THE FAILURE OF DUCTILE PLATES CONTAINING THIN LIGAMENTS

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SUMMARY

A series of tensile tests on mild steel plates containing partially penetrating defects is described. In the tests both the thickness of the ligament and the length of the crack were varied. The experiments determine the load and crack-opening-displacement (COD) at ligament failure, and the subsequent mode of plate failure. The loading was displacement controlled and three modes of failure were observed,

1. Ligament failure followed by plate failure at higher load.
2. Ligament failure followed by plate failure at lower load.
3. Simultaneous failure of ligament and plate.

A relation is established between the COD at ligament failure and the ligament thickness. This relation is used in conjunction with a reference stress technique to predict the load at which ligament failure occurs. The mode of failure and the loads at failure predicted in this way are in good agreement with the experimental results.

1. INTRODUCTION

Recent research on the integrity of pressurized components has been directed at developing a 'leak-before-break' criterion for the assessment of defects (Irwin and Srawley [1], Irwin [2], Begley et al. [3]). This ensures that a defect will propagate through the wall leading to a small but detectable leak rather than cause extension of the crack which could lead to catastrophic failure. The development of such a criterion requires investigation into crack propagation through the wall which may depend on creep, fatigue and environmental factors. In addition, it is necessary to establish the conditions for failure of the remaining ligament. The latter consideration forms the subject for the present investigation and in particular ligament failure in a ductile material is studied.

2. EXPERIMENTAL DETAILS

The specimen geometry chosen for the programme of tests is shown in Fig. 1a and consists of a flat plate containing a double notch. Each notch was manufactured using a spark erosion process which generates the notch cross section illustrated in Fig. 1b. The specimens were manufactured from a mild steel (BS 970 EN3) and then stress relieved for 3 hours at 650°C. After stress relief the material had the stress/strain relationship shown in Fig. 2.

In order to determine the crack opening displacement as a function of load two types of measuring technique were used. The first technique was infiltration of the crack with a hardening silicone rubber (Robinson and Tetelman [4]) which allowed the full profile of the crack to be determined while the specimen was still on load. The replica of the notch was subsequently examined at a magnification of x10 for signs of crack initiation. A thin section was then cut from the rubber at the notch mid-point and its dimensions obtained using a travelling microscope. The second technique used a standard clip gauge mounted between two hardened steel knife edges which gave a continuous measure of COD as a function of load, including the COD at the instant prior to fracture.

Displacement controlled tensile tests were conducted at room temperature on the mild steel plates containing various geometries of defect; details of which are given in Table I. All specimens were tested in a Denison machine of 500 kN capacity. Records of clip gauge displacement with increasing load were obtained for each test. The load at which ligament failure occurred was, in most cases, indicated by a sharp change in the compliance measured by the clip gauge. However, in a few cases, ligament failure consisted of a slow fibrous tearing at approximately constant load. The ligament failure load and COD were taken at the stage at which light first became visible through the crack. After ligament failure, the test was continued until final rupture of the plate occurred.

3. OBSERVATIONS ON FAILURE OF THE LIGAMENT

The experiments establish a relationship between the crack opening displacement at ligament failure and the ligament thickness. Ignoring the effect of notch radius, it is apparent from geometric considerations that for ductile rupture (failure at a critical strain state) the relationship between the crack opening displacement and the ligament thickness should be linear for ligaments small compared with the plate thickness. In addition, for thin ligaments (ℓ <1.2mm) it is to be expected that the ligament will be in a condition of plane strain at fracture. The results obtained from the experiments are shown in Fig. 3 and for thin ligaments it may be seen that the variation of COD with ligament thickness is approximately linear. In the intermediate range of ligament thickness ($1.2\text{mm} < \ell < 2.5\text{mm}$) the stress distribution and deformation pattern change, and the COD increases more rapidly with ligament thickness. However for thick ligaments ($\ell > 2.5$ mm), failure is governed by the critical value of the COD for transverse crack extension, δ_u, and the COD becomes insensitive to ligament thickness.

4. A THEORETICAL MODEL FOR LIGAMENT FAILURE AND SUBSEQUENT PLATE FAILURE

The plate behaviour may be related to that of pressurized components. Failures of type 1 may be regarded as 'leaks' as additional load is required to fail the rest of the structure after the ligament has failed. Failures of types 2 and 3 should be regarded as 'breaks'. It is clear, however, that failures of type 2 and 3 may also produce a leak in situations where the leak rate is sufficiently high to cause rapid depressurization. Studies of such dynamic effects are beyond the scope of the present method but it is important that any theory should predict the load at failure and also the mode of failure.

