ABSTRACT

ALEXANDER, M. ERIN. The Progressive Income Tax and the Macroeconomy. (Under the direction of Dr. John J. Seater.)

This dissertation presents a series of three essays that examine the functional form of the U. S. federal income tax and its implications. In the first essay we introduce the convex functional form of the income tax which we believe is superior to the standard income-proportional form. We also describe the parameters within this function and their construction over the years from 1913 to 2005. The second essay discusses the characteristics of the time series of these parameters, the relation of these series to other tax series in the literature, the relation of the intertemporal variation in the tax parameters to the sharp reduction in volatility of macroeconomic time series after about 1950, and the interrelation of the tax parameters with other federal fiscal variables. In chapter three, we use a standard dynamic stochastic general equilibrium model and insert our tax function. We explore the implications that different tax policies will have on the macroeconomy by changing parameter values within this tax function. Specifically we compare the steady state values, second moments, and impulse response functions, of the usual

variables, generated by these policies.

The Progressive Income Tax and the Macroeconomy

by
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DEDICATION

I would like to dedicate this work to my parents and grandmother. Without your love and support this would have only been a dream.

BIOGRAPHY

Erin Alexander was born to Margaret and Christopher Alexander on 28 January 1974 in Atlanta, Georgia. She attended Summit Country Day School in Cincinnati, Ohio, St. Andrew's Episcopal School in Jackson, Mississippi, and graduated high school from Brentwood School in Sandersville, Georgia. She graduated from the University of Georgia in 1997 with a BBA and a major in Finance. She worked at Smith Barney until she returned to school. At the University of South Carolina she earned a Master of International Business Studies degree, which includes a concentration in french, in 2002. While earning her MIBS degree she worked as a graduate assistant. In 2001 she moved to France and worked for Schlumberger-Sema which was just outside of Paris in Nanterre. Immediately after graduating from the University of South Carolina, she moved to Raleigh and began her studies at North Carolina State University. Here she earned a Master of Economics in 2004. As a graduate teaching assistant, she taught at NCSU for three years. The following year she taught three economics courses and a finance course at Meredith College. She will graduate from NCSU in the summer of 2009 with a Doctor of Philosophy degree in Economics.

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Chapter 1

The Tax Function

The U.S. income tax has grown wildly since its inception in 1862 (the Sixteenth Amendment to the Constitution made it permanent in 1913). Once a very managable three page form with only one page of instructions has developed into an epic length tome with multiple pages of forms and, as of 2005, 142 pages of instructions (161 pages of instructions in 2008). Income tax law has become so challenging to understand and comply with that it has spawned a whole industry of tax preparers. The effects of the income tax on the economy and how it should be changed are hotly debated topics among politicians and pundits. Virtually everyone has an opinion - with no concensus in sight.

Although widely studied, the exact effects of of the U.S. income tax on our economy are not obvious. No immediate answers jump from economic theory or from existing empirical work. Countless people and organizations have weighed in on the debate. We believe that the answers can be found through the correct measurement and modeling of the income tax. Although the study of taxes is too broad of a subject to discuss in detail here, there are some standard practices, patterns, and stand-out papers. These will be discussed in the following chapters. Conventional economic models include, at worst, simply a lump sum tax on the household, and at best, some combination of income-proportional and lump sum taxes. We know that the U.S. income tax is progressive. And, certainly after examining a graph of income versus tax generated, this should be obvious. So, why not model taxes as such? This is exactly what we intend to do - to use an appropriate model of the income tax and generate results from actual data.

This paper is divided into three chapers. Chapter One introduces the form of

our of our income tax function and its parameters. It includes a time series, from 1913 to 2005, of values for each of the function's parameters. A description of the data and the process used to create each of these times series is also contained in this chapter. Chapter Two examines the relationship between these parameters in our tax function and possible explanatory variables. We also discuss the characteristics of the parameters' time series and relate them to other tax series in the literature. In Chapter Three we insert our income tax function into a standard Real Business Cycle model. We detail the perturbation technique used to solve for the state space dynamics of the model which offers us a solution to the system of equations and allows us to perform certain experiments. Specifically, we vary the parameter values within our tax function and compare steady state values, second moments, and impulse response functions.

1.1 Tax Function

We begin this discussion with a statement of our tax function and then proceed with the definitions of the parameters in it. The construction of these parameters over time is then explained. Admittedly, this process is a bit tedious, but it renders a data set with many possibilities.

The following formula describes the federal, personal income tax function:

$$T = \alpha[(1-\theta)\phi Y - eN]^{\beta}$$

where T is equal to the total income tax liability, α and β are parameters set by the government, θ is the aggregate deduction amount divided by adjusted gross income (AGI), ϕ is adjusted gross income divided by gross domestic product (GDP), Y is gross domestic product, e is the individual exemption amount allowed by the tax law, and N is the aggregate number of exemptions claimed. The θ ratio will always be between zero and one, and N is exogenous to the government. The value inside the brackets approximates taxable income. Most macro models discuss income taxes as either lump sum or proportional to income; however, we know that in actuality the U.S. tax system is progressive. This characteristic is captured by the α and β parameters. Although GDP is commonly used as a measure of income, only a small proportion of it is subject to income taxes. Consequently, a more

suitable measure, taxable income, is used in our tax function. The quantity in brackets details the conversion of and explains the relationship between GDP and taxable income. Taxable income, as defined by the tax law, is:

$$Y_t = AGI - D - E$$

where Y_t is taxable income, D is aggregate deductions, and E is aggregate exemptions. As an identity, we also know that:

$$E = eN.$$

We rewrite deductions as a proportion of AGI, θ, include the above transformation for E, and get:

$$Y_t = (1 - \theta)AGI - eN.$$

Because AGI can be expressed as a proportion of GDP, the following formula for taxable income is used in our tax function above:

$$Y_t = (1 - \theta)\phi Y - eN.$$

This document details the construction of a time series for each parameter in our tax function. The series includes a value for each parameter in each year from 1913 to 2005.

1.1.1 α and β

The process resulting in the α and β parameter values is the most involved and will be addressed first. All information required to formulate these values originates from (1) the tax law and, more specifically, from each years 1040 income tax form and its instructions and (2) the Statistics of Income, Individual Income Tax Return Collection. The time series of the α and β used in our tax function is created using the following formula:

$$T = \alpha(Y_t)^{\beta},$$

where T is the amount of tax paid, and Y_t is a taxable income amount called a breakpoint. The α and β are estimated for each individual year using the set of breakpoints, Y_t, and their corresponding amounts of tax paid, T, from each of these specific years. Breakpoint is

the term used to describe the taxable income levels at which the marginal tax rate changes. These figures will be explicitly listed in each years tax table(s) as ranges, and the boundaries of these ranges denote the precise levels of taxable income when the tax rate changes. These boundaries also signify the points where the brackets within each schedule meet. A schedule is the set of tax rates facing a filer. In early tax law there is only one schedule which is used in the tax computation for all returns; however, at present, filers are categorized into four different groups based on marital status, each with its own schedule of tax rates. Brackets denote the different ranges of income within a schedule that are subject to the same tax rate. Because the distribution of income within brackets is unknown, we assume it is uniform. Unless otherwise noted, when there is a tax table (for any type of tax: normal, surtax), the breakpoints and amounts of tax paid are constructed in the same manner. The following steps detail the process used.

1.1.2 1913-1947

For the years 1913 to 1947, there is a single tax rate schedule faced by all tax filers. During this span of years, because there is only one schedule, the calculation of tax paid is less complex than it is in the subsequent years. Other than specific issues occurring in certain years, which will be discussed in the following paragraphs, this computation includes simply taking each breakpoint, or taxable income amount, and calculating its corresponding tax payable with the tax rates given in the tax rate schedule. This results in a set of pairs, breakpoints and tax paid at each, which is used in the estimation of α and β. The following example illustrates the calculation used to find the tax paid amount for one breakpoint, when there is only one schedule. The breakpoints in this case include: \$2000, \$4000, \$6000, and \$8000. In this demonstration, we focus on the \$6000 breakpoint. Using an excerpt from the 1947 tax rate schedule from the 1040 tax form:

Calculations: Tax paid at a breakpoint of \$6,000 = (\$2,000*.20) + (\$2,000*.22) + (\$2,000*.26) = \$1360. The first \$2,000 of income is taxed at the lowest rate of 20%, the next \$2,000 is taxed at the higher rate of 22%, and the final \$2,000 is taxed at 26%. In this example the Y_t used in the estimation of the α and β parameters is \$6,000, and the T used is \$1,360.

Table 1.1: 1947: Tax Rate Schedule from 1040 Form

Taxable Income Is Over	But, Not Over	Tax Rate
0	2,000	.20
2,000	4,000	.22
4,000	6,000	.26
6,000	8,000	.30
8,000	8,000+	.34

1.1.3 1948-2005

From 1948 to 2005, all tax filers do not face the same tax rate schedule, and the segregating criterion is marital status. Filers are separated into two groups - single and married filing separately combined as one group and married filing jointly as the other - during the years from 1948 to 1951. Although there continues to be only one tax rate schedule in these years, the calculation of tax payable is different for each group. From 1952 to 1954, the tax law explicitly lists two distinct tax rate schedules one for head of household filers and the other for non-head of household filers. During these years, the joint filers use a slightly different method of tax calculation than the other filers. Essentially there are three groups: the single and separate filers using the non-head of household schedule, the joint filers using the non-head of household schedule along with a slightly altered method of calculation, and the head of household filers using their own, separate schedule. From 1955 to 1970, filers are further separated into three groups: single and separate combined, joint, and head of household, each with its own tax rate schedule. Finally in 1971, filers are divided into the four groups: single, separate, joint, and head of household that we see today with a specific, individual tax rate schedule for each. When there is more than one tax rate schedule, the information must be combined and used to form one set of breakpoints and their resulting amounts of tax paid for each year. This combination process is the same regardless of the number of tax rate schedules used. The first step of this combination process is to find the union of the breakpoints in all schedules. The amount of tax paid at each breakpoint, even if the breakpoint is not one specific to that schedule, is then calculated for each group of filers. At this point in the process, the number of tax paid amounts at each of the breakpoints is equal to the number of different tax rate schedules.

Because the estimation will use only one amount of tax paid per each breakpoint, and, because the amounts of tax paid differ according to the tax rate schedule being used to calculate them, they are averaged together. A weighted average of the tax paid at each breakpoint across the schedules is used. Because there is limited data available in some years, two different methods of calculating the weights are employed. The most accurate approach uses adjusted gross income amounts. The relationship between taxable income and adjusted gross income is that: taxable income is equal to adjusted gross income minus exemptions and deductions. Given this connection, there is an amount of adjusted gross income that leads to each breakpoint. The weight applied to each amount of tax paid is the ratio of the cumulative adjusted gross income for that schedule that is associated with that breakpoint's range of income, over the sum of these amounts from each schedule:

$$Weight_{i,j} = \frac{AGI_{i,j}}{\sum_{j}[AGI_{i,j}]}$$

where i describes the breakpoint that determines the adjusted gross income amount, and j denotes the schedule. When the table of adjusted gross income is not divided such that it specifically lists the necessary breakpoint values, extrapolation is used.

For the years in which the tables that include the adjusted gross income amounts are not available (1948-1953, 1974, 1978, and 1980-1982), the number of returns in each schedule is used as an alternative. In these years, the weight applied to each amount of tax paid is the ratio of the number of returns using that schedule over the total number of returns filed:

$$Weight_{j} = \frac{Number of Returns_{j}}{\sum_{j} Number of Returns}$$

where j denotes the schedule.

When using this process, the weight does not vary according to breakpoint, as it does with the other weighting scheme, but only varies according to schedule. There are groups of years when the tax law and tax tables do not change. In these years, however, the α and β parameters may still change due to the different weights across the years.

1.2 Earned Income Tax Credit, Alternative Minimum Tax

1.2.1 Earned Income Tax Credit: 1975 - 2005

Two other matters, the earned income tax credit (EITC) and the alternative minimum tax (AMT), may be addressed before moving to other parameters and year-specific issues, as they both affect the α and β of a large group of years. The earned income tax credit began in 1975 (with the Tax Reduction Act of 1975, and was made a permanent part of tax legislation with the Revenue Act of 1978) and continues until present, but because of its calculation, only affects the following years in our study: 1975, 1977-1987, 1996-2000, and 2002-2005. The main purpose of this credit is poverty reduction, and its aim is to benefit low-income families. The earned income tax credit reduces the amount of tax owed, and because it is a refundable credit, may reduce a filers tax liability below zero. If the tax owed does fall below zero, the filer will receive a payment from the government and essentially face a negative income tax. For the purposes of this study, if the calculated tax liability is negative, we use an amount of zero. This is necessary as the estimation of our α and β parameters includes log-linearization. Starting with an initial income level of \$0, this tax credit will increase as income increases, then reach its maximum and begin to decline and continue to fall back \$0. In 1975 the basic requirements for eligibility to receive this credit were: (1) that the filers earned income or adjusted gross income was below a certain level, (2) the filer had at least one dependent child, and (3) the individual filed a return classified as either joint or head of household. These requirements have changed slightly over the years: with the maximum level of income being raised in several years, in 1991 the single filers became eligible, and in 1994 filers with no children also became eligible. Information reported by the Treasury is different from year to year causing differences in availability, and consequently, differences in our methods of calculation for the earned income tax credit.

For this study, we need to find the earned income tax credit amount that is applicable to each breakpoint. This process is repeated for each schedule in each year. Because the amount of the credit depends on adjusted gross income and our breakpoints are taxable income amounts, we first find the adjusted gross income amounts that correspond to each breakpoint. Using the calculations from the 1040 tax form, the following formula again describes this relationship:

$$Y_t = AGI - D - E.$$

Although the calculation in every year begins the same, as just described, the subsequent steps vary.

1975 - 1978

From 1975 to 1978 eligibility requires that adjusted gross income is less than $8000, and a maximum credit amount of $400 is specified. In these years there is a worksheet in the 1040 form that details the computation of the credit which ultimately results in the formula:

$$EITC = 400 - .1[(AGI - 4000)].$$

Because the credit only applies to low income households, and we are estimating the tax payable for a representative individual, the correct proportion of this credit must be applied to the amount of tax payable. To find this proportion, divide the number of returns that receive the credit in each schedule by the total number of returns in each schedule. This proportion multiplied by the amount of credit found for each breakpoint in each schedule is then subtracted from the tax payable at each breakpoint before the weighted average of taxes payable is taken across schedules.

$$Taftercredit_{i,j} = Tbeforecredit_{i,j} - \frac{\#ofreturnsgettingcredit_j}{total\#ofreturns_j} amountofcredit_{i,j}$$

where i denotes the breakpoint and j denotes the schedule. The weighted average that gives the final amount of tax paid for each breakpoint is found using each of these amounts of tax paid after the credit for each schedule at each breakpoint and applying the appropriate weight for each schedule as detailed above.

1979 - 1983

The method for calculating the tax credit changes slightly and then remains the same for years 1979 through 1983. In these years, to receive the credit, adjusted gross income must be less than $10,000, and the maximum amount of credit is $500. To find the amount of the credit, use the table provided which lists credit amounts according to adjusted gross income. Again the adjusted gross income amount that corresponds to each breakpoint must be calculated. After the credit amount is found, the subsequent steps are the same as in the previous years.

1984 - 1987

The tax law does not change in 1984; however, the data reported by the Treasury Department do which causes our calculation to change. The number of returns that receive the credit is no longer reported. The proportion of returns in each schedule receiving the credit is now calculated using the average proportion of returns receiving the credit in each schedule from 1975 to 1983. The other steps in the process remain the same as in the years from 1979 to 1983.

From 1985 to 1987, the calculations do not change and are the same as in 1984. The only things that change in the tax law are the amount of the maximum credit allowed and the maximum amount of adjusted gross income that a filer can have and still be eligible for the credit. The maximum credit amount in 1985 and 1986 was increased to $550. In 1987 it is again increased to $851. The maximum adjusted gross income amount allowed in 1985 and 1986 was also increased to $11,000. In 1987 this allowable amount was increased again to $15,432.

1988 - 1995

The breakpoints in the years from 1988 to 1995 result in adjusted gross income levels that are above the maximum allowable amount for the tax credit. Therefore, for our purposes, in these years the tax credit is, in effect, $0 and does not affect our amounts of tax payable.

1996 - 2005

In 1996, the calculations become a little more involved as the tax credit table now includes single filers and different amounts of credit depending on the number of dependent children. The methods used to calculate the credit from 1996 to 2000 do not change. The maximum credit amounts and adjusted gross income levels eligible for the credit steadily increase, regardless of number of children, over these years. The only breakpoints that result in an adjusted gross income small enough to be eligible for the credit are the lowest breakpoints in the single schedule assuming two dependent children for each year. The following steps are used for this calculation: (1) again, use the average proportion of returns

that receive the credit from years 1975-1983 for the joint and head of household filers; there is no average for single filers as they were not eligible for the credit in those years; (2) multiply these proportions with the total number of returns in each respective schedule, resulting in the number of returns in each schedule that receive the credit; (3) subtract the number of joint and head of household returns receiving the credit from the total number of returns receiving the credit, resulting in the number of single filers who receive the credit; (4) divide this number of single filers receiving the credit by the total number of single filers to find this schedules proportion of returns receiving the credit; (5) find the credit amount for each number of dependent children in the earned income tax credit schedule that corresponds to the breakpoint; (6) find the average of the credit amounts corresponding to the different possible numbers of children 0,1, and more than 1; (7) multiply this average credit amount by the proportion of returns in that schedule that receive the credit, and this is the amount used to reduce the tax payable for that breakpoint, that schedule.

The breakpoints in 2001 do not result in adjusted gross income amounts low enough to be eligible for the credit. Again, for our purposes, the tax credits in this year are essentially $0 and do not affect the tax payable amounts.

The calculation is exactly the same for years 2002 through 2005 as it was for years 1996 to 2000. In this last set of years, the maximum credit amount and the highest amount of adjusted gross income eligible for the credit gradually increase each year. The only difference between this group of years and the years from 1996 to 2000 is that the breakpoints in the joint and head of household schedules yield adjusted gross income amounts small enough to be eligible for the credit, and more than just the lowest breakpoints in the single schedule receive the credit.

1.2.2 Alternative Minimum Tax: 1970 - 2005

The Alternative Minimum Tax (AMT) began in 1979, however, its predecessor, the Minimum Tax, began in 1970. The two taxes were in affect concurrently during the years 1979 and 1980, before the AMT replaced the Minimum Tax completely. After being established by the Tax Reform Act of 1969 this tax became effective the following year. The purpose of this tax was to reduce the incidence of high income filers having disproportionately small tax liabilities. Filers in question must determine their tax liability using both the regular tax method and the AMT method, and, then are obligated to use the

computation which results in the higher amount of tax payable. The AMT may lead to the higher tax amount because it excludes many deductions and exemptions that are allowed in the regular computation. General indicators that a filer may be subject to the AMT are: that his adjusted gross income is over $100,000, he claims a large amount of itemized deductions, or he has a large amount of personal exemptions. Since inception, the number of filers required to use the AMT has been growing. This increase has occurred for two reasons: (1) the Tax Reform Act of 1986 broadened the law to include a collection of deductions which most filers claim, and (2) this method has only been revised for inflation purposes twice during its existence, resulting in bracket-creep. Although this tax may have a larger impact in future years, its impact on the years in this study is minimal and is thusly not included. Three ratios were used to gauge the effect of this tax on our study: (1) the proportion of returns which were compelled to use the AMT computation, (2) the ratio of the amount of AMT paid to total adjusted gross income, and (3) the percentage of AMT in the total tax liability. These ratios had been steadily growing each year until 2005 when there was a large and abrupt increase. The largest ratios to date previously had occurred in 2004 with the respective ratios as listed above equaling: 2.34%, 0.19%, and 1.57%. In 2005, the ratios became: 16.93%, 0.57%, and 4.54%. The most dramatic increase occurs in the first ratio as the number of returns filing with the AMT computation increases from 3,096,299,000 to 22,751,904,000 slightly more than a 700% increase. Although this is a significant amount, the other two ratios remain relatively low, giving justification for the omission of this specific nuance of the tax law.

