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Strain-Softening Model for Concrete Under Dynamic Loading

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Abstract

A time-dependent continuum model of strain-softening in plain concrete under multiaxial stress is proposed. The model is similar to the type of viscoplastic models of Perzyna and others, which include a yield surface capable of contracting in a time-dependent fashion. The relationship of this model to recent work in equivalent and non-local continuum theories, and implications for finite element applications are discussed.

1. Introduction

Recent work in constitutive modeling of plain concrete, Willam [1], points out that apparent strain-softening in quasistatic experiments is the result of inhomogeneous deformation within the specimen. It is no longer regarded appropriate to represent such behavior by continuum constitutive models. This is because, contrary to the assumptions of continuum behavior, inhomogeneity introduces characteristic dimensions and hence scale effects which make information derived from one size of specimen of doubtful value for predicting behavior at a different scale. Experimental work in rock mechanics reported by Hudson et al. [2] and Brady et al. [3] support the idea that apparent strain-softening is due to progressive structural breakdown accompanied by growth and coalescence of microcracks whose dimensions eventually become large on the scale of specimen dimensions. The effect is to transform the specimen from a material which might be regarded as a continuum into a structure with characteristic dimensions. Studies of cracking in concrete by means of microscopic inspection, Hsu et al. [4], and x-radiography, Isenberg [5], suggest that strain-softening follows after cracks extend beyond several characteristic coarse aggregate spacings. Hence, continuum models for strain-softening in concrete are suspect as they are in rock.

Several mainstreams of research have emerged which attempt to address the problem of inhomogeneous stress and strain states. One of these, proposed by the University of Colorado group, Willam et al. [6,7], is to define a composite damage model in which fracture is confined to a localized crack band. An equivalent continuum element is defined having the same rate of energy dissipation as the initial fracture energy release of the local crack band. This model exhibits a pronounced size effect. However, application of the model in static finite element examples indicate mesh insensitivity if certain conditions are imposed on tension fracture energy and softening modulus. Another approach, proposed by Belytchko and Bazant and Bazant et al. [8,9], uses the concept of non-local continuum in which the stress at any point depends on the deformation history within a characteristic finite distance of the point. The non-local law with strain-softening is superposed on a local law without strain-softening. The resulting composite local, non-local model is termed an imbricate continuum. Finite element examples are presented which indicate slow convergence to a solution that does not depend on discretization.

Many attempts have been made to use continuum strain-softening models in the context of standard numerical techniques. In general, these solutions suffer from localization of strain in the first element where strain-softening develops. Deformation tends to become concentrated there. The location of the softening zone and the magnitude of the deformation tend to be unduly sensitive to element size. Examples of such difficulties in quasistatic applications are reported by Pietruszczak and Mroz [10].

An additional problem presents itself in dynamic analysis involving rate-independent, strain-softening constitutive models. On the strain-softening branch, the equations of motion are elliptic instead of hyperbolic. The consequences of this are discussed by Sandler [11], Sandler and Wright [12] and Perzyna [13], where it is demonstrated that a valid solution for wave propagation in a bar with rate-independent softening properties cannot be obtained in the softening regime. It is also shown that adding rate-dependence to the constitutive model makes it possible to obtain a mathematically valid solution.

Constitutive models satisfying requirements for valid mathematical solutions that account for inhomogeneous deformation, represent the main features of experimental data and are suitable for computations are not yet available. However, there are problems for which practical solutions are needed in the near term. The present paper gives the outline of a rate-dependent (viscoplastic) model that satisfies at least the mathematical requirements mentioned above and appears to represent some features of experimental data.

2. Rate- or Time-Dependent Plasticity Model

We propose a rate-dependent, strain-softening plasticity model in which an internal damage variable is assumed to represent strength degradation. This type of model is related to viscoplasticity as proposed by Perzyna [13]. In that model, the rate of plastic straining is assumed to be proportional to the difference between a stress function and a quasi-static yield limit. This leads to stress relaxation from a dynamically induced peak back to the quasistatic yield surface at a rate that depends on strain rate. Such a model may describe relaxation from a dynamically enhanced strength to a quasistatic strength but it requires modification to describe total loss of strength.

The model proposed here assumes that the locus of maximum stress states for a concrete specimen defines an initial yield surface, Fig. 1. Within the surface, behavior is assumed to be elastic; this is not a fundamental restriction, however. Upon satisfying the initial yield condition, the yield surface begins to contract at a rate that increases with time spent on the yield surface and with growth of a damage parameter that is an invariant of cumulative plastic strain. A function behaving in this manner is

$$f = \sqrt{J_2'} - B = 0 \tag{1a}$$

where

$$B = B_{1\Delta} \exp\left(\frac{-Qt^*}{\bar{t}}\right) + B_{10} - B_{1\Delta} \tag{1b}$$

and B_{10}, $B_{1\Delta}$ = initial, residual effective cohesion, respectively; $Q = g(\varepsilon^p_{ij})$; t^* = total accumulated times on yield surface; and $\bar{t}$ = empirical strain-softening coefficient. Thus, the initial yield surface may contract during some phase of a complicated strain rate path, may become stationary during an unloading phase and may resume contraction. Underlying the model, which is at the moment speculative, are the ideas that strain-softening is due to development and coalescence of cracks (role of Q in eq. 1b) and that cracking progresses at an inherent rate governed by viscous forces acting on a microscopic level.