The 'reference stress' is a concept that has proved valuable in describing the non-linear response of structures operating in the creep range (Penny and Marriott [5]). Although the concept was originally developed for describing creep deformation it can be readily adapted to take account of the effects of strain hardening in structural response. Sim [6] proposed that an approximate reference stress level can be obtained for any structure in which the limit load is known. This reference stress, $\bar{\sigma}$, is given by

$$\bar{\sigma} = \frac{L\,\sigma_y}{L_y} \tag{1}$$

where L is the actual loading applied to the structure, and L_y is the rigid-plastic collapse load for the yield stress, σ_y. It should be noted that since the ratio $\frac{\sigma_y}{L_y}$ is dependent only on the geometry for a given pattern of loading, an arbitrary value of σ_y may be used in the calculation of L_y. By adapting the work of Sim [6], the deformation, Δ, in a strain hardening material may then be estimated for a particular point on the component from the relation,

$$\Delta = \frac{\Delta e\ \bar{\varepsilon}(\bar{\sigma})}{(\bar{\sigma}/E)} \tag{2}$$

where Δ is the actual displacement under consideration, Δe is the elastic deformation of the same point due to the same load L, E is Young's Modulus and $\bar{\varepsilon}(\bar{\sigma})$ is the total uniaxial strain in the <u>actual</u> material at a reference stress $\bar{\sigma}$.

For the particular plate structure under consideration the limit load is given by,

$$L_y = 2ht\,(1-\gamma + c\beta\gamma)\,\sigma_y$$

where $\gamma = {}^{a}/h$, the normalized crack length

$\beta = \frac{\ell}{t}$, the normalized ligament thickness

and c is the plastic constraint factor for the ligament in plane strain which for a notch tip of radius r may be obtained from the lower value of

$$c = \left(1+\frac{2r}{\ell}\right) \ln \left(1+\frac{\ell}{2r}\right) \tag{3}$$

and

$$c = 1+ \ln \left(\frac{1+\beta}{2\beta}\right) \tag{4}$$

which are given by Hill [7] and Ewing and Hill [8] respectively.

Defining a normalized load $\lambda = \frac{L}{2ht\sigma_y}$, the reference stress defined by equation (1) becomes

$$\frac{\bar{\sigma}}{\sigma_y} = \frac{\lambda}{(1-\gamma + c\beta\gamma)} \tag{5}$$

The crack opening displacement can be derived from equation (2)

$$\delta = \delta_e \; E \; \frac{\bar{\varepsilon}(\bar{\sigma})}{\bar{\sigma}} \tag{6}$$

where $\bar{\varepsilon}(\bar{\sigma})$ is the total strain corresponding to the stress $\bar{\sigma}$ on the uniaxial stress-strain curve; δ_e is the elastically constrained crack opening displacement due to the applied load, λ. The value of δ_e may be obtained from Erdogan and Bakioglu [9] who obtained the solution for the COD in a plate of finite width under tension. The solution for small plastic zones and $\gamma \leq 0.5$ is given to a good approximation by:

$$\delta_e = o \quad \text{for } \lambda < c\beta \tag{7a}$$

$$\delta_e = \frac{4a\sigma_y}{E} \; (\lambda - c\beta) \; \text{for } \lambda \geq c\beta \tag{7b}$$

The reference stress method may be compared with the analytical solution obtained by Erdogan and Bakioglu [9] for an elastic perfectly plastic material. In Fig. 4 a comparison is made for the variation of crack opening displacement with load. It can be seen that the reference stress prediction underestimates the value of the COD in the elastic plastic transition region but that the error based on load is never greater than 12%. It should be noted that the error will be less for thicker ligament sizes and also in strain hardening materials. Indeed it is the general applicability of the reference stress technique to strain hardening material and also arbitrary shaped structures which makes its use attractive.

5. COMPARISON OF THEORY WITH EXPERIMENT

Ligament failure is assumed to occur at a value of δ which is not just a material property but is also a function of the ligament geometry. The failure criterion is therefore based on the data given in Fig. 3 which may be expressed by

$$\delta_p/t = 0.18\beta + 1.62\beta^2 \text{ or } \delta_p = 2.2\text{mm whichever is less} \tag{8}$$

The variation of crack opening displacement (δ) with applied load may be predicted by combining equations (5) and (6) with the uniaxial stress-strain properties in Fig. 2. Combining this procedure with the failure criterion of equation (8), a numerical method was developed to derive the variation of ligament failure load with ligament thickness. These predictions are compared with the experimental results in Table II.

