An Impedance Function Approach for Soil-Structure Interaction Analyses Including Structure-to-Structure Interaction Effects

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SUMMARY

The dynamic soil-structure and structure-to-structure interaction effects may be determined in one of the two ways: by modeling the entire soil-structure system by a finite-element model, or by using a frequency-dependent (or frequency-independent) impedance function approach. In seismic design of nuclear power plant structures, the normal practice is to use the first approach because of its simplicity and easy availability of computer codes to perform such analyses. However, in the finite-element approach, because of the size and cost restrictions, the three-dimensional behavior of the entire soil-structure system and the radiation damping in soil are only approximately included by using a two-dimensional finite-element mesh.

In using the impedance function approach, the soil-structure analyses can be performed in four steps: (a) determination of the dynamic properties of the fixed base superstructure, (b) determination of foundation and structure impedance matrices and input motions, (c) evaluation of foundation motion, (d) analysis of the fixed base superstructure using computed foundation motion.

In this paper, the standard frequency-dependent impedance function approach is developed further to study the effects of soil-structure and structure-structure interaction on the response of structures. The development is based on the derivation of frequency-dependent coupling matrices using a flexibility approach. This procedure essentially consists of evaluating complex displacements resulting from unit dynamic loads applied on a half-space representing the soil medium, for each input frequency, along directions corresponding to the degrees of freedom of the two foundations at their interface with the supporting soil. These displacements are used to develop the compliance matrix of the complete soil-structure system, which is then inverted to obtain the impedance matrix. The numerical procedures employed for this development are described in detail. Procedures for approximately including the soil nonlinearity and embedment effects in conjunction with the above described approach, as well as a procedure for including the foundation flexibility, are also discussed. Detailed computational steps required to perform the soil-structure analyses by this modified impedance-function approach are also described.

Results are then presented for analyses performed to determine the influence of interaction effects between two structures, subjected to horizontal ground accelerations. Numerical results are also presented for impedance coefficients, obtained using the above approach for footings of various shapes and compared with available solutions.

1. Introduction

In the past decade, the effects of flexibility of foundation soil on the response of structures, known as soil-structure interaction effects, and the effects of dynamic characteristic of a structure on the response of adjacent structures through the soil medium, known as structure-to-structure interaction effects, have attracted considerable attention by engineers and researchers, especially in regard to their application to the seismic design of nuclear power plant structures. These interaction effects have been included by modeling the entire soil-structure system by finite elements or by using a frequency-dependent (or frequency-independent) impedance function approach. Although the finite element method is applicable to a wide range of problems, its scope is usually limited to the analysis of soil-structure systems modeled by two-dimensional finite elements.

In this paper, the standard frequency-dependent impedance function approach is developed further to study the effects of soil-structure and structure-to-structure interaction on the response of structures. The development is based on the derivation of frequency-dependent coupling matrices using a flexibility approach. The procedures for approximately including the soil nonlinearities and foundation embedment effects are also discussed. Numerical results are presented demonstrating the application of these procedures.

2. Theoretical Formulation

2.1 Equilibrium Equations

At frequency ω, the equilibrium equations for structure-foundation-soil system may be written as

$$\left\{-\omega^2 (\underline{M}_B + M_0) + K_S\right\} \ddot{\underline{U}} = \omega^2 \underline{F}_D \tag{1}$$

where M_B = Equivalent mass matrix of the superstructures, M_0 = Mass matrix of the foundations, K_S = Impedance matrix of the soil-foundation system, $\underline{F}_D$ = Driving force vector, and $\ddot{\underline{U}}$ = Total generalized acceleration vector of the foundation. Assuming seismic excitation to be represented by vertically incident shear waves, the driving force vector, $\underline{F}_D$, is given by

$$\underline{F}_D = \underline{K}_S \ddot{U}g/\omega^2 \tag{2}$$

where $\ddot{U}g$ = Foundation input motion under the action of seismic waves. Once $\underline{M}_B$, $\underline{M}_0$, and $\underline{K}_S$ are computed, the total foundation acceleration response in the frequency-domain may be obtained using eq. (1) and (2).