1.3 θ: Aggregate Deductions / Adjusted Gross Income

This ratio, explained above, is used to capture the effects of deductions within the calculation of taxable income and is a determinant of the amount of tax payable. A filer may take only one of the two types of deductions, either the standard (in some years called the zero bracket amount) or the itemized. As this amount is subtracted from adjusted gross income in the tax computation process and ultimately lowers the taxable income amount, the choice between the types is made according to which method provides the larger quantity. The standard deduction is a fixed amount according to the filers marital status. There is an additional standard deduction, beyond what the basic standard deduction provides,

which is available if the filer is over 65 years old or blind. This additional deduction is not included in our study. If a filer elects the itemized deduction, he must submit with his tax return the sum of specific expenses that he incurred during the year. The qualified categories of expenses which make up this deduction have slightly changed over the years, but basically include: medical expenses, mortgage interest, state and local taxes, charitable contributions, casualty and theft losses, and job expenses. Although this study only requires the total aggregate deduction amount - the sum of the itemized and standard deductions taken - each year, the data reported by the Treasury change and necessitate different calculations for this total across different years.

1913 - 1915

For these years no information was available either for the aggregate deduction amount itself or for any data that could be used to calculate it. Consequently, these amounts were estimated. The tax function was rearranged as:

$$\frac{(T/\alpha)^{1/\beta} + eN}{Y} = (1-\theta)\phi.$$

The parameters on the left had side of the equation were substituted in for each year, and the resulting number was used as the value for the right had side. Due to limitations in the existing data, the θ and ϕ parameters were estimated concurrently. Thus in this group of years the two parameters, θ and ϕ, were combined, and there is one less parameter for each year.

1916 - 1943

From 1913 to 1943 there was only a general deduction whose treatment appears to correspond with the itemized deduction in later years as the filer was required to list out specific expenses. The standard deduction was not yet in existence. The total aggregate amount of this general deduction was reported beginning in 1916, therefore, no calculations were necessary.

1944 - 1953

The standard deduction was introduced in 1944. During these years, because the

data available has changed, some calculations are necessary. First sum the number of returns from all types of forms (the 1040 short form, 1040 long form, 1040A, and/or the W-2 Form) that elect the standard deduction. Next, multiple this total number of returns by the standard deduction amount, found on the 1040 form. This results in the total amount of standard deductions claimed. The amount of total itemized deductions claimed is reported. The total aggregate deduction amount is then found by adding the total standard and itemized deduction quantities.

1954 - 1974

In these years, the total number of returns claiming the standard deduction is reported. This number is multiplied by the standard deduction amount resulting in the total amount of standard deduction received. Again, the total amount of itemized deductions is reported. To find the total aggregate deduction amount, simply add the total standard and itemized deduction quantities.

1975 - 1977

The total aggregate deduction amount is reported; therefore, no calculations are necessary.

1978 - 1982

This group of years is a subset of the years in which the standard deduction was replaced by the zero bracket amount. The sum of the total zero bracket amount and the total itemized deductions claimed yields the total aggregate deduction amount.

1983 - 1991

No calculations are required during these years, as the total aggregate deduction amount is reported.

1992 - 2005

In this final group of years, to find the total aggregate deduction amount, add the total standard deduction and total itemized deduction amounts.

1.4 ϕ: Adjusted Gross Income / Gross Domestic Product

This ratio is used as part of the conversion of Gross Domestic Product to taxable income.

1.4.1 AGI: Adjusted Gross Income

The adjusted gross income amounts, which make up the numerator in this ratio, used were explicitly listed in the respective Statistics of Income books for each year, with four exceptions. First, no data were available from any source for the AGI from 1913 to 1915; therefore, these were estimated concurrently with the θ parameter as explained above. Secondly, AGI for the years from 1916 to 1929 were listed in the 1929 edition of the Statistics of Income. Next, from 1916 to 1943 AGI, as we are using it, was called total income. Finally, from 1938 to 1942 the AGI from two groups: returns with net income and returns with no net income, must be added to find the total AGI amount. The denominator portion of the f variable, Gross Domestic Product, is described below.

1.4.2 Y: Gross Domestic Product

As this parameter is so often used in macro models, it is our starting point in the calculation of taxable income. The values for this parameter for the years 1929 to 2005 were obtained from the National Income and Product Account Table 1.1.5 from the Bureau of Economic Analysis. The figures for the years 1913 to 1928 come from the Resources for Economists (RFE) division of the American Economic Association. Specifically they come from the AEAs list of primary macro and regional sites that generate data. This time series was constructed by Louis Johnston and Samuel Williamson (2005) using the estimates of John Kendrick (1969) for these years.

1.5 eN: Aggregate Exemptions

The aggregate exemption amount for each year is the product of e, the exemption amount found on the 1040 form, and N, the total number of exemptions taken. Similar to the aggregate deduction ratio, this parameter is an element in the taxable income calculation and is also a determinant of the amount of tax payable. Because the available data

for these parameters varies widely, this series can be broken down into four subsets, each with its own estimation. Due to the differing degrees of data reported across the years, the most straightforward method of extracting the needed information is actually in reverse chronological order.

1944 - 2005

The e and N parameters for most years in this final group require no calculations. There is only one exemption amount, regardless of marital status or if the exemption is taken for a dependent. Thus, this value may be taken directly from the 1040 form. The number of exemptions claimed is explicitly listed in each years Statistics of Income volume.

The only calculations needed for the e parameter are due to the phase out of exemptions which occurs in the years from 1991 to the present. This phase out reduces the exemption amount allowable after a filers adjusted gross income reaches a threshold amount. This threshold amount is determined by the filers marital status. If the filers adjusted gross income grows sufficiently large, the exemption amount could ultimately be reduced to zero.

The instructions for calculating the correct exemption amount are found on the Deduction for Exemptions Worksheet included in the instructions for the 1040 form. The data needed for these calculations include: the full exemption amount before the deduction, the exemption amount if the full deduction is taken - which for this group of years is zero, the adjusted gross income level where the reduction starts, the level of adjusted gross income when the exemption is reduced to zero, and the total adjusted gross income amounts for each type of filer. All of this information can be found either on the Deduction for Exemptions Worksheet or in the Statistics of Income series. In the years from 1991 to 2002 there is available data on adjusted gross income amounts for each type of filer; however, there is an extra group included: surviving spouses. Following the category specifications in the 1040 form, this group is combined with the joint filers.

The following are the steps used to calculate the e parameter for the years 1944 to 2005. First, for each type of filer, find the cumulative amount of adjusted gross income accounted for under the initial threshold levels. If this specific value does not appear in the tables, extrapolate from the quantities just above and below it. Because the separate, head of household, and surviving spouse categories are combined in the Statistics of Income table,

extra steps are needed for their separation. The cumulative adjusted gross income amounts relative to each groups threshold are found, then multiplied by each groups proportion of total adjusted gross income:

$$CAGI_j = (CAGI_{j,fromcombinedtable})\frac{TAGI_j}{\sum\limits_{j} TAGI_j},$$

where j = separate filers, head of household filers, or surviving spouse filers. CAGI is cumulative adjusted gross income, and TAGI is total adjusted gross income.

The cumulative adjusted gross income amounts found for the joint and surviving spouse returns must then be combined. Finally, these cumulative adjusted gross income amounts are divided by the total adjusted gross income for each group. This final value is a proportion that will be multiplied by the exemption amount before any reduction has occurred and is the quantity in the first bracket of the formula that calculates the final exemption amount for each group:

$$e_j = \frac{CAGI_{j,1}}{TAGI_j}\varepsilon_{j,1} + \frac{CAGI_{j,2}}{TAGI_j}\frac{\varepsilon_{j,1} + \varepsilon_{j,2}}{2}$$

where j = a single, joint, separate, or head of household filer. The ε values are exemption amounts, the 1 subscript denotes data that relate to the initial threshold amount, and the 2 subscript indicates data that relate to the adjusted gross income amount at which the exemption becomes zero. Therefore, $\varepsilon_{j,1}$ is the maximum exemption amount for each group of filers, and $\varepsilon_{j,2}$ will equal zero.

Next, the same process is repeated for each group, except the adjusted gross income amounts at which the exemptions become zero are used instead of the initial threshold amounts. When making the cumulative calculations for this second step, the corresponding cumulative amounts from the threshold group must be subtracted from those found for the zero exemption group. Because a linear relationship describes the movement from $\varepsilon_{j,1}$ to $\varepsilon_{j,2}$, the final piece of the second bracket above is divided by two. These calculations yield the quantity in the second bracket in the formula above.

To find the final, average exemption amount, multiply the e_js as found above by

their respective proportions of total adjusted gross income:

$$e = \sum^{j} (e_j) \frac{TAGI_j}{\sum^{j} TAGI}.$$

The same basic process is used to calculate the exemption amounts for the years 2003 to 2005. The only difference is that the separate, head of household, and surviving spouse categories are not combined in the tables, and consequently the steps that are used to divide them are now unnecessary.

1934 - 1943

In this group of years, the exemption is divided into two categories: the personal exemption and the credit for dependents. The personal exemption amount is further divided into two more categories and depends on the filers marital status. The single and separate filers are grouped together and have an exemption amount usually valued at slightly less than half of the amount that the other group, the joint and head of household filers, receives. Because there are three possible exemption amounts, all listed on the 1040 form, a weighted average is used for the final parameter value. These weights must be calculated, and the process is detailed below.

During these years the aggregate amounts of personal exemptions and credits claimed for dependents are reported separately. The information needed for the weights are the numbers of exemptions and credits which apply to each individual credit or exemption amount. To find the total number of credits received for dependents simply divide the aggregate amount received by the individual credit amount listed on the 1040 form:

$$D = \frac{aggregatecreditamount}{individualcreditamount},$$

where D is the total number of credits claimed for dependents.

The number of returns filed under each type of marital status is reported and used as a proxy for the number of each type of exemption taken. There is one issue with the data reported. In 1934, the number of joint returns and returns of men filing separately from their spouses are combined. The group of separate filers only includes women. Because there should be the same number of men and women filing separately, the correction involves

2 steps: (1) subtract the number of women filing separately from the joint amount as this should eliminate the men filing separately from the joint group, and (2) multiply the number of women filing separately by two to account for the men also filing separately. These numbers plus the value for the total number of credits for dependents just found are used to calculate the weights as follows.

$$\begin{aligned} Weight_{S,P} &= \frac{NS + NP}{N + D} \\ Weight_{J,H} &= \frac{NJ + NHOH}{N + D} \\ Weight_{D} &= \frac{D}{N + D}, \end{aligned}$$

where NS denotes the number of single filer returns, NP is the number of separate returns, NJ is the number of joint returns, and NH is the number of head of household returns. N, alone, is the total number of returns filed that year. These weights are multiplied by their respective exemption or credit amount, and the resulting values are then summed. This final value is the average exemption amount and used in our data set as the e parameter:

$$e = e_S \frac{NS + NP}{N + D} + e_J \frac{NJ + NHOH}{N + D} + e_D \frac{D}{N + D},$$

where e_S is the exemption amount that single and separate filers are allowed, e_J is the exemption amount that joint and head of household filers may claim, and e_D is the amount of credit that can be taken for each dependent.

Because the total aggregate amount of exemptions and credits combined, E, is also reported, the total number of exemptions and credits, N, can be found with the following formula:

$$N = \frac{E}{e}.$$

The same steps are used for each year in this subset.

1921 - 1933

The data reported for this group of years includes the number of each type of return filed, the three different exemption amounts as explained above, and the total aggregate

exemption amount. For all of these years, the same steps as detailed above for 1934 must be taken to separate the number of joint returns from the number of returns for men filing separately. Because in these years the aggregate personal exemption amounts and the aggregate credit for dependents amounts are not listed individually, the number of credits taken for dependents must be calculated differently from the previous group of years. Several assumptions must be made. First, we believe that single filers do not have any dependents. This stems from the fact that a single individual with a child would be better off, and therefore, choose to file as head of household. As described in the tax law, a head of household filer has at least one dependent. We further assume that the joint filers and head of household filers will each have the average number of dependents, Z. A separate filer is one half of a married couple which we assume also has the average number of dependents. Because the dependents may only be claimed by one of these filers, the number of dependents per separate filer is half of the average number of dependents: $\frac{Z}{2}$. As Z is part of the formula for the average exemption amount, using these assumptions, we first calculate Z. This is accomplished by substituting in variables in the total aggregate exemption formula:

$$E = NS\{e_S\} + NP\{e_S + \frac{Z}{2}e_D\} + \{NJ + NHOH\}\{e_J + Ze_D\}.$$

Rearranging the above formula yields:

$$Z = \frac{E - e_S\{NS + NP\} - e_J\{NJ + NHOH\}}{e_D\{NP/2 + NJ + NHOH\}}.$$

Using the calculated value for Z, find the average exemption amount with the following formula:

$$e = \frac{e_S\{NS + NP\} + e_J\{NJ + NHOH\} + e_D Z\{NP/2 + NJ + NHOH\}}{NS + NP + NJ + NHOH + Z\{NP/2 + NJ + NHOH\}}.$$

Again, N, the total number of exemptions and credits, can be found with the following formula:

$$N = \frac{E}{e}.$$

1913 - 1920

In this earliest group of years the available data is quite limited and includes only: the individual exemption amounts and the credit for dependent amounts found on the 1040 form and the total number of returns filed each year (Historical Statistics of the United States, Table Ea748). Because the number of returns using each type of exemption is needed for the weighted average, as in the previous two subsets of years, these values must be estimated. For this calculation, the total number of returns each year is scaled by the number of returns for each type of filer in 1921 with the following formula:

$$N_{i,t} = R_t \frac{N_{i,1921}}{R_1 921},$$

where i = S, P, J, HOH, and R is the number of tax returns filed in year t. The only remaining piece of data needed is the value for Z in each year. Due to the lack of existing information, use the value of Z in 1921 as an approximation for the Z values from 1913 to 1920. All of the data necessary to calculate the average exemption amount, with the formula previously used for the years 1921 to 1933, is now available or constructed. Finally, use the following formula to calculate the total number of exemptions taken:

$$N = NS + NP + NJ + NHOH + Z\{NP/2 + NJ + NHOH\}.$$

1.6 Tax Law Specific to each Year: 1913 - 2005

The tax law does not change from 1913 to 1915. For these years there is only one tax bracket, but taxation occurs in two parts. The normal tax, a flat amount of 1%, is imposed on taxable income (For the purpose of this study, in these years, taxable income equals gross income minus deductions and exemptions). The additional or super tax applies to all returns with net income (gross income minus deductions) of $20,000 or higher. Although this additional tax is levied on net income, we assume that taxable income, in this case, is a close approximation for net income and then use taxable income when calculating the additional tax. This tax ranges from 1% to 6% over 6 levels of income. The total tax liability is the sum of the normal tax and the additional tax.

In 1916, there is again only one tax bracket, and taxation takes the same structure,

two parts. The normal tax increases to 2% of taxable income. The additional tax is applicable when net income exceeds $20,000 and ranges from 1% to 13% over 13 levels of income. Again we substitute taxable income for net income in the calculations of this additional tax. The total tax liability is again the sum of the normal tax and the additional tax. In this year and continuing until 1943 the equivalent of adjusted gross income, which is used to calculate taxable income as noted previously, is called total income.

In 1917, there is only one tax bracket, but taxation occurs in three parts: a normal tax (which itself has two parts), a surtax, and an excess profits tax. For the first time (and forward until 1944) the tax computation begins with total net income. To find taxable income, simply subtract the applicable exemptions from this amount. The first part of the normal tax is a flat amount of 2% levied on taxable income. For the second part of the normal tax, subtract $2000 from taxable income and take another 2% as the tax. The surtax uses net income (we again substitute in taxable income) as its base and is applicable only if the net income exceeds $5000. The surtax extends from 1% to 63% over 19 ranges of income. If the taxpayers income from the sum of: salaries, wages, commissions, bonuses, directors fees, pensions, and professions surpasses $6000, then that taxpayer is subject to the excess profits tax. We substitute taxable income as the base for this tax also. The amount of this sum, or for our calculations - the amount of taxable income, which is over $6000 is taxed at a flat rate of 8%. The total tax liability includes the normal tax, surtax, and excess profits tax.

The tax from 1918 involves only one bracket and includes a normal tax and a surtax with each having two parts. The normal tax includes the tax at 1918 rates which is 6% of taxable income up to $4000 and 12% for all taxable income above $4000. It also includes a tax at 1917 rates which is 4% of the other income category of income from partnerships, personal service corporations, and estates and trusts minus any amount of the personal exemption that exceeded the net income on which normal tax was calculated at the 1918 rates. The surtax includes a tax at 1918 rates and applies if the sum of: net income, dividends on stock of corporations in the US, and taxable interest on bonds and other obligations of the US issued after 1 September 1917 (called the total net income subject to surtax at 1918 rates) exceeds $5000. This sum is taxed at rates from 1% to 65% which occur over 54 ranges of this type of income. There is also a surtax which uses prior year (1917, 1916, and 1913-1915) rates. The surtax for these prior years is calculated in

the same way, but the surtax for 1917 is applicable if net income is greater than $5000, and the surtax for 1916 and 1913-1915 applies if net income exceeds $20,000. The steps to find these surtax amounts are as follows: (1) find total net income subject to surtax at the appropriate years rates, (2) take the lesser of: (a) the amount in the net income column of the surtax table that is just greater than the total net income from (1) or (b) the sum of: the net income subject to surtax at that years rates, the amount of stock dividends accrued in that year, and income from partnerships, personal service corporations, and estates and trusts from that year, (3) find the difference between (1) and (2), (4) find the tax rate in the surtax table that corresponds to the net income amount found in (2a), (5) take the product of (3) and (4). If (2b) is greater than (2a) do the following: (6) find the amount of net income in the surtax table that is just smaller than (2b) and subtract (2b) from it, (7) find the tax rate in the surtax table that is just higher than the rate that corresponds to (6), (8) take the product of (6) and (7). (9) Find the sum of the amounts of surtax in the table that correspond to the tax rates which are between the tax rates from (4) and (7). (10) Find the sum of (5), (8), and (9) this is the total surtax for the corresponding year. When surtax is computed in this way for 1917, 1916, and 1913-1915 and then added together, they make up the total surtax at prior year rates. The 1917 surtax rates range from 1% to 63% and span 19 net income ranges. The 1916 surtax rates range from 1% to 13% and span 13 net income ranges. The 1913-1915 surtax rates range from 1% to 6% and cover 6 net income ranges. The total tax liability includes the normal tax at the 1918 and 1917 rates and the surtax at the 1918, 1917, 1916, and 1913-1915 rates. For our purposes, we assume the total tax liability includes only the normal tax and surtax under the 1918 rates and that the base for both of these taxes is taxable income.

The 1919 tax again has only one bracket and two parts: a normal tax and a surtax. The normal tax includes a tax at 1919 rates and a tax at 1918 rates. The tax at the 1919 rates, which is levied on taxable income, is 4% up to $4000 and 8% on the balance of taxable income. The normal tax at 1918 rates is levied on other income, which includes interest on tax free bonds by partnerships and personal service corporations, minus any amount of the personal exemption that exceeded the net income on which normal tax was calculated at the 1919 rates. This amount is taxed at a rate of 6% up to $4000, and any amount higher than that is taxed at 12%. The surtax includes a tax at 1919 rates and a tax at prior rates (1918, 1917, 1916, and 1913-1915). The surtax at the 1919 rates is applicable if net income

exceeds $5000, ranges from 1% to 65%, and covers 54 ranges of income. The calculation of surtax at 1918 rates is calculated in exactly the same way and uses the same tax rates and ranges of income as the 1919 surtax. The remainder of the surtax at prior year rates is found in the same way as it was for prior year rates in 1918 except that (2b) is now equal to only the net income subject to surtax at that years rates plus the amount of stock dividends that accrued in that year. The surtax rate tables for 1917, 1916, and 1913-1915 are the same as they were in 1918. The total tax liability includes the normal tax at 1919 and 1918 rates and the surtax at 1919 and prior year rates. Again, for this study, we assume that the total tax liability includes only the normal tax and surtax under the 1919 rates and that the base used for these taxes is taxable income.

