Transactions of the **14th International Conference on Structural Mechanics in Reactor Technology** (SMiRT 14), Lyon, France, August 17-22, 1997

J09/4

Derivation and application of a flow-solid coupling dynamic similarity criteria for NPP components

Yao W., Shi G., Jiang N., Yang R.
Shanghai Nuclear Engineering Research & Design Institute, China

ABSTRACT In this paper, the flow-solid coupling dynamic similarity criteria are derived from a flow-solid fundamental equation, taking account of interface continuity and interactive conditions. In terms of different study objects, some of the similarity can be relaxed.
The similarity criteria are applied in flow-induced vibration test of a reactor core barrel, seismic test of a reactor internals and vibration test of a oxygen top-blown steel making turning furnace.
Key words:
Flow-solid coupling
Dynamic similarity criteria

1. INTRODUCTION

Owing to flowing fluid existing in many nuclear power plants' systems and components, Vibration problem of the flow-solid interaction is very conspicuous. The design code[1]. require to minimize the flow-induced vibration under operating conditions. If analytic method is applied to resolve the flow-induced vibration, then, two difficulties exist: the first is that the structure is probably very complicated so that the flow-solid coupling calculating model can be hardly built up. The second is that the force function of the flow to the structure can be solved more difficulty by use of a theoretical method[2]. For a prototype reactor (or other trade's equipment), the vibration problem should be solved, in the phase of design, using simulating tests in proportional models. For examples, flow-induced vibration of the reactor core barrel, seismic tests of the reactor internals and some other components, rock of the storing tank and pool and etc. All of these require a similarity criteriion which satisfies the flow-solid coupling conditions, so as to design the models and convert test parameters by means of the similarity.

In the model design and tests it is of importance that the essential problems containing in the objects should satisfy the similarity for the model designed, namely, the two satisfies the second similarity theorem. In this way, the results obtained from the model tests can only be converted into the objects.

2. FLOW DYNAMIC SIMILARITY

2.1 Fundamental equation of flow dynamics

Suppose that is Newtonian viscous fluid, its dynamic equation is described by Navier-Stokes equation.

$$\frac{\partial V}{\partial t} + (V \cdot grad)V = F_f - \frac{1}{\rho_f} grad p + \nu \nabla^2 V \qquad (2\text{-}1)$$

Where: $V = V_x i + V_y j + V_z k$, (2-2)

V_x, V_y, V_z are velocities of the fluid micro elements in x, y, z directions.

$Ff = F_{fx} i + F_{fy} j + F_{fz} k$; (2-3)

F_{fx}, F_{fy}, F_{fz} are mass forces of the fluid microelements in x, y, z directions.

When gravity g is only considered, $F_f = gk$ (2-4)

p is the pressure in the fluid. $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$, grad is the gradient. ρ_f, ν are the fluid density and dynamic viscosity.

2.2 Derivation of similarity

There are six parameters in eq.(2-1): velocity, dimension, time, mass force, pressure and viscosity. The proportional relation between a object and a model is supposed as C_V, C_e, C_t, C_{Ff}, C_p, C_v. If the fluid movement in the model test is similar with that in the objet, the fluid elements both in the model and the object satisfy eq.(2-1). then the relation is obtained as:

$$C_v/C_t = C_v^2/C_l = C_p/C_{\rho f}C_l = C_{Ff} = C_v C_v/C_l^2 \qquad (2\text{-}5)$$

From eq. (2-5), four basic similarities are obtained. Their physical meanings are:[5][7]

(1) $C_v C_t / C_l = 1$, the criterion indicates that the ratio of the inertial force (C_v^2 / C_l). related to the fluid micro element movement and the nonstationary inertial force (C_V/C_t) is equal, called as Strouhal number

$S_t = Vt / l$ (2-6)

(2) $(C_v)^2/C_l C_{Ff} = 1$, the criterion indicates that the ratio of the inertial force (C_v^2/C_l) related to the fluid micro element movement and the mass force C_{Fr} is equal, also called as Froued number

$F_r = V^2 / l g$ (2-7)

(3) $C_p / C_{\rho f} C_v^2 = 1$, the criterion indicates that the ratio of the pressure ($C_p/C_{\rho f}C_l$) related to the fluid micro element and the inertial force (C_v^2/C_l) is equal, also called as Euler number

$Eu = p / \rho_f V^2$ (2-8)

