

4. Constitutive equations for reinforced concrete

The explicit scheme facilitates the implementation of non-linear constitutive equations. The time steps and thereby apparent load steps are small and immediately an action can be taken when the stress pattern violates a yielding criterion or another failure criterion. The stress redistribution is accounted for when calculating the internal forces (9), to be used in the equation of motion (8) for the next time step. In SLOOFSAN various types of non-linear material behaviour are implemented:

i) plasticity of reinforcement and concrete;
ii) cracking of concrete in tensile regime;
iii) crushing of concrete and thereby buckling of reinforcement in compressive regime;
iv) non-linear secant biaxial constitutive relation for concrete.

4.1 Plasticity

The plasticity model of SLOOFSAN is explained in detail in ref. [4] and has the great advantage that it is not needed to memorize the stresses: only the deformation history is memorized.

4.2 Cracking of concrete

The model for concrete cracking in SLOOFSAN is based on the concept of zero tensile strength. In concrete structures where impact and impulsive loads are expected, reinforcement or even prestressing is present and the tensile strength is taken up by the reinforcement. The great advantage of the method is the ease of implementation, in particular when the explicit time integration scheme is used. Moreover, nearly no additional calculations and no additional storage is required. The model is illustrated by means of Mohr's circles in Fig. 2. More details are given in [4,5].

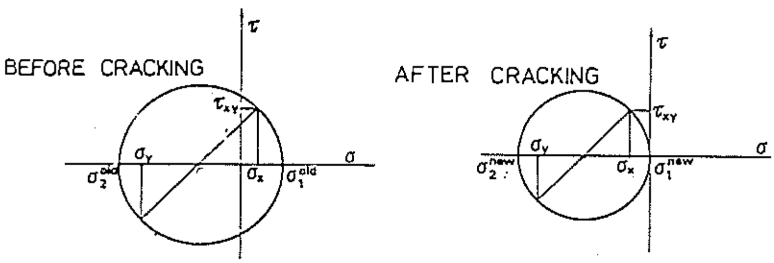

Fig. 2 - Mohr's circles.

4.3 Crushing of concrete and buckling of reinforcement

For compressive stresses of the order $|\sigma| > 35$ MPa, concrete starts to crush and looses also its compressive strength. A kind of strain softening takes place. Explicit solution strategies do not have any difficulties to implement strain softening. Apart from loosing its load bearing capacity, the concrete cannot fulfill its other task, i.e. to keep the reinforcement straight. The reinforcement has gone already plastic - depending on the amount of prestressing - and has lost some load bearing capacity in the compressive regime, but when concrete crushing takes place it is assumed that local buckling will occur. In SLOOFSAN an option is introduced to cancel completely the contribution of the reinforcement to the internal forces (9) in the steel compressive regime as soon as crushing in concrete has taken place.

4.4 Secant biaxial constitutive relation for concrete

The description of the non-linear behaviour of concrete using classical plasticity theory based on Von Mises' stress and Drucker's normality rule, does not account for the non-linear volumetric strain change as experimentally observed during strain softening of concrete. Recently a constitutive relation has been formulated for triaxial [6] and for the special case of plane stress biaxial monotonic loading [7]. This variable secant constitutive relations, in which the stress is defined explicitly from the total strains, might give a better description of the structural behaviour in dynamics.
A vast area of research, however, remains open, i.e. the definition of rate dependent constitutive equations for concrete for arbitrary load paths. The variable secant explicit constitutive relation for concrete plane stress situations as described in [7] can be summarized as follows:

$$\begin{Bmatrix}\sigma_1\\ \sigma_2\end{Bmatrix} = K_S\begin{bmatrix}1+f & 1+f\\ 1+f & 1+f\end{bmatrix}\begin{Bmatrix}\epsilon_1\\ \epsilon_2\end{Bmatrix} + \tfrac{2}{3}G_S\begin{bmatrix}2-f & -1-f\\ -1-f & 2-f\end{bmatrix}\begin{Bmatrix}\epsilon_1\\ \epsilon_2\end{Bmatrix} \qquad (10)$$

