Design Spectra Development Considering Short Time Histories

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Summary

The need for generation of seismic acceleration histories to prescribed response spectra arises several ways in structural dynamics. For example, one way of obtaining floor spectra is to generate a history from a foundation spectra and then solve for the floor motion from which a floor spectrum can be obtained. In other instances, there may be a need for modifying existing histories. A given history may have narrow peaks in its response spectrum, and these peaks may have to be broadened. In another case, a given history may lack some component such as torsion, and its spectrum can be adjusted for this component.

Two separate programs, MODQKE and MDOF, were written to provide a capability of obtaining equipment spectra from design spectra. MODQKE generates or modifies acceleration histories to conform with design spectra pertaining to, say, a foundation. MDOF is a simple linear modal superposition program that solves for equipment support histories using the design spectra conforming histories as input. Equipment spectra, then, are obtained from the support histories using MODQKE.

MODQKE was written to modify or provide new histories with special attention paid to short seismic records. A technique from the open literature was borrowed to generate an initial history that approximates a given response spectrum. Further refinement is done with smoothing cycles in which several correction signals are added to the history in a way that produces a least squares fit between actual and prescribed spectra. Provision is made for history shaping, a baseline correction, and final scaling. MODQKE performance has been demonstrated with seven examples having zero to ten percent damping ratios, and 2.5 seconds to 20 seconds durations and a variety of target spectra. The examples show the program is inexpensive to use.

MDOF is a simple modal superposition program. It has no eigensolver, and the user supplies mode shapes, frequencies, and participation factors as input. Floor spectra can be generated from design spectra by using a history from MODQKE that conforms to the design spectrum as input to MDOF. Floor motions from MDOF can be fed back to MODQKE without modification to obtain the floor spectra. A simple example is given to show how equipment mass effects can be incorporated into the MDOF solution. Any transient solution capability can be used to replace MDOF. For example, a direct transient approach may be desirable if both the equipment and floor structures are to be included in the model with different damping fractions.

1. Introduction

Input ground motion to nuclear power plant structures is defined by response spectra, and the seismic analysis for the design of the structure is generally based on response spectrum methods. Floor response spectra are also required for the design of piping and equipment. These may be generated using a time-history analysis of the structure provided an input history to the structure is available.

Various procedures have been suggested [1-4] for simplifying the analysis of equipment mounted on a supporting structure which responds to a seismic event. One of the simplest of these, developed by Gelman [1], uses interaction factors determined on the basis of a two degree-of-freedom dynamic system. Each equipment modal response is multiplied by an appropriate interaction factor, and the total component response is determined by combining the modal contributions using the root-sum-square method. Another procedure by Biggs and Roesset [3] determines the equipment seismic response from the ground motion response spectra on a mode-to-mode basis with the support structure. The procedure is based on the assumption that the support structure is much more massive than the equipment and is, therefore, unaffected by the equipment response. The method by Peters, Schmitz, and Wagner [4] offers some improvement over the above techniques in the case where the equipment and support structure frequencies are in resonance. In this method, a random vibration is used to determine the equipment magnification factors.

The need for generation of artificial seismic acceleration histories to prescribed response spectra arises several ways. For example, one way of obtaining equipment spectra is to generate a history from a foundation spectra and then solve for the floor motion from which the equipment spectrum can be obtained. In other instances, there may be a need for modifying existing histories. For example, a given history may have narrow peaks in its response spectrum, and these peaks may have to be broadened. In another case, a given history may lack some component such as torsion, and its spectrum can be adjusted for this component.

References [5] through [12] discuss ways of generating artificial histories. A program was desired that could produce and modify histories with short durations less than five seconds. This is attractive for expensive nonlinear transient analyses. The approach taken in MODQKE is to provide a means of generating initial histories as is done in Reference [12] or to allow the user to read in an existing history. Then the history is modified toward the desired target spectrum with a technique motivated by the spectrum raising technique given in Reference [6]. In each smoothing cycle, several sinusoids are added to the history in a manner that produces a least squares fit between the target spectrum and the spectrum resulting from the history.

