Analysis of the Creep Rupture of Structural Members by the Finite Element Method

L. Gadomski, W. Wojewodzki
Instytut Mechaniki Konstrukcji Inzynierskich, Politechnika Warszawska, Aleja Armii Ludowej 16, PL-00-637 Warsaw, Poland

Summary

The finite element method is applied to the analysis of creep rupture of structural members. The members operate at constant temperature and are subjected to time-independent load.

The total strain rates are assumed to be a sum of the elastic and the creep strain rates. The elastic strain rates are given by Hooke's law. The creep strain rates and the damage rate of material are expressed by the Leckie and Hayhurst's equations, [2].

The set of differential-integral equations describing the problem is reduced after dividing the structure into finite elements and numerical integrations with respect to space, to the set of nonlinear first-order differential equations with respect to time. This set is integrated by the Euler's method. The time- step which ensures the numerical stability of the solution is determined. The program written for the computer RIAD-32 may by used to analyse the structures in plane stress, plane strain as well as in the axisymmetrical stress state. Using the eight-node isoparametric elements the following examples of structures are computed in details: a two-bar structure, a flat strip with a hole, a thick cylinder and a sphere-cylinder shell container made of BS 1501-271 steel. In particular, the propagation of front damage, the stress redistributions, the variation of displacements, the rupture life and influence of the elasticity of material are given. The obtained numerical results are compared with the available results obtained by other methods and by bounding techniques. Fairly good agreement can by noted.

Unlike in the existing solution, [4] a different and more convenient procedure of numerical realization of creep rupture process is proposed in the paper.

The presented application of the finite element method to the creep rupture offers the possibility to analyse more complex structures encountered in reactor technology.

1. Introduction

Many structural members, for example in nuclear reactors and in chemical plants have to operate above the creep threshold temperature, approximately $0.3\theta_m$ where θ_m is the absolute melting temperature. In such conditions the process of material deterioration is accelerated. As a result the stress redistribution occurs and the strain rates increase with time leading to the rupture in the tertiary phase of creep. Most engineering structures, which have a design life of 10^5h, are stressed to the level of 20 to 30 per cent of the high-temperature yield stress of the material. At such low stresses and high temperature the failures which occur due to creep do so at relatively low strains and are of brittle type, [1].

In order to determine the creep deformations, the stress redistribution and the rupture life of complex components it is necessary to employ the finite element method. The method will be applied to the structures which operate at constant elevated temperature and are subjected to time-independent load.

2. Basic equations

Consider the boundary-value problem for a body occupying the volume Ω in the three-dimensional Euclidean space. On a part of the surface Γ_u are given the displacements $\bar{u}_i$ and on a part $\Gamma_t = \Gamma - \Gamma_u$ the tractions $\bar{t}_i$ are given. The problem is described by the following equations.

- The constitutive equations. The total strain tensor ε_{ij} is assumed to be the sum of the elastic strain ε^e_{ij} and the creep strain ε^c_{ij} so that

$$\varepsilon_{ij} = \varepsilon^e_{ij} + \varepsilon^c_{ij} \,. \tag{1}$$

The elastic strain ε^e_{ij} is related to the stress σ_{ij} according to the equation

$$\varepsilon^e_{ij} = C_{ijkl}\sigma_{kl} \,, \qquad \text{where } C_{ijkl} \text{ is a tensor of constants.} \tag{2a}$$

The creep strain rate $\dot{\varepsilon}^c_{ij}$ and the damage rate of material $\dot{\psi}$ are given by the equations, [2]

$$\dot{\varepsilon}^c_{ij} = K\psi^{-n}\varphi^n \partial\varphi/\partial\sigma_{ij} \,, \qquad \dot{\psi} = -A\psi^{-\nu}\varphi^{\nu} \tag{2b,c}$$

where K, n, A, ν are material constants, $\varphi(\sigma_{ij})$ is a homogeneous and convex function of degree one and is equal to σ when the applied stress is uniaxial. The damage (or continuity) parameter ψ remains equal to unity when the material is undamaged and becomes zero when rupture occurs. In the calculations we assume $\varphi = (3I_2)^{1/2}$, where $I_2 = s_{ij}s_{ij}/2$ stands for the second invariant of the stress deviator s_{ij}.