where

σ_1,σ_2 principal stress components

ϵ_1,ϵ_2 principal total strain components

$K_S = K_S(\epsilon,\gamma)$	variable secant bulk modulus	(11)
$G_S = G_S(\gamma)$	variable secant shear modulus	(12)
$\epsilon = (\epsilon_1 + \epsilon_2 + \epsilon_3)/3$	octahedral normal component of strain	(13)
$\gamma = \frac{2}{3}$ SQRT $((\epsilon_1 - \epsilon_2)^2 + (\epsilon_2 - \epsilon_3)^2 + (\epsilon_3 - \epsilon_1)^2)$	octahedral shear component of strain	(14)

The function f relates the transverse principal strain ϵ_3 to the in-plane principal strains through the relation

$$\epsilon_3 = (\epsilon_1 + \epsilon_2)\, f\,(\epsilon_1, \epsilon_2) \qquad (15)$$

The relations (11), (12) and (15) have been defined by curve fitting on a number of experimental results carried out by different laboratories in a benchmark exercise and are described in [7].

5. Applications

The response of a concrete nuclear containment building and a chimney due to an aircraft crash is analysed using SLOOFSAN with various constitutive equations. The results are compared to those published by Rebora et al. [8], using a degenerated solid element to model the shell.

5.1 Load due to impact of an aircraft

The aircraft can be considered as a soft missile. It deforms plastically and the load vs time history [9] is defined by:

i) the speed of the aircraft and
ii) the mass distribution such as wings and engines along its length.

Figure 3 gives two typical load vs time histories for a military aircraft (Phantom) and a civil aircraft (Boeing), respectively. These loading histories are used throughout the present analyses.

5.2 Response of a concrete nuclear containment building

The mesh (Fig. 4) (26 SEMILOOF ELEMENTS) and the linear dynamic analysis have been presented in a previous paper [5]. The problem has also been studied by Rebora et al. [8] using a complete different modelling with degenerated solid elements, which turns out to be more stiff than the present rather coarse mesh with SEMILOOF. The non-linear response is summarized in Fig. 5. From these results it is seen that the SEMILOOF model is more flexible than the degenerated solid element model. The SEMILOOF linear elastic analyses have been carried out with two different meshes from which it can be concluded that the SEMILOOF mesh had converged. The non-linear analyses with SEMILOOF consist of three cases:

i) cracking at a tensile strength of $\sigma_t = 5.6$ MPa
ii) cracking at a tensile strength of $\sigma_t = 0.0$ MPa but including prestressing due to dead weight
iii) cracking at a tensile strength of $\sigma_t = 0$.

The results of Rebora et al. only correspond to case (i) and a fairly good comparison is obtained, with the SEMILOOF results a bit more flexible as compared to the degenerated solid element results. The importance of the constitutive equations in particular the tensile strength of concrete, is clearly shown. The effect of dead weight, giving a kind of prestressing at the bottom of the containment building is noted. The crack pattern at interior and exterior surface is shown in Fig. 6.

5.3 Response of a concrete chimney

A typical concrete chimney (height 100 m, diameter 8 m at bottom, 5 m at top) has been analysed with SLOOFSAN using a mesh as depicted in Fig. 7. The results for a Boeing aircraft using the linear analysis (Fig. 7a) shows as response the first fundamental mode with a deflection at the top of H = 5 m. The use of non-linear theory both for constitutive equations as well as geometrical non-linearities (Fig. 7b) causes only damage at the top with a horizontal displacement of H > 45 m. The base of the chimney is hardly loaded. This phenomenon of local disintegration is even more pronounced for the faster load of a Phantom (however with less integrated impact) (Fig. 7c). These (preliminary) results are important in the sense that aircraft crashes onto a chimney do not cause the chimney to fall down as a rigid mass on nearby buildings, such as a reactor containment building.