Generation of artificial seismic acceleration histories is a key element to obtaining equipment spectra from design spectra with a transient approach. MODQKE, along with MDOF, a simple modal superposition, provides such a capability. Given a design spectrum at a basement location, for example, a corresponding artificial history may be generated with MODQKE. If the user can supply mode shapes, frequencies, participation factors and modal damping fractions for the floor structure and possibly the equipment, then MDOF will efficiently solve for the floor history. This history, in turn, can be converted to the floor spectra using MODQKE again without modification options.

2. MODQKE Theory

2.1 History Modification

The governing equation for a simple oscillator subjected to a base motion history $\ddot{x}_0$ is

$$\ddot{x} + 2\zeta p\dot{x} + p^2x = -\ddot{x}_0 \qquad (1)$$

where x is the relative displacement in the oscillator, ζ is the fractional damping, and p is the natural circular frequency. Quiescent initial conditions are assumed. If a correction

$$\ddot{x}_0 = a\cos(\omega t + \phi) \qquad (2)$$

is to be applied to the base motion, then its effect on the response of the oscillator is found by solving Equations (1) and (2). The solution is given in Reference [13]. When $\omega = p$, it is

$$\frac{\ddot{z}}{a} = \cos\phi \left\{ \begin{array}{l} \cos pt + \frac{1}{2\zeta}\sin pt - \\ \frac{e^{-\zeta pt}}{2\zeta}\left(2\zeta\cos p\sqrt{1-\zeta^2}\,t + \frac{1-2\zeta^2}{\sqrt{1-\zeta^2}}\sin p\sqrt{1-\zeta^2}\,t\right) \end{array} \right\} \qquad (3)$$

$$+ \sin\phi \left\{ \begin{array}{l} -\sin pt + \frac{1}{2\zeta}\cos pt - \\ \frac{e^{-\zeta pt}}{2\zeta}\left(\cos p\sqrt{1-\zeta^2}\,t - \frac{\zeta}{\sqrt{1-\zeta^2}}\sin p\sqrt{1-\zeta^2}\,t\right) \end{array} \right\}$$

where $\ddot{z} = \ddot{x} + \ddot{x}_0$ is the absolute acceleration. The choice of $\omega = p$ is made to minimize the effect of the correction on oscillators with other natural frequencies.

If the first order damping terms are correctly retained in Equation (3), the following expression is obtained.

$$\ddot{z}/a = \cos\phi \left\{ (1-e^{-\zeta pt})\cos pt + \frac{1-e^{-\zeta pt}}{2\zeta}\sin pt \right\}$$
$$+ \sin\phi \left\{ \frac{1-e^{-\zeta pt}}{2\zeta}\cos pt - (1 - \tfrac{1}{2}e^{-\zeta pt})\sin pt \right\} \qquad (4)$$

Now, as the damping approaches zero, it is easy to show that

$$\ddot{z}/a = \frac{pt}{2}\sin pt\cos\phi + \left(\frac{pt}{2}\cos pt - \frac{1}{2}\sin pt\right)\sin\phi \qquad (5)$$

Suppose that the response spectrum of a given history is known. This means that for each natural frequency of an oscillator subjected to the base motion, one has the maximum acceleration and the time at which it occurs. An obvious way then to correct this maximum acceleration towards the desired maximum acceleration is to choose the phase angle ϕ in Equation (3) so that a maximum effect is had at the time of maximum acceleration. With the time and phase set then, the appropriate magnitude of the correction could be determined from Equation (3). This is like the spectrum raising technique in [6]. Clearly for long duration histories, the response of the correction itself involves a very narrow peak and therefore will not influence response of oscillators having other natural frequencies. However, in the case of short duration histories, this is not so. This is easily seen by observing the response spectrum, for example, for a history consisting simply of a half sine pulse.

This frequency coupling effect is treated by considering the effect of a correction or adjusting signal on all the spectral accelerations at frequencies under consideration. Several adjusting signals are considered simultaneously in one spectrum smoothing or modification cycle. Their frequencies are chosen as those where the worst discrepancies between the actual and target spectra of the previous cycle occur. Each adjusting signal phase is set using Equation (3) so that the signal will peak at the time of maximum response at the same frequency. The magnitudes of the adjusting signals are determined in a way that minimizes the sum of squares of the differences between the actual spectrum and the target spectrum at all spectral frequencies.

