OSCAR: a New Compute Code for the Dynamic Analysis by Substructuration

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Abstract

This paper describes a substructuration procedure used in the computer code OSCAR (from CASTEM 2000 system) to perform vibration or seismic analysis of complex coupled fluid-structure systems.

The main originality of OSCAR is to allow in the same formalism both free and blocked substructure procedure and even "intermediate" modal configuration using special "connection substructures" ; the efficiency of the substructuration method being, in fact, strongly connected to a good choice of the substructure modal basis.

In this paper, a chapter is devoted to the particular case of substructure which consists of fluid-structure coupled system where fluid is defined by pressure variables.

I. Introduction

The dynamic analysis of complex structures is easier and cheaper if they can be divided into simpler substructures, with a small number of connection points. Then the procedure is the following. Each substructure is characterized independently from the others by a family of eigen modes and static solutions. Next, these substructures are assembled by links between different variables. This representation limits the number of variables and allows a better physical knowledge of the problem.

The code OSCAR which is a part of the general computer system CASTEM 2000 includes both principal variants of substructuration : ("free" substructure technic and "blocked" substructure technic) inside a single formalism. It allows an adaptable and optimizated utilisation of substructuration technic.

II. Definition of a substructure and its links

(S) is a substructure characterized by its stiffness and mass matrix K and M. X is the D.O.F. vector of (S) and PX the D.O.F. vector which are used in the links (this vector components can be more generally linear combinations of the D.O.F.). X_L represents the displacement of the link nodes ; F_i are the forces applied by the substructure on the links. These variables are joined together by the relation :

$$F_L = K_L \ (PX-X_L)$$

where K_L is a stiffness link matrix ; K_L is supposed diagonal to make easy the writting (which doesn't limit the conclusions).

For each substructure, the associated problem is defined by the system of equilibrium and link equations :

$$(1)\qquad \begin{cases} KX + M\ddot{X} + P^T F_L = Fe & \text{(Fe : external forces)} \\ F_L = K_L(PX - X_L) \end{cases}$$

The connection with another structure is written by the continuity of the variables X_L and F_L.

Let's introduce a new link variable R_L and its dual variable T_L

$$(2)\qquad R_L = AX_L - BK_L^{-1} F_L \qquad\qquad T_L = BK_L X_L + AF_L$$

where A and B are two diagonal matrix as $A^2 + B^2 = I$ (their coefficients are for example positive).

The above equations can be written with the new variables :

$$(3)\qquad \begin{cases} K'X + M\ddot{X} - P^T (A+B)^{-1} K_L R_L = Fe \\ K_L PX = (A+B)T_L + (A-B) K_L R_L \\ \text{with } K' = K + P^T A (A+B)^{-1} K_L P \end{cases}$$

K' is symmetrical because A, B, K_L are diagonal. K' is a new stiffness matrix for the system ; it associates the link stiffness with the substructure stiffness K. With this presentation, the substructuration formalism becomes very general. Later,particular cases will be examined.

III. Projections on an eigen mode and static solution basis

To resolve (3), displacement vector X is chosen to be represented by a combination of static solutions and the contribution of the N first eigen modes :

. Let's consider the matrix U, static solutions of (3) with the impedance relations $R_L = R_{LU} = I$ (identity matrix)

$$(4)\qquad K'U - P^T (A+B)^{-1} K_L = 0$$

. The matrix $\overline{X}$ of eigen modes, is calculated with the conditions $R_L = \overline{R}_L = 0$ on links :

$$(5)\qquad K'\overline{X} - M \overline{X}\Omega = 0 \qquad (\Omega \text{ is the diagonal matrix of } \omega^2,\ \omega = \text{eigen pulsations})$$

The solution of (3) is represented on the $\overline{X}$ and U base by :

$$X(t) = \overline{X}\,\alpha(t) + U\,\beta(t)$$

Written on the basis, the variable $R_L(t)$ associated to the solution, is the following :

$$R_L(t) = \overline{R}_L\,\alpha(t) + R_{LU}\,\beta(t) \quad \text{with } \overline{R}_L = 0 \text{ and } R_{LU} = I$$

so that :$\beta(t) = R_L(t)$ so that :

$$(6)\qquad \boxed{X(t) = \overline{X}\,\alpha(t) + U\,R_L(t)}$$

Equation (3) is written again and projected on $\overline{X}$ and U vectors :

$$(7)\quad \begin{cases} \bar{X}^T K'X\alpha + \bar{X}^T M\bar{X}\ddot{\alpha} + \bar{X}^T MU \ddot{R}_L = \bar{X}^T Fe \\ U^T K'\bar{X}\alpha + U^T M\bar{X}\ddot{\alpha} + U^T MU \ddot{R}_L = U^T Fe \end{cases}$$

Some terms can be simplified :

. $\bar{X}^T K'\bar{X}$ and $\bar{X}^T M\bar{X}$ are the diagonal matrix K'_G and M'_G of modal stiffnesses and masses.

