OSCIL AND OSCVERT:COMPUTER CODES TO EVALUATE THE NON-LINEAR SEISMIC RESPONSE OF AN HTGR CORE

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SUMMARY

OSCIL and OSCVERT are FORTRAN codes which simulate the effects of seismic input on a High Temperature Gas Cooled Reactor (HTGR) core. The analysis is to be used to determine safety standards and licensing regulations. OSCIL models the core as a one-dimensional non-linear spring-mass system and OSCVERT is the two-dimensional extension, with each mass in OSCIL expanded to a vertical column of blocks. Either code can be used more generally for any system represented by such a model.

Springs are attached to each block with clearances between neighboring blocks and the spring constants simulate the elastic properties of the core blocks. In OSCIL the dynamics of the system is determined by the solution of the set of non-linear differential equations:

$$m_i\ddot{x}_i + k_i(x_i - x_{i-1}) + c_i(\dot{x}_i - \dot{x}_{i-1}) + k_{i+1}(x_i - x_{i+1}) + c_{i+1}(\dot{x}_i - \dot{x}_{i+1}) + FR_i = 0 \qquad \text{(A)}$$

x_0, x_{N+1} left and right walls displacements,
x_i displacement of ith block $1 \leq i \leq N$,
m_i mass of ith block,
k_i non-linear spring constants with $k_i \neq 0$ during block collision, $k_i = 0$ blocks separated, $k_i \rightarrow \infty$ when block edges overlap,
c_i non-linear damping coefficients,
FR_i frictional force. The usual friction step function at zero velocities is approximated by a continuous linear curve so that it can be numerically integrated.

OSCVERT allows vertical and rotational motion. The additional degrees of freedom cause a 3-fold increase in the numbers of equations, and a 9-fold increase in execution time over OSCIL for equivalent problems.

The equations (A) fall into the class of stiff differential equations. OSCIL and OSCVERT use a Gear multistep integration package for stiff equations written by A. C. Hindmarsh, to ensure convergence and numerical stability. This is a major improvement over other seismic reactor codes which have been developed and which are limited to Runga-Kutta and other single step methods. The equations (A) are stiff for two reasons.

1. They have widely varying time constants. In a realistic run $0[K] \sim 10^6 - 10^7$ lbs/in and hence a time constant $T \sim [m/k] \sim 10^{-4}$ sec, whereas the time of interest is commensurate with the time constant corresponding to the seismic oscillations $0[1/w] \sim 1$ sec.

2. As there are gaps between the blocks, the forces $m\ddot{x}_i$ have discontinuous first derivatives but are Lipshitz continuous.

Program runs produce numerical and graphical output and 16 mm animation movies of the time evolution of the system can be produced.

Results. — 1. Extensive parametric analysis was made for various configurations and the resonance frequencies and other features for different inputs were determined for both codes.

2. The codes have been demonstrated to be able to successfully handle cases involving multiple solutions and transients. The main test uses a single cubic spring (restoring force $= ax^3$) with forcing function $A \sin(\omega t)$. For a range $\omega_L < \omega < \omega_u$ the response amplitude has triple solutions. In sweeping ω through this range, the computed values followed the correct solution branch and the transients $\Delta\omega$ satisfied $\Delta\omega/(\omega_u - \omega_L) \ll 1$ as required for a valid model of non-linear springs.

Conclusions. — By using Gear's integration procedure for stiff differential equations, OSCIL (OSCVERT) produce accurate and numerically stable time histories of core response to arbitrary inputs, and give correct solutions for cases involving multiple solutions and transients. Friction has been successfully incorporated into the models. Extensive analysis of the time histories for realistic ranges of parameters have been done and resonance frequencies have been determined.

1. Introduction

OSCIL and OSCVERT are FORTRAN codes which simulate the effects of seismic input on a High Temperature Gas Cooled Reactor (HTGR) core. The analysis, in conjunction with results of experimental scale models, is to be used to determine safety standards and licensing regulations for HTGR. Much interest has been arising in seismic analysis due to the obvious hazards [1], [2], [3].

OSCIL models the core as a horizontal one-dimensional non-linear spring-mass system with friction. OSCVERT is the two-dimensional extension, with each mass in OSCIL expanded to a vertical column of blocks. An important feature of OSCVERT is that large rotations are allowed.

The codes use a powerful integration scheme due to GEAR[5]. This permits very stiff springs, as well as transients('jump' phenomena)due to multiple solutions that arise in non-linear systems, to be incorporated in the codes.