2.2 Derivation of Impedance Matrices of Soil-Foundation System

For a force vector, $\underline{P}$ (X_0), acting at an arbitrary point, X_0, on the surface of an elastic half-space, the displacement vector, $\underline{U}$, that results at another point, $\underline{X}$, is given by

$$\underline{U}\,(X) = \underline{G}\,(\omega, \underline{X} - \underline{X}_0)\,\underline{P}\,(X_0) \tag{3}$$

where $\underline{G}\,(\omega, \underline{X} - \underline{X}_0)$ is the Green's function matrix of the order 3x3 and ω is the frequency of excitation of the applied force. Each term of the Green's function matrix is a function of Poisson's ratio, ν, and shear modulus, G, of the elastic medium [1].

Once the Green's function matrix of an elastic half-space is known, the compliance matrix of the soil-foundation system may be obtained by the following numerical procedure.

Consider two rigid footings of arbitrary shape (Fig. 1), supported on the surface of an elastic half-space. The expressions for the dynamic displacement amplitudes of the footings, in terms of the corresponding dynamic force amplitudes, may be expressed as

$$\begin{Bmatrix} \underline{d}_1 \\ \underline{d}_2 \end{Bmatrix} = [F] \begin{Bmatrix} \underline{f}_1 \\ \underline{f}_2 \end{Bmatrix}$$

where $\underline{d} = (U_x, U_y, U_z, U_{xx}, U_{yy}, U_{zz})$ are displacement and rotation amplitudes at a footing, and $\underline{f} = (f_x, f_y, f_z, f_{xx}, f_{yy}, f_{zz})$ are corresponding forces and moments at the footing. The subscripts 1 and 2 represent the corresponding quantities for Footings 1 and 2, respectively.

and $[F]$ = Compliance matrix defined by

$$\begin{bmatrix} \underline{F}_{11} & \underline{F}_{12} \\ \underline{F}_{21} & \underline{F}_{22} \end{bmatrix}$$

where $\underline{F}_{11}$ and $\underline{F}_{22}$ are 6x6 uncoupled compliance matrices of each footing and $\underline{F}_{12} = \underline{F}_{21}^T$ is the 6x6 coupled compliance matrix.

The numerical procedure employed for the computation of coupling compliance matrices basically consists of evaluating the displacements resulting from unit dynamic loads at each frequency ω corresponding to each degree of freedom of the foundations 1 and 2. Under the application of unit force or moment at one of the footings (in the three translational and three rotational directions), the corresponding rigid base pressure distribution, as shown in Figure 2, is assumed. The footings are subdivided into a rectangular grid of elements. It is assumed that the pressure distribution under each element is uniform, and the total force at the center of the element is computed by multiplying the area of that element by the pressure at the center of the element of one footing; the displacements at the center of each element of the other footings are computed using Green's function matrix described previously for the loads on the surface of an elastic half-space.

The displacements at the center of each element of the second footing are then averaged and normalized by the total reaction force or moment applied at the first footing to obtain the corresponding compliance coefficients. The rotation-translation coupling coefficients are computed by applying a unit translational force at one footing and computing the average rotation at the second footing. Using a similar procedure, rotation-rotation coupling coefficients are also obtained.

The uncoupled compliance matrices, $\underline{F}_{11}$ and $\underline{F}_{22}$, are computed considering a similar numerical procedure in which each footing is assumed to rest independently on the surface of an elastic half-space. Each footing is divided into a rectangular grid of elements. The pressure distribution under each footing due to a unit load corresponding to each degree of freedom of the footing is assumed. The force at the center of each element is obtained by assuming uniform pressure distribution under the element. Due to a concentrated force applied at the center of each element, the displacements at the centers of the remainder of the elements are computed. The corresponding compliance coefficients of the footing are obtained by averaging the displacements at the footing due to the load on each element, in turn, and normalizing them by the total reaction force or moment. The compliance matrix of the soil-structure system so obtained is inverted to obtain the impedance matrix of the system.