- The geometric equations

$$\varepsilon_{ij} = (u_{i,j} + u_{j,i})/2 \,, \quad \text{where } u_i \text{ denotes a displacement vector.} \tag{3}$$

- The equilibrium equations

$$\sigma_{ji,j} + f_i = 0 \,, \qquad \text{where } f_i \text{ is a vector of body forces.} \tag{4}$$

The above formulated problem will be solved using the finite element method.

3. The finite element method

In the finite element procedure the entire volume of the body is approximated by the sum of small discrete elements interconnected at the finite number of nodes, $\Omega \cong \bigcup_{(E)} \Omega_{(E)}$. At the instant of time t the nodal displacement vector $\underset{\sim}{a}$ satisfies the set of equations

$$\underset{\sim}{K}\underset{\sim}{a} = \underset{\sim}{F} + \int_{\Omega} \underset{\sim}{B}^T \underset{\sim}{D} \underset{\sim}{\varepsilon}^c d\Omega \tag{5}$$

where $\underset{\sim}{K} = \int_{\Omega} \underset{\sim}{B}^T \underset{\sim}{D} \underset{\sim}{B} d\Omega$ is the stiffness matrix, $\underset{\sim}{F}$ denotes the load vector, $\underset{\sim}{B}$ is the matrix of derivatives of the shape functions ($\underset{\sim}{\varepsilon} = \underset{\sim}{B}\underset{\sim}{a}$), $\underset{\sim}{D}$ stands for the matrix of elastic constants ($\underset{\sim}{\sigma} = \underset{\sim}{D}\underset{\sim}{\varepsilon}^e$) and $\underset{\sim}{\varepsilon}^c = \int_0^t \dot{\underset{\sim}{\varepsilon}}^c(\underset{\sim}{\sigma}, \psi) d\tau$ are the creep strains. Getting $\underset{\sim}{a}$ from the solution of eqs.(5) and accounting for the eqs.(1 ÷ 4) the problem can be described by the following set of differential -integral equations:

$$\dot{\underset{\sim}{\sigma}} = \underset{\sim}{D}\dot{\underset{\sim}{\varepsilon}}^e = \underset{\sim}{D}(\underset{\sim}{B}\dot{\underset{\sim}{a}} - \dot{\underset{\sim}{\varepsilon}}^c) = -\underset{\sim}{D}\dot{\underset{\sim}{\varepsilon}}^c + DBK^{-1}\int_{\Omega} B^T D \dot{\varepsilon}^c d\Omega \;, \tag{6a}$$

$$\dot{\psi} = -A\psi^{-\nu}\varphi^{\nu}(\underset{\sim}{\sigma}) \tag{6b}$$

with the unknown quantities $\underset{\sim}{\sigma}(\underset{\sim}{x}, \tau)$ and $\psi(\underset{\sim}{x}, \tau)$ where $\tau \in \langle 0, t \rangle$.

The volume integrals can be evaluated numerically (using the Gauss quadratures) in the form of a sum $\int_{\Omega} g(\underset{\sim}{x})\, d\Omega = \sum_{i=1}^{G} c_i\, g(\underset{\sim}{x}_i)$ where c_i denotes the weights of integration at the points $\underset{\sim}{x}_i \in \Omega$. Thus, the time variation of $\underset{\sim}{\sigma}_i$, $\underset{\sim}{\varepsilon}_i^e$, $\underset{\sim}{\varepsilon}_i^c$ and ψ_i will only be investigated at the integration points $\underset{\sim}{x}_i$, $i = 1, 2, \ldots, G$.