Either code can be used more generally to analyze the non-linear dynamic response of a spring-mass system excited by boundary motions.

2. Model Description

OSCIL determines the dynamics of the system depicted in Fig. 1. Springs are attached to each rigid block with clearances between neighboring blocks, and the spring constants simulate the elastic properties of the core blocks. The dynamics of the system is determined by the solution of the set of non-linear differential equations.

$$m_i \ddot{x}_i + k_i(x_i - x_{i-1}) + c_i(\dot{x}_i - \dot{x}_{i-1}) + k_{i+1}(x_i - x_{i+1}) + c_{i+1}(\dot{x}_i - \dot{x}_{i+1}) + fr_i = 0 \quad 1 \le i \le n \qquad (1)$$

x_o, x_{n+1} – left and right wall displacements

x_i – displacement (from equilibrium) of ith block $1 \le i \le n$

m_i – mass of ith block

k_i – non-linear spring constant

a) $k_i = 0$ blocks separated $x_i - x_{i-1} > -A_{1i}$

b) $k_i = k_{i1}$ collision $-A_{2i} \le x_i - x_{i-1} \le -A_{1i}$

c) $k_i = k_{i2} >> k_{i1}$ edges overlap $x_i - x_{i-1} < -A_{2i}$

(see Fig. 2a)

c_i – non-linear damping coefficients

fr_i – frictional force (Fig. 2b); the exact delta function is approximated by a continuous piece-wise linear curve to satsify the conditions of the numerical integration scheme (see sec. 3)

OSCVERT permits vertical and rotational block motions. A typical block in the array is depicted in Fig. 3. The additional degrees of freedom cause a 3-fold increase in the number of equations. Each equation has many terms arising from the base springs, dowell pins, corner springs and gravity, and are of the form:

$$p_i \ddot{u}_i - \Sigma f_i^u (x,y,\theta) = 0 \qquad (2)$$

p_i = mass or moment of inertia of ith block

f_i = force or torque in direction $u \in \{x,y,\theta\}$

3. Numerical Methods

The equations (1) and (2) fall into the class of stiff differential equations as defined by Gear [4]. OSCIL and OSCVERT use a Gear multistep integration package for stiff differential equations written by A.C. Hindmarsh [5]. This is necessary to ensure convergence and numerical stability of the solutions and this is a major improvement over other reactor seismic codes [1], [2], [3], which are limited to Runga-Kutta and other single step methods.

The equations (2) are stiff for two reasons:

1. They have widely varying time constants. In a realistic run $O[K] \sim 10^6 - 10^7$ lbs/in and hence a time constant $T \sim O[m/k] \sim 10^{-4}$ secs, whereas the time of interest is commensurate with the time constant corresponding to the seismic oscillations $O[1/\omega] \sim 1$ sec. During block collisions when the spring constants are effective, the time integration step $h < 10^{-8}$ secs. for convergence, and hence the problems could not be completed using single step integration methods.

2. As there are gaps between the blocks, and there is friction, the forces $m\ddot{x}_i$ have discontinuous first derivatives (Fig. 2). However, the spring forces and the approximated frictional forces are Lipschitz continuous [8], i.e.,

$$\frac{|F(x_1,t) - F(x_2,t)|}{|x_1 - x_2|} \leq K \qquad \text{For some } K>0 \text{ all } x_1, x_2 \epsilon R. \qquad (3)$$

R - real line

In [8] it is shown that the solutions will converge with known local truncation error if the functions being integrated are Lipschitz continuous.

4. Results

1. The reliability of the solutions was determined using simple test cases with known analytic solutions. Cases included 2 and 3 blocks with simple harmonic forcing functions. In all cases the computed solution could be determined with a relative error $<10^{-4}$ compared to the exact solution.

The execution times varied widely depending on the problem configuration. Substantial storage and execution time savings could be achieved in OSCIL by using GEARB [7], which utilizes the banded structure of the Jacobian of equations (1). For equivalent problems OSCVERT is approximately nine times slower than OSCIL due to the additional degrees of freedom.

2. The codes were demonstrated to be able to successfully handle cases involving multiple solutions and transients, a major requirement for this model.

The main test consisted of a single cubic spring (restoring force = $\alpha x + \beta x^3$) with a forcing function A sin (ωt). For a range $\omega_L < \omega < \omega_u$, the response amplitude has triple solutions (solid lines in Fig. 4). In sweeping ω up (O's in Fig. 4) and down (x's in Fig. 4) through this range, the computed values followed the expected solution branches and exhibited the transient ('jump' phenomena) to a different branch as predicted. The transient $\Delta\omega$ satisfied $\Delta\omega/(\omega_u - \omega_L) \ll 1$, indicating a true 'jump' effect. Further details can be found in [9].