The tax law does not change from 1920 to 1921, and the law continues to specify only one tax bracket and the same two parts: a normal tax and a surtax. The normal tax is the same as in 1919 and is levied on taxable income at a rate of 4% for the first $4000 and at 8% on the balance. The surtax is levied on net income (we substitute taxable income) when it is above $5000, at rates from 1% to 65%, and occurs over 54 income ranges. The total tax liability is the normal tax plus the surtax.

The tax law does not change between 1922 and 1923. There is one tax bracket, and there are two parts: a normal tax and a surtax. The normal tax remains unchanged from that in 1919. The surtax, however, changes. It is levied on net income (again, we substitute taxable income) when it is greater than $6000, at rates from 1% to 50%, and occurs over 48 income ranges. The total tax liability remains the normal tax plus the surtax.

The tax for 1924 becomes a little more complicated because a credit is added to the computation. The normal tax is levied on taxable income at a rate of: 2% for the first $4000, 4% for the second $4000, and 6% on the amount over $8000. The surtax is levied on net income (again, substitute taxable income) at rates from 1% to 40% over 40 ranges of income. The total tax liability is the normal tax plus the surtax minus the credit. The base of the credit is earned income (compensation for personal services rendered) minus the personal exemption and credit for dependents. In our calculations, however, we continue to use taxable income as the base. The credit is 25% of: 2% of the base described above, up to $4000, 4% of the next $4000, and 6% of the balance above $8000.

The 1925 tax is the same as the 1924 tax except that the amount of earned net income which is used to calculate the credit is capped at $10,000.

The tax computation for 1926 is much the same as for 1924 and 1925 there is a normal tax, surtax, and a credit. The normal tax is levied on taxable income at a rate of 1.5% for the first $4000, 3% for the next $4000, and 5% on the balance above $8000. The surtax is levied on net income (again, substitute taxable income) above $10,000, at rates from 1% to 20%, over 20 income ranges. The credit uses the same base as 1924 and 1925: earned income minus the personal exemption and credit for dependents, except in 1926 the earned net income is capped at $20,000. We, however, will continue to use taxable income as the base for the credit. The credit is now 25% of: 1.5% of the base up to $4000, 3% of the next $4000, 5% of the remainder of the base, and also 25% of the surtax. The total tax liability is the normal tax plus the surtax minus the credit.

The tax computation in 1927 is the same as that of 1926.

The tax from 1928 to 1931 is the same. It is computed the same way as the tax in 1926 except that earned net income is capped at $30,000. In 1929 what had previously been called earned net income is now, and will continue to be, called earned income.

The 1932 tax includes only a normal tax and surtax. The normal tax is levied on taxable income at a rate of 4% for the first $4000 and at a rate of 8% for the balance. The surtax is now levied on net income for tax computation which is net income minus the net loss for 1931. We, however, will substitute taxable income as the base. The surtax applies if the net income for tax computation is over $6000 and covers rates from 1% to 55% over 53 ranges of income. The total tax liability is the sum of the normal tax and the surtax.

The 1933 tax is the same as the 1932 tax except that the surtax is again levied on what is called net income instead of net income for tax computation.

The tax for 1934 and 1935 is the same and includes a normal tax and a surtax. The normal tax is levied on taxable income at a rate of 4% on the entire amount. The surtax is, essentially, also imposed on taxable income. It applies if taxable income exceeds $4000 and covers rates from 4% to 59% over 29 income ranges. The total tax liability is the normal tax plus the surtax.

The normal tax for 1936 is the same as for 1934 and 1935, but the surtax has changed. The surtax is still levied on taxable income, but, now applies if taxable income exceeds $4000 and includes rates from 4% to 75% over 32 ranges of income. The total tax liability is the sum of the normal tax and surtax.

The tax for 1937, 1938, and 1939 is the same as the 1936 tax.

In 1940, the normal tax is the same as in the years 1934 to 1939 (levied on taxable income at a rate of 4% over the entire amount). The surtax is imposed on taxable income when it exceeds $4000 at rates from 4% to 75% over 30 income ranges. The total tax liability is the sum of the normal tax and surtax amounts.

In 1941, the normal tax remains the same as it has since 1934, but the surtax has changed. The surtax is levied on taxable income and has no minimum limit (it begins at $1). It includes rates from 6% to 77% and covers 32 income ranges. The total tax liability remains the sum of the normal tax and surtax.

The 1942 tax includes a normal tax and a surtax. The normal tax is levied on taxable income at a rate of 6% over the entire amount. The surtax, also imposed on taxable income, includes rates from 13% to 82%, covers 24 income ranges, and has no minimum limit (it begins at $1). The total tax liability is still the normal tax plus the surtax.

The tax in 1943 includes a normal tax and surtax which make up the income tax, and a victory tax (for the first and only time). The normal tax is levied on taxable income at a rate of 6% over the entire amount. The surtax is levied on taxable income, has no minimum limit, includes rates from 13% to 82%, and covers 24 income ranges. The victory tax computation is as follows: (1) start with the victory tax net income which is essentially the same as the income tax net income without the deductions, (2) subtract the specific victory tax exemption: $624 if single or $1248 if husband and wife are filing together (3) this remaining amount is the income subject to the victory tax, (4) take 5% of this amount and get the victory tax before the credit, (5) to find the credit: if single, take 25% of the victory tax before the credit (plus 2% for each dependent) up to $500 (plus $100 for each dependent); if married and filing separately, take 40% of the victory tax before the credit (plus 2% for each dependent) up to $500 (plus $100 for each dependent); if married and filing jointly or filing head of a family, take 40% of the victory tax before the credit (plus 2% for each dependent) up to $1000 (plus $100 for each dependent), (6) the net victory tax is the victory tax before the credit minus the credit. The total tax liability is the sum of the normal tax, the surtax, and the net victory tax. There is a forgiveness feature in this year, but this only applies if the tax payer is liable for the payment during 1943 of the taxes due for both 1942 and 1943.

The tax in 1944 includes a normal tax and a surtax. For the first time the tax computation begins with adjusted gross income (equivalent to, but replacing total income).

From this year forward, taxable income equals adjusted gross income minus deductions and exemptions. This year the surtax and normal tax exemptions are listed separately, but they equal the same amount. The normal tax is 3% of taxable income. The surtax is levied on what is called surtax net income, but this is essentially taxable income. The surtax has no minimum limit, includes rates from 20% to 91%, and covers 24 income ranges.

In 1945, the tax is the same as it was in 1944.

The 1946 tax includes a normal tax, surtax, and a reduction. The tax rate table for this year (for the first time) combines the normal tax and surtax. This combined tax is levied on taxable income, has no minimum limit, includes rates from 20% to 91%, and covers 24 income ranges. The tentative tax is the amount of tax calculated from the tax table. This amount is then reduced by 5% to leave what is the total tax liability.

The 1947 tax is the same as the 1946 tax.

1948 is the first year that there is a distinction between the tax calculation of a single or married-filing-separate return and a joint return. The single and married filing separate filers face the same tax rates, so essentially there are two types of filers. Again, the normal tax and surtax are combined, levied on taxable income, have no minimum limit, consist of rates from 20% to 91%, and include 24 income ranges. The tentative tax for both types of returns is found using the same tax table. For a single or married filing separate return, use the tentative tax and the following steps: take 17% of the first $400 dollars of this amount, take 12% of the amount between $400 and $100,000, and take 9.75% on the balance. Subtract the sum of these amounts from the tentative tax to get the total tax liability for the single or married filing separate individual. For a joint return, use the following steps. First, find the tentative tax in the tax table using one half of the taxable income amount. Again, take 17% of the first $400 dollars of this tentative tax, take 12% of the amount between $400 and $100,000, and take 9.75% on the balance. Subtract this sum from the tentative tax and then multiply this new amount by two to get the total tax liability for the joint return. From 1948 to 1953, the Statistics of Income lists the adjusted gross income amounts used as weights in this study independently for the single and separate filers. Therefore, for the purpose of calculating our α and β we have effectively three types of filers in the years from 1948 to 1951 and four types of filers (head of household is added) in the years 1952 and 1953.

The 1949 tax is calculated in the same way, with the same rates as the 1948 tax.

In 1950, there is only one difference from the tax of 1948 and 1949. The normal tax and surtax are again: combined, levied on taxable income, have no minimum limit, consist of rates from 20% to 91%, and include 24 income ranges. The tentative tax is found using the same tax table as 1948 and 1949. The difference is in the calculation of the reduction. For a single or married filing separate return, use the tentative tax and the following steps: take 13% of the first $400 dollars of this amount, take 9% of the amount between $400 and $100,000, and take 7.3% on the balance. Subtract the sum of these amounts from the tentative tax to get the total tax liability for the single or married filing separate individual. For a joint return, use the following steps. First, find the tentative tax in the tax table using one half of the taxable income amount. Again, take 13% of the first $400 dollars of this tentative tax, take 9% of the amount between $400 and $100,000, and take 7.3% on the balance. Subtract this sum from the tentative tax and then multiply this new amount by two to get the total tax liability for the joint return.

In 1951 there is again the distinction between the single or married filing separate return and the married filing jointly return. The normal tax and surtax are combined in the same tax rate table, and this table is used for both types of returns. The combined tax has no minimum limit, is levied on taxable income, covers rates from 20.4% to 91%, and includes 24 income ranges. A single or married filing separate individual can find his total tax liability directly from the tax table. A married filing jointly individual divides his taxable income in half, calculates his tax using this amount and the tax table, then multiplies this amount of tax by two to find his total tax liability.

In 1952, and for the first time, there is the distinction between three types of filers: single or married filing separate, head of household, and married filing jointly. The tax form, also for the first time, has 2 tax rate schedules. The tax rate schedule for all taxpayers except head of household has no minimum limit, covers rates from 22.2% to 92%, and includes 24 income ranges. The tax rate schedule for head of household filers also has no minimum limit, includes rates from 22.2% to 92%, but now covers 26 income ranges. The single, married filing separate and head of household filers simply use their taxable income amount and calculate their total tax liability using the appropriate tax table. The joint filers take half of their taxable income, use this amount and the for all taxpayers except head of household tax rate schedule to find the amount of tax to be paid, and then multiply this amount of tax by 2 to find their total tax liability. Although the tax forms no longer

(from this year forward) specify that the tax being paid is the normal tax plus the surtax, there continues to be this combined tax until 1970.

The tax for 1953 is calculated in the same way, with the same rates as the 1952 tax. The only change is that the names of the tax rate schedules have slightly changed to: for all taxpayers except unmarried (or legally separated) persons qualifying as head of household and only for unmarried (or legally separated) taxpayers who qualify as head of household.

The calculation in 1954 is the same as in 1952 and 1953, and there are still three groups of filers. The single, married filing separate and head of household filers can find their tax directly from their amount of taxable income (which is actually called taxable income on the form for the first time) and their appropriate tax schedule. The joint filers must still follow the same steps of dividing taxable income in half, finding the tax on that amount, then multiplying this tax by 2 to arrive at the final tax. But, this year the tax rate schedules have changed. The for all taxpayers except unmarried (or legally separated) persons qualifying as head of household schedule has no minimum limit, applies rates from 20% to 91%, and includes 24 income ranges. The only for unmarried (or legally separated) taxpayers who qualify as head of household schedule also has no minimum limit and uses rates from 20% to 91%, but has 26 income ranges. From 1954 to 1970 the Statistics of Income combines the single and separate filers in the table that covers the adjusted gross income amounts used as weights. Therefore, in these years the effective number of different types of filers is reduced (from four in the years 1952 and 1953) to three.

The 1955 tax includes three tax rate schedules for the first time: I. (A) single taxpayers who do not qualify for rates in tables II and III, and (B) married persons filing separate returns, II. (A) married taxpayers filing joint returns, and (B) certain widows and widowers, and III. unmarried (or legally separated) taxpayers who qualify as head of household. The addition of table II does not change the end result of the tax calculation, and it remains essentially the same as it was in the previous 3 years. Table II simply replaces the steps of dividing the taxable income in half and multiplying the tax amount by two for the joint filers. This is evident because table II has the same tax rates as table I, but its income ranges have been doubled. For all filers, the tax is found using the amount of taxable income and the tax rate table. Table I has no minimum limit, rates from 20% to 91%, and 24 income ranges. Table II also has no minimum limit, rates from 20% to 91%,

and 24 income ranges. The only difference between table I and II is that the income ranges in table II are double that of table I. Table III has no minimum limit, rates from 20% to 91%, and 26 income ranges. Continuing from this year to the present the tax rate schedules have no minimum limits.

From 1956 to 1960, the 1040 form does not change its look from that in 1955, the tax computation stays the same as in 1955, and the tax rate schedules for each type of filer also do not change (same as in 1955).

Although the tax form changes its look in 1961, the computation remains the same. The three tax rate schedules also remain the same, with no change since 1955. For all filers the tax continues to be found using the amount of taxable income and the appropriate tax rate schedule.

The tax form, tax computation, and tax rate schedules remain the same from 1961 to 1963. This means that the tax rate schedules have not changed over the years: 1955-1963.

In 1964, the tax computation remains the same, but the tax rate schedules change. Table I has rates from 16% to 77% and 26 income ranges. Table II also has rates from 16% to 77% and 26 income ranges. Although there are the same number of income ranges in Table I and Table II of the tax rate schedule, the amounts in the ranges are different. This is true also for the years following. Table III has rates from 16% to 77% but has 36 income ranges. The total tax liability for each filer is found using their taxable income amount and the appropriate tax rate schedule.

The 1965 tax computation does not change from that of prior years; however, the tax rate schedules do. The tax is computed using the taxable income amount and the correct tax rate table. Table I has rates from 14% to 70% and 25 income ranges. Table II also has rates from 14% to 70% and 25 income ranges. Table III also has rates from 14% to 70% but has 33 income ranges.

The tax computation and tax rate schedules in 1966 and 1967 are the same as those in 1965.

The 1968 tax includes the normal tax and a tax surcharge. The calculation for the normal tax does not change: the taxable income amount and the appropriate tax rate schedule are used. These tax rate schedules are the same as they were in the years: 1965-1967. The tax surcharge is calculated using the amount of normal tax and the tax surcharge tables. There are 3 tax surcharge tables which correspond to the 3 normal tax

rate schedules. The surcharge tables show the amount of surcharge that is associated with each range of normal tax. Each table ends at a normal tax of $733 and specifies that if the normal tax amount is $734 or larger for any type of filer, multiply the normal tax by 7.5% to find the tax surcharge. The total tax liability is the normal tax plus the surcharge. This surcharge appears in the years from 1968 to 1970 and, interestingly, causes the tax payable amount to become non-monotonically increasing.

The 1969 tax includes the normal tax and tax surcharge. Although the instructions for the tax computation are now on schedule T that accompanies the 1040 form and not specifically listed on the 1040 form itself, the calculations have not changed. The normal tax is still calculated using the taxable income amount and the appropriate tax rate schedule. The tax rate schedules are the same as they were in years 1965-1968. The tax surcharge is found the same way as in 1968, with the normal tax amount and the surcharge tables. Again, there are 3 tax surcharge tables (each associated with a normal tax schedule) which show the surcharge amounts that correspond to different ranges of normal tax. Each table uses different increments of normal tax than its counterpart in 1968, and the tables end at a normal tax of $734. The table indicates that if the normal tax amount is $735 or larger for any type of filer, multiply the normal tax by 10% to find the tax surcharge. The total tax liability is the normal tax plus the surcharge.

The 1970 tax also has a normal tax and a tax surcharge. The tax computation instructions have moved back to the 1040 form, as they were before 1969. The normal tax is still calculated using the taxable income amount and the appropriate tax rate schedule. The tax rate schedules are the same as they were in the years from 1965 to 1969. The tax surcharge is found the same way as in 1968 and 1969, with the normal tax amount and the surcharge tables. Again, there are 3 tax surcharge tables (each associated with a normal tax schedule) which show the surcharge amount that corresponds to different ranges of normal tax. Each table uses different increments of normal tax than its counterparts in 1968 and 1969, and the tables end at a normal tax of $2019. If the normal tax amount is $2020 or larger for any type of filer, the table stipulates that the filer should multiply his normal tax by 2.5% to find the tax surcharge. The total tax liability is the normal tax plus the tax surcharge.

The surcharge is eliminated in 1971, so the total tax liability includes only the normal tax. Also in 1971, a fourth and final tax rate schedule is explicitly stated for

the first time. The schedules for the married taxpayers who file separately and the single taxpayers (which had previously been combined into one schedule) are now individually listed. Now, the schedules include: Schedule X single taxpayers not qualifying for rates in schedule Y or Z, Schedule Y married taxpayers and certain widows and widowers, but this schedule has two tables: (1) married taxpayers filing joint returns and certain widows and widowers and (2) married taxpayers filing separate returns, and Schedule Z unmarried (or legally separated) taxpayers who qualify as heads of household. Although the tax rates range from 14% to 70% in each schedule, the rates rise at different increments. Schedule X and both parts of schedule Y include 25 income ranges, and schedule Z has 33 income ranges. From 1971 to the present, the tax computation does not change, but the tax rate schedules do change. Using the appropriate tax rate schedule and taxable income, the filer is able to find his total tax liability. In this year and all years following, the single and separate filers are listed independently in the Statistics of Income tables that include the adjusted gross income amounts used as weights. The effective number of different filers is, therefore, again four.

The tax computation and tax rate schedules from 1972 to 1975 are the same as those in 1971.

The only change in 1975 is the introduction of an extra tax credit. An additional personal exemption credit equal to the product of $30 and the number of exemptions (not including those for age of blindness) taken by the tax payer is subtracted from the tax payable to arrive at the final tax liability. This credit is not used in the calculations of this study.

In 1976, the tax computation does not change, but the tax rate schedules do change. However, this change is in appearance only as the rates themselves do not change. In this year, each tax rate schedule begins at a taxable income amount of $20,000. As a result, the tax rates on income below this amount must be obtained somewhere other than the Tax Rate Schedule. These rates that apply to income below $20,000 can be found in the Tax Table which is part of the Instructions for the 1040 form. Alternatively, these rates may also be found in Table 3.12 in the 1976 Statistics of Income publication. Schedule X includes rates from 38% to 70% and covers 12 income ranges. The part of schedule Y that includes married taxpayers filing joint returns and qualifying widows and widowers includes rates from 32% to 70% and contains 17 income ranges. The other part of schedule

Y, which includes married taxpayers filing separately, covers rates from 48% to 70% over 12 income ranges. Schedule Z includes rates from 35% to 70% over 22 income ranges. Also, the tax credit that was added in 1975 was expanded for 1976 (and continued unchanged until 1978) and renamed the general tax credit. This credit is now equal to the larger of (1) the personal exemption credit which was continued from 1975 and increased to $35 per exemption or (2) the new taxable income credit which is equal to 2% of taxable income up to $180 ($90 for married filing separately). Again, this credit is not used in the calculations of this study. The total tax liability for this year is the tax payable found with the taxable income amount and the tax rate schedules minus the tax credit.