Introducing the compound vectors and matrices

$$\mathbb{\Sigma} = \begin{Bmatrix} \underset{\sim}{\sigma}_1 \\ \vdots \\ \underset{\sim}{\sigma}_G \end{Bmatrix} \quad \mathbb{E} = \begin{Bmatrix} \underset{\sim}{\varepsilon}_1 \\ \vdots \\ \underset{\sim}{\varepsilon}_G \end{Bmatrix} \quad \mathbb{E}^c = \begin{Bmatrix} \underset{\sim}{\varepsilon}_1^c \\ \vdots \\ \underset{\sim}{\varepsilon}_G^c \end{Bmatrix} \quad \mathbb{C} = \begin{bmatrix} c_1 \underset{\sim}{I}_6 & \cdots & \underset{\sim}{0} \\ \vdots & \ddots & \vdots \\ \underset{\sim}{0} & & c_G \underset{\sim}{I}_6 \end{bmatrix} \quad \mathbb{D} = \begin{bmatrix} \underset{\sim}{D}_1 & \cdots & \underset{\sim}{0} \\ \vdots & \ddots & \vdots \\ \underset{\sim}{0} & & \underset{\sim}{D}_G \end{bmatrix} \quad \mathbb{B} = \begin{bmatrix} \underset{\sim}{B}_1 \\ \vdots \\ \underset{\sim}{B}_G \end{bmatrix} \tag{7}$$

where I_6 is the identity 6×6 matrix, the set (6) can be reduced to the following set of nonlinear differential equations of the first order with respect to time:

$$\dot{\mathbb{\Sigma}} = \mathbb{S}\mathbb{C}\dot{\mathbb{E}}^c, \qquad \dot{\psi}_i = -A\psi_i^{-\nu}\varphi^{\nu}(\underset{\sim}{\sigma}_i), \qquad i = 1, 2, \ldots G. \tag{8a,b}$$

In these equations the matrix $\mathbb{S} = \mathbb{D}\mathbb{B}\mathbb{K}_G^{-1}\mathbb{B}^T\mathbb{D} - \mathbb{D}\mathbb{C}^{-1}$ is symmetric and non-positive definite, see [3], $\mathbb{K} = \mathbb{B}^T\mathbb{C}\mathbb{D}\mathbb{B} = \sum_{i=1}^{G} c_i \underset{\sim}{B}_i^T \underset{\sim}{D}_i \underset{\sim}{B}_i \cong \int_{\Omega} \underset{\sim}{B}^T \underset{\sim}{D} \underset{\sim}{B} d\Omega = \underset{\sim}{K}$, $\dot{\mathbb{E}}^c$ is given by eq. (2b). The set (8) can be integrated by the Euler's method. If the values of stress $\mathbb{\Sigma}_{(n)}$ and parameter $\psi_{i(n)}$ are known at the instant of time t_n then the corresponding values at the time $t_{n+1} = t_n + \Delta t_n$ are given by the expressions

$$\begin{aligned} &\mathbb{\Sigma}_{(n+1)} = \mathbb{\Sigma}_{(n)} + \Delta t_n \dot{\mathbb{\Sigma}}_{(n)} \;, \\ &\psi_{i(n+1)} = \psi_{i(n)} + \Delta t_n \dot{\psi}_{i(n)} \;, \qquad \mathbb{E}^c_{(n+1)} = \mathbb{E}^c_{(n)} + \Delta t_n \dot{\mathbb{E}}^c_{(n)} \;. \end{aligned} \tag{9}$$

Accuracy of the integration of eqs. (8) depends upon the time step Δt_n especially prior to rupture when the creep process becomes accelerated. The empirical methods of determination of Δt, see [4, 5], should be confirmed by many numerical tests. In this paper an analytical formula is proposed, use being made of the result given in [3] for the elastic / viscoplastic problem

$$0 < \Delta t < 2/|\lambda|_{max} \tag{10}$$

where $|\lambda|_{max}$ denotes the maximum modulus from eigenvalues of the Jacobian $\mathbb{J}$.

Satisfying the inequality (10), the stability is ensured of the solution of differential equations integrated by the Euler's method. Accounting for the linearized form of the equations (8a) and the relation

$$\mathbb{J} = \mathbb{S}\mathbb{C}\mathbb{H} = \frac{\partial \Sigma}{\partial \Sigma} \quad where \quad \mathbb{H} = \begin{bmatrix} \underset{\sim}{H}_1 & \cdots & \underset{\sim}{0} \\ \vdots & \ddots & \vdots \\ \underset{\sim}{0} & \cdots & \underset{\sim}{H}_6 \end{bmatrix} \qquad \underset{\sim}{H}_i = \frac{\partial \dot{\varepsilon}_i^c}{\partial \underset{\sim}{\sigma}}, \quad i=1,2,\ldots,6. \quad (11)$$

the value of $|\lambda|$ max was obtained. Inserting $|\lambda|$ max into (10) gives the critical time step

$$\Delta t_{cr} = min\left(\frac{4(1+\bar{\nu})}{3EK}\,\frac{\psi_i^n}{n\,\varphi^{n-1}(\underset{\sim}{\sigma}_i)}\right) \tag{12}$$

where E is Young's modulus and $\bar{\nu}$ denotes Poisson's ratio.

