

A Precise and Cost-Effective Method to Calculate the Stresses and Deformations of Pipe Bends with Realistic Boundary Conditions

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Abstract

The transfer matrix of a pipe bend is derived by the semi-bending theory. Thereby realistic boundary conditions of the bends like connected flanges, cylindrical and conical pipes and concentrated or distributed loads will be considered in pipeline calculations.

Furthermore, this transfer matrix of the pipe bend is converted into a stiffness matrix and is incorporated into an existing FE program. This way the exact stiffness as well as realistic boundary conditions of pipe bends may be considered for the stress and deformation analysis of complex piping systems.

Several comparisons between analytical and test results of pipe bends demonstrate a very good agreement. As it can be shown this transfer matrix method is highly economical and is very precise as well.

1. Introduction

Pipe bends have always been of special interest in piping calculations. The reason for this is that their stresses and deformations differ extremely from those obtained by calculations according to the elementary beam bending theory. Furthermore parameters of a pipe bend have to be varied so that no general practical solution can be stated to cover all cases of boundary conditions.

Today piping systems with pipe bends are calculated as beam structures with Finite-Element methods. The special behaviour of the bends is considered by empirical or semi-empirical flexibility and stress-intensification factors. But this method gives only an unsatisfying approximation for the real boundary conditions. To calculate the stresses and deformations of a cross-section of the pipe bends it is necessary to make a highly sophisticated model of the bends for a further FE calculation which takes much time and is very expensive.

Now we present a transfer matrix method for the stress and deformation analysis of pipe bends. This method is based on the semi-bending theory of thin-walled shells formulated for cylindrical shells by SCHNELL [1]. It permits the consideration of realistic boundary conditions of the pipe bends. These realistic boundary conditions may be connections between pipe bends and real flanges, cylindrical and conical pipes and concentrated or distributed loads as well. Furthermore, the transfer matrix of the pipe bend can be converted into a condensed stiffness matrix and incorporated into an existing FE program. This way the exact

stiffness as well as the real boundary conditions of the pipe bends are considered for the analysis of a complex piping system as a beam structure. So an economical and very precise stress and deformation analysis of complex piping systems (including branched ones) is possible by this new procedure combining the transfer matrix method with FE methods.

Several comparisons between analytical and test results show a very good agreement. A typical example of a branched piping system as part of a high pressure reactor is the so-called HDR piping system. This piping system was tested at the MPA, Stuttgart, and calculated with our new method in cooperation with the KfK, Karlsruhe, under sponsorship of the Bundesminister für Forschung und Technologie, FR Germany.

2. The transfer matrix of a pipe bend

Similar to the yet existing transfer matrix of the thin-walled cylindrical shell we derived the transfer matrix W of a pipe bend on the basis of the semi-bending theory developed by SCHNELL [1]. The semi-bending theory states that the stresses and deformations of a loaded flexible shell like a pipe bend will change more intensively in circumferential direction than in axial direction. Thereby the complete 8th order differential equations of the shell can be reduced to a system of linear differential equations of 4th order. This reduction is obtained by the approximation that a membrane state of stress is defined in the axial direction \bar{x} whereas a complete state of stress may exist in the circumferential direction φ of the pipe bend (Fig. 1).

If we replace the external loads, the displacements and the forces by their Fourier series in circumferential direction φ -and after some transformations- we finally get a set of linear differential equations of first order of the amplitudes of the circumferential and axial displacements (v, u) and the membrane forces ($n_x, n_{x\varphi}$) as well:

$$U' = A \cdot U \quad (1)$$

where U and U' are the so-called state vectors containing the amplitudes of the above mentioned displacements and membrane forces. The matrix A is the 'coefficient' matrix with components which are constant in \bar{x} . According to the theory of linear vectorial differential equations the transfer matrix W of a curved pipe with the length \bar{x} can be calculated numerically as follows:

$$W = I + A\bar{x} + \frac{(A\bar{x})^2}{2!} + \frac{(A\bar{x})^3}{3!} + \dots \quad (2)$$

Then the solution of eq.(1) is

$$U_1 = W U_0 \quad (3)$$

Eq.(3) gives us the relation between the state vectors at the beginning and at the end of a pipe bend. Altogether half of the elements in both of the state vectors have to be known from the boundary conditions of the bend. Now the unknown elements can be calculated from eq.(3) itself. If one of the state vectors of eq.(3) is completely known we are able to compute the stresses and deformations at any point of the bend according to the semi-bending theory.