The Tax Reduction and Simplification Act of 1977 resulted in a change of the tax law which affects the computations in this study. This act introduced the zero bracket amount which replaced the standard deduction. This substitution occurred only during the years from 1977 to 1986. The zero bracket amount is a flat amount determined by marital status. This amount was incorporated into the tax rate schedules as that lowest amount of income, taxed at a rate of 0%. This changed the tax computation because it altered the calculation of taxable income. By eliminating the step of reducing adjusted gross income by the deduction amount, it made taxable income equal to adjusted gross income minus exemptions only. Although the calculation is different, taxable income is still essentially equal to adjusted gross income minus deductions and exemptions because the amount of deductions to be subtracted is included in the tax rate schedules. Therefore, it is as if there is no difference between this group of years and the other years. However, adjustments to the calculation of breakpoints were necessary to make the data from this group of years (1977-1986) comparable to the other years in the study. To essentially reverse the effects of the zero deduction amount, each breakpoint in each schedule was reduced by the amount of the zero deduction. These values, which are different for each schedule, could be found in the instructions for the 1040 form. Not only was each breakpoint lowered, but this change also brought the beginning values of each initial bracket back to zero. Due to these changes in breakpoints, adjustments to the calculation of the weights must also be made. The calculation remains the same, as detailed above, with one exception. For the years: 1977, 1979, and 1983-1986 the adjusted gross income amounts listed in the Statistics of Income for the first two brackets of each schedule must be combined to form the weights for the new, initial breakpoints. Although this breakpoint correction is also made in the years:

1978 and 1980-1982, no changes to the weights are necessary as the numbers of returns in each schedule are used as the weights and do not vary across brackets.

Additionally in 1977, the Tax Computation Schedule was introduced (and continued until 1980). This simply moved the calculations for the taxable income amount and for the general tax credit to Schedule TC of the 1040 form instead of being on the actual 1040 form itself. Taxable income is computed as detailed above, and the general tax credit is similar to that of 1976 except for 3 changes: (1) the number of exemptions used in the personal exemption credit now includes the exemptions for age and blindness, (2) when calculating the taxable income credit, the amount of taxable income must be reduced by the appropriate zero bracket amount before calculating the 2% credit, and (3) married persons filing separately were permitted to use only the personal exemption method of calculating this credit. This tax credit is, again, not used in the computations of this study.

In this year, schedule X includes rates from 14% to 70% and covers 26 income ranges. The part of schedule Y that includes married taxpayers filing joint returns and qualifying widows and widowers includes rates from 14% to 70% and contains 26 income ranges. The other part of schedule Y, which includes married taxpayers filing separately, covers rates from 14% to 70% over 26 income ranges. Schedule Z includes rates from 14% to 70% over 34 income ranges. Although the rates and income ranges seem to have the similar amounts and limits from this description, they are different across schedules. This will hold true in the following years also.

The tax computation and tax rate schedules for 1978 are the same as those in 1977.

The tax computation for 1979 is the same as that of 1977 and 1978 except that the general tax credit was discontinued. Schedule X includes rates from 14% to 70% and covers 17 income ranges. The part of schedule Y that includes married taxpayers filing joint returns and qualifying widows and widowers includes rates from 14% to 70% and contains 16 income ranges. The other part of schedule Y, which includes married taxpayers filing separately, covers rates from 14% to 70% over 16 income ranges. Schedule Z includes rates from 14% to 70% over 16 income ranges.

The tax computation and tax rate schedules for 1980 are the same as those in 1979.

The tax computation and tax rate schedules for 1981 are the same as those in

1979 and 1980, with one exception. The change is that Schedule TC is no longer used: the taxable income computation is returned to the actual 1040 form.

In 1982, the tax computation remains the same as in 1981, but the tax rate schedules change. Schedule X includes rates from 12% to 50% and covers 14 income ranges. The part of schedule Y that includes married taxpayers filing joint returns and qualifying widows and widowers includes rates from 12% to 50% and contains 13 income ranges. The other part of schedule Y, which includes married taxpayers filing separately, covers rates from 12% to 50% over 13 income ranges. Schedule Z includes rates from 12% to 50% over 13 income ranges.

In 1983, the tax computation remains the same as in 1981 and 1982, but again, the tax rate schedules change. Schedule X includes rates from 11% to 50% and covers 14 income ranges. The part of schedule Y that includes married taxpayers filing joint returns and qualifying widows and widowers includes rates from 11% to 50% and contains 14 income ranges. The other part of schedule Y, which includes married taxpayers filing separately, covers rates from 11% to 50% over 14 income ranges. Schedule Z includes rates from 11% to 50% over 14 income ranges. Even though the schedules appear the same because they have the same number of income ranges and the same minimum and maximum tax rates, they are each different (which continues to be true when the schedules appear the same in the following years).

The tax computation in 1984 does not change from that in the years from 1981 to 1983, but the tax rate schedules this year do change. Schedule X includes rates from 11% to 50% and covers 16 income ranges. Both parts of schedule Y and schedule Z all contain rates from 11% to 50% over 15 income ranges.

The tax computation in 1985 is the same as the computation from 1981 to 1984, but the tax rate schedules do change. Schedule X includes rates from 11% to 50% and covers 16 income ranges. Both parts of schedule Y and schedule Z all contain rates from 11% to 50% over 15 income ranges. Again, the schedules seem to be unchanged from the previous year given the number of income ranges and lowest and highest tax rates; however, they are different.

Again, the tax computation in 1986 does not change, but the tax rate schedules do. Schedule X contains rates from 11% to 50% over 16 income ranges. Both parts of schedule Y and schedule Z include rates from 11% to 50% and 15 income ranges. Again,

the schedules appear to be the same as previous years given the number of income ranges and lowest and highest tax rates; however, they are different.

The Tax Reform Act of 1986 produced many changes that affected the tax law in 1987. The changes that have the most significance to this study include changes in: tax rates, exemptions, and deductions. This year there is a reversal that brings back the standard deduction, and consequently, eliminates the zero bracket amount. Therefore, the calculation for taxable income returns to the original (as in years before 1977): adjusted gross income minus deductions and exemptions. The tax rate schedules are also considerably different. This ends the period of years (1977-1986) which have a 0% tax rate on the lowest taxable income range in each schedule; the rates themselves are lowered; and the number of brackets within each schedule is reduced. Each schedule includes 5 income ranges and includes tax rates from 11% to 38.5%. Although the schedules include the same tax rates and number of income ranges, the amounts in the ranges of income are different which makes the amounts of tax payable also different.

The tax computation for 1988 begins the same as the computation for 1987: taxable income equals adjusted gross income minus deductions and exemptions. This taxable income amount is then used with the tax rate schedules to find the tax payable. The difference occurs in how this year handles the computation for the tax payable on the highest income range for each schedule. In the years 1988 through 1990 there is a worksheet which is part of the tax rate schedules for this calculation. The worksheet computes the tax payable on the highest increment of taxable income by taking the sum of: 28% of the amount of income not already taxed and the smaller of: 5% of the amount of income not already taxed or the product of $546 and the number of exemptions claimed. Because the number of exemptions is specific to each taxpayer and this study is interested in the representative taxpayer, we use the 5% option. This leaves the final tax bracket with a rate of 33%, which is the same as the previous bracket. In effect, this combines the last 2 tax brackets, and each schedule shrinks by 1 bracket. Each schedule contains rates from 15% to 33% and 3 income ranges. Although the schedules would appear to be the same given this information, the amount of tax payable in each schedule is different because the amounts in the income ranges differ.

The 1989 and 1990 tax computations do not change from that of 1988: taxable income is calculated in the same way and the worksheet follows the same steps. In both

years, each schedule, again, has tax rates which range from 15% to 33% over 3 income levels. The income levels taxed at each rate, however, are different for all three years.

From 1991 to 2005, the tax computations remain the same as taxable income is calculated by subtracting deductions and exemptions from adjusted gross income. The worksheet is no longer used, and the only things changing over these years are the tax rates and income ranges. In 1991, each schedule has 3 income ranges taxed at rates from 15% to 31%. As before, the income levels change across schedules while the tax rates do not.

In 1992, all 4 schedules include tax rates from 15% to 31% over 3 income ranges. The income levels will continue to change across schedules and are also different between years.

In 1993 the number of income ranges being taxed begins to increase. This year, there are 5 ranges in each schedule with tax rates from 15% to 39.6

Again, from 1994 to 2000, each schedule has 5 income ranges taxed at rates from 15% to 39.6%. Although the rates are the same in these years, and in 1993, the ranges of income being taxed at these rates differ.

In 2001, there are 5 income ranges being taxed in each schedule, but the tax rates that apply to each change from those in the previous years. This year, the rates vary from 15% to 39.1%.

For the years 2002 through 2005, the number of income ranges increases to 6, but the income levels included in these ranges vary between years. In 2002, the tax rates extend from 10% to 38.6%. The tax rates from 2003 to 2005 continue the downward trend and range from 10% to 35%.

Although a bit tedious to construct, the time series just detailed offers countless possibilities for new experiments. In the next chapter we first look at possible explanatory variables for these parameters.

Chapter 2

Large Analysis of the Tax Parameter Series

2.1 Introduction

The previous chapter explained the construction of the time series for the five parameters of the federal personal income tax function. This chapter discusses (1) the characteristics of the series, (2) the historical context of major changes in the series, (3) the relation of these series to some other tax series in the literature, (4) the relation of the intertemporal variation in the tax parameters to the sharp reduction in volatility of macroeconomic time series after about 1950, and (5) the interrelation of the tax parameters other federal fiscal variables.

2.2 Characteristics of the Tax Parameter Series

Recall that the tax function is

$$\begin{aligned} T_t &= \alpha_t \left[\phi_t \left(1-\theta_t\right) Y_t - \varepsilon_t N_t\right]^{\beta_t} \\ &= \alpha_t \left[\tilde{Y}_t\right]^{\beta_t} \end{aligned} \tag{2.1}$$

where T is the tax paid by a taxpayer, Y_t is gross domestic product per person, ϕ is the fraction of gross domestic product subject to the personal income tax, ϕY is adjusted gross

income per person, θ is the fraction of adjusted gross income taken in deductions, ε is the dollar value of each exemption, N is the average number of exemptions claimed by each taxpayer, and $\tilde{Y}$ is taxable income. The tax parameter ε differs from the others in that it is expressed in dollars and so will be rendered non-stationary by economic growth. It therefore is convenient for the subsequent analysis to write the exemption amount ε relative to income: $e = (\varepsilon/Y)$, which is stationary. The tax function then is

$$T_t = \alpha_t \left[\phi_t \left(1 - \theta_t\right) Y_t - e_t N_t Y_t\right]^{\beta_t} \tag{2.2}$$

The tax function thus has five parameters: α, β, ϕ, θ, and e.

The marginal tax rate can be defined with respect to gross income Y or taxable income $\tilde{Y}$. Each has its uses. The usual concept is with respect to gross income:

$$\begin{aligned} MTR_t|_Y = \frac{\partial T_t}{\partial Y_t} &= \beta_t \phi_t \left(1 - \theta_t\right) \alpha_t \left[\phi_t \left(1 - \theta_t\right) Y_t - \varepsilon_t N_{tt}\right]^{\beta_t - 1} \\ &= \beta_t \phi_t \left(1 - \theta_t\right) \alpha_t \left[\phi_t \left(1 - \theta_t\right) Y_t - e_t N_t Y_t\right]^{\beta_t - 1} \end{aligned} \tag{2.3}$$

This concept often would be the one to use in a general equilibrium macro model, in which the gross level of income would be GDP, an endogenous variable determined by the equilibrating mechanism of the economy, and the variable that one would be interested in studying. The other concept for the marginal tax rate is with respect to taxable income:

$$MTR_t|_{\tilde{Y}} = \frac{\partial T_t}{\partial \tilde{Y}_t} = \beta_t \alpha_t \left[\tilde{Y}_t\right]^{\beta_t - 1}$$

This concept is especially useful in defining the progressivity of the tax law. In particular, the parameter β is the elasticity of tax paid with respect to taxable income:

$$\begin{aligned} \beta_t &= \frac{\partial T_t}{\partial \tilde{Y}_t} \frac{\tilde{Y}_t}{T_t} \\ &= \left(MTR_t|_{\tilde{Y}}\right) \left(\frac{\tilde{Y}_t}{T_t}\right) \end{aligned}$$

so that the single parameter β captures the curvature of the tax function. The tax is progressive, linear, or regressive as β is greater than, equal to, or less than 1.

Figure 2.1 presents the time plots of the five parameters along with the HP filtered series for each. There is both low frequency and high frequency variation in the series, discussed in more detail in the next section. Here we give a brief overview of the meaning of the parameters and the implications of their values as shown in the Figures.

The value of β always has exceeded 1, which is unsurprising because the income tax was intended from the start to be progressive. The degree of progressivity varies dramatically over the sample with a maximum of 1.8253 in 1929 and a minimum of 1.2723 in 1970. For the period 1941-2000, the value of β fluctuated narrowly around a value of about 1.3. It rose considerably in 2002 to 1.4345, the first time since 1940 that it was above 1.3555. One might have expected the high degree of progressivity in the 1920s to have serious adverse incentive effects. However, incentive effects work through the marginal tax rate, and in the 1920s the high value of β was counterbalanced by low values of α and ϕ, with the result that the average marginal tax rate was actually quite low in the 1920s (see the discussion of average marginal tax rates below for details).

The parameter α is somewhat like a proportional tax rate. If β were 1, then α indeed would be the proportional tax rate on taxable income, $[\varphi(1-\theta)-eN]Y$. Notice that α is important in determining the amount of distortion introduced by progressivity. Distortion depends on the marginal tax rate MTR. The response of MTR to a change in β, taking income as given, is

$$\begin{aligned}\frac{\partial MTR}{\partial \beta} &= \phi(1-\theta)\alpha\left[\phi(1-\theta)Y-eNY\right]^{\beta-1} \\ &\quad +\beta\phi(1-\theta)\alpha\left[\phi(1-\theta)Y-eNY\right]^{\beta-1}\ln\left[\alpha\phi(1-\theta)Y-eNY\right] \\ &= {}^{\beta-1}\Big\{\phi(1-\theta)\left[\phi(1-\theta)-eN\right]^{\beta-1} \\ &\quad +\beta\phi(1-\theta)\left[\phi(1-\theta)-eN\right]^{\beta-1}\ln\left[\alpha\phi(1-\theta)Y-eNY\right]\Big\}\end{aligned}$$

The distortion is low if the value of α is low even if the value of β is high. That is precisely the situation that prevailed in the early years of the income tax. The value of β reached a peak value of 1.8253 in 1929, but the value of α was tiny at 0.000012. The resulting distortion was small. The macroeconomic effect of the distortion caused by β is examined in detail in Chapter 3.

The parameters φ and θ affect the tax function the same way, though in opposite directions. Taxable income, $\tilde{Y_T} = [\phi\,(1-\theta) - eN]\,Y$, rises with ϕ and $1-\theta$. The effect of ϕ and θ on tax collected is similar to that of α. If β were 1 (proportional tax) and e were 0 (no exemptions), then the effects of the three parameters would be exactly the same. In the general case of $\beta > 1$ and $e > 0$, the effects of ϕ and θ still are similar to that of α, and changes in one parameter can reinforce or offset changes in another in a straightforward way. That sort of interaction figures prominently in the history of the tax function, as we see in the next section.

Because exemptions are inside the nonlinear part of the tax function, there is no lump-sum element to the income tax. In particular, a change in the exemption value, e, alters the marginal tax rate as well as the average tax rate. This point bears on some previous discussions in the literature, which often discuss the income tax as if it were proportional (i.e., $\beta = 1$):

$$T = \alpha Y - \varepsilon N \tag{2.4}$$

In this form, ε has no effect on the marginal tax rate, which here is simply α. Seater (1993) uses this form in discussing Ricardian Equivalence. If the government reduces current taxes by issuing bonds, Ricardian results can emerge if the tax reduction arises through ε rather than α. In that case, the refinancing scheme only changes the timing of lump-sum tax collections with no changes in either the present value of taxes or the timing of incentive effects. As a result, Ricardian Equivalence holds. If, however, the proportional tax function (2.4) is replaced by the progressive income tax functions (2.1), then the refinancing scheme alters the time path of marginal tax rates even if the only tax parameter to change is ε. Using debt to shift taxes intertemporally also shifts the accompanying distortions, implying that Ricardian Equivalence cannot hold. Given that the tax actually is and always has been progressive, it is an interesting question why Ricardian Equivalence generally is not rejected by statistical tests. Similarly, Romer and Romer (2007) make the following remark in their description of the Tax Reform Act of 1969:

> Like the revenue effects, the nature of the tax change was complicated. All of the changes were legislated to be permanent. The reduction and end of the tax surcharge was a reduction in marginal tax rates. *However, the increase in the personal exemption was a reduction in average tax rates.* (p.43, emphasis added.)

The last sentence is correct for a proportional tax but not a progressive tax such as the federal income tax, for which exemptions are inside the nonlinear part of the tax function.

2.3 Salient Events in the History of the Income Tax

The low frequency variation in the tax parameters is easily summarized. The low frequency variations in the five parameters show little obvious correlation, with major changes in the five parameters mostly coming at different times. The parameter α starts very low, remains low for 30 years, rises abruptly in 1941, stays high for another 40 years, and then declines over the remaining 20 years of the sample. The parameter β starts at a moderate level, rises steeply for 20 years, then falls just as steeply over the next 10 years, remains at a low level for 60 years, and rises to a moderate level at the very end of the sample. The parameter ϕ starts at a low level, rises for 10 years, falls somewhat over the next 15 years, rises abruptly in 1941, and remains high with little trend over the rest of the sample. The parameter θ starts high, immediately drops to a low value, stays low for 20 years, rises steeply in the 1940s, and remains high over the rest of the sample. The parameter e starts extremely high (nearly 10 times higher than average income per person), immediately drops precipitously, rises slowly over the 20 years from 1920 to 1940, drops quickly to a very low value by 1945, then continues to drop very slowly over the rest of the sample, nearly fading out completely by the end of the sample period. The entry of the United States into the Second World War is the only time when all five parameters do something notable simultaneously, when four of the five (α, ϕ, θ, and e) either rise or fall sharply and the fifth (β) abruptly stops falling.

The high frequency variation in the parameters' time paths is best discussed in historical context. As is clear from Romer and Romer's (2007) excellent history of federal taxes after 1945, changes in taxes occur continually. We discuss here the episodes in the history of the income tax in which the tax parameters were changed by large amounts. There are eight major episodes.

2.3.1 World War I

The federal income tax started in 1913 after passage of the Sixteenth Amendment to the United States Constitution that same year. Initially, the tax had a tiny value of α (0.000063), a moderate value of β (1.5148), and an enormous value of e (8.92). We do not have data on the values of ϕ or θ until 1916. In 1916, the value of α had risen to 0.0005, substantially higher than its 1913 value but still an order of magnitude below its sample-period median of 0.0094 and far below its maximum of 0.0192 in 1943. The value of β had fallen a good bit to 1.3782, and e had fallen to a still large value of 7.38. The values of ϕ and θ in 1916 were 0.1683 and 0.2457, respectively. This combination of values produced a tax that was very progressive but that applied to hardly anyone. Especially notable is the huge value of e, the ratio of the value of an exemption to income per person. The value indicates that an exemption was worth over seven times as much as the average income per person. This parameter alone guaranteed that only people with very high incomes would pay any tax, which the history of the Sixteenth Amendment suggests was the intention of many of the amendment's supporters. See Jensen (2002) and the references cited therein.

The United States entered the First World War in early April of 1917. Four of the five tax parameters (β, ϕ, θ, e) change abruptly in 1917. The remaining parameter (α) changes abruptly in 1918. A series of armistices were signed in the fall of 1918, and all combat stopped by mid-November. However, the end of the war did not immediately bring substantial reversals of the tax parameters' values, except for α which fell to pre-war levels. This pattern of war's end bringing a reduction in α but little or no change in the other tax parameters appears after the Second World War and the Korean War as well.