(4) C_vC_l/C_v=1, the criterion indicates that the ratio of force (C_v^2/C_l) related to the fluid micro element and the viscous force C_vC_ν/C_l^2 is equal, also called as Reynolds number

$$Re=Vl/\nu \tag{2-9}$$

The four similarity criteria obtained form (2-6) to (2-9) are:

$$S_t=Vt/l,\ F_r=V^2/lg,\ Eu=p/\rho_f V^2,\ Re=Vl/\nu \tag{2-10}$$

For the four similarity criteria, we need to note as :

(1) in the fluid micro element movement, the velocity V and the pressure p consist of average and pulse, namely:

$$V=V+V',\quad p=p+p' \tag{2-11}$$

substitute into eq.(2-1), we have $C_V=C_{V'}$, $C_p=C_{p'}$. For simplifying , using of symbol V, p here means the similarity criteria for the average and the fluctuation have been satisfied and equal.

(2) It is difficult to satisfy all of the similarity criteria in eq. (2-10) in model tests. In terms of test requirements, major similarity criteria should be satisfied and others can be relaxed.

3. SOLID DYNAMIC SIMILARITY

3.1 Fundamental equation of solid dynamics

Suppose that the displacement of a solid particle is very small, neglecting nonlinear item, then the fundamental equation of solid dynamics can be expressed as:

$$\frac{\partial^2 u}{\partial t^2} = \frac{(\lambda+\mu)}{\rho_s} grad\ div\ u + \frac{\mu}{\rho_s}\nabla^2 u + Fs \tag{3-1}$$

where: u is the particle displacement vector.

$$u=u_x i + u_y j + u_z k \tag{3-2}$$

ρ_s is the solid density. λ, μ are the Lamme constants. E, ν_0 are the modules of elasticity and Poisson's ratio.

$$\lambda=E/(1+\nu_0)(1-2\nu_0),\quad \mu=E/2(1+\nu_0)$$
$$F_s=F_{sx} i + F_{sy} j + F_{sz} k \tag{3-3}$$

When only the gravity is considered,

$$F_s=gk \tag{3-4}$$

The relation of stress σ_{ij} and strain is

$$\sigma_{ij}=\lambda\ div\ u(\delta_{ij}) + 2\mu\varepsilon_{ij} \tag{3-5}$$

where strain

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial j} + \frac{\partial u_j}{\partial i}\right)$$
$$i、j = x、y、z;\ \delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} \qquad (3\text{-}6)$$

3.2 Derivation of similarity

There are five parameters in eq. (3-1): displacement, dimension, time, mass force, physical parameter. The proportional relation between a object and a model is assumed as C_u, C_l, C_t, C_{Fs}, C_E, $C_{\rho s}$, C_{V0} . If the solid movement in the model and the object is similar, all the solid elements of the model and the object must satisfy eq.(3-1). Then the equation is obtained:

$$C_u / C_t^2 = C_E C_u / C_{\rho s} C_l^2 = C_{Fs} \qquad (3\text{-}7)$$

$$C_\sigma = C_E C_u / C_l,\quad C_\varepsilon = C_u / C_l \qquad (3\text{-}8)$$

From eq.(3-7), two basic similarity criteria are obtained. their physical meanings are:

(1) $C_E C_t^2 / C_{\rho s} C_l^2 = 1$, the criterion indicates that the ratio of the inertial force $C_E/C_{\rho s}C_l$ related to the solid micro element movement and the nonstationary inertial force C_u/C_t^2 is equal.

(2) $C_u / C_t^2 C_{Fs} = 1$, the criterion indicates that the ratio of the nonstationary inertial force C_u/C_t^2 related to the particle and the mass force C_{Fs} is equal.