In the case of a series of curved and straight pipes and ring frames as well the end vector of one interval (bend, ring frame etc.) is the beginning vector of the next one. Now the intermediate state vectors can be eliminated by matrix multiplications. Then we find e.g.

$$U_4 = (W_3 S_2 W_1 S_0) U_0 \quad (4)$$

which is the proper transfer equation. S_0 and S_2 is the yet existing transfer matrix of the ring frames whereas W_1 is the transfer matrix of a cylindrical shell and W_3 is the transfer matrix of a pipe bend. The stresses and deformations at any point of this series of elastic structures may be obtained analogous to the above described manner.

Stress and deformation calculations with this transfer method were compared with own test results, test results of other authors and results obtained by other theories. All comparisons show a good agreement. Moreover, the superiority of the transfer method for straight and curved pipe elements with realistic boundary conditions can be demonstrated in being economical and simultaneously precise as well.

You will find a detailed description of this above mentioned transfer method with many examples of boundary conditions in WILCZEK [2] .

3. The calculation of piping systems

Generally the calculations of a complex piping system -which is mostly branched- is not expedient with the previously mentioned proper transfer method.

But very often it is possible to calculate the straight piping intervals of those systems as beam structures -not indeed the pipe bends. The FE methods of today use this fact for the calculation of piping systems. However the modelling of the pipe bends with its realistic boundary conditions by this FE computations is mostly very expensive, on the contrary very inexact by so-called flexibility factors and stress intensification factors.

Now we have developed a new method which is combining our transfer method with FE methods. Thereby a precise calculation of complex piping systems as beam structures is possible by usual FE programs whereat the diminished stiffness of the bends and their connections to other pipe elements are now considered in the right manner.

The first step for this combined transfer matrix FE method is the condensation of the transfer matrix of the pipe bends. In fact it is a reduction of all transfer matrix elements to only those elements which are coefficients of the beam parts of the state vectors. This is easy to manage in the case of pipe bends which are connected to long pipes, flanges and/or have one free end as well.

After this reduction of the transfer matrix of the pipe bend considering its realistic boundary conditions as integrated parts of a piping system we transform this reduced transfer matrix into the equivalent stiffness matrix in the usual manner. This new stiffness matrix of the pipe bend easily can be incorporated into FE programs (like SAP IV) idealizing piping systems as beam structures. The computation with these FE programs delivers the internal forces and deformations of the beam structure.

In a second step we return to the original transfer matrix of the pipe bend to calculate the stresses and deformations of a discrete cross-section. Now we use those above mentioned internal forces as part of the boundary conditions at one end of the bend while the center of the other end is assumed as fixed. If there are long straight pipes joint to the bends these cylinders will be modelled as so-called elastic supports of the bends in the transfer equation.

4. Test results and calculations of a branched piping system

Fig. 2 shows the principle sketch of a branched piping system. This piping system was part of a high pressure reactor and was tested at the MPA (Materialprüfungsanstalt)Stuttgart.

The general dimensions of this piping system are about 6.5 m from point ① to point ②⑤ and about 2.5 m between point ①④ and ⑥ . The upper branch has a pipe diameter of about 80 mm while the lower branch consists of pipes with a diameter of about 260 mm. The wall thicknesses are about 6 mm and 20 mm respectively. The material of this piping system is steel with an E-modulus of about 200,000 N/mm².

This test system was loaded at the point ①④ by a force F_y or a moment M_y .

Special interest was given to the strains in the pipe bends. As an example we present a comparison of these test results (solid circles) with calculations (solid lines) according to the combined transfer matrix FE method in Fig. 3. Fig. 3a shows the outside circumferential strains of the central cross-section of bend NW 250/2 while Fig. 3b presents the results of the central cross-section of bend NW 80/1. In these diagrams the strains are plotted about the circumferential angle φ of the cross-section. This comparison of test and calculation results demonstrates the quality of the combined transfer matrix FE method. (For further examples see WILCZEK [2] , ÖRY/WILCZEK [3] .)