3. Extensive parametric analysis was made for various configurations. Resonance frequencies and other qualitative features were determined for different inputs. Parameters that were varied included:

i) Number of masses

ii) Input frequency

iii) Gap size

iv) Spring constants

Fig. 5 shows the results of a typical OSCIL run. Detailed results of this analysis using OSCIL are recorded in a series of BNL reports [9].

Fig. 6 shows a typical OSCVERT run. Further discussion of the OSVERT results can be found in [10].

5. Conclusions

OSCIL and OSCVERT successfully incorporate all the features needed for a two-dimensional array model of a reactor core. The solutions have been shown to converge and are numerically stable for any configuration. The codes have been extensively used for seismic analysis using realistic material parameters, and match the results of experimental scale models [9], [10].

The codes can be used to analyze the dynamic response of any system based on a spring-mass model.

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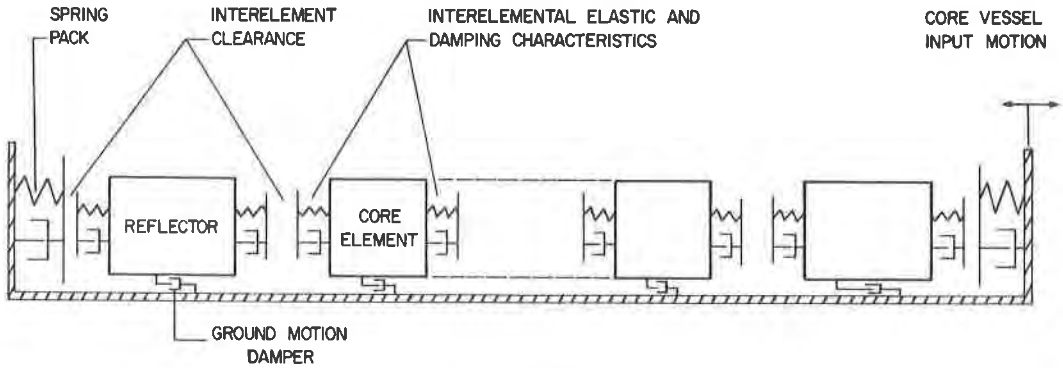