. $U^T M\bar{X} = U^T(K'\bar{X}\Omega^{-1}) = (U^T K')\bar{X}\Omega^{-1} = (A+B)^{-1} K_L P\bar{X}\Omega^{-1} = \bar{F}\Omega^{-1}$

where $\bar{F}$ represents the modal reactions matrix at the links.

. $U^T K'\bar{X}\alpha = (U^T K')(\bar{X}\alpha) = (A+B)^{-1} K_L P.(X-U_{RL}) = T_L + (A+B)^{-1} [(A-B)K_L - K_L PU] R_L = T_L - GR_L$

with $G = -(A+B)^{-1}(A-B)K_L + (A+B)^{-1} K_L PU$

G is symmetrical because K' is symmetrical and K_L, A,B are diagonal.

(7) can be written with symmetrical matrix :

$$(8)\quad \begin{bmatrix} K'_G & 0 \\ 0 & -G \end{bmatrix} \begin{bmatrix} \alpha \\ R_L \end{bmatrix} + \begin{bmatrix} M_G & \Omega^{-1}\bar{F} \\ \bar{F}\Omega^{-1} & U^T MU \end{bmatrix} \begin{bmatrix} \ddot{\alpha} \\ \ddot{R}_L \end{bmatrix} = \begin{bmatrix} \bar{X}^T Fe \\ U^T Fe \end{bmatrix} + \begin{bmatrix} 0 \\ -T_L \end{bmatrix}$$

Terms of equation (8) will be explicited in chapter IV in particular cases.

IV. Connection of two substructures :

The system (8) is written for each substructure. The continuity of the F_L and X_L gives the link :

$$\begin{cases} (X_L)_1 - (X_L)_2 = 0 \\ (F_L)_1 - (F_L)_2 = 0 \end{cases}$$

These relations can be expressed with the variables $(R_L)_1$, $(R_L)_2$, $(T_L)_1$, $(T_L)_2$. Then, the following symmetrical system is obtained, with the modal variables α and the link-variable R_L :

$$(9)\quad \begin{bmatrix} K_{G1} & 0 & 0 & 0 \\ 0 & K_{G2} & 0 & 0 \\ 0 & 0 & (C_{11}-G_1) & C_{12} \\ 0 & 0 & C_{12} & (C_{22}-G_2) \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ R_{L1} \\ R_{L2} \end{bmatrix} + \begin{bmatrix} M_{G1} & 0 & (\Omega_1^{-1}\bar{F}_1^T) & 0 \\ 0 & M_{G2} & 0 & \Omega_2^{-1}\bar{F}_2^T \\ \bar{F}_1^T\Omega_1^{-1} & 0 & U_1^T MU_1 & 0 \\ 0 & \bar{F}_2^T\Omega_2^{-1} & 0 & U_2^T MU_2 \end{bmatrix} \begin{bmatrix} \ddot{\alpha}_1 \\ \ddot{\alpha}_2 \\ \ddot{R}_{L1} \\ \ddot{R}_{L2} \end{bmatrix} = \begin{bmatrix} \bar{X}_1^T & Fe_1 \\ \bar{X}_2^T & Fe_2 \\ U_1^T & Fe_1 \\ U_2^T & Fe_2 \end{bmatrix}$$

V. Study of two extreme cases :

1. Link on free nodes

Let's have A = 0 and B = I. It gives in (2) :

$R_L = -K_L^{-1} F_L$ and $T_L = K_L X_L$ and $K' = K$.

Taking again F_L and K_L as link variables (which corresponds to the variable change $R'_L = F_L$ and $T'_L = X_L$), the following system is found :

$$(10)\quad \begin{cases} \begin{bmatrix} K_G & 0 \\ 0 & -(K_L^{-1} - PU') \end{bmatrix} \begin{bmatrix} \alpha \\ F_L \end{bmatrix} + \begin{bmatrix} M_G & -\Omega^{-1}\bar{X}^T P^T \\ -PX\Omega^{-1} & U'^T MU' \end{bmatrix} \begin{bmatrix} \ddot{\alpha} \\ \ddot{F}_L \end{bmatrix} = \begin{bmatrix} \bar{X}^T Fe \\ U'^T Fe \end{bmatrix} + \begin{bmatrix} 0 \\ X_L \end{bmatrix} \\ X(t) = \bar{X}\alpha(t) + U' F_L(t) \end{cases}$$

where the modes are the ones calculated for the structure with free link nodes :
$F_L = 0$.