To minimize the lease squares of the differences, let b_k be the responses at k spectral frequencies after adjustment and let a_k be the responses before adjustment. Then

$$b_k = a_k + S_{kj}\ \delta_j \tag{6}$$

where S_{kj} is sensitivity matrix giving the effect on the response by the j^{th} adjusting signal. Also, δ_j is the amplitude of the adjusting signal. These amplitudes are to be chosen to minimize

$$e = (t_k - b_k)^2 w_{\underline{k}} \tag{7}$$

where t_k is the target response at the k^{th} spectral frequency and w_k is a weight assigned to this frequency. In the last two equations, a repeated subscript implies summation while an underlined subscript goes along with the summation, if any. Now, if Equation (6) is substituted into Equation (7) and the derivative of e with respect to δ_i is set to zero, the following equation results.

$$w_{\underline{k}}^2\ S_{ki}\ S_{kj}\ \delta_j = w_{\underline{k}}^2\ S_{ki}\ (t_k - a_k) \tag{8}$$

This system of equations then is solved for the δ_j adjusting signal amplitudes. Normally t_k is positive. If a_k is negative, then t_k in Equation (8) has its sign changed. In general, the weight w_k is simply the reciprocal of the target response so that the sum of the squared percentage errors is minimized. However, the maximum weight is limited to the reciprocal of the zpa (zero period acceleration) so that low responses at low frequencies will not dominate the smoothing process. Also, a row is added to the S_{kj} sensitivity matrix for the zpa, and the weight assigned to it is five times the weight that would be assigned to a response of the same value.

Consider now the times that correspond to the maximum response accelerations. Clearly, these are the times that are used in Equation (3) when evaluating the sensitivity matrix. After the adjustments are made, the maximum accelerations may not occur at these times anymore. This is a problem that may lead to instabilities. A relaxation factor has been provided in MODQKE which is multiplied with all adjusting amplitudes to help avoid a series divergent smoothing cycles.

2.2 Initial History

An initial history may read into MODQKE in two ways. The first way is simply to read in the history at each uniform time increment. The second way is to let the program produce a constant large sinusoidal history and then shape this history with a few terms that give a piecewise linear shaping envelope versus time. This approach has been used successfully for short duration histories which cannot be generated with the SIMQKE approach described below. Given a target response spectrum with a particular maximum response and corresponding frequency and a zpa, the coefficient of the first sin pt term in Equation (4) can be used to determine the number of cycles in a simple sine history that would produce these characteristics of the target spectrum. Smoothing then, in MOOQKE can be effectively used to bring the actual spectrum to the target.

A SIMQKE approach given in Reference [12] is borrowed in MODQKE to generate initial histories. There, random vibration concepts lead to a power spectral density function $G(\omega)$, which is determined from the target response spectra. The history is computed with

$$\ddot{z}(t) = \sum_{n=1}^{N} 2G(\omega_n)\Delta\omega \cos(\omega_n t + \phi_n) \tag{9}$$

where the ω_n correspond to the spectral frequencies which are evenly spaced on the log axis from the minimum to the maximum spectral frequencies selected by the user. The ϕ_n in Equation (9) are randomly selected in the interval from 0 to 2π.

2.3 Response Spectrum Evaluation

The basic task here is to integrate Equation (1) and obtain the maximum acceleration $\ddot{x} + \ddot{x}_0$. A reliable method is the so-called exact integration approach in which the right side of Equation (1) is assumed linear from one time to the next. Equation (1) then may be integrated exactly. The solution in matrix form is written

$$\begin{Bmatrix} x_2 \\ \dot{x}_2 \end{Bmatrix} = C \begin{Bmatrix} x_1 \\ \dot{x}_1 \end{Bmatrix} + P \begin{Bmatrix} \ddot{x}_{01} \\ \ddot{x}_{02} \end{Bmatrix} \tag{10}$$

where the subscripts 1 and 2 refer to the beginning and end of a time increment. The C and P matrices are the complimentary and particular solutions given by

$$C = \frac{e^{-nt}}{q} \begin{bmatrix} q \cos qt + n \sin qt & \sin qt \\ - p^2 \sin qt & q \cos qt - n \sin qt \end{bmatrix} \tag{11a}$$