2.3.2 Mellon Tax Cuts

Treasury Secretary Andrew W. Mellon wanted to reduce the huge federal debt accumulated during World War I. Mellon was an early supply-side economist, believing that high tax rates reduced revenue collected. He, therefore, proposed a plan that reduced the highest income tax rate, 73%, to 24% and the lowest rate from 4% to 1/2%. Mellon wanted the tax to remain progressive. Congress enacted Mellon's proposed tax cuts in stages with the Revenue Acts of 1921, 1924, and 1926. The top marginal rate dropped

from 73% to 58% in 1922, to 50% in 1923, to 46% in 1924, to 25% in 1925, and to 24% in 1929. Commensurate with Mellon's plan of reducing the tax, both α and ϕ fell, and θ and e rose. In fact, in 1929 α reached its lowest level in the history of the income tax, 0.000012. However, in that same year, the degree of progressivity rose to its highest level in the history of the tax, as β reached a value of 1.8253. So, although tax rates were reduced, progressivity was increased. This pattern of trading off overall tax rates against progressivity generally continues throughout the history of the income tax.

2.3.3 Great Depression and New Deal

The Great Crash hit the stock market the same year that the last of the Mellon tax cuts took effect, heralding the beginning of the Great Depression. Both α and ϕ rose through the 1930s, and θ fell. Progressivity dropped during the 1930s even more than it had risen in the 1920s. The exemption parameter, e, remained about constant through the period, first rising and then falling back to where it started the decade.

2.3.4 World War II

The United States entered the Second World War at the end of 1941. That year, α rose enormously from 0.0014 in 1940 to 0.0126 in 1941, ϕ rose substantially from 0.3996 to 0.5060, θ dropped somewhat from 0.1115 to 0.0916, and e dropped considerably from 1.9830 to 1.2940. All of those changes raised taxes substantially. At the same time, progressivity dropped, with β falling from 1.4672 in 1940 to 1.3056 in 1941, a change that reduced taxes. The net effect was to raise taxes a great deal. The war ended in 1945. The next year, α dropped from 0.0154 to 0.0104. The other tax parameters, however, hardly changed.

2.3.5 Korean War

The United States entered the Korean War in the summer of 1950. Taxes did not change significantly until the next year, when α jumped from 0.0099 to 0.0125. The other tax parameters hardly changed. The armistice was signed in the summer of 1953. The following year α dropped from 0.0160 to 0.0107. Once again, the other tax parameters changed little.

2.3.6 Kennedy/Johnson Tax Cuts

The Revenue Act of 1964 reduced tax rates for the explicit purpose of raising the growth rate of output. The parameter α rose from 0.0105 in 1963 to 0.0190 in 1964 and then dropped back somewhat to 0.0166 in 1965, where it approximately remained for some time. The net increase in α raised tax rates, of course, but that effect was more than offset by a reduction in β, which fell from 1.3400 in 1963 to 1.2744 in 1964, where it remained for many years.

2.3.7 Nixon/Ford/Carter Tax Changes

A series of back-and-forth changes in the tax law over the years 1974-1980 led to a net reduction of α from 0.0171 in 1973 to 0.0119, where it remained for several years. Other tax parameters changed little in this period.

2.3.8 Reagan Tax Cuts

President Reagan tinkered with the tax law early in his administration, but in 1986 Congress adopted his Tax Reform Act, which phased in changes in tax rates. By 1988, α had dropped by nearly one-half from 0.0115 in 1985 to 0.0061. At the same time, there was a partial offset in ϕ, which rose from its 1985 value of 0.5464 to 0.6041 in 1988. The net effect, however, was a reduction in tax rates. Progressivity increased, with β rising from 1.2829 in 1985 to 1.3241 in 1988. Again we see the typical opposing movements in overall tax rates and progressivity.

2.3.9 Clinton Tax Increases

The tax on income increased over the course of the Clinton administration, virtually entirely through an increase in ϕ, which rose from 0.5593 in 1993 to 0.6484 in 2000.

2.3.10 Bush Tax Cuts

The income tax changed twice in President George Bush's first term. Congress passed two acts, the Economic Growth and Tax Relief Reconciliation Act of 2001 and then

the Economic Growth and Tax Relief Reconciliation Act of 2003. The second act partly undid the effects of the first act. The net effect of the two acts was to reduce α from its value of 0.0069 in 2000 to 0.0017 in 2005, reduce ϕ from 0.0069 in 2000 to 0.5970 in 2005, raise θ from 0.2032 to 0.2294, leave e approximately unchanged, and raise β from 1.3035 to 1.4074. Yet again we have an example of progressivity moving in the opposite direction as tax rates themselves.

2.3.11 A Comment

An interesting aspect of the history of the income tax is that events with the same name can have very different characteristics. In particular, many changes in the tax law have been called "tax cuts." Though they all resulted in a lower average tax rate and a lower average marginal tax rate, they had very different effects on progressivity. The Mellon, Reagan, and Bush tax cuts all raised progressivity, whereas the Kennedy/Johnson tax cuts reduced progressivity. It also is interesting that the large increases and decreases in progressivity occurred during Republican and Democrat administrations, respectively, which is the opposite of what one would expect from both parties' rhetoric regarding redistribution of income.

2.4 Parameter Volatility

Figure 2.2 plots the first differences of the five parameters. Four of the five parameters (β, ϕ, θ, and e) show a marked decrease in volatility after about 1950. The fifth parameter, α, shows an increase in volatility around 1940 followed by a decrease after about 1980, with the volatility after 1980 being substantially more than before 1940. These volatility patterns may have some bearing on the post-war stabilization of most macroeconomic time series that has been discussed in the literature. Figure 2.3(a) plots the growth rate of real GDP over our sample period. Obviously, volatility is much lower after 1950 than before. Much of the volatility before 1950 arose from the two World Wars, but volatility during the inter-war period also is much higher than after 1950, implying that real GDP's volatility was higher before 1950 even apart from the effects of the wars. The higher volatility of four of the five tax parameters before 1950 than after is consistent

with the hypothesis that GDP calmed down after 1950 because tax policy became more stable. It is interesting that real federal government purchases also show a marked decline in volatility after 1950 (see Figure 2.3(b)), suggesting that a less erratic fiscal policy may explain the drop in volatility of output. Notice that the volatilities of the four parameters and of purchases are not consistent with the hypothesis that the macroeconomy stabilized because of improved countercyclical policy. That hypothesis implies that the volatility of the fiscal variables would have increased after 1950 as those variables were used to counteract shocks to the economy, in the same way that a constant interior house temperature requires the heating/cooling unit to work harder when the outside temperature fluctuates. The volatility of the remaining parameter α runs counter to the first hypothesis that policy caused stabilization by itself, becoming less erratic because α has much higher volatility after 1940 than before. All the data are consistent with a third hypothesis, that policy is endogenous but not countercyclical. A convincing evaluation of these various hypotheses requires a detailed study with statistical and simulation methods. This dissertation does not take up that issue, deferring it to future study.

2.5 Relation to Other Tax Series

The five parameters of the tax function capture the income tax law extremely accurately, as shown in the previous chapter. They therefore can be used to study other measures of taxes. Here, we examine three such measures: the average marginal income tax rate; the maximum marginal tax rate; and the Romer and Romer (2007) series based on their narrative history.

2.5.1 Average Marginal Tax Rate

Economic theory has long recognized the central role of the marginal tax rate in distorting economic decisions. With a progressive tax, there is no single marginal tax rate but rather one for each level of income. Consequently, for aggregate analysis it is necessary to have some sort of marginal tax rate index. The most popular construct is an average marginal tax rate, which is a weighted average of individual marginal tax rates. Akhand and Liu (2002) have shown that the most accurate construct is the series

constructed by Seater (1982, 1985) and extended by Stephenson (1998) that measures the average marginal tax rate on personal income. We have updated the Seater-Stephenson series to 2005. For purposes of comparison, we can also compute a marginal tax rate on average GDP using equation (2.3). Figure 2.4(a) shows both the computed series and the updated Seater-Stephenson series. The Seater-Stephenson series is an income-weighted average of different marginal rates on personal income, whereas our computed series is the marginal rate computed at the average GDP per person. The two are different because of the non-linear nature of the tax and because the type of income involved is different. Nonetheless, a comparison is useful to see how closely the two series are related. Our computed series is higher than the Seater-Stephenson series immediately after 1940 and then shows a downward trend after 1940, whereas the Seater-Stephenson series shows no trend. Otherwise, the movements of the two series are very similar. The correlation between them is 0.90. If we regress our computed series on the Seater-Stephenson series, we obtain

$$\begin{aligned} AMTRComp_t = \quad & -0.0072 + \quad 1.1333 AMTRPI_t \\ & (0.0029) \qquad (0.0701) \end{aligned}$$

$$\bar{R}^2 = 0.8069$$

The constant is statistically significantly greater than zero, but the magnitude is not economically significant. The average value of $AMTRComp$ is 0.1074, implying that an addition of one percentage point (i.e., 0.01) to $AMTRPI$ would raise $AMTRComp$ by (0.007)(0.01)=0.00007, which is negligible. The slope coefficient is not significantly different from 1 at the 5% level. These results show that there is no significant bias in the relation between the two series. The two series seem to contain much the same information.

As noted above in the discussion of the major events in the income tax's history, progressivity typically moves in the opposite direction as the average tax rate and the average marginal tax rate. Table 2.1 confirms that impression, reporting the correlation matrix for the five tax parameters and also for $AMTRPI$. The progressivity parameter β is strongly negatively correlated with $AMTRPI$. It also is negatively correlated with both α and ϕ, which play the major role over the sample period in determining both the average and marginal tax rates.

Table 2.1: Tax Parameters Correlation Matrix

	α	β	ϕ	θ	e	$AMTRPI$
α	1	-0.772224	0.753642	0.463398	-0.617093	0.827776
β	-0.772224	1	-0.821703	-0.557569	0.624610	-0.865657
ϕ	0.753642	-0.821703	1	0.671277	-0.858967	0.959222
θ	0.463398	-0.557569	0.671277	1	-0.454778	0.633277
e	-0.617093	0.624610	-0.858967	-0.454778	1	-0.343448
$AMTRPI$	0.827776	-0.865657	0.959222	0.633277	-0.343448	1

We explore the relation between the tax function parameters and the observed average marginal tax rate by regressing $AMTRPI$ on the five tax parameters. The tax function analogous to equation (2.2) for personal income would have the same values of α and β as the GDP tax function but different values for ϕ, θ, and e because those were constructed specifically for GDP. Nonetheless, the analogous parameters applying to personal income should be closely related to those for GDP, so the regression should be informative. The results are shown in Table 2.2. In the regression with all five parameters included, only α, β, and ϕ are individually significant. A joint test cannot reject the joint hypothesis that the coefficients of θ and e are zero. When those are excluded, the constant also becomes insignificant. A joint test also fails to reject the hypothesis that the constant and the coefficients of θ and e are zero. A regression retaining only α, β, and ϕ has both a higher adjusted R^2 and a lower Schwarz-Bayes criterion value indicating that the excluded parameters add more noise than information. However, the coefficients for the three retained tax parameters are roughly the same in the two regressions.

As one would expect, AMTRPI depends positively on both α and ϕ. The surprise is that AMTRPI depends negatively on β. Equation (2.3) unambiguously predicts a positive relation, *given* Y, so the negative relation emerging in the regressions reflects one of two things: a sufficiently strong negative effect on income, enough to reduce the marginal tax rate or a choice by policy makers to reduce progressivity when tax rates are high. The latter possibility is consistent with Lindert's (2003, 2004) suggestion that governments recognize the disincentive effects of fiscal policies and offset changes in one policy's incentive effects with countervailing effects in some other policy. Indeed, Lindert's suggestion may explain the observation in the preceding section concerning progressivity and the two major

Table 2.2: Regression of AMTRPI on the Five Tax Parameters

Variable	Coefficient	
Const.	0.057411	-
	(0.029702)	
α	1.636481	1.776045
	(0.542703)	(0.534611)
β	-0.060908	-0.035611
	(0.016528)	(0.003116)
ϕ	0.231868	0.272495
	(0.027255)	(0.013312)
θ	0.008104	-
	(0.064642)	
e	-0.007044	-
	(0.005656)	
$\bar{\mathrm{R}}^2$	0.949680	0.949735
SBC	-5.742827	-5.858836

political parties. Most increases in "social spending," and transfer programs in particular, have occurred during Democrat administrations. Those types of spending programs are redistributive by design. Perhaps those advocating them have recognized the need to offset the incentive effects of the spending programs with less redistributive taxation.

2.5.2 Maximum Marginal Tax Rates

Sometimes the burden of the income tax is discussed by referring to the highest marginal tax rate in the law. In principle, that is not a particularly useful standard because the highest rate may apply to very little income. For the same reason, the maximal marginal rate may not be closely related to the degree of progressivity. For example, a flat tax with a tax rate of 90% is not progressive by construction. We analyze the relation between the maximal marginal tax rate and the five parameters of the tax function by means of a straightforward linear regression of the former on the latter. We also regress the difference between the highest and lowest marginal tax rates on the five tax function parameters.

Figure 2.4(b) shows the maximal marginal rate, the minimal marginal rate, and the difference between them. The maximal rate and the difference between the maximal and minimal rates move fairly closely together, and they generally seem to reflect the movement in the parameter α. The regression results, reported in Table 2.3, confirm the visual impression. The only tax parameter with significant explanatory power for the maximal marginal rate is α. The other four parameters are individually and jointly insignificant. Similar results hold for the difference between the maximal and minimal marginal tax rates, except that ϕ is marginally significant. In particular, β has no explanatory power in either regression. These results strongly suggest that there is little relation between progressivity on the one hand and either the maximal marginal tax rate or the difference between the maximal and minimal rates on the other.

2.5.3 Romer and Romer Narrative Tax Variable

Romer and Romer (2007) construct a measure of exogenous tax changes for the period 1947-2005. Their method of construction is first to read the historical record to identify changes in any tax law, then identify those that are exogenous, and finally to assign

Table 2.3: Maximal Marginal Tax Rate Regressions

	Dependent Variable	
	Maximal Tax Rate	Maximal less Minimal
Const.	1.390169	1.377439
	(0.888722)	(0.825235)
α	23.84624	21.94376
	(7.641295)	(7.107003)
β	-0.341278	-0.292143
	(0.471299)	(0.445506)
ϕ	-0.662545	-0.967980
	(0.587851)	(0.517763)
θ	-0.687465	-0.719950
	(0.733882)	(0.694781)
e	-0.021059	-0.024971
	(0.056560)	(0.052089)
$\bar{\mathrm{R}}^2$	0.280233	0.238318

Table 2.4: Correlation between Romer and Romer measure and tax parameter changes

$\Delta\alpha$	$\Delta\beta$	$\Delta\phi$	$\Delta\theta$	Δe
0.048721	0.066582	0.172449	-0.326640	-0.285102

a magnitude to each exogenous change equal to the change's revenue effect. Exogenous changes are those that are not motivated by current or projected economic conditions. The motivation for a tax change is taken to be whatever Congress said it was at the time the change was enacted. The revenue effects are taken to be the consensus of estimates contemporaneous with the enactment of the change. Magnitudes are expressed as annual flows relative to the level of GDP at the time the change was enacted. Romer and Romer do not report their tax measure in a convenient table, but Mertens and Ravn (2008) have collected Romer and Romer's tax measure data in a convenient table, their Table A.1. We take our data on the Romer and Romer tax measure from the Mertens and Ravn table. We are interested here only in tax changes that pertain to the income tax, so we exclude all other changes in taxes. The resulting series is reported in Table 2.5 at the end of the chapter and shown in Figure 2.4(c).

Comparison of the Romer and Romer measure with the parameters of the income tax function raises doubts about the information content of the Romer and Romer measure. The tax parameters define the income tax and are measured with little (ϕ, θ, e) or no (α, β) error. So, one would expect their movements to have a high level of explanatory power for the Romer and Romer measure if the latter is meaningful. They do not have such power. Table 2.5 reports the first differences in the five tax parameters that were displayed in Figure 2.2. First-differences of the tax parameters are relevant here because the Romer and Romer measure concerns changes in the tax law. It is difficult to see in either the Table or in the Figures much relation between the Romer and Romer variable on the one hand and the changes in the tax parameters on the other. Table 2.4 reports the correlation between Romer and Romer's measure and the first-difference of the tax function parameters. The correlations range from low to very low. Furthermore, the two highest correlations are with the deduction and exemption parameters, which have the least influence on the income tax's time path. The exemption parameter in particular is of virtually no importance in the evolution of the income tax after 1950, after which it is so low that its movements, apart

from a steady downward trend, are negligible. Regressing the Romer and Romer measure on the five tax parameters gives the following results:

$$
\begin{array}{lccccccc}
R\&RTax = & -0.223 & +2.031\Delta\alpha & & +.292515\Delta\beta & +.992\Delta\phi & -.147303\Delta\theta & -2.65438\Delta e \\
& (0.111) & (73.852) & & (7.118) & (11.286) & (4.278) & (7.436)
\end{array}
$$

$$
\begin{array}{lll}
\bar{R}^2 = 0.168 & F_{\Delta\alpha,\Delta\beta,\Delta\phi=0} & = 0.389 \\
 & & [0.762] \\
 & F_{\Delta\theta,\Delta e=0} & = 17.093 \\
 & & [0.000]
\end{array}
$$

Standard errors are in parentheses, and p-values are in brackets. The three most important parameters for explaining the time path of the tax (α, β, ϕ) are individually and jointly insignificant. The remaining two parameters are strongly jointly significant but poorly determined. Finally, the adjusted R^2 is only 0.168, even though the explanatory variables define the income tax. There is indeed little relation between the Romer and Romer measure of income tax law changes on the one hand and the parameters that define those changes on the other.

It is not clear what the problem with the Romer and Romer measure is. One can argue that the series is not exogenous. Romer and Romer decided whether a change in the tax law was endogenous or exogenous on the basis of the reason stated by Congress for making the change. Changes said to be made in response to current or expected economic conditions were treated as endogenous, and all other changes were treated as exogenous. Such a classification scheme, however, is subject to political fads. A cynic might argue that politicians will say whatever is convenient to justify what they want to do for other reasons. Romer and Romer themselves note that there were many endogenous changes in the law before 1980 and almost none after that. That dichotomy lines up nearly perfectly with the state of macroeconomic theory. Before the appearance of endogenous growth theory in the late 1980s, macroeconomics treated growth as a *deus ex machina*, imposed as an exogenous constant and consequently not amenable to policy actions. With the advent of endogenous growth, the perspective changed entirely and growth became a legitimate target for policy. The discussion in the macroeconomic literature shows the change. Given that small changes in growth rates have large welfare implications, it may not be surprising that Congress changed its rhetoric in justifying what it was doing. Moreover, one can argue

with some of Romer and Romer's choices in classifying the tax changes. For example, Romer and Romer explain that the Revenue Act of 1951 was motivated by the increased spending on the Korean War, surely an exogenous event. Nonetheless, they classify the Revenue Act as endogenous because Congress mentioned the need to stabilize the economy. Despite these problems and ambiguities, though, it remains somewhat puzzling that Romer and Romer's measure has so little relation to the parameters that define the law.

Romer and Romer's series also shows no relation to the Seater-Stephenson marginal tax rate series. Regressing the Romer and Romer measure on the Seater-Stephenson series gives the following results:

$$\begin{array}{rll} R\&RTax = & -0.813 & + 4.794 AMTRPI \\ & (1.410) & (10.145) \end{array}$$

$$\bar{R}^2 = -0.011$$

where again standard errors are in parentheses. Both the constant and the slope coefficient are statistically insignificant, and the adjusted R^2 is minuscule and negative.

The Romer and Romer measure reflects neither the parameters that define the law nor the marginal tax rates emerging from the law.

2.6 Relation to Government Spending

Barro (1979) has proposed the hypothesis that taxes are set to finance the normal level (relative to GDP) of government spending and that transitory movements in the ratio of spending to GDP are financed by issuing or retiring government debt. The idea is that the distortions arising from taxation are non-linear functions of the tax rates, so social optimality requires that variation in tax rates be minimized. In particular, tax rates should not vary at all in response to transitory fluctuations in other fiscal variables because those variations can be accommodated by issuing or retiring government debt, keeping the tax rates constant. As a result, the tax rates also should not vary in response to changes in

government debt outstanding because those changes arise for the purposes of keeping tax rates constant in the first place.