When the rupture condition $\psi = 0$ for an element is satisfied, the material is assumed to be no longer capable of supporting stress. In [4] the failed elements were removed from the structure. In this paper a different method is proposed which seems to be easier in a numerical realization. The ruptured element is left in the structure but its mechanical properties are changed by an alteration of its stiffness matrix $\bar{\underset{\sim}{K}}_{(E)} = \alpha \underset{\sim}{K}_{(E)}$, $0 < \alpha \ll 1$. This results in changing the overall stiffness matrix of the structure and in an instantaneous elastic change of the equilibrium state (usually small). In further analysis the weakened element is treated in a special way. First, it does not influence the magnitude of Δt and second, the increment of creep strain in the sequent increment of time is taken as a difference between the actual total strain and creep strain. It causes that the influence of this element on the behaviour of the remaining structure is not cumulative and is kept invariably small. The lifetime of the structure is reached when the overall structure is no longer capable of withstading load due to creep rupture.

4. The numerical results

The numerical computations were carried out using the RIAD-32 Computer. The program MINIFEM worked out by R. Taylor and published in [6], was suitably extended, modified and adopted.

Some of the results are presented for a two-bar structure, Fig. 1, a thick cylinder, Fig. 3, and for a sphere-cylinder shell container, Fig. 5. The analysed atructures were aubjected to constant loading and temperature. The material parameters n, K, ν and A as the functions of temperature in the range from 500 to 575°C were taken as for the steel BS 1501-271, see [7]. In Fig. 2 the time-variation of the damage parameter and the stress redistribution are shown in the two bars for the temperature 550°C. At the instant $t_{r(2)} = 43045$ h the bar No. 2 ruptured first, followed by the bar No. 1 at $t_{r(1)} = 45114$ h. In the case of thick cylinder the thickness of the wall was divided into 5 elements. The rupture was initiated at the inner boundary at the instant t_{r1} and the front damage reached the outer element at t_r. For example, for the temperature 575°C the results were $t_{r1} = 5686$ h, $t_r = 5992$ h. In Fig. 4 the stress redistribution and the propagation of the damage front are illustrated.

The obtained results for the two-bar structure and the thick cylinder were compared with available results obtained by other methods, [7]. Good agreement was observed. Also, the elastic strains were found to have small influence on the rupture life.

The formation and propagation of fracture in the sphere-cylinder shell container are shown in Fig. 6. The determined damage regions seem to be in accordance with the experimental observations disscused in paper [8]. The influence of the reinforcement shown in Fig. 5 is found to be relevant. In the investigated cases the lifetime was about 3 ÷ 4 times longer than for the container without reinforcement.

The prepared program enables one to obtain the steady-state creep solution which may be used to find the bounds on the rupture life of structures and, as a result, to make their design more rational and simpler, [9].

References

[1] PENNY, A.K., MARRIOTT, D.L., "Desing for Creep", McGraw-Hill Book Campany (UK) Limited, London (1971)

[2] LECKIE, F.A., HAYHURST, D.R., "Creep Rupture of Structures", Proc. Roy. Soc., A 340, 325, London (1974)

[3] CORMEAU, I.C., "Numerical Stability in Quasi-Static Elasto/Visco-Plasticity", Int. J. Num. Engng, 9, 109-127 (1975)

[4] HAYHURST, D.R., DIMMER, P.P. CHERNUKA, M.W., "Estimates of the Creep Rupture Lifetime of Structures Using the Finite Element Method", J.Mech. Phys. Solids, 23, 335-355 (1975)

[5] ZIENKIEWICZ, O.C., CORMEAU, I.C., "Viscoplasticity Solution by Finite Element Method", Arch. Mech., 24, 5 - 6, 873-888 (1972)