The static solutions U' are obtained imposing unity forces on link nodes :

$F_L = I \qquad KU' + P^T = 0$.

In system (10), the "stiffness" of the link happens only in the stiffness term of the link equation : $- PU' + K_L^{-1}$. K_L^{-1} is an "external" flexibility which is added to the local stiffness of S, -PU', defined by the static calculation $KU' + P^T = 0$.

2. Link on blocked nodes

Let's have A = I B = 0. It gives in (2) :

$R_L = X_L$ and $T_L = F_L$ and $K' = K + P^T K_L P$.

System (7) becomes :

$$(11) \quad \begin{cases} \begin{bmatrix} K'_G & 0 \\ 0 & -G \end{bmatrix} \begin{bmatrix} \alpha \\ X_L \end{bmatrix} + \begin{bmatrix} M_G & \Omega_1^{-1} \bar{F}^T \\ \bar{F}\Omega_1^{-1} & U^T MU \end{bmatrix} \begin{bmatrix} \ddot{\alpha} \\ \ddot{X}_L \end{bmatrix} = \begin{bmatrix} \bar{X}^T & Fe \\ U^T & Fe \end{bmatrix} + \begin{bmatrix} 0 \\ -F_L \end{bmatrix} \\ X(t) = \bar{X}\,\alpha(t) + UX_L\,(t) \end{cases}$$

Static solutions are obtained imposing unity displacement on link nodes : $X_L = I$. G are the static reactions on link nodes. $K'U - P^T K_L = 0 \quad G = K_L\,(PU-I)$. Displacements are blocked at link nodes to calculate the modes ($X_L = 0$) and $\bar{F}$ are the modal reactions at link nodes. $K'\bar{X} + M\bar{X}\Omega = 0 \qquad \bar{F} = K_L\, P\bar{X}$.

Link stiffness does not need to be explicit in equation (11) : this stiffness is wholly included in the substructure (S) by the change of its stiffness matrix K'.

VI. Application to the fluid

The above formalism can be extended at the study of a coupled fluid-structure system. In the "acoustical" formulation motions of such systems are defined by double variables : displacements X for the structure, pressure p for the fluid.

By analogy with the mechanical structure let us consider two kinds of connections between fluid volumes :

- "pressure" connection (↔"blocked" link)
-"normal fluid displacement connection"(↔ "free" link).

The subsystems are separated by the following boundary conditions for the fluid :

- at pressure connection $\vec{\text{grad}}\, p . \vec{n} = 0$ is assumed
- at flow rate connection $p = 0$ is assumed.

The solution is written down as a linear combination of acoustical-mechanical modes and some static solutions. These solutions correspond to an incompressible fluid problem, limited by rigid walls. The source terms act at the fluid connection level.

More precisely three families of static solutions are computed for the following source terms :

- unit pressure imposed at the pressure connections
- unit normal fluid displacement imposed at the displacement connections
- external sources.

Then, the subsystems are connected by assuming the pressure continuity and the normal fluid displacement continuity.

VII. Discussion

The above presentation shows that there is an "infinity" of ways to describe a substructure connected to some link points L.

An arbitrary impedance on link nodes can be chosen. Then the corresponding modes and static solutions basis are built. It leads to insert partly in (S) the link stiffness K_L ; then the other part of the stiffness takes place outside in link equation between substructures

Theoretically, the modal basis which provides the best representation of the structure movement of the assembled structures, must be chosen. But this movement is a priori unknown. However, in most cases, an approximative choice will be practically possible (between free or fixed or eventually intermediate conditions).

Another practical requirement is to limit, in parametrical analysis, the number of computations. So, it is interesting to define some unchanged substructures and some substructures and connections to be varied. Let's remark that only free proceeding allows to conserve some symmetry characters disturbed by the links (axisymmetrical geometry for instance).

VIII. Conclusion

OSCAR Code uses the above formalism. Then OSCAR allows to realize mixed connections between substructures in mechanical or fluid modes.

Link stiffness can be introduced :

- inside the substructures
- outside as particular substructures defined by a set of displacement and force link variables. These substructures are characterized by a stiffness matrix which is usually non diagonal.

Intermittent links as shocks and friction which settle naturally in such a formalism can be treated by OSCAR.