$$\begin{aligned} p^2hP = {} & \left(t - \frac{1}{q} e^{-nt} \sin qt\right) \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} \\ & + \left\{1 - e^{-nt} \left(\cos qt - \frac{n}{q} \sin qt\right)\right\} \begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix} \\ & + \left\{1 - e^{-nt} \left(\cos qt + \frac{n}{q} \sin qt\right)\right\} \begin{bmatrix} -(h+\xi) & \xi \\ 0 & 0 \end{bmatrix} \\ & + \frac{p^2}{q} e^{-nt} \sin qt \begin{bmatrix} 0 & 0 \\ -(h+\xi) & \xi \end{bmatrix} \end{aligned} \tag{11b}$$

Here **h** is the time increment and

$$n \equiv \zeta p \qquad q \equiv p\sqrt{1-\zeta^2} \qquad \xi \equiv 2\zeta/p \tag{12}$$

3. MDOF Procedure

Given modes, frequencies, participation factors, system histories, and damping fractions, MDOF solves the usual uncoupled equations for the modal histories using the exact integration operator. Time steps are constant. The modal histories are then summed in the manner of modal superposition to obtain absolute structure accelerations.

4. Programming and Special Considerations

MODQKE was written for a UNIVAC 110D in ASCII Fortran and also run on a CDC7600. The random number generator was written so that initial histories generated by the SIMQKE method will be the same from machine to machine. Other program features include the ability to zero damping, allowing for the shaping of a history with an envelope, a baseline correction, adjusting the final history so that the spectrum bounds the target spectrum from above, and provision for a printer plot that allows the user to quickly assess the overall fit of the spectra.

Clearly, there are spectra for which histories do not exist. For example, a prescribed spectrum with a maximum response to maximum acceleration ratio greater than that given by steady harmonic response is unlikely to have a history coverge to it in the smoothing process. The idea may be extended to short duration histories with the first sin pt coefficient in Equation (4).

5. Examples

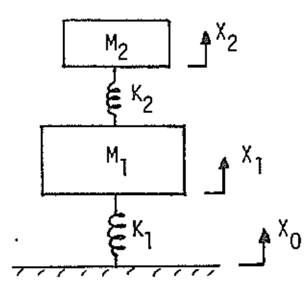
5.1 MODQKE Examples

Seven MODQKE examples are summarized in Table 1. They involve damping fractions from zero to 10% and durations ranging from 2.5 seconds to 20 seconds. Most of the cases required relaxation factors less than 1 to avoid a divergent smoothing process. Figures 1 and 2 show typical last smoothing cycle spectra compared to the target spectra.

The last four rows in Table 1 give an indication of MODQKE's performance. The large differences from the minimum to the maximum percentage difference (PCT) between the actual and target spectra is attributed to a few localized peaks in the actual spectra. The final rms of the percentage differences are much less. The UNIVAC central processor seconds reflect the run cost at about 3¢/second. This shows the program is inexpensive to use.

5.2 Combined MODQKE/MDOF Example

A simple example with both MODQKE and MDOF is given for a two degree of freedom system shown at the right. The 2-system should be viewed as equipment and the 1-system as a floor structure. By definition and choice:



$\mu \equiv m_2/m_1 = .05 \qquad \omega_1^2 \equiv k_1/m_1$

$\alpha \equiv \omega_2/\omega_1 = 1 \qquad \omega_2^2 \equiv k_2/m_2$

Note that the floor frequency ω_1 is matched to the equipment frequency in this example. Reference [1] gives details of the modal solution. The natural angular frequency ω is given through

$$\lambda \equiv \left(\frac{\omega}{\omega_1}\right)^2 = 1 + \frac{\mu}{2} \mp \sqrt{\mu(1 + \mu/4)}, \quad \lambda_1 = 0.8, \quad \lambda_2 = 1.25$$

and the participation factors Ψ_i corresponding to a unit rigid body displacement and modes normalized to $x_1 = 1$ are shown below.