In addition the method predicts subsequent plate failure. Assuming that ligament failure has occurred at a load λ_p, the geometry to be considered is that of a centre cracked plate. The reference stress for this structure is given by

$$\frac{\bar{\sigma}'}{\sigma_y} = \frac{\lambda}{1-\gamma} \tag{9}$$

and thus the COD at the tip of the crack δ' is given approximately by

$$\delta' = \frac{4a\sigma_y\lambda}{\bar{\sigma}'}\ \bar{\varepsilon}(\bar{\sigma}') \tag{10}$$

Plate failure is assumed to occur at a constant value of COD, δ_u, which from Fig. 3 is given by δ = 2.2mm ($\pm$ 0.2mm). Combining this failure criterion with equation (9) enables the load at subsequent plate failure to be calculated, Table II. It may be seen that predictions are accurate allowing the mode of failure (leak or break) to be reliably determined.

6. CONCLUSIONS

The ductile fracture of symmetric partially penetrating cracks in plates loaded in tension has been examined. It has been shown that the crack opening displacement at ligament separation increases with the ligament thickness until it reaches a value determined by transverse fracture of the remaining plate. Reference stress techniques have been used to give good predictions of the load at which the ligament separates and the mode of failure (leak or break). General conclusions must await studies currently being pursued on asymmetric cracks, varying notch radius and larger plates.

ACKNOWLEDGEMENTS

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Table I

Geometry of Specimens and Test Results

Specimen no.	Ligament Dimensions		Failure Load			
	Width (2a) mm	Thickness (ℓ) mm	L_p kN	L_u kN	δ_p mm	δ_u mm
A1	12.7	1.0	85	106	-	-
A2	12.7	0.7	80	103	-	-
A3	NONE	†	-	144	-	-
A4	NONE	†	-	138	-	-
A5	12.7	2.6	128	105	2.18	2.36
A6	12.7	3.5	143	98	2.10	2.29
A7	12.7	1.5	101	108	0.93	2.10
A9	25.4	2.5	113	113	2.02	2.02
A10	25.4	1.3	84	74	0.73	2.25
B1	25.4	0.6	57	73	0.18	2.40
B2	25.4	1.0	66	56	0.37	2.01
B4	12.7	1.8	112	101	1.28	2.07

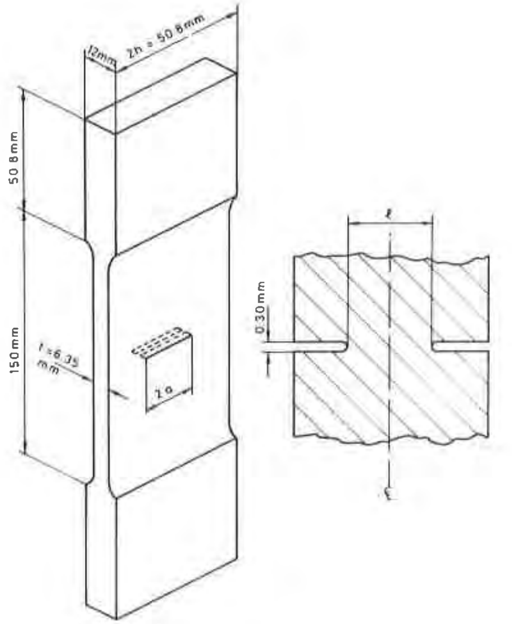
† Tests to establish stress-strain properties of plate material.

Table II

Comparison of Test Results with Reference stress Predictions

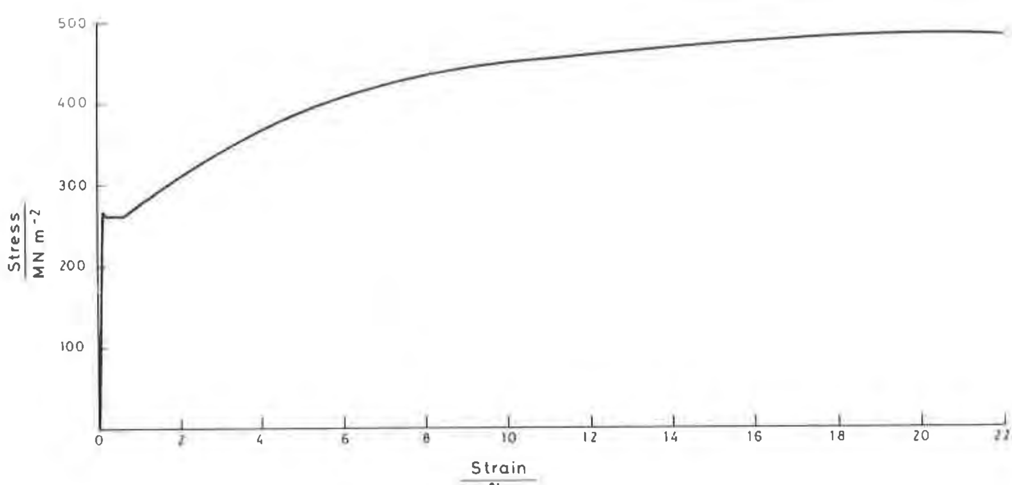
Specimen	Normalised Ligament Dimensions		Constraint factor	Ligament failure Load λ_p		Plate failure Load λ_u	
	A	B	F	Experimental	Theoretical	Experimental	Theoretical
A1	0.25	0.16	2.29	1.06	1.08	1.26	1.29
A2	0.25	0.11	2.62	0.95	0.95	1.23	1.29
A5	0.25	0.41	1.54	1.54	1.65	1.25	1.29
A6	0.25	0.55	1.34	1.71	1.71	1.17	1.29
A7	0.25	0.24	1.95	1.20	1.31	1.29	1.29
A9	0.50	0.39	1.58	1.35	1.32	*	0.82
A10	0.50	0.20	2.10	1.00	0.93	0.88	0.82
B1	0.50	0.10	2.70	0.68	0.73	0.87	0.82
B2	0.50	0.16	2.29	0.79	0.82	0.67	0.82
B4	0.25	0.28	1.83	1.34	1.40	1.20	1.29