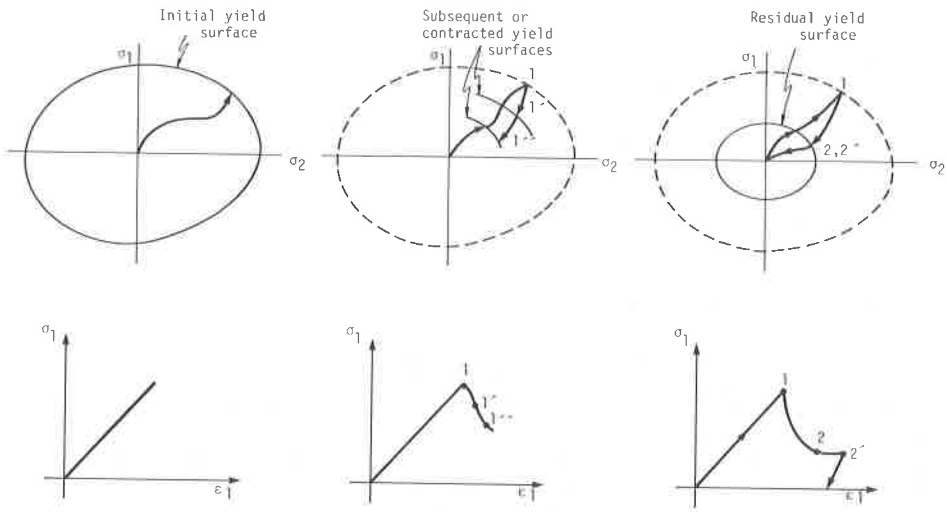

Figure 1. Rate- or time-dependent viscoplastic model for strain-softening in concrete.

To illustrate the behavior of a specialized version of the model, example stress-strain relations for prescribed strain rates are shown in Fig. 2. The yield surface is specialized to two non-zero stress variables σ_{11} and σ_{12}. The yield surface is

$$f = \sigma_{11}^2 + B_1 \sigma_{11} + B_2 \sigma_{12}^2 + B_3 = 0 \tag{2}$$

which is an ellipse in $\sigma_{11} - \sigma_{12}$ plane.

Fig. 2a shows the case of uniaxial compressive strain. The material is elastic until the unconfined compressive strength $f_c' = 5000$ psi is reached. The residual compressive stress is $J_1 = 500$ psi and it is reached at $\varepsilon_{11} = 0.6\%$. No special attempt is made here to fit this stress-strain relation to a particular suite of data, which might require adjusting or other adjustments. Fig. 2b shows the results of imposing combined shear and compressive strain.

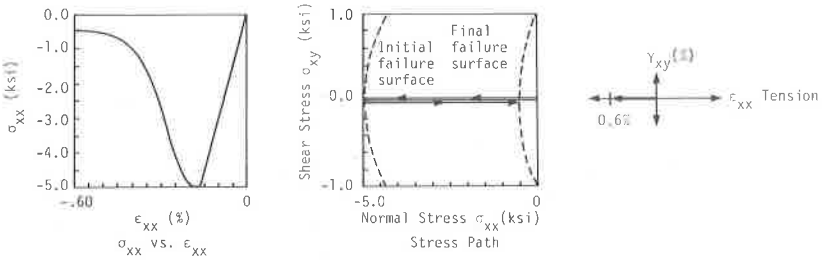

Figure 2a. Uniaxial compression ($\dot{\varepsilon}_{xx} = 0.02\%$ per second).

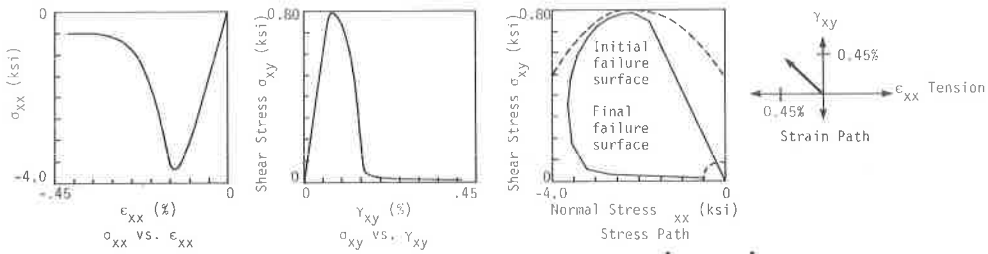

Figure 2b. Combined shear and extensional strain $\dot{\gamma}_{xy}$ and $\dot{\varepsilon}_{xx} = 0.01\%$ per second.

3. Finite Element Application

The model described above was incorporated into the finite element program described by Ghaboussi et al. [14] and a series of numerical experiments was performed involving dynamic loading of moderately deep beams. The results were compared with physical experiments and with results of previous simulations using a perfectly plastic (rate-independent) model. It is to be expected that the strain-softening model better represents the concentrated shear hinges illustrated in Fig. 3; the deformed shapes in Fig. 4 appear to confirm this. Further numerical experiments, also described by Isenberg et al. [15], demonstrate the insensitivity of computed deformations to changes in mesh discretization over a range of 9 to 27 elements for a half span.

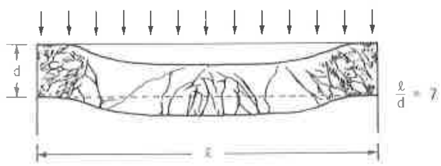

Figure 3. Deformed shape of member exhibiting shear hinge.

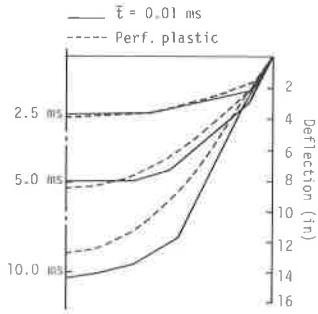

Figure 4. Comparison of calculated deformed shapes using perfectly plastic and strain-softening models.

4. Summary

A time-dependent, strain-softening model is proposed which satisfies theoretical requirements for dynamic applications. This constitutive model has been incorporated in a finite element analysis of moderately deep beams.

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