2.3 Inclusion of Foundation Flexibility

The above procedure for the computation of impedance coefficients for the soil foundation considered only the rigid body degrees of freedom of a foundation on the surface of an elastic half-space. The procedure is also applicable to the derivation of the impedance coefficients assembled in the form of a substructure matrix for a flexible footing supported on the surface of the soil medium, without making any rigid body approximations for the footing. The flexible footing may be idealized by finite elements with discrete nodes and included in the superstructure model. A unit dynamic load is applied corresponding to each degree of freedom of the finite element nodes at the foundation-soil interface, and the corresponding complex displacement vector associated with all the degrees of freedom at the interface is obtained. These displacement vectors are assembled to obtain the half-space substructure compliance matrix, which is then inverted to obtain the corresponding impedance matrix. The frequency-dependent substructure impedance matrix so obtained may be used in conjunction with the superstructure model (which includes the flexible footing), and interaction analysis may then be performed in frequency domain to compute the response of the superstructure.

2.4 Inclusion of Foundation Embedment Effects and Soil Nonlinearities

The embedment effects may be approximately included, using frequency-dependent side layer impedance coefficients. These springs are obtained using relationships given in [2], where the soil is modeled using a side layer resting on the same half-space as the structure. These side layer impedance coefficients are functions of properties of the soil on both sides of the building (shear modulus G, and Poisson's ratio ν), the depth of embedment, and the distance of the center of gravity of the foundation from the bottom of the base-mat. The impedance coefficients of the side layer so obtained may be directly added to the diagonal terms of the impedance matrices computed for the half-space.

The nonlinear behavior of the foundation soil may be taken into consideration by using an iterative equivalent linear procedure based on the strain-dependent characteristics of the soil. A first estimate of the strains in the soil may be obtained using frequency-independent soil springs in conjunction with the fixed base models of the superstructures. From a time history analysis of this soil-structure model, the shear forces are computed, from which "effective average" soil strains are obtained. On the basis of these preliminary soil strains, the corresponding values of the shear moduli are computed for each soil case from the shear modulus versus strain curves. These values of soil shear moduli are used for the soil-structure interaction analyses. The new shear strains are obtained from the base shear forces obtained from the soil-structure interaction analyses, from which the new values of shear moduli are then computed. This procedure may be repeated until convergence on soil properties is obtained.

2.5 Computational Steps in Performing Soil-Structure Interaction Analyses

The computational steps required to perform the complete soil-structure interaction analyses are as follows:

1. Development of the fixed-base superstructure models.

2. Analyses of the fixed-base models for frequencies, modeshapes, and mass matrices.

3. Preliminary soil-structure interaction analyses and computation of initial values of average soil strain and shear modulus.

4. Generation of Green's function table at specified frequencies for linear elastic isotropic half-space.

5. Development of frequency-dependent coupled and uncoupled foundation compliance matrices using Green's function table generated in Step 4, and the average soil shear modulus computed in Step 3.

6. Inversion of the complete compliance matrix, consisting of uncoupled and coupled terms, to obtain the complete impedance matrix of the soil-foundation system.

7. Development of the complete impedance matrix, $\underline{M}_B$, of the superstructures.

8. Formation and inversion of the matrix, $[-\omega^2 (\underline{M}_B + \underline{M}_0) + \underline{K}_S]$, at each frequency step ω, where $\underline{M}_B$ is the impedance matrix of the superstructures, $\underline{M}_0$ is the mass matrix of the foundation, and $\underline{K}_S$ is the impedance matrix of the soil-foundation system.

9. Computation of foundation driving force vector.

10. Deconvolution of the time history of acceleration at the grade level in free-field to obtain time histories of input accelerations at the foundation levels of the structures.

11. Transformation of input acceleration time histories from time to frequency domain.

12. Soil-structure interaction analyses and computation of the acceleration time histories at the base of each footing, as follows:

$$\ddot{\underline{U}} = [-\omega^2 (\underline{M}_B + \underline{M}_O) + \underline{K}_S]^{-1} \underline{F}_D$$

where $\underline{F}_D$ = driving force.

13. Transformation of the foundation motion, obtained in Step 12, from frequency to time domain.

14. Time history dynamic analyses of the fixed base structures and computation of average soil strain and the shear modulus.

15. Repetition of Steps 5 through 14 until convergence is obtained on soil properties.

16. Computation of maximum response quantities and floor response spectra for fixed-base superstructures for the final soil shear modulus.