Four floor frequencies $f_1 = 2\pi\omega_1$ are considered, and they are shown along with the natural frequencies below. Fractional damping is 2% throughout.

$$\psi_i = \frac{1+\mu a_i}{1+\mu a_i^2}, \quad a_i \equiv \frac{1}{1+\lambda_i}$$

$$\psi_1 = 0.5556 \qquad \psi_2 = 0.4444$$

FLOOR FREQUENCY (hz)	NATURAL FREQUENCIES (hz)	
7.20	6.44	8.05
9.92	8.88	11.10
15.04	13.45	16.81
19.82	17.72	22.16

MODQKE is used to generate an artificial ten second history $\ddot{x}_0$. Then MDOF is executed to solve for $\ddot{x}_1$. This is done four times in a multiproblem loop, once for each choice of floor frequency. The four histories for $\ddot{x}_1$ are used as MODQKE input, and their response spectra are evaluated by executing MODQKE without smoothing cycles.

The results are presented in Figure 3 superimposed on the design spectra. Frequency splitting, that is, the manifestation of two distinct natural frequencies from matched floor and equipment frequencies is obvious for the two lower floor frequencies.

References

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TABLE 1: MODQKE Example Summary

CASE NAME	ZERO	248	SIHZ	SD20	10	5	DAMP
Spectral Frequencies	101	101	121	121	101	101	61
Damping Fraction	0	.02	0	.02	.02	.02	.10
Max Spectral Freq (Hz)	30	30	50	50	30	30	30
Smoothing Cycles	8	8	0	2	4	6	5
Adjusting Frequencies	20	20	36	36	30	30	18
Max Adjusting Freq. (Hz)	20	20	50	50	15	15	30
Relaxation Factor	.8	.6	1	1	1	.8	.7
Duration (Sec) *	2.48	2.48	10	20	10	5	10
Max PCT - Min PCT**	15	18	74	23	25	22	10
RMS PCT (Cycle 0)	30.6	35.1	27.1	14.1	33.8	21.1	10.7
RMS PCT (Final Cycle)	2.8	4.3	14.3	4.6	4.9	3.3	1.9
UNIVAC 1100 User CP Sec	44	55	154	95	74	75	47
7600 CP Sec	4.2	--	--	--	--	7.7	--

* All time increments are .01 sec.

** Largest positive percent error minus largest negative percent error between actual and target spectra.

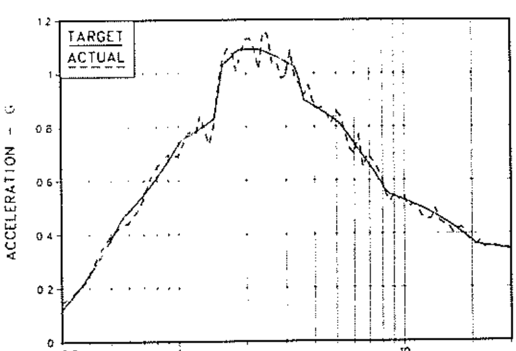

FIGURE 1: CASE 248 SPECTRA

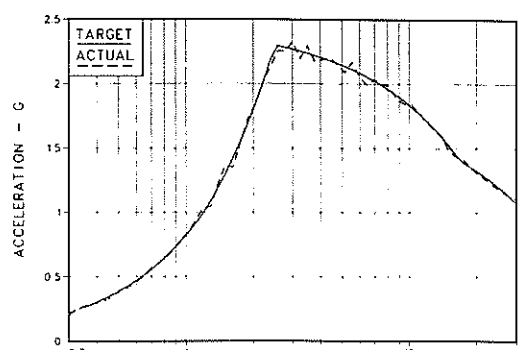

FIGURE 2: CASE DAMP SPECTRA

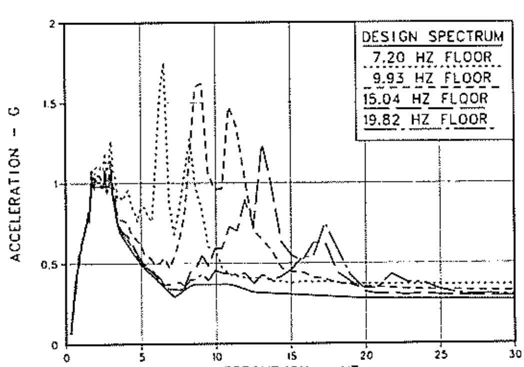

FIGURE 3. MODQKE/MDOF EXAMPLE RESULTS