There are two difficulties with Barro's theoretical treatment. First, Barro defines the cost of taxes to be only the direct collection costs (administrative, enforcement, etc.) of running the tax system. The usual concern in the public finance literature is not collection costs but rather welfare losses, which are a non-linear function of the marginal tax rate. To some extent, Barro mitigates this problem by assuming that collection costs are a convex function of revenue collected, which does introduce a type of non-linearity. Still, Barro leaves unexplored the connection between welfare costs of taxation and optimal tax smoothing. Second, Barro does not explore the structure of the tax function at all. He simply treats losses as functions of revenue collected. There is no examination of how the structure of the tax law could affect losses and therefore affect optimal tax smoothing. His discussion sheds no light on how the structure of the tax should change in response to various changes in spending. That is an important issue because many types of government expenditure are transfers to people or to state and local governments rather than purchases of goods and services. Transfers are a kind of negative tax, so even transitory fluctuations in transfers may call for pro-cyclical or counter-cyclical fluctuations in the income tax function's parameters. Barro's theory of debt as a tax smoothing device seems unaffected by these concerns and should still be used to minimize changes in tax parameters in the face of transitory spending fluctuations.

We use our tax parameter series to examine the relation between fluctuations in various kind of spending on the one hand and the structure of the income tax on the other. Lacking any tight theory on that relation, we must be circumspect in drawing conclusions about the validity of Barro's tax smoothing hypothesis.

We run regressions of the form

$$\tau_t = A_0 + A_{GD}\frac{\Delta D_t}{Y_{N\,t}} + A_{GN}\frac{G_{N\,t}}{Y_{N\,t}} + A_{GT}\frac{G_{T\,t}}{Y_{N\,t}} + A_{YT}\frac{Y_{T\,t}}{Y_{N\,t}} + u_t \tag{2.5}$$

where τ is one of the five tax parameters, the A's are constants to be estimated, D is the real market value of the outstanding government debt, Y_N is the normal level of real GDP, G_N is a vector of normal levels of various kinds of government expenditures, G_T is

a vector of transitory levels of the government expenditure variables, Y_T is transitory (or business cycle) income, and u is the residual. The motivation for this distinction between normal and transitory expenditures comes from Barro's (1981) theory of the output effects of government purchases. Barro argues that the relevant income concept in solving the representative household problem in a general equilibrium model is permanent *disposable* income, which is permanent gross income less permanent taxes. By the government budget constraint, one can substitute permanent purchases for permanent taxes. Barro shows that permanent and transitory changes in purchases have different effects on the economy. We follow Barro's idea here. We think of the government as an agent for the representative household so that the government takes into account the difference between permanent and transitory changes in its expenditure when setting tax and debt policy. We extend Barro's treatment of purchases to include all types of expenditure, so that we include in the analysis the distinction between permanent and transitory transfers. The reason for this extension is that our tax function is the piece of the total government budget constraint that generates the revenue to finance all types of expenditures, both purchases and transfers. An interesting issue worthy of further study is the theoretical relation between transfer programs, which are a kind of negative tax, and the structure of the income tax. For example, when the government decides to increase transfers to persons, should it match that with an increase in progressivity (viewing both the transfer spending and the progressivity of the income tax as complementary ways to redistribute income), or should it match increased transfer spending with reduced progressivity (viewing the two ways of redistributing income as substitutes rather than complements)? We have no idea. We do not perform a theoretical analysis here but restrict ourselves to an empirical analysis. Our results show many regularities but also some intriguing irregularities for theorists to chew on.

We divide government spending into several categories that seem to have different motivation: military purchases ($GMil$), non-military purchases ($GCiv$), transfers to persons (TrP), transfers to state and local governments ($TrSL$), interest payments on the national debt (Int), and the issuing or retiring of debt ($DDebt$, the change in debt outstanding). For each of those, we construct "normal" and "transitory" series. With the exception of military purchases, all normal series are simply the Hodrick-Prescott filtered values of the raw series.

The idea is that each variable follows a stochastic trend with transitory fluctuations about the trend. The HP-filtered series is a way to measure the stochastic trend. Military expenditures are different because they clearly violate the conditions necessary for the HP filter to be an adequate measure of their trend. In particular, it is clear from visual inspection of the data, as well as from reading US military history, that there have been permanent shocks to the level of military expenditure, most notably sometime between the end of the Second World War and the beginning of the Korean War. At this time the United States became involved in the Cold War, and military purchases jumped to a "peacetime" level far higher than they ever had been and then stayed there for a few decades. It seems, from reading US history, that awareness of the Cold War first appeared in the US government with Harry Truman's succession to the presidency upon the death of President Franklin Roosevelt. We therefore splice together series constructed for the pre-WWII and post-WWII eras to obtain a normal level for military expenditure. Details are in the Appendix. All transitory series are constructed as the difference between the raw series and the normal series. We do not construct normal and transitory series for the change in government debt. According to Barro's tax smoothing hypothesis, debt changes to smooth taxes, that is, it changes one-to-one with transitory changes in expenditures. Consequently, all its changes are transitory in nature. We scale everything by normal GDP, Y_N, which again is taken to be the HP-filtered series for GDP. The ratio of expenditure to GDP will have transitory fluctuations if expenditure is constant but GDP fluctuates. To capture those business cycle effects, we also construct a business cycle indicator variable, the ratio of transitory GDP Y_T to normal GDP Y_N, that is, Y_T/Y_N.

We estimate the following regression:

$$\begin{aligned}\tau_t &= A_0 + A_1\frac{\Delta D_t}{Y_{N\,t}} + A_2\frac{TrP_{N\,t}}{Y_{N\,t}} + A_3\frac{TrP_{T\,t}}{Y_{N\,t}} + A_4\frac{TrSL_{N\,t}}{Y_{N\,t}} + A_5\frac{TrSL_{T\,t}}{Y_{N\,t}} \\ &\quad + A_6\frac{GC\,iv_{N\,t}}{Y_{N\,t}} + A_7\frac{GC\,iv_{T\,t}}{Y_{N\,t}} + A_8\frac{GMil_{N\,t}}{Y_{N\,t}} + A_9\frac{GMil_{T\,t}}{Y_{N\,t}} + A_{10}\frac{Y_{T\,t}}{Y_{N\,t}} + u_t\end{aligned}$$

where, as before, τ is any of the five tax function parameters, α, β, ϕ, θ, or e. Table 2.6 at the end of the chapter presents the regression results.

Parameters α and β have almost identical sets of explanatory variables, the only difference being a significant impact of normal interest payments on β but not α. All

statistically significant coefficients have opposite signs across the two regressions, indicating that increases or decreases in the level of taxation brought about by changes in the parameter α are matched by changes in progressivity in the opposite direction through changes in the parameter β. That behavior is consistent with Lindert's (2003, 2004) hypothesis about countervailing distortions, mentioned above. Parameters ϕ and θ enter the tax function (2.2) in mathematically identical ways except for opposite signs. Nonetheless, the two are determined by sets of variables different from each other and also from the sets determining α and β. We see no obvious explanation for this difference. Parameter e is poorly determined. The fit of the regression is the worst of the five, by far. The set of individually insignificant variables is large, as it is for θ, but for e the F-test strongly rejects their joint insignificance, in contrast to the result for θ. The F-test continues to reject joint insignificance even after the variable closest to individual significance is omitted from the test. We thus are left with a regression containing a large number of coefficients whose magnitude cannot be determined at all accurately, making it difficult to say much more about the determination of the parameter e.

Federal military and non-military purchases have different effects on tax structure. Normal military expenditure is the only variable to have a statistically significant coefficient for all five parameters. Transitory military expenditure is statistically significant for four of the five parameters, the exception being θ. The statistically significant coefficients for the two types of military expenditure have the same sign pattern across tax parameters, indicating qualitatively identical effects on tax structure. The results for federal non-military purchases are quite different from those for military purchases and also from each other. First, normal non-military purchases are statistically significant only for α and β, whereas transitory non-military purchases are significant for α, β, θ, and e. Second, the sign patterns for the two types of purchases are opposite across tax parameters except for ϕ, for which both types of purchases are statistically insignificant.

The coefficients for transitory federal transfers to persons and to state and local governments have the same signs across the five tax parameters, though they are not always statistically significant. Transitory transfers reduce both α and ϕ and increase β, θ, and e, implying that transitory increases in transfers of either type are matched with increased but less progressive taxes. This result is again consistent with Lindert's (2003, 2004)

hypothesis of countervailing distortions: transfers are distorting, and the taxes used to finance them are changed in a way that minimizes the total distortion. Unfortunately, this neat consistency with Lindert's hypothesis does not carry over to normal transfers. The signs of the coefficients for normal transfers to persons and to state and local governments differ across tax parameters, even if we restrict attention to just those that are statistically significant. There is no obvious unified interpretation of the effects of normal transfers on the tax parameters. This outcome may not be surprising because the two kinds of transfers are very different in nature and motivation. Transfers to state and local governments mostly are meant to fund purchases by the state and local governments, such as building schools or roads. Thus, in a sense, most transfers to state and local governments actually are a type of purchase, with the state and local governments acting as the purchasing agents for the federal government. Before 1964, transfers to state and local governments virtually always were for purchases. After 1964, presidents Johnson and Nixon both invented transfers to state and local governments that were not tied to purchases and sometimes were intended explicitly for ultimate payment to individuals. However, even after 1964 the bulk of federal transfers to state and local governments were for purchases. In contrast, federal transfers to persons are just what they say and so are a kind of negative tax. Transfers to persons thus are themselves a part of the tax system and so unsurprisingly have different effects on the structure of the income tax. What is difficult to explain is the difference across normal and transitory transfers. The latter is consistent with Lindert's hypothesis, but the former is not. Transitory transfers, which are just as different in nature as permanent transfers, have qualitatively identical effects on the income tax's structure. It therefore seems that the permanence of the transfers matters for the effect the transfers have on the structure of the tax, a result that begs a theoretical explanation. In any case, normal and transitory transfers do have very different effects on tax structure.

Barro's theory of tax smoothing is supported in part and rejected in part. The business cycle variable never is significant, which is consistent with Barro's theory. The change in the market value of debt outstanding is marginally significant overall, being individually significant for ϕ and θ, though not jointly significant for θ. This result is not strong, but it is somewhat at variance with Barro's theory, which implies insignificance. Transitory changes in spending are often significant, and every parameter has at least one

individually significant transitory spending variable. This result is strongly inconsistent with Barro's theory as he states it in terms of overall taxation and spending. However, the result could be consistent with a more completely developed theory that relates the structure of the tax to types of spending.

2.7 Conclusion

This chapter has explored some of the properties of the time series on the income tax parameters that were constructed in the previous chapter. There is a great deal of variation in the values of the parameters over the history of the income tax. Some systematic elements of that variation are interesting. The volatility of four of the five tax parameters decreased markedly around 1950, and the volatility of the remaining parameter decreased around 1980. It is well known that the volatility of output decreased at both those times, so the observed reductions in tax parameter volatility suggests the hypothesis that output volatility decreased because the tax function became more stable. That hypothesis has not been tested here and awaits further study. Another interesting, and perhaps surprising, finding is that the degree of progressivity of the income tax is strongly negatively correlated with the observed average marginal tax rate. That behavior is consistent with Lindert's (2003,2004) hypothesis that governments recognize the overall distortions of their policies and act to counteract distortions arising from one set of policies (such as spending on income transfers) by reducing distortions in other policies (such as the progressivity of the income tax). Again, this observation merits further study.

The tax parameters are closely related to the observed average marginal tax rate, not very closely related to the observed maximum marginal tax rate, and almost unrelated to Romer and Romer's measure of tax changes based on their narrative approach.

The behavior of the tax parameters is consistent in some ways with Barro's theory of income tax smoothing but inconsistent in others. The business cycle has no significant effect on any of the five parameters, as Barro's theory suggests. Changes in the outstanding debt have some weak effects on the tax parameters. Barro's theory suggests no relation at all, so there is some disagreement here, though it is not strong statistically. A much stronger

disagreement with Barro's theory is the effect of transitory changes in government spending on the tax parameters. The theory suggests that transitory changes in expenditures should be financed entirely by issuing or retiring debt with no changes in taxes. In fact, however, transitory expenditures of every type except interest on the debt have statistically significant effects on at least two of the five parameters, and every parameter is affected by at least one type of transitory expenditure. Barro's theory was stated in terms of total net taxes collected and did not address the issues of how the structure of the tax system, such as the income tax, would change in response to changes in purchases or transfers. The data show a strong relation between the tax structure and both purchases and transfers, so an extension of Barro's theory would be worthwhile.

Table 2.5: Comparison of the Romer and Romer Measure with our Parameters

Year	RandR	$\Delta\alpha$	$\Delta\beta$	$\Delta\phi$	$\Delta\theta$	Δe
1947	0.00	0.000000	0.000000	0.010009	-0.009785	-0.021
1948	-1.92	-0.000985	-0.010160	-0.005755	0.111266	0.032
1949	0.00	-3.10E-05	-4.00E-05	-0.006690	0.004299	0.008
1950	0.00	0.000507	-0.001930	0.009038	-0.022986	-0.024
1951	0.00	0.002660	-0.011940	-0.013428	-0.018495	-0.038
1952	0.00	0.003474	-0.016620	0.004530	-0.009479	-0.010
1953	0.00	-2.20E-05	2.00E-05	0.001951	-0.007957	-0.011
1954	-0.76	-0.005303	0.023610	-0.000236	-0.002928	0.004
1955	0.00	-8.60E-05	0.000800	-0.003423	-0.008280	-0.016
1956	0.00	5.90E-05	-0.000690	0.012784	-0.010250	-0.008
1957	0.00	-2.90E-05	0.000290	-0.004002	-0.003462	-0.008
1958	0.00	3.20E-05	-0.000330	-0.006154	0.000769	0.001
1959	0.00	2.10E-05	-0.000260	0.000455	-0.005225	-0.014
1960	0.00	-2.20E-05	0.000170	-0.002950	0.000341	-0.005
1961	0.00	-0.000130	0.001160	0.006293	-0.004117	-0.003
1962	0.00	-4.20E-05	0.000440	-0.010123	-0.002680	-0.012
1963	0.00	6.00E-06	-5.00E-05	0.001558	-0.001498	-0.007
1964	-1.30	0.008468	-0.065580	0.000721	-0.006805	-0.010
1965	-0.65	-0.002374	0.005590	-0.000880	-0.004106	-0.012
1966	0.00	7.00E-05	-0.000380	-0.002228	-0.005358	-0.012
1967	0.00	0.000479	0.007170	0.011674	-0.004152	-0.007
1968	0.00	0.001831	-0.014350	0.002948	-0.001039	-0.011
1969	0.00	-0.001244	0.009910	0.003733	0.000639	-0.008
1970	0.00	0.000678	-0.009970	-0.004712	0.000496	0.000
1971	-0.09	-0.001427	0.004460	-0.010617	0.032730	0.001
1972	-0.77	3.20E-05	-0.000100	0.004761	0.030882	0.003
1973	0.00	0.000118	-0.000620	-0.004206	-0.008420	-0.012
1974	0.00	-0.001320	0.012910	0.005470	-0.006191	-0.008
1975	0.00	-0.003783	0.020060	-0.025165	-0.013955	-0.008

Table 2.5 Continued

Year	RandR	$\Delta\alpha$	$\Delta\beta$	$\Delta\phi$	$\Delta\theta$	Δe
1976	0.13	0.005299	-0.033830	-0.001135	-0.000497	-0.010
1977	-0.04	5.20E-05	-0.000230	-0.006949	0.003408	-0.008
1978	0.00	-0.001592	0.014460	-0.002844	-0.004765	-0.008
1979	-0.77	-0.002090	0.001680	0.004094	-0.006410	0.015
1980	0.00	-0.001812	0.018100	0.006819	-0.001649	-0.007
1981	-0.28	-2.00E-06	-1.00E-05	-0.011885	0.000751	-0.008
1982	-1.53	0.001047	-0.023830	0.002395	0.003001	-0.002
1983	-1.69	-0.001039	-0.001850	-0.019746	0.001646	-0.005
1984	-1.28	-0.000419	-0.000670	-0.005204	0.002498	-0.006
1985	0.00	3.10E-05	-2.00E-05	0.002333	0.007105	-0.001
1986	0.50	-0.000600	0.003880	0.009687	0.005756	-0.001
1987	-0.73	-0.000347	-0.009870	0.029175	-0.027410	0.039
1988	0.00	-0.004301	0.047240	0.018807	0.003582	-0.004
1989	0.00	-0.000131	0.000960	-0.010315	0.004879	-0.003
1990	0.00	-0.000114	0.000320	-0.006920	0.004592	-0.002
1991	0.60	-6.60E-05	-0.000580	-0.009013	0.004362	-0.010
1992	0.00	-0.000151	0.000870	-0.005190	-0.002531	0.001
1993	1.61	0.001531	-0.023590	-0.013348	0.000546	-0.001
1994	0.19	-0.000270	0.002750	-0.006760	-0.006382	-0.001
1995	0.00	1.90E-05	-0.001010	0.013787	-0.003354	-0.002
1996	0.00	-0.000102	0.000670	0.013973	-0.004446	-0.003
1997	0.00	-4.00E-06	-0.000690	0.018201	-0.006375	-0.003
1998	0.00	-8.20E-05	0.000360	0.020702	-0.004051	-0.003
1999	0.00	1.40E-05	-0.000590	0.012586	-0.003887	-0.003
2000	0.02	-2.50E-05	-8.00E-05	0.016636	-0.002690	-0.004
2001	0.01	0.000443	-0.007210	-0.039141	0.018284	0.005
2002	-0.80	-0.005903	0.138240	-0.032966	0.008953	0.002
2003	-2.86	0.000333	-0.025690	-0.009995	0.004439	-0.002
2004	1.70	-7.10E-05	0.002810	0.014639	-0.005167	-0.005

Table 2.5 Continued

Year	RandR	$\Delta\alpha$	$\Delta\beta$	$\Delta\phi$	$\Delta\theta$	Δe
2005	0.56	7.00E-05	-0.004280	0.016016	-0.000290	-0.004

Note, Table 2.6: Regressions of tax parameters on fiscal variables

Numbers in parentheses are standard errors. Those in brackets are p-values. $F_{i,j,...,k}$ is the joint significance test for variables $i, j, ..., k$. Numbers in bold are significant at the 10% level or less. Those with an asterisk are individually insignificant but jointly significant. Those in italic are individually significant but jointly insignificant.