3. Sample Analyses

3.1 Impedance Functions for a Rigid Rectangular Footing

The impedance functions for a rigid rectangular footing resting on the surface of an elastic half-space were computed by the flexibility approach described in this paper. The half-space soil properties used were: Poisson's ratio, ν = 0.35, shear modulus G = 12,500 ksf, and mass density ρ = 0.00388 K-sec^2/ft^4. A 5x10 grid was used for the footing of dimension 158' x 340'. The results obtained for ω = 0 were compared with frequency-independent solution obtained using relationships given in [2]. The results are presented in Table I.

3.2 Compliance Functions for a Rigid Circular Footing

The horizontal compliance functions for a rigid circular footing resting on the surface of an elastic half-space with Poisson's ratio, ν = 0.33, and shear modulus G = 12,500 ksf, were computed for several values of dimensionless frequencies and compared with solution given in [1]. The results are presented in Table II.

3.3 Interaction Analysis of Two Buildings

The soil structure interaction analyses of two buildings were performed for several values of distance D between the buildings. The first five natural frequencies of Building A, which has a circular footing, are 3.97, 5.24, 5.45, 5.94 and 8.07, and those of Building B, which has a rectangular footing, are 2.95, 4.50, 7.32, 7.83, 10.50. The two footings were assumed to be on an elastic half-space with shear wave velocity, V_S = 2,000 fps, Poisson's ratio, ν = 0.35, and soil shear modulus, G = 13,000 ksf. The plots of spectral acceleration responses at the top of Building A for several distances D between the buildings are shown in Figure 3.

4. Summary and Conclusions

A flexibility approach was described in this paper to compute frequency-dependent impedance matrices for rigid, as well as flexible footings resting on the surface of an elastic half-space. Results were obtained using this approach for rigid footings of rectangular and circular shapes. The influence of building-to-building interaction of two buildings through soil on their responses was also investigated. Results were presented for various distances between the buildings. For the example shown in the paper, it was found that the influence of building-to-building interaction on structural response was negligible. However, this is not true in general. The importance of building-to-building interaction depends on the relative dynamic characteristics of the adjacent structures, the properties of the soil foundation, and the characteristics of the input motion. The procedure described in this paper is also applicable to more than two footings resting on the surface of an elastic half-space. Analyses were also performed using impedance functions computed using a flexible footing supported on a half-space for four adjacent buildings. The results for these analyses will be presented in a future paper.

References

[1] WONG, H. L., LUCO, J. E., "Dynamic Response of Rigid Foundations of Arbitrary Shape," Int. J. Earthq. Engng Struct. Dyn. 4, 579-587 (1976).

[2] NOVAK, M., "Vibrations of Embedded Footings," Proc. of American Society of Civil Engineering, National Structural Engineering Meeting, San Francisco, California, April, 1973.

[3] Engineering Decision Analysis Company, Inc. (EDAC), "Soil-structure Interaction Analyses Using an Impedance Function Approach," Prepared for General Electric Company, EDAC-134-340.02, March 1980.

Table I. Impedance Functions of a Rigid Rectangular Footing on Elastic Half-Space

Spring Constant	Units	Present Approach	Novak's Approach Ref. [2]
K_X	kips/ft	7.73×10^6	7.82×10^6
K_Y	kips/ft	8.45×10^6	8.21×10^6
K_Z	kips/ft	10.20×10^6	9.36×10^6
K_{XX}	kip-ft/rad	6.35×10^{10}	6.86×10^{10}
K_{YY}	kip-ft/rad	1.80×10^{11}	1.93×10^{11}
K_{ZZ}	kip-ft/rad	1.52×10^{11}	1.88×10^{11}

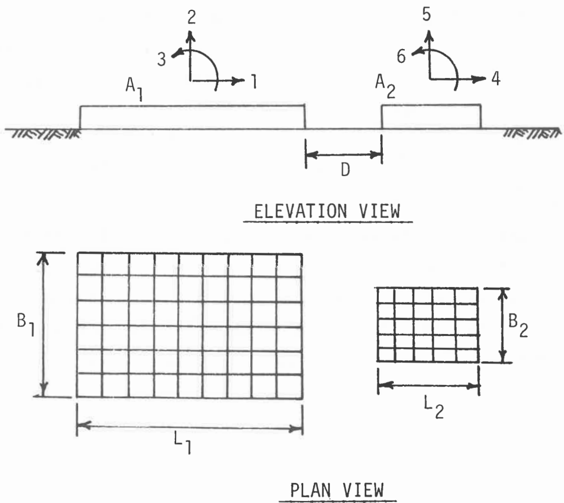

Figure 1. Mathematical Model for the Computation of the Compliance Matrix for the Soil Foundation