6. Acknowledgement

The authors are indepted to Prof. Cedolin of Politecnico di Milano for fruitful discussions on the constitutive equations of concrete.

7. References

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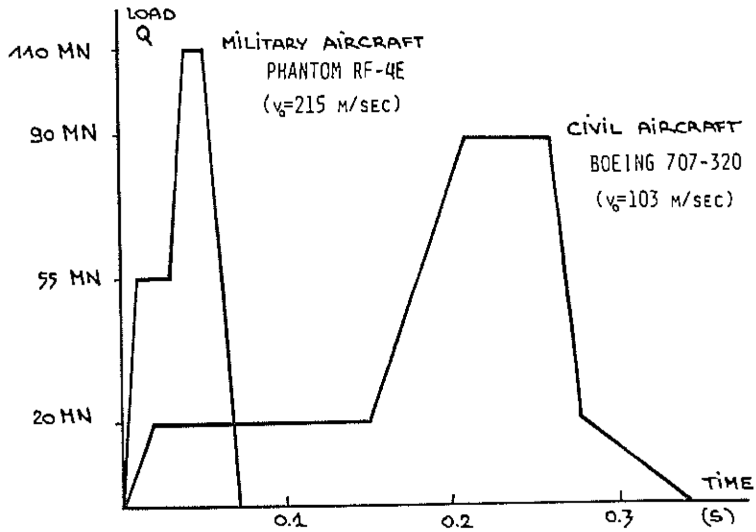


Fig. 3 - Aircraft loading functions.

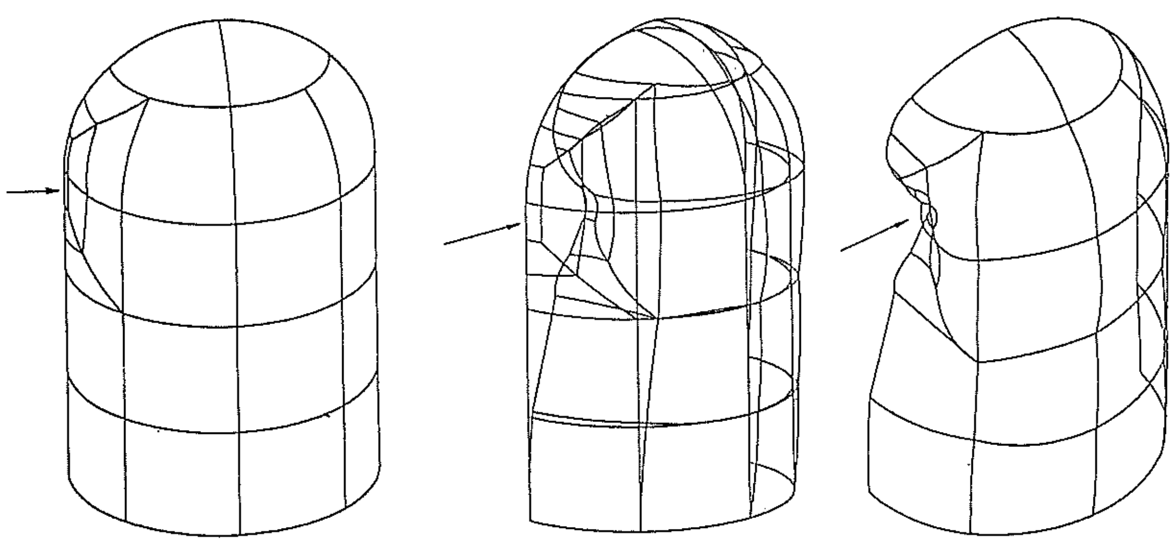


Fig. 4 - Original and deformed mesh (26 SEMILOOF elements).