Table 2.6: Regressions of tax parameters on fiscal variables

Var.#	Explanatory Var.	Dependent Var.				
		α	β	ϕ	θ	e
1	Constant	**-0.0120**	**2.0291**	**0.3021**	**0.1323**	0.8628*
		(0.0046)	(0.0413)	(0.0528)	(0.0344)	(0.9687)
		[0.0111]	[0.0000]	[0.0000]	[0.0002]	[0.3758]
2	$\Delta D/Y_N$	0.0051	-0.0570	**-0.1409**	*0.0797*	3.1492*
		(0.0065)	(0.1724)	(0.0814)	(0.0454)	(2.3500)
		[0.4344]	[0.7417]	[0.0874]	[0.0829]	[0.1841]
3	Int_N/Y_N	-0.0506	**-3.7916**	**-3.3955**	-0.5287	21.7532*
		(0.1997)	(1.4481)	(1.2346)	(0.6091)	(32.8195)
		[0.8007]	[0.0106]	[0.0074]	[0.3881]	[0.5094]
4	Int_T/Y_N	0.0162	3.2465	-2.5276	1.1146	-33.7555*
		(0.2937)	(4.6710)	(5.4675)	(2.6633)	(65.0651)
		[0.9561]	[0.4891]	[0.6452]	[0.6767]	[0.6054]
5	TrP_N/Y_N	-0.0451	-0.0797	**5.0162**	**0.9434**	-37.2365*
		(0.1337)	(1.0466)	(0.8745)	(0.4832)	(29.5833)
		[0.7369]	[0.9395]	[0.0000]	[0.0545]	[0.2118]

Table 2.6 Continued

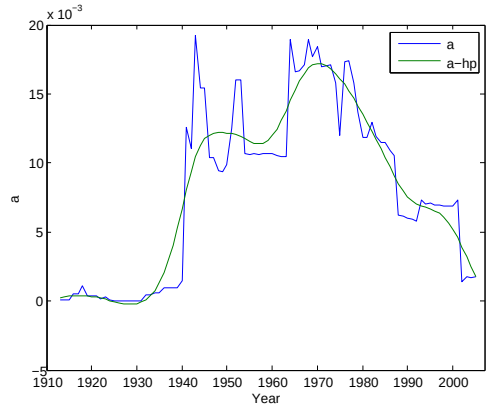
Var.#	Explanatory Var.	Dependent Var.				
		α	β	ϕ	θ	e
6	TrP_T/Y_N	**-0.1903**	**3.6336**	**-3.0098**	0.8814	22.4879*
		(0.0945)	(1.8190)	(1.5376)	(1.1085)	(18.1200)
		[0.0474]	[0.0492]	[0.0539]	[0.4290]	[0.2183]
7	$TrSL_N/Y_N$	**0.6011**	**-9.8215**	-3.1157*	-0.1234	15.6242*
		(0.2480)	(1.9060)	(1.8945)	(1.2055)	(80.9927)
		[0.0176]	[0.0000]	[0.1041]	[0.9187]	[0.8475]
8	$TrSL_T/Y_N$	**-0.1938**	**2.7660**	-2.1099	0.0471	17.7165*
		(0.0870)	(1.2967)	(1.7062)	(0.8926)	(20.5530)
		[0.0287]	[0.0360]	[0.2200]	[0.9581]	[0.3913]
9	$GC\,iv_N/Y_N$	**0.2371**	**-9.5153**	-1.4142	0.1202	66.8174*
		(0.1071)	(1.2692)	(1.4657)	(0.9909)	(41.8248)
		[0.0297]	[0.0000]	[0.3376]	[0.9037]	[0.1141]
10	$GC\,iv_T/Y_N$	**-0.05993**	**1.1247**	-0.2654	**-0.4369**	**-33.0188**
		(0.0229)	(0.3164)	(0.3406)	(0.2610)	(18.8032)
		[0.0105]	[0.0006]	[0.4383]	[0.0982]	[0.0830]
11	$GMil_N/Y_N$	**0.0939**	**-1.9014**	**1.6403**	**0.5622**	**-10.0566**
		(0.0113)	(0.1411)	(0.1210)	(0.0873)	(2.95135)
		[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0010]
12	$GMil_T/Y_N$	**0.0116**	**-0.3802**	**0.4193**	-0.1031	**-4.8110**
		(0.0069)	(0.1386)	(0.0742)	(0.0656)	(1.6803)
		[0.0987]	[0.0075]	[0.0000]	[0.1199]	[0.0054]
13	$Y_{T\,t}/Y_N$	*0.0222*	0.2200	-0.2567	0.0593	1.4528*

Table 2.6 Continued

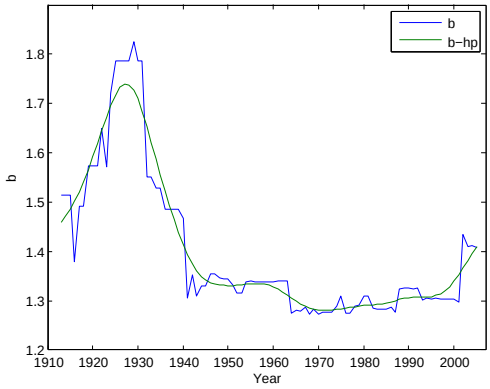
Var.#	Explanatory Var.	Dependent Var.				
		α	β	ϕ	θ	e
		(0.0126)	(0.1836)	(0.2183)	(0.1219)	(3.2415)
		[0.0821]	[0.2343]	[0.2433]	[0.6281]	[0.6552]

Table 2.7: Regressions of tax parameters on fiscal variables, $\bar{R}^2$

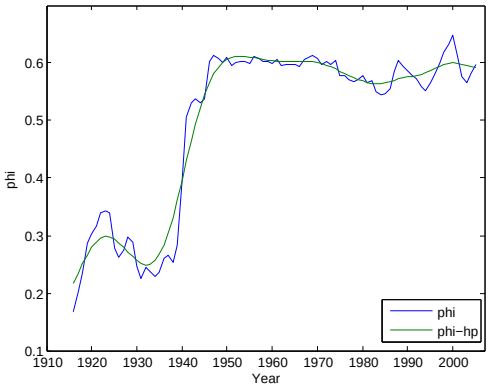
	Dependent Var.				
	α	β	ϕ	θ	e
$\bar{R}^2$	0.7521	0.9011	0.9180	0.7041	0.5895
	$F_{2,3,4,5,13}$= 1.4761 [0.2070]	$F_{2,4,5,13}$= 0.5912 [0.6700]	$F_{4,7,8,9,10,13}$= 2.2727 [0.0451]	$F_{2,3,4,6,7,8,9,12,13}$= 0.9273 [0.5065]	$F_{1,2,3,4,5,6,7,8,9,13}$= 32.3657 [0.0000]
	$F_{2,3,4,5,12,13}$= 10.43001 [0.000000]		$F_{4,8,9,10,13}$= 1.6452 [0.1582]		$F_{1,2,3,4,5,6,7,8,13}$= 24.3368 [0.0000]
			$F_{2,4,8,9,10,13}$= 6.5304 [0.0000]		



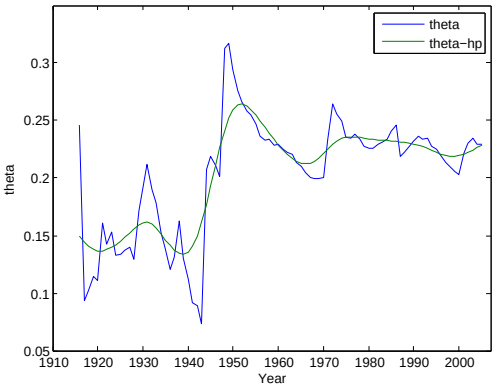
(a) Tax Parameter alpha



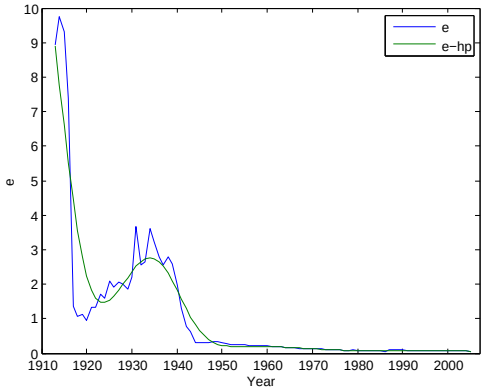
(b) Tax Parameter beta



(c) Tax Parameter phi

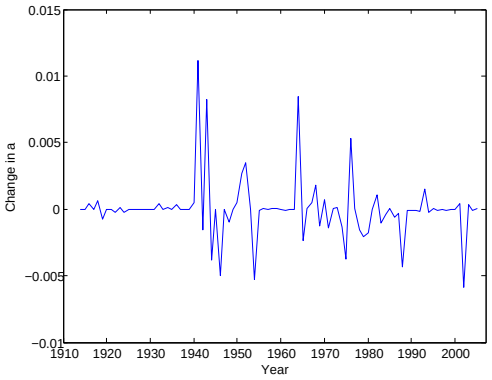


(d) Tax Parameter theta

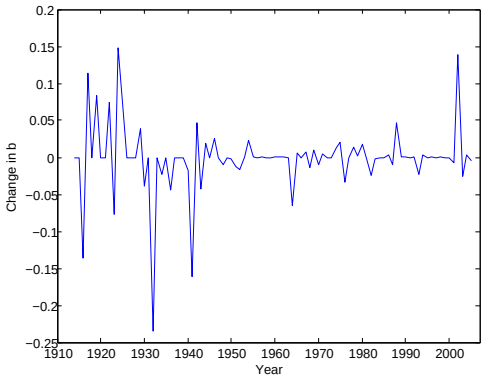


(e) Tax Parameter e

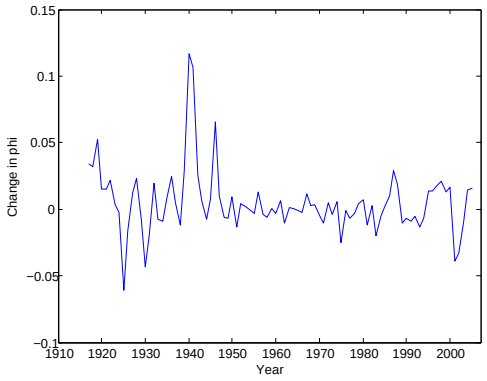
Figure 2.1: The Tax Parameters Over Time



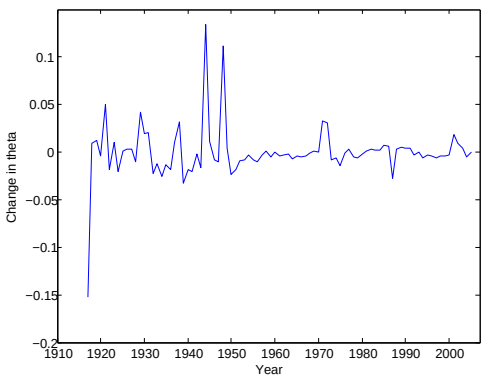
(a) First Difference of alpha



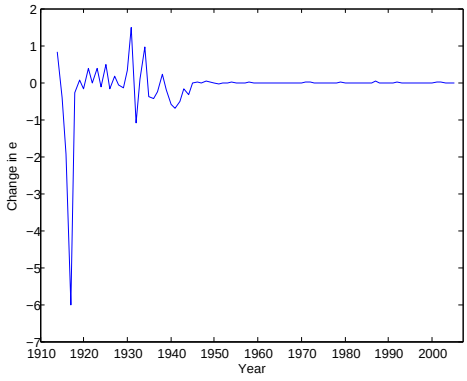
(b) First Difference of beta



(c) First Difference of phi

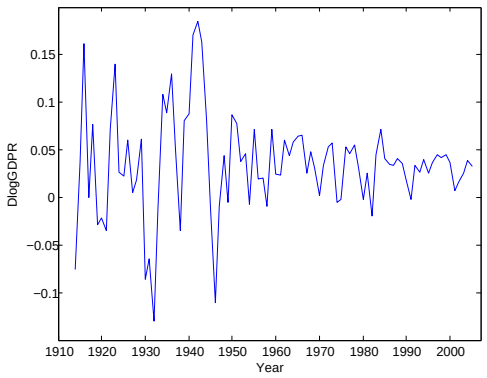


(d) First Difference of theta

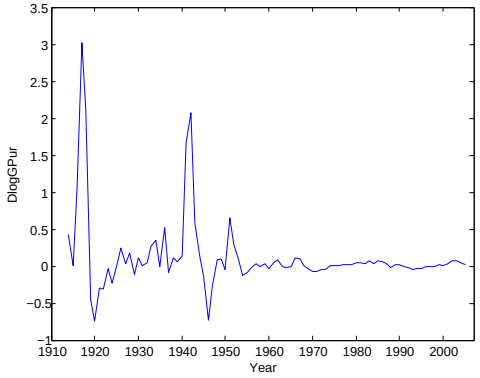


(e) First Difference of e

Figure 2.2: Tax Parameters' First Differences

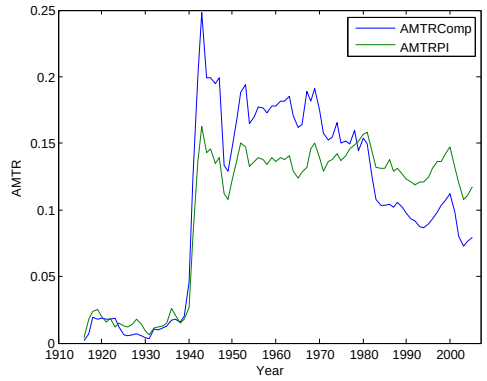


(a) Growth Rate of Real GDP

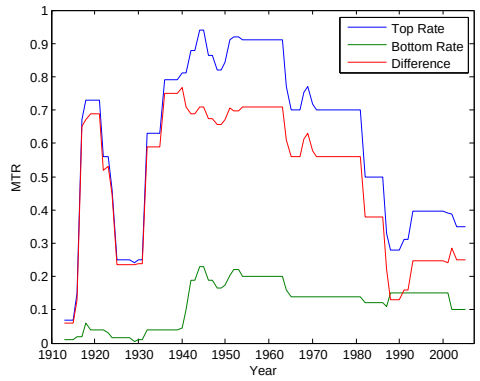


(b) Growth Rate of Real Federal Purchases

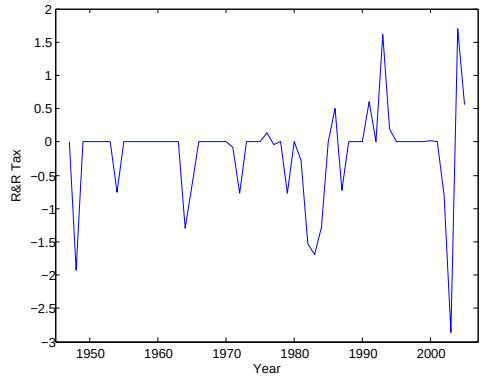
Figure 2.3: Growth Rates



(a) AMTRComp & AMTRPI



(b) Extreme Marginal Tax Rates



(c) Romer & Romer Tax Change Measure

Figure 2.4: Tax Rates

Chapter 3

The Model and Experiments

This chapter begins with a description of our dynamic stochastic general equilibrium (DSGE) model that includes our income tax function. From this model we have a system on nonlinear difference equations which can be solved with Schmitt-Grohé and Uribe's (2004) perturbation technique. This method is outlined, and we have the basis for our experiments. In these experiments we vary the parameter values in our income tax function which allows us to compare a range of different tax policies. In particular we examine the steady state values, second moments, and impulse response functions generated by these different parameters. The chapter closes with our results and conclusions.

We are using the basic neoclassical model. We assume a large, closed economy and therefore have no international borrowing or lending and no stock of net foreign investment. Although this is not realistic, we chose this type of economy for two reasons. First, we are using U.S. data which is comparable to that of a large, closed economy. Secondly, Schmitt-Grohé and Uribe (2003) state that the standard small, open economy model will have a stationarity problem. After the log-linearization of the equations in our model, some variables would have exhibited a random walk with this type of economy. This problem in the transformation will create variables whose expectation will not converge on some value as time approaches infinity; therefore, the usual descriptive statistics will be incalculable. Although Schmitt-Grohe and Uribe do submit five approaches to correct this issue while keeping the economy small and open, we believe that none will be universally acceptable. We, therefore, adopt the closed economy model with an interest rate that is time varying and endogenously determined by the general equilibrium conditions each year. Chadha and

Dimsdale (1999) and Blanchard and Summers (1984) provide evidence that the real interest rate moves around a long-term average, of about .04, with a slow reversion to the mean, which is in fact what our model produces.

3.1 The Firm

A representative firm hires labor and purchases capital in perfectly competitive input markets, and therefore, faces given input prices. The firms intention is to maximize the present value of all future profits:

$$E_t[\sum_{j=o}^{\infty}[(1+r_{t+j})^{-j}\{Y_{t+j} - s_{t+j}H_{t+j} - \psi\frac{(K_{t+j}-K_{t+j-1})^2}{2K_{t+j}} - I_{t+j}\}]] \tag{3.1}$$

where r is the real interest rate, Y is output or gross domestic product, s is the real wage rate, H is the proportion of time that the houshold spends on labor, ψ is the cost of capital adjustment parameter, K is the capital stock, and I is gross investment. The real wage rate and the interest rate are the mechanisms that move to bring our markets into equilibrium. There is no price level because there is no money. The model assumes that the price of new capital, gross investment, is one unit of output at all times. This firm produces the one final good present in the economy, which acts as the numeraire, according to the production function:

$$Y_{t+j} = exp(z_{t+j})AK_{t+j-1}^{\omega}H_{t+j}^{1-\omega} \tag{3.2}$$

where ω represents capitals share of output and is set equal to .36 leaving labors share of output equal to .64. We find these parameter values in Prescott (1986). Z is an exogenous, stochastic productivity shock and follows an AR(1) process:

$$z_{t+1} = \varrho z_t + \nu \tag{3.3}$$

where ν is white noise distributed normally, IID, with a mean of zero and a variance of .007632. This variance value is taken from Prescott (1986) who uses Solow's (1957) technique for measuring technological change. Solow determines a variance of .00902 for the process that the technology shock follows. Prescott believes that Solows method overstates this value because he miss-measures the labor input. Although Prescott believes that his

variance value is a more accurate measurement, he acknowledges that this quantity is difficult to estimate and may still not be precise. As in Kydland and Prescott (1982) and Long and Plosser (1983) the productivity shock adds uncertainty to our model. Although the value of ϱ could fall between zero and one, we choose a value that provides high serial correlation among the shocks. Kydland and Prescott (1982), Long and Plosser (1983), Hansen (1985), and Prescott (1986) state that a high serial correlation, such as with ϱ equal to .7 (on an annual basis), will produce business cycles in the model which approximate those in U.S. postwar macro data. The agent will have complete information about each shock at the beginning of the period and will make decisions for that period accordingly. The shock and the firms input decisions, which take place at the beginning of each period, dictate the amount of output that will be available at the beginning of the following period. Because this production technology satisfies all of the Inada conditions and requires both factors for production King, Plosser, and Rebelo (1988) describe it as neoclassical. Specifically, the Cobb Douglas form is used, which is homogenous of degree one and, therefore, provides constant returns to scale. We also assume, as Long and Plosser (1983) do, that the inputs can readily be used as substitutes and that the marginal productivity of either input, at any level of employment, is strictly diminishing. The level of capital that enters our production function each period is set in the preceding period, and the initial level of capital stock in our model is given. The firm is owned by the household, which is a convenient assumption because it makes the firms maximization problem entirely static. The household is passive in all firm decisions. The firm also does not perceive taxes which means that we are omitting corporate income taxes. We assume that all taxes are passed through to the owner of the firm, the household. The final good in our economy may be consumed, used to pay taxes, or invested. Investment evolves according to:

$$I_{t+j} = K_{t+j} - K_{t+j-1} + \delta K_{t+j-1} \tag{3.4}$$

where $0 \leq \delta \leq 1$ represents the rate of depreciation of capital. With this specification we assume that depreciation is a constant percentage of capital stock. Although Griliches (1963) believes that this proportional measurement may not be accurate, it is commonly used and further investigation into its structure is beyond the scope of this study. Long and Plosser (1983) believe that, with this structure, an annual depreciation rate of 10% is

reasonable. Capital adjustment costs may be treated in one of two ways. They may be viewed as a reduction of final output as resources are used in the adjustment process and not in the actual production process (Treadway, 1969). Alternatively, they may be thought of as payment for installation costs which are earned by some other sector of the economy (Lucas, 1967). In our model, we take the first approach, and these adjustment costs enter as a quantity in the firms value function which reduces the firms output:

$$\psi \frac{(K_{t+j} - K_{t+j-1})^2}{2K_{t+j}} \tag{3.5}$$

This function satisfies the following restrictions: (1) when investment is zero, $\psi(0)=\psi\prime(0)=0$, which guarantees that adjustment costs in the steady state are zero and (2) for positive levels of investment, $\psi(\mathrm{I})>0$, $\psi\prime(\mathrm{I})>0$, and $\psi\prime\prime(\mathrm{I})>0$. The last inequality describes a function where higher costs are linked with a faster adjustment of the capital stock. Cooper and Haltiwanger (2006) state that these types of adjustment costs are complex and challenging to measure. We use the usual specification of convex costs with a quadratic function, which complies with both of our restrictions listed above. Although Caballero (1999) asserts that this specification is not an adequate approximation for actual costs, Gould (1968) believes that although it varies across industries, this function is a good general estimate for actual costs in many cases. In fact, Holt, Modigliani, Muth, and Simon (1960) believe that this quadratic can adequately describe a variety of costs including: hiring and layoff costs, overtime costs, inventory costs, and machine setup costs. They also state that even non-trivial errors in this estimation will not produce large differences in firm behavior. Using this quadratic cost function also lends tractability to our problem. Gould (1968) notes that this cost structure in combination with our linearly homogenous production technology will yield an exact solution to the models Euler equations. We chose a cost of adjustment function that is dependent on net investment following Lucas (1967) and Strotz and Eisner (1963). The appropriateness of selecting net investment, as opposed to gross investment which Abel and Blanchard (1983) choose, is determined by which of the two capital uses yields higher costs at higher levels of investment. A capital purchase may be used to expand the capital stock or used as replacement for depreciated capital. We believe that net investment is the more appropriate choice because as a firm replaces existing capital, no preparation or extra learning, and therefore no extra cost, is necessary. Because it replicates

the movement in actual investment data, we use the established value, .5, for the cost of adjustment parameter. The following are the necessary conditions for the firms problem, which are found by constructing and solving the Lagrangian. The first condition is the derivative of the Lagrangian with respect to capital:

$$(1+r_{t+j})^{-j}[\frac{\psi}{K_{t+j}}(K_{t+j}-K_{t+j-1})+\frac{\psi}{2K_{t+j}^2}(K_{t+j}-K_{t+j-1})^2-1] \tag{3.6}$$
$$+(1+r_{t+j})^{-(j+1)}[exp(z_{t+j+1})\omega(\frac{H_{t+j+1}}{K_{t+j}})^{1-\omega}+(1-\delta)+\frac{\psi}{K_{t+j+1}}(K_{t+j+1}-K_{t+j})]=0$$

The second condition is the derivative of the Lagrangian with respect to labor:

$$(1+r_{t+j})^{-j}[exp(z_{t+j})(1-\omega)K_{t+j-1}^{\omega}H_{t+j}^{-\omega}]=s_{t+j} \tag{3.7}$$

which has the expected interpretation where the marginal product of labor equals the wage rate.