[6] ZIENKIEWICZ, O.C., "Finite Element Method for Engineers", McGraw-Hill, London (1977)

[7] WOJEWÓDZKI, W., "Creep Rupture of Structures at Elevated Temperature and Estimates of Rupture Life", Eng. Trans., 26, 3, 515-540, (1978)

[8] LECKIE, F.A., HAYHURST, D.R., MORRISON, C.J., "The Creep Behaviour of Sphere-Cylinder Shell Intersection Subjected to Internal Pressure", Proc. Roy. Soc. A 349, 9-34, London (1975)

[9] LECKIE, F.A., WOJEWÓDZKI, W., "Estimates of Rupture Life-Constant Load", Int. J. Solids Structures, 11, 1357-1365 (1975)

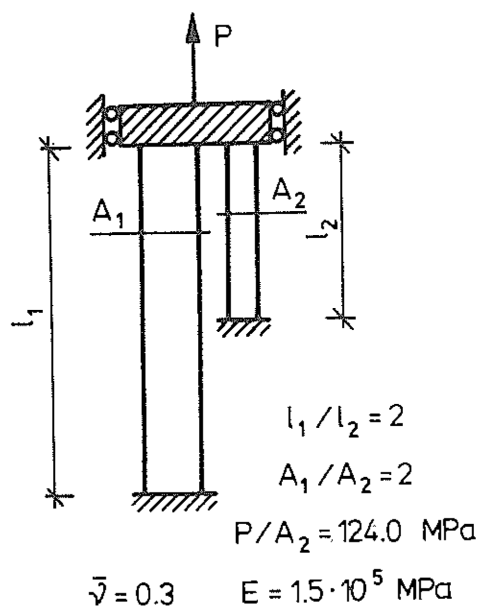


Fig. 1 Two-bar structure

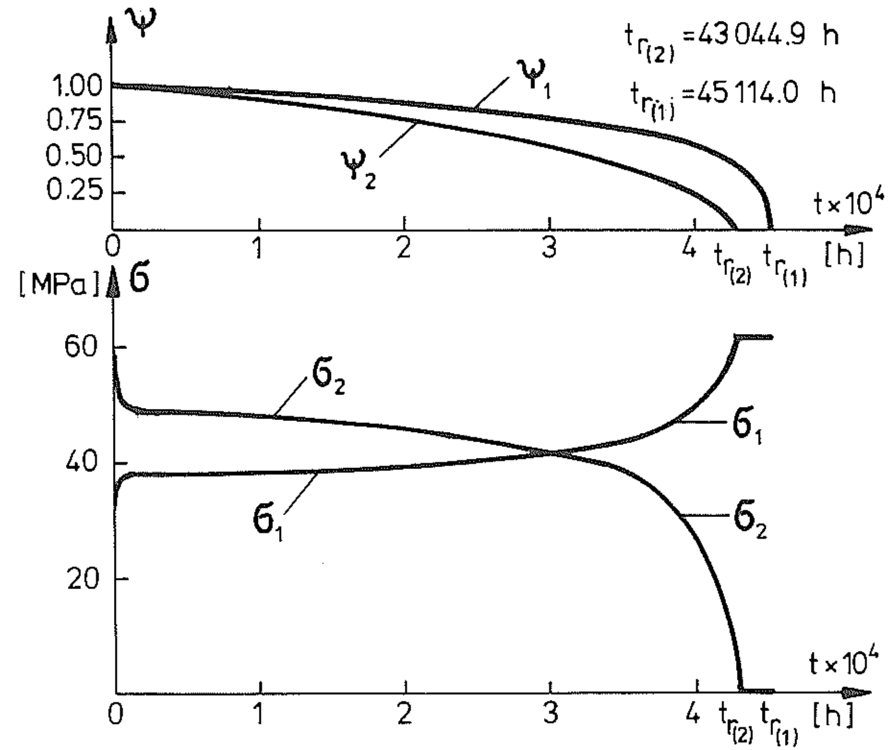
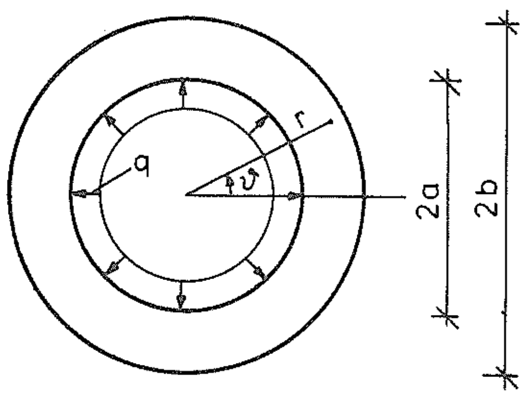