REACTOR BUILDING ANALYSIS	SLOOFSAN (CM)	REBORA (CM)
LINEAR ELASTIC	4.03/4.08	3.32
NON-LINEAR (CRACKING) σ_t=5.6 MPa	4.47	4.29
NON-LINEAR (CRACKING) σ_t=0.0 + DEAD WEIGHT	7.21	-
NON-LINEAR (CRACKING) σ_t=0.0	9.13	-

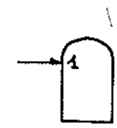


Fig. 5 - Comparison of horizontal displacement in point 1 for linear and non-linear analysis (σ_t tensile strength of concrete).

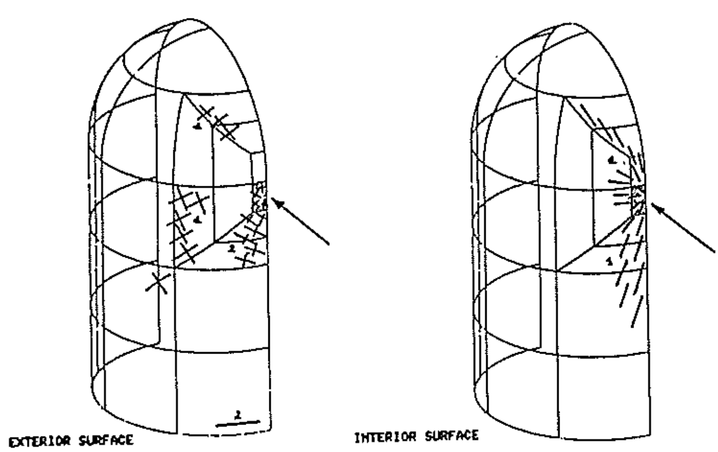
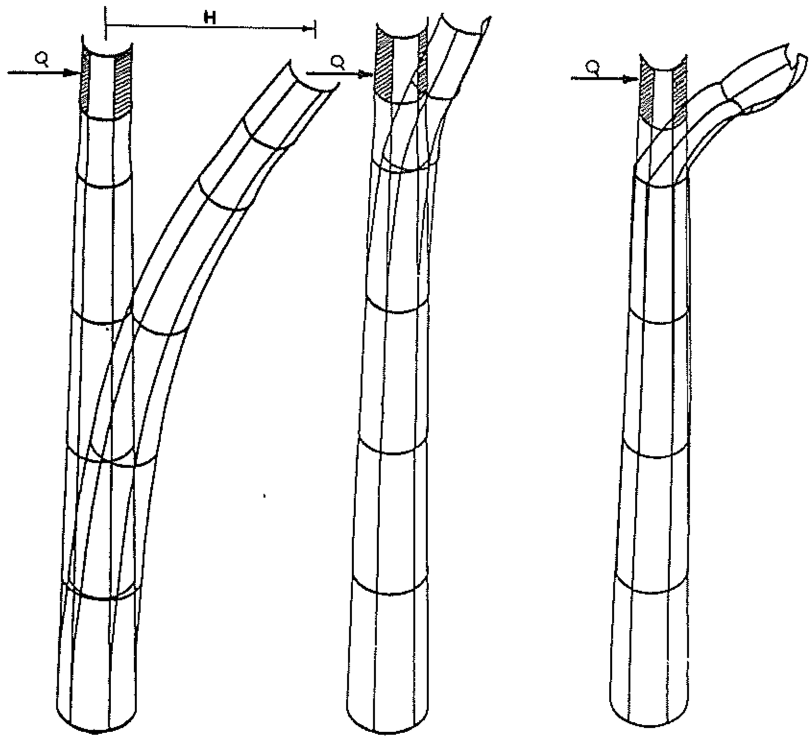


Fig. 6 - Crack pattern at exterior and interior surface.



a) Elastic linear Boeing b) Non-linear Boeing c) Non-linear Phantom

Fig. 7 - Chimney deformations due to aircraft inpingement.