* Simultaneous fast fracture of ligament and plate occurred.

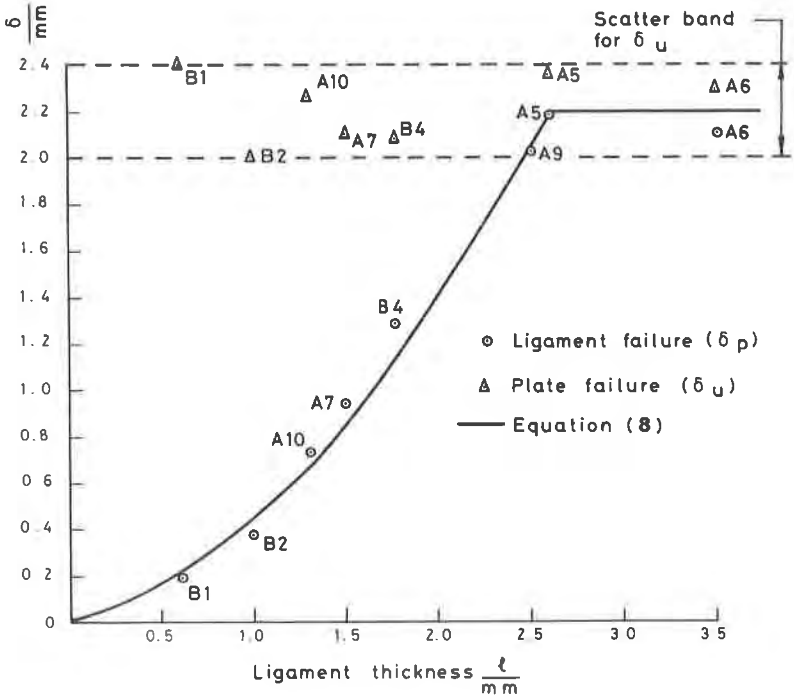


a) Isometric view of plate
b) Cross-section of plate showing initial notch profile

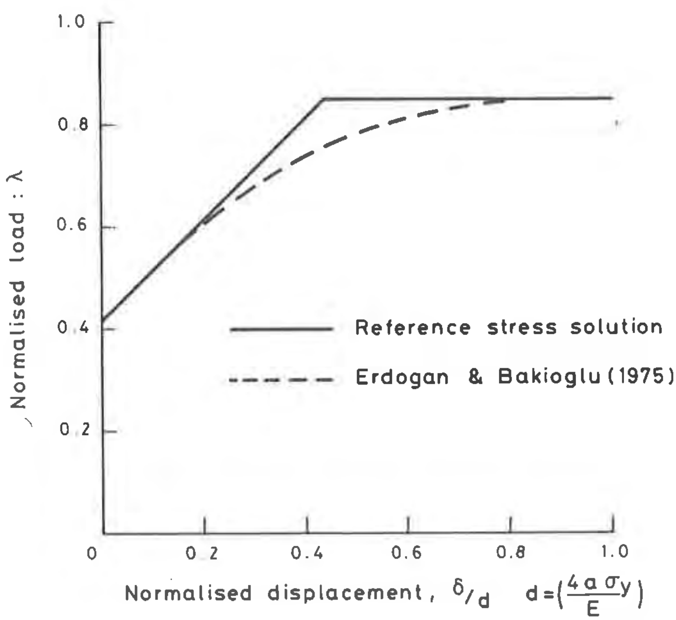
Specimen Geometry



Tensile Properties of Specimen Material



Variation of C.O.D. at Failure with Ligament Thickness



Load-Displacement Curve for Elastic, Perfectly Plastic Material; β=0.2, γ=0.25.