3.2 The Household

Our economy is populated by one infinitely lived individual. As in most real business cycle models, we, therefore, are using a representative agent. Our individual has a given amount of initial resources and has tastes that are assumed to remain constant over time. He faces the same choices at the beginning of each period which are: (1) the amount of the final good to be consumed that period, (2) the amount of capital to be supplied, or rented to the firm, in the following period, and (3) the amount of time dedicated to leisure, and consequently the amount given to labor. These choices are dependent on the information available that period. The agent maximizes his lifetime utility given the following additively separable, logarithmic preferences over consumption and leisure:

$$U_t=\sum_{j=0}^{\infty}[\Gamma^j E_t[ln(C_{t+j})+\mu ln(L_{t+j})] \tag{3.8}$$

where Γ, which is set at .961539 as in Cardia (1997) and Prescott (1986), is the effective discount factor, C is the expenditure on the consumption good, and L is the amount of time allocated to leisure. E_t denotes the conditional expectation in period t given the

information available at the beginning of that same period, and μ is the weight given to leisure in our utility function. We again follow Prescott (1986) and Cardia (1997) and set μ to 2.11. These authors refer to a study by Ghez and Becker (1975) who determine that an individual will designate roughly one third of his active time to market activities and the remaining two-thirds to non-market activities. This 2.11 parameter value gives our model a steady state value of time allocated to labor of about one third. King, Plosser, and Rebelo (1987) however, set the μ parameter such that the steady state value of the time allocated to labor is equal to approximately one fifth. They believe that this fraction matches the average amount of time allotted for work found in data from the U.S. from 1948 to 1986. This modification does not significantly change our results. Cochrane (2001) states that the expected value for labor should fall between one quarter and one third, and our value does. The functional form used yields zero cross-elasticities, a unitary own elasticity in consumption, and an own elasticity in leisure that is determined by the specified value μ. We assume that the consumption good and leisure are both normal goods, and consequently, the utility function is increasing in both. The utility function is strictly concave, is twice continuously differentiable, and satisfies the Inada conditions. Although not entirely accurate, we have chosen to omit government services from the utility function. We understand that the representative agent would only impose taxes if he percieved some welfare increase in return. Also in theory, these services belong in the utility function because we hope that the government is using at least some tax revenue to benefit the household. And, in many ways they are. There are government services, for example roads, which increase the productivity of the household's resources. Also, government services should be included because in some cases they are complements or substitutes for the other arguments in the utility function and would, therefore, have an impact on their marginal utilities. There are, however, other cases where government services may not add much in utility for the household. Some portion of tax revenue is simply used for redistribution purposes - between households or geographic areas. Because we are using a representative agent, this type of redistribution would have no effect on our agent's utility. Also, we have seen many well documented instances where the government has used money in ways, and taken on projects, that have offered no gain in social welfare to the public. Consequently, we acknowledge that our utility function is not thoroughly complete, but believe that the inclusion of government services will not materially change our results. The representative

agent is also subject, in each period, to two resource constraints. First, his choice set is limited by his endowment of time which is allocated between leisure and work and is normalized to one:

$$L_{t+j} + H_{t+j} = 1 \tag{3.9}$$

where H is time allocated to labor activities and L is time allocated for leisure activities. Second, he is also restricted by his budget constraint which equates his sources and uses of funds:

$$\begin{aligned} B_{t+j+1} =& (1+r)\{B_{t+j} + Y_{t+j} - Y_{t+j}(\tau_{t+j}) - \alpha_{t+j}[(1-\theta_{t+j})\phi_{t+j}(Y_{t+j}) - \varepsilon_{t+j}(N_{t+j})]^{\beta_{t+j}} \\ & - T_{t+j} - C_{t+j}\} \end{aligned} \tag{3.10}$$

where W is household wealth, τ is the tax rate associated with the portion of taxes that are proportional to Y, and T is lump sum taxes. Income, Y, captures labor income as well as dividend and capital gains income. The household owns the firms; therefore, it receives all the dividend payments and earns all of the capital gains. We assume that the firm pays a dividend to the household everytime there is one available. Although we have chosen to include capital gains in the model, we are ignoring a capital gains tax that would be specified differently from the three taxes which face our household. Using data from, and the tax categories of, the Bureau of Economic Analysis' National Income and Product Accounts, we calculate τ by first finding the governments total tax receipts which are proportional to Y. They include the following: (1) state personal current taxes, (2) taxes on corporate income, (3) contributions for government social insurance, and (4) taxes on production and imports. Because we do not have a foreign sector in this model, customs duties are subtracted from the taxes on production and imports. To find τ, these quantities are summed and then divided by Y for each year. The NIPAs data begins in 1929; therefore, we use data from the Historical Statistics of the United States for the construction of τ in the years from 1913 to 1928. Data is available for the years 1922, 1927, and 1932 which we use for those years respectively and which we also use to extrapolate for data in the years in between. We find τ by subtracting the individual income tax (because it is accounted for elsewhere) from the governments general revenue and dividing this quantity by Y. There is no data available for the years from 1913 to 1921, however, the calculated τ from 1922 to 1928 is approximately constant. We therefore use the average τ over the period from 1922 to 1928 as the τ for the

earliest group of years. The following portion of the agents budget constraint corresponds to the final tax faced by our agent: the federal tax on personal income:

$$\alpha_{t+j}[(1-\theta_{t+j})\phi_{t+j}(Y_{t+j}) - \varepsilon_{t+j}(N_{t+j})]^{\beta_{t+j}} \tag{3.11}$$

where α and β are parameters set by the government, θ is the aggregate deduction amount divided by adjusted gross income (AGI), ϕ is adjusted gross income divided by gross domestic product (GDP), Y is gross domestic product, ε is the individual exemption amount allowed by the tax law, and N is the aggregate number of exemptions claimed. Data for these values come from the U.S. tax law and are described in a previous section of this paper. They can be found, or are derived from, either the 1040 Individual Income Tax Return form or the Statistics of Income Individual Income Tax Return Collection. There are only two differences between the parameters in the previous description of our income tax function and the ones used in the actual simulations: (1) each ε is divided by GDP and then by total population and (2) N is also divided by total population. These new values for ε and N are at the appropriate scale, which is in per person terms, for our model as it is describing a representative agent. The equations of the model where ε appears will also be modified - ε will be multiplied by the model-generated value of Y in each year. This change essentially cancels out the GDP which e is initially divided by and leaves only the ratio of ε to total population. The θ ratio will always be between zero and one, and N is exogenous to the government. The value inside the brackets approximates taxable income. Most macro models discuss income taxes as either lump sum or proportional to income; however, we know that in actuality the U.S. tax on income is progressive. This characteristic is captured by the α and β parameters.

Although GDP is commonly used as a measure of income, only a small proportion of it is subject to income taxes. Consequently, a more suitable measure, taxable income, is used in our tax function. The quantity in brackets details the conversion of and explains the relationship between GDP and taxable income. Taxable income, as defined by the tax law, is:

$$Y_T = AGI - D - E \tag{3.12}$$

where Y_T is taxable income, D is aggregate deductions, and E is aggregate exemptions. As

an identity, we also know that:

$$E = \varepsilon N. \tag{3.13}$$

We rewrite deductions as a proportion of AGI, θ, include the above transformation for E, and get:

$$Y_T = (1 - \theta)AGI - \varepsilon N. \tag{3.14}$$

Because AGI can be expressed as a proportion of GDP, the following formula for taxable income is used in our tax function above:

$$Y_T = (1 - \theta)\phi Y - \varepsilon N. \tag{3.15}$$

The household does not forcast changes in the income tax structure, which would essentially be a change in the α or β parameter, in any other way than to expect that it will not change. Although this is not entirely correct, we chose to handle forcasting this way because: (1) we do not know the precise method that a household would use to forcast taxes, and (2) most tax changes are not anticipated in years before the year that they occur. Total taxes are comprised of three parts: the two aready described - the tax rate, τ and the tax on personal income, and the third which is the lump sum tax, T. The lump sum tax parameter, T, from the budget constraint is used in the calibration of our model. We believe that, in its steady state, the ratio G/Y should equal .165. We found this number by examining the time series for this ratio which becomes mean reverting and quite stable after 1953. We find the average of the ratio from 1953 to 2005 and use this as our steady state value, .165. Of the three taxes, the only open parameter is T which we set at .0118 so that the ratio G/Y is approximately equal to its steady state value found in the data. To solve the agents problem, we write the value function in the recursive form of Bellmans equation and find the two first order conditions. The first condition is the Euler equation:

$$\frac{1}{c_{t+j}} = \Gamma(1 + r_{t+j})\frac{1}{c_{t+j+1}} \tag{3.16}$$

which implies that consumption can be predicted optimally by its own lagged value. The

second condition is:

$$H_{t+j} = 1 - \mu\{c_{t+j}/s_{t+j}[1 - \tau_{t+j} - \alpha_{t+j}\beta_{t+j}[(1-\theta_{t+j})\phi_{t+j}(Y_{t+j} - \psi\frac{(K_{t+j} - K_{t+j-1})^2}{2K_{t+j}} - I_{t+j}) - \varepsilon_{t+j}(N_{t+j})]^{\beta_{t+j}-1}(1-\theta_{t+j})\phi_{t+j}]\} \tag{3.17}$$

which illustrates the agents optimal consumption leisure tradeoff. At the maximum the marginal rate of substitution between consumption and leisure is equal to the wage net of taxes.

3.3 The Government

The constraint facing the government is:

$$G_{t+j} = [Y_{t+j} - \psi\frac{(K_{t+j} - K_{t+j-1})^2}{2K_{t+j}} - I_{t+j}]\tau_{t+j} + \alpha_{t+j}[(1-\theta_{t+j})\phi_{t+j}(Y_{t+j} - \psi\frac{(K_{t+j} - K_{t+j-1})^2}{2K_{t+j}} - I_{t+j}) - \varepsilon_{t+j}(N_{t+j})]^{\beta_{t+j}} + T_{t+j} \tag{3.18}$$

With this construction, government spending, G, must equal government receipts forcing a balanced budget in each period. Government spending is exogenous. We understand that because we do not allow for government debt, this model is not able to answer many interesting questions, and we do intend to make this addition in future work.

3.4 The National Accounts Identity

Because the economy is closed, there is no current account equation. The standard national income equation, with the addition of the cost of capital adjustment term, ensures market clearing in our economy and is essentially our economys resource constraint:

$$Y_{t+j} = C_{t+j} + I_{t+j} + G_{t+j} + \psi\frac{(K_{t+j} - K_{t+j-1})^2}{2K_{t+j}} \tag{3.19}$$

3.5 The Simulation

In order to find the state space dynamics of this model, we used the perturbation techniques of Martin Uribe and Stephanie Schmitt-Grohe. I specifically follow the steps outlined by Martin Uribe in his Lectures in Open Economy Macroeconomics (2007). These steps can be found in Appendix B: Solving Dynamic General Equilibrium Models, of Chapter Four: The Real Business Cycle Model.

The equations from the model which are calibrated and used in the simulation create a system of nine nonlinear stochastic difference equations which includes: the two necessary conditions from the firms problem, the two necessary conditions from the households problem, the production function, the time path of our exogenous productivity shock, z, the governments budget constraint, the national income identity, and an equation which relates capital in different time periods. We, therefore, have a system of nine equilibrium conditions with nine unknown variables. These equations may be expressed as the following vector:

$$E_t f(y_{t+1}, y_t, x_{t+1}, x_t) = 0, \tag{3.20}$$

where E_t is the expectations operator given the information available at time t; y_t is the vector of control variables: C_t, H_t, s_t, Y_t, K_t, r_t, G_t; and x_t is the vector of state variables: K_{t-1} and z_t. The variables in the state vector may be further divided with x_1 denoting the endogenous state variable, which is k_{t-1}, and x_2 denoting the exogenous state variable, z_t. We know the functional form of f and all of the parameter values which appear in our equations. As there is no closed form solution to this type of problem, we approximate a solution with a perturbation technique that includes using the implicit function theorem and taking a first order Taylor series expansion. The solution to our system of equations is the following pair of functions:

$$y_t = g(x_{t+1}, \sigma) \tag{3.21}$$

$$x_{t+1} = h(x_t, \sigma) + \eta\sigma\varepsilon_{t+1} \tag{3.22}$$

The values within the parentheses of these functions explicitly tell us that theses solutions depend on the state variables and σ which is the scalar parameter, .006213, that determines the level of uncertainty in our model. The vector η contains all zeros except in

the position that corresponds to the exogenous state variable, x_2, where there is the value one. The scalar ε_{t+1} corresponds to ν in the process that our productivity shock follows and is independently and identically distributed as N(0,I). We find a local approximation for the g and h functions by taking a first order Taylor series expansion around the steady state values of our nine variables and around $\sigma = 0$. This lets our process begin at the steady state, which is a known solution of our system, and then permits uncertainty to increase over time. We use Newton's Method to solve for the steady state values of our variables. We were not able to solve for these values analytically due to the nonlinearities in our tax function. The expansions of the functions g and h evaluated at the steady states and at $\sigma = 0$ take the following forms:

$$g(x,\sigma) = g(\bar{x},0) + g_x(\bar{x},0)(x-\bar{x}) + g_\sigma(\bar{x},0)(\sigma-0) \tag{3.23}$$

$$h(x,\sigma) = h(\bar{x},0) + h_x(\bar{x},0)(x-\bar{x}) + h_\sigma(\bar{x},0)(\sigma-0) \tag{3.24}$$

where the bar over a variable indicates specifically the steady state value of that variable. Equations (2.21) and (2.22) evaluated at the steady states yield the following:

$$\bar{y} = g(\bar{x},0) \tag{3.25}$$

$$\bar{x} = h(\bar{x},0) \tag{3.26}$$

The first quantity in equations (2.23) and (2.24) can be replaced with $\bar{y}$ and $\bar{x}$, respectively. Uribe provides the following proof which explains why the third quantities in equations (2.23) and (2.24) are equal to zero. By substituting equations (2.21) and (2.22) into equation (2.20), we have:

$$F(x,\sigma) \equiv E_t f[g(h(x,\sigma) + \eta\sigma\varepsilon_{t+1}), g(x,\sigma), h(x,\sigma) + \eta\sigma\varepsilon_{t+1}, x] = 0 \tag{3.27}$$

We then know that for all values of x and σ, F(x,σ) equals zero. Therefore, the derivatives of any order of F must also equal zero which leaves:

$$F_\sigma(\bar{x},0) = 0 \tag{3.28}$$

By taking the derivative of (2.27) with respect to σ we have:

$$F_\sigma(\bar{x},0) = E_t\{f_{y_{t+1}}[g_x(h_\sigma + \eta\varepsilon_{t+1}) + g_\sigma] + f_y g_\sigma + f_{x_{t+1}}(h_\sigma + \eta\varepsilon_{t+1})\} \tag{3.29}$$

Because the expectation of ε_{t+1} is equal to zero, (2.29) can be rewritten as:

$$F_\sigma(\bar{x},0) = f_{y_{t+1}}[g_x h_\sigma + g_\sigma] + f_y g_\sigma + f_{x_{t+1}} h_\sigma \tag{3.30}$$

Given (2.28), writing (2.30) in matrix form gives the following:

$$[f_{y_{t+1}} g_x + f_{x_{t+1}} \quad f_{y_{t+1}} + f_y] \begin{bmatrix} h_\sigma \\ g_\sigma \end{bmatrix} = 0 \tag{3.31}$$

Because there is no singularity in our system, we have a unique solution and

$$h_\sigma = 0 \tag{3.32}$$

$$g_\sigma = 0 \tag{3.33}$$

By substituting (2.25) and (2.33) into (2.23) and substituting (2.26) and (2.32) into (2.24), we are then left with the following functions:

$$h(x,\sigma) = \bar{x} + h_x(\bar{x},0)(x - \bar{x}) \tag{3.34}$$

$$g(x,\sigma) = \bar{y} + g_x(\bar{x},0)(x - \bar{x}) \tag{3.35}$$

From (2.34) and (2.35) it is evident that, because we have a first order approximation where h_σ and g_σ equal zero, the size of uncertainty does not directly impact the state space dynamics of the system. This is the certainty equivalence issue which arises because the first order approximation ignores the higher moments which capture the amount of risk present and leaves the expected values of the endogenous variables equal to their non-stochastic steady state values. Schmitt-Grohe and Uribe (2004) believe that although first order approximations are not sufficient for answering problems that deal with risk premia or comparisons of welfare levels across different policy structures, they can appropriately answer problems dealing with local existence, determinacy of equilibrium, and the size of endogenous variables second moments. We believe that the first order approximations are

suitable for our study because we are examining how different tax structures affect our economy, not how uncertainty affects our economy. Finally substitute (2.35) and (2.34) into (2.21) and (2.22) respectively to obtain our solutions:

$$y_t = \bar{y} + g_x(\bar{x}, 0)(x_{t+1} - \bar{x}) \tag{3.36}$$

$$x_{t+1} = \bar{x} + h_x(\bar{x}, 0)(x_t - \bar{x}) + \eta\sigma\varepsilon_{t+1} \tag{3.37}$$

Therefore, the preceding two equations together with the equation describing the evolution of our productivity shock determine the linearized version of our model.

3.6 Experiments and Model Parameterization

We use the preceding solution to our system of equations and now vary certain parameter values in the household's income tax function, $\alpha_t[(1-\theta_t)\phi_t(Y_t) - \varepsilon_t(N_t)]^{\beta_t}$, and its proportional tax rate, τ_t. This allows us to compare the effects of different tax structures on our economy. We will center our discussion around changes in the α and β parameters as they hold the most meaning in our tax function. The other parameters present in this function simply correspond to the conversion of taxable income to Gross Domestic Product. The specific variables