4. SIMILARITY CRITERIA IN FLOW-SOLID COUPLING INTERFACE

In the flow-solid interface, normal stress of the solid is balanced with the flow pulse pressure and the normal velocity continues, namely,

$$-p = (\sigma_{ij})_n \qquad (4\text{-}1)$$

$$(V_x,\ V_y,\ V_z)_n = \left(\frac{\partial u_x}{\partial t},\ \frac{\partial u_y}{\partial t},\ \frac{\partial u_z}{\partial t}\right)_n \qquad (4\text{-}2)$$

where $(\dots\dots)_n$ means the projection fo boundary normal n. Two similarity criteria can be obtained from eq.(4-1):

$$C_p = C_\sigma = C_E C_u / C_l,\quad C_v = C_u / C_t \qquad (4\text{-}3)$$

5. SUMMARY OF FLOW-SOLID COUPLING SIMILARITY CRITERIA

By using the relation of the nonstationary movement time C_t and the frequency C_f, $C_f = 1/C_t$, the flow-solid coupling similarity criteria can then be summarized as shown in Table 5.1

6. APPLICATION OF SIMILARITY CRITERIA

6.1 Flow-induced vibration model test of the reactor core barrel .[9][10][11]

In this test, the St dynamic similarity was mainly concerned when Re is self-similar and ranged from 10^3--10^7, it can be relaxed. If the influence of solid and flow gravity to movement is neglected, Fr can also be relaxed.

Suppose that Cv=1, $C_{\rho s}=C_{\rho f}=C_E=1$, $C_l=5$, then the corresponding similarities are obtained and listed in Table 6.1.

6.2 Seismic test for the reactor internals.

If the flow-solid coupling effect is taken as a dominant factor, neglecting the influence of the gravity velocity similarity is $C_V=1$ and acceleration similarity is $C_a=1/C_l$. But for a model seismic test made in a same gravity, the similarity criteria of C_a and C_g are contradictory. The similarity criterion of $C_g=1$ can relaxed so long as the model's right movement caused by vertical seismic excitation, or flow rock caused by horizontal seismic excitation are involved. In the case, the similarity criteria in Table 6.1 are still available, or else the method described in paragraph 6.3 will be used.

6.3 Modeling technique for a pure oxygen top-blown steelmaking turning furnace.

This example is a typical coupling vibration for gas (oxygen-blown), fluid (steel liquid) and solid (furnace). Owing to steel liquid disturbance induced by top- or bottom-blown oxygen, effect of the steel liquid's mass force can not be neglected, namely, it is convinced for acceleration similarity in the flow and the solid to satisfy:

$$C_a=C_g=1.$$

Reasonably selecting model's material can satisfy eq.(6-1). In the test, hard PVC plate is selected to manufacture the furnace model. its modules of elasticity $E^{(m)}=4\times 10^3$ MPa density $\rho_s^{(m)}=1.56\text{kg/dm}^3$. for the object, $E^{(p)}=2\times 10^5$ Mpa, $\rho_s^{(p)}=7.8\text{kg/dm}^3$. Then there are $C_E=50$, $C_\rho=5$. Further, the dimension dcale are $Cl=CE/C\rho s=50/5=10$, steel liquid. $\rho_f^{(p)}=6.9\text{kg/dm}^3$. Reasonably disposing iron thichloride solution with certain concentration, its density is $\rho_f^{(m)}=6.9/5=1.38\text{kg/dm}^3$. Other parameters are listed in Table 6.2.

7. CONCLUSION

(1) In the paper, the similarity criteria is derived through the flow-solid coupling. In engineering application of modeling techniques, some "primary" factors should be stressed and some "secondary" factor can be relaxed. Meaning of the "primary" is that it can ensure similarity for non-stationary internal item of the flow-solid coupling. The "secondary" means that in the modeling techniques, some parameters' similarity is very difficult to be achieved or cause contradictor results, but that givesless influence to the "primary" similarity criteria.

(2) The similarity criteria of three engineering examples derived in the paper have been applied and satisfactory test results have been obtained. We consider the

similarity criteria can be extended to other relevant engineering application, such fluid rock, magnetic-flow coupling, and etc.

References

[1] ASME, Boiler and Pressure Vessel code, Section III, Rules for construction of Nuclear Power Plant Components, Division 1-Subsection NG, Core Support Structures, NG-3112.3(c), 1989, P38.

[2] Regulatory Guide 1.20, Comprehensive Vibration Assessment Program for Reactor Internals During Proportional and Initial Startup Testing, U.S. NRC, 1978.

[3] F.M.White, Viscous Fluid Flow, McGraw-Hill, New York, 1974.

[4] Kouge.H.E, Fluid Theory, People's Education Press, 1962.

[5] W.Kaufmann, Fluid Theory, High Press Educational, 1964

[6] Dou Guoren, Turbulence Theory, People's Education Press,1981.

[7] Zhang Zhongyin, Viscous Fluid Theory, 1980.