5. Outlook

Further development of the present transfer method for pipe bends is now under way to consider surface loads or to take into account the stiffening by internal pressure by a second order theory. In addition, it is intended to extend the investigations in order to include shell vibrations and elastic-plastic behaviour of the pipes.

6. References

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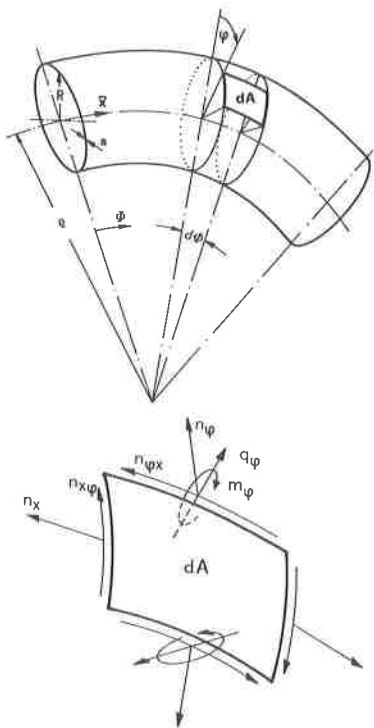


Fig. 1. The pipe bend and the internal forces at its element dA .

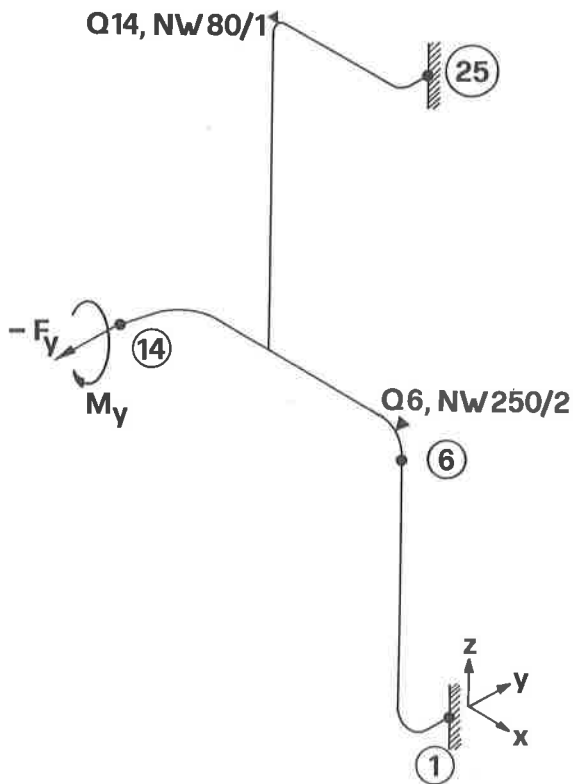


Fig. 2. The test piping system at the MPA Stuttgart (principle sketch).

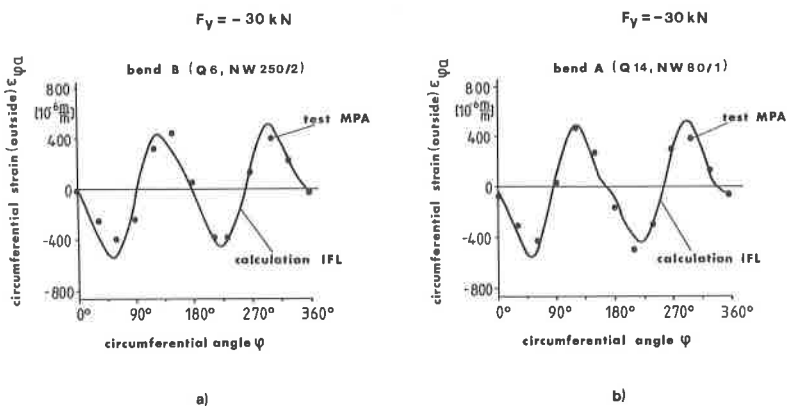


Fig. 3. Comparison between calculated and measured strains of the MPA test piping system.