Fig. 2 Time-variation of the damage parameter and the stress redistribution, $\theta = 550^oC$



$b/a = 1.5 \qquad q = 31.0\ \mathrm{MPa}$

$E = 1.5 \cdot 10^5\ \mathrm{MPa} \qquad \bar{\nu} = 0.3$

Fig. 3 Thick cylinder

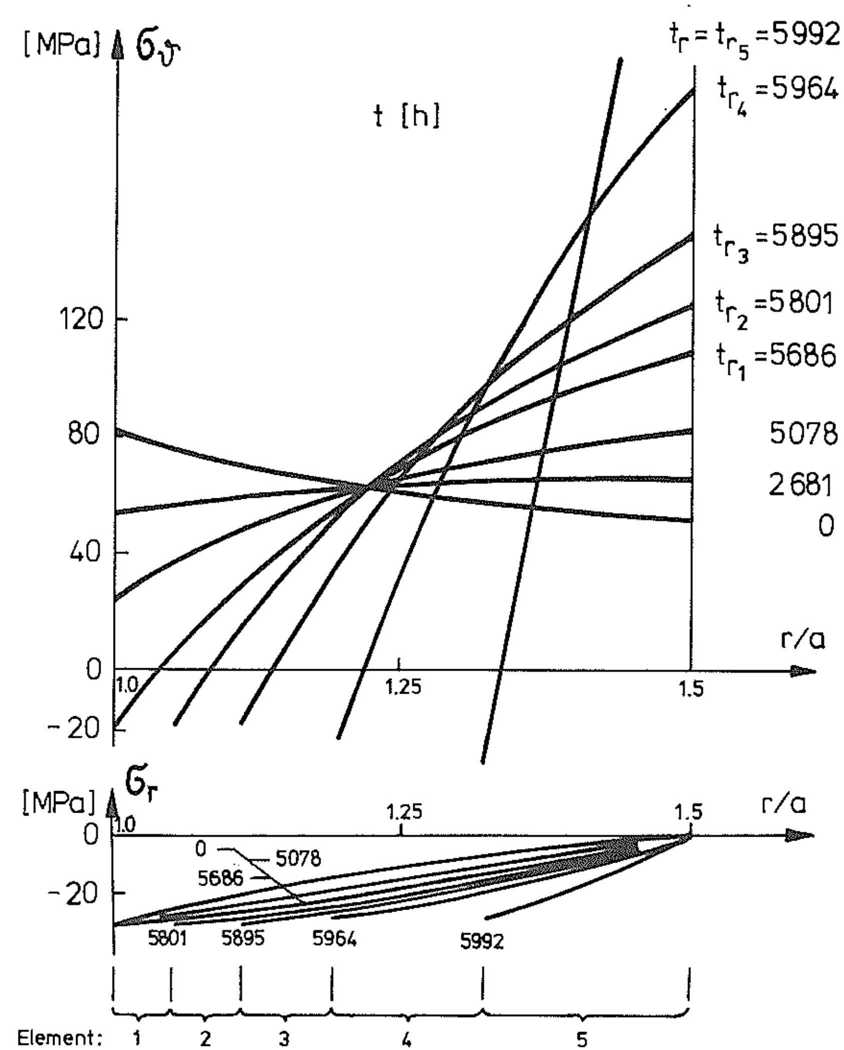


Fig. 4 Stress redistribution and propagation of the front damage in the thick cylinder, θ = 575°C