Figure 1 "N" MASS MODEL

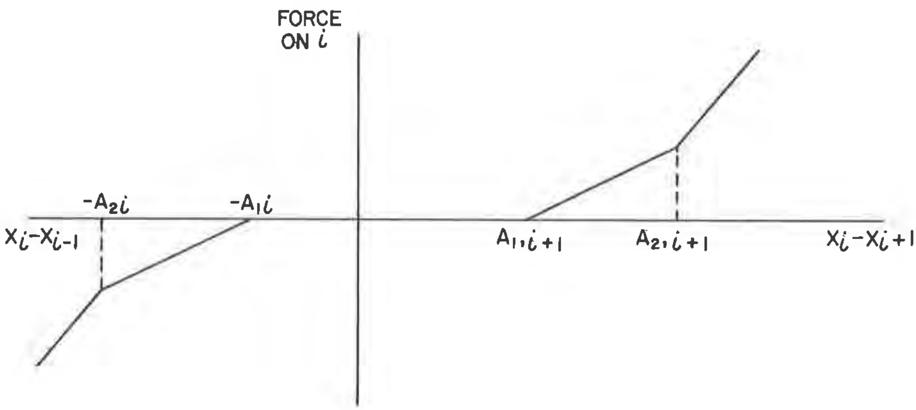

Figure 2a SPRING FORCES ON ith BLOCK

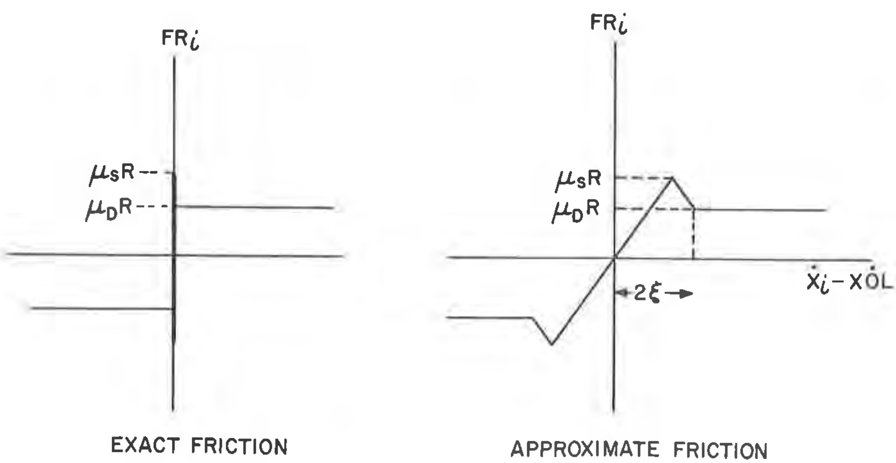

Figure 2b FRICTION FORCES

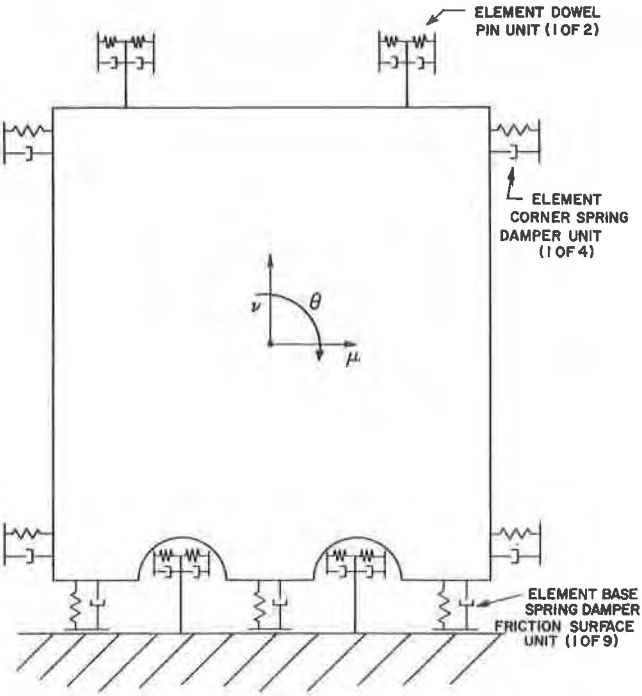

Figure 3 BLOCK ELEMENT, VERTICAL ARRAY CODES