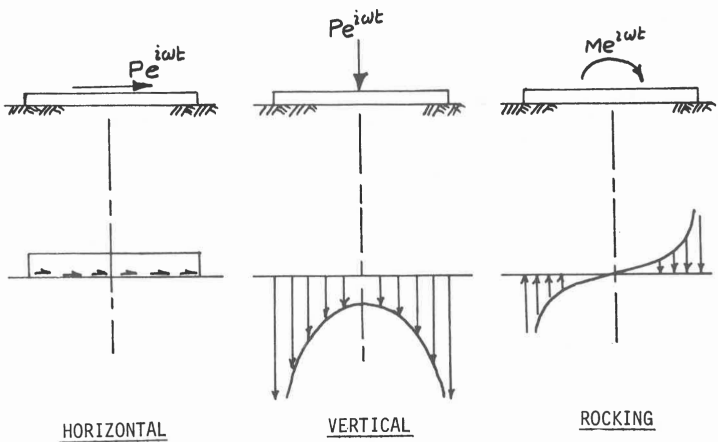

Figure 2. Rigid Base Pressure Distributions

Table II. Compliance Function for a Rigid Circular Footing On Elastic Half-Space

Dimensional Frequency	Stiffness* Present Approach	Stiffness* Reference [1]	Damping* Present Approach	Damping* Reference [1]
0	0.2120	0.2110	0	0
0.4	0.2030	0.2010	-0.0570	-0.0472
0.8	0.1670	0.1750	-0.1250	-0.0848

*Multiplication factor, $\mu a \Delta/P$, where μ = shear modulus of elastic half-space, a = radius of circular footing, Δ = deflection, P = force

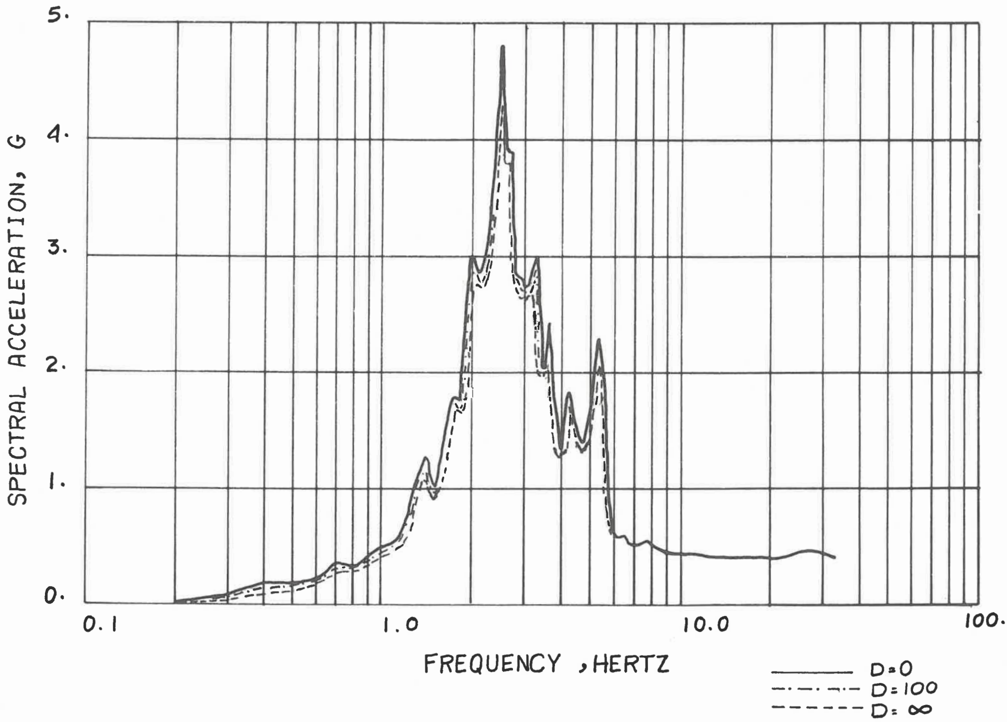

Figure 3. Spectral Acceleration Responses at the Top of Building A