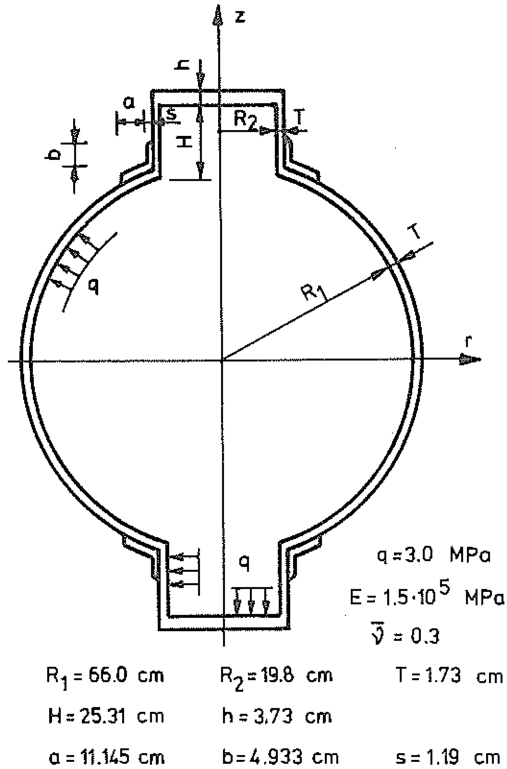


Fig. 5 Sphere-cylinder shell container with reinforcement

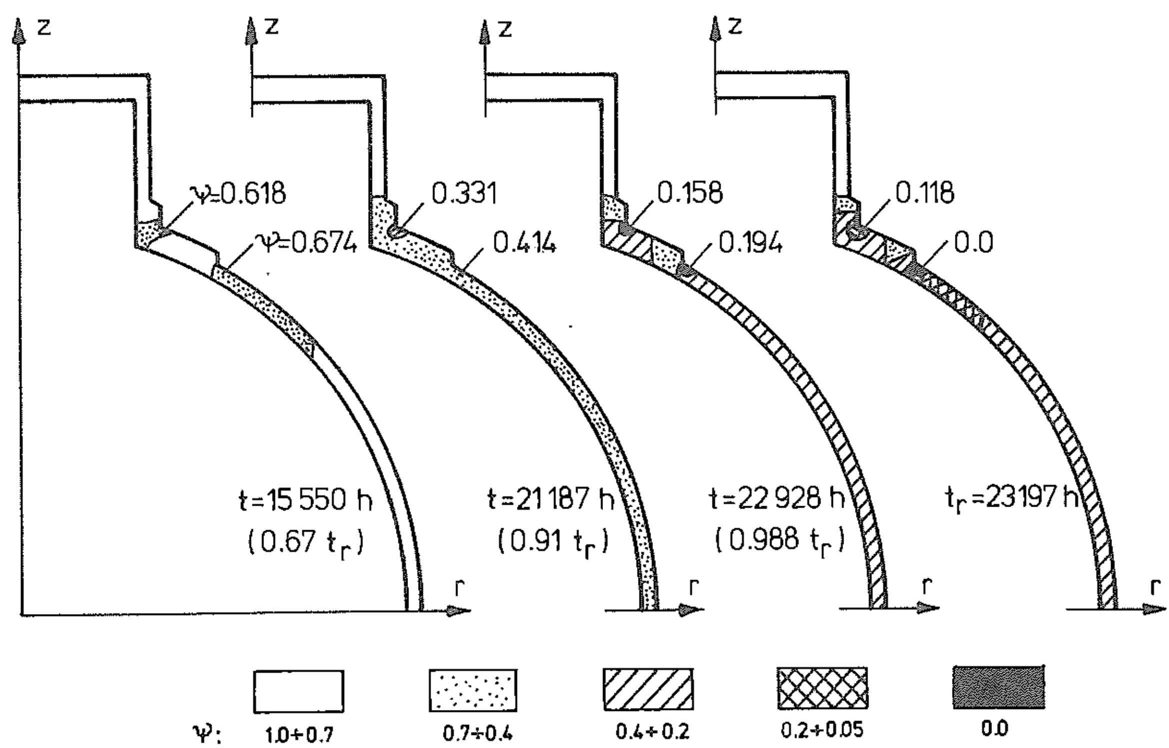


Fig. 6 Formation and propagation of fracture in the sphere-cylinder shell container, $\theta = 550^{\circ}C$