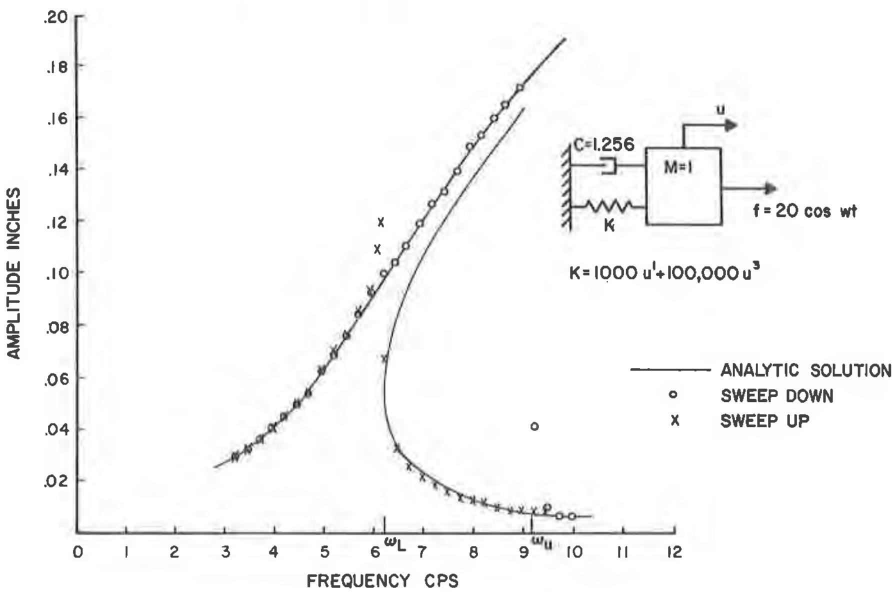

Figure 4 TRANSIENTS

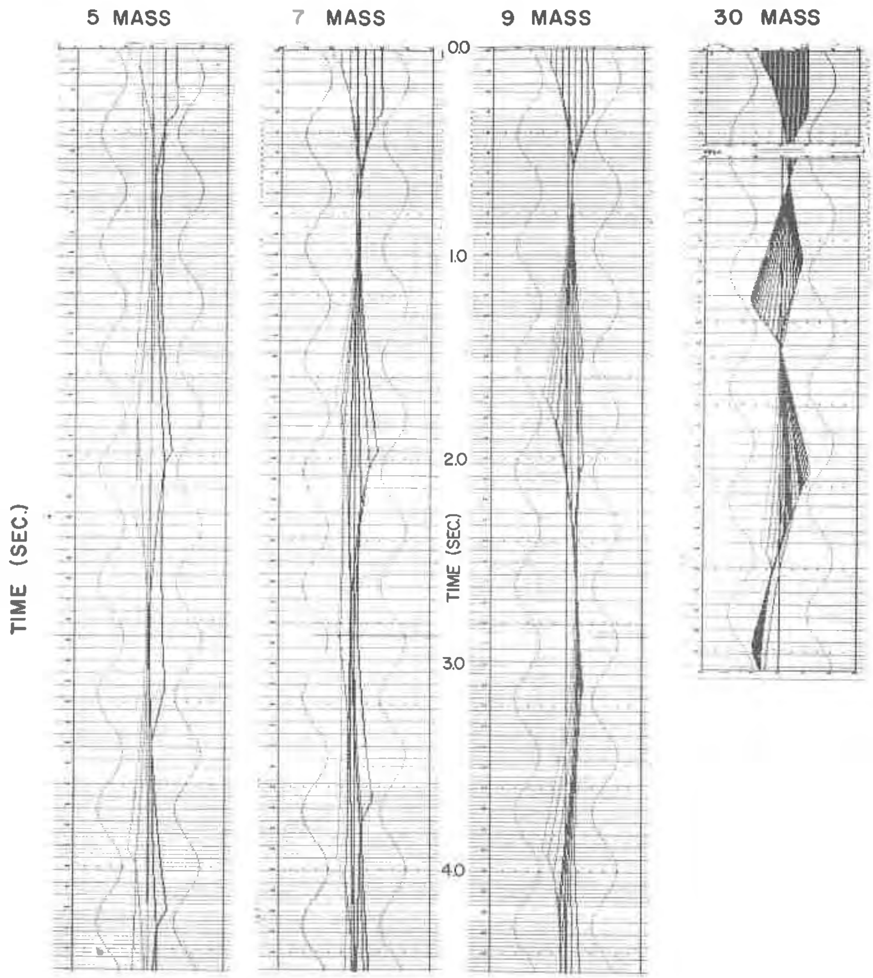

Figure 5 DYNAMIC RESPONSE OF DIFFERENT MASS MODELA AT 11.4 $\frac{rad}{sec}$.

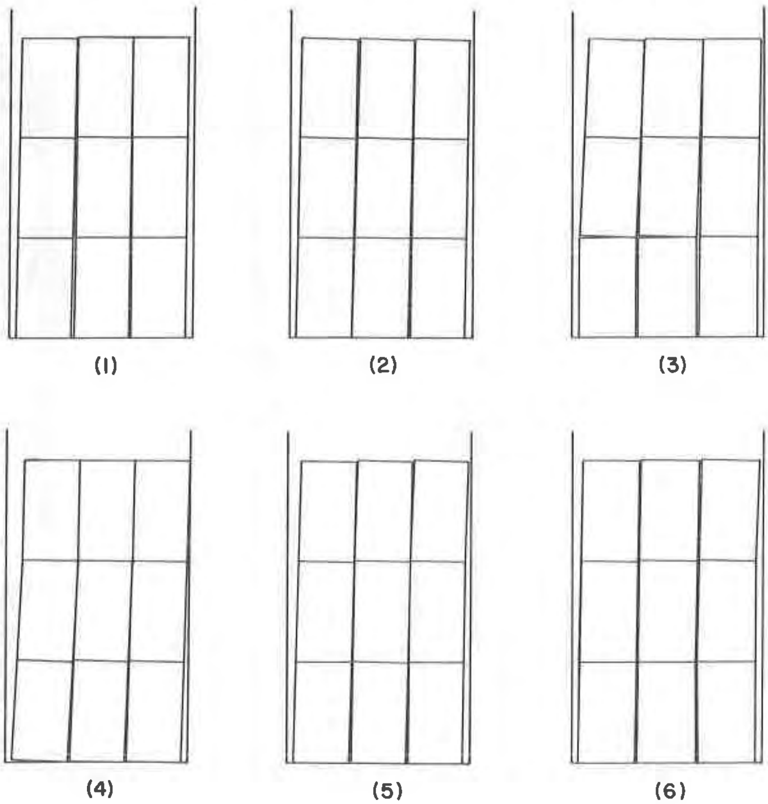

Figure 6 RESPONSE NINE BLOCK SQUARE ARRAY