[8] Redwood, Mechanical Wave Guides Pergamon Press, 1960.

[9] Yao Weida, Shi Guolin: Research of Flow-induced Vibration for Reactor Core barrel, Journal of Nuclear Science and Engineering, 1989.

[10] Yao weida, Shi Guolin: Research of flow-induced Vibration for Reactor Core barrel, Nuclear Science Report, CNIC-296, 1987.

[11] Yao Weida, Shi Guolin, Study and Analysis for the Flow-Induced Vibration of the Core Barrel of PWR, 10th SMiT, Division T, 1989.

[12] Shi Guolin, Seismic Research for a Reactor Internals, SNERDI, 1992.

[13] Wang Yixin, Vibration Research for A Top & Botton Oxygen Blown Steelmaking turning furnace, Shanghai, 1992.

Table 5.1 flow-solid coupling similarity criteria

flow	solid	flow-solid interface
$\frac{V}{lf}$ (St) $\frac{p}{\rho_f V^2}$ (Eu) $\frac{V^2}{lg}$ (Fr) $\frac{Vl}{\nu}$ (Re)	$\frac{\rho_s l^2 f^2}{E}$ $\frac{uf^2}{g}$	$\frac{Eu}{pl}$ $\frac{V}{uf}$

Table 6.1 similarities of model and object

physical parameters	symbol	unit	similarity	scale
dimension	l	m	C_l	5
density	ρ	kg/m^3	$C_\rho=1$	1
modulus of elasticity	E	MPa	$C_E=1$	1
mass	M	kg	$C_m=C_l^3$	125
amplitude	u	m	$C_u=C_l$	5
velocity	V	m/s	$C_v=1$	1
acceleration	a	m/s^2	$C_a=1/C_l$	1/5
force	F	N	$C_F=C_l^2$	25
pressure	p	pa	$C_p=1$	1
stress	σ	MPa	$C_\sigma=1$	1
strain	ε	m/m	$C_\varepsilon=1$	1
time	t	sec	$C_t=C_l$	5
frequency	f	HZ	$C_f=1/C_l$	1/5
volume flow rate	Q	m^3/s	$C_Q=C_l^2$	25
pressure PSD	G_p	$(Pa)^2/HZ$	$C_{Gp}=C_l$	5
stress PSD	G_σ	$(MPa)^2/HZ$	$C_{G\sigma}=C_l$	5
acceleration PSD	G_a	$(m/s^2)^2/HZ$	$C_{Ga}=1/C_l$	1/5

Table 6.2 Similarity criteria of model and object

physical parameters	symbol	unit	similarity	scale
dimension	l	m	$C_l=C_E/C_{\rho s}$	10
density	ρ_s、ρ_f	kg/m^3	$C_{\rho s}=C_{\rho f}$	5
modulus of elasticity	E	MPa	C_E	50
mass	M	kg	$C_m=C_\rho C_l^3$	5×10^3
amplitude	u	m	$C_u=C_l$	10
velocity	V	m/s	$C_v=\sqrt{C_l}$	$\sqrt{10}$
acceleration	a	m/s^2	$C_a=1$	1
force	F	N	$C_F=C_M C_g$	5×10^3
pressure	p	pa	$C_p=C_E$	50
stress	σ	MPa	$C_\sigma=C_E$	50
strain	ε	m/m	$C_\varepsilon=C_\sigma/C_E$	1
time	t	sec	$C_t=\sqrt{C_l}$	$\sqrt{10}$
frequency	f	HZ	$C_f=1/\sqrt{C_l}$	$1/\sqrt{10}$
oxygen flow rate	Q	m^3/s	$C_Q=(C_{\rho f}/C_{\rho g})C_l^{5/2}$	1581
pressure PSD	G_p	$(Pa)^2/HZ$	$C_{Gp}=C_E^2\sqrt{C_l}$	$50^2\sqrt{10}$
stress PSD	G_σ	$(MPa)^2/HZ$	$C_{G\sigma}=C_E^2\sqrt{C_l}$	$50^2\sqrt{10}$
acceleration PSD	G_a	$(m/s^2)^2/HZ$	$C_{Ga}=\sqrt{C_l}$	$\sqrt{10}$
force PSD	G_f	$(N)^2/HZ$	$C_{Gf}=C_E^2 C_l^{7/2}$	$50^2(\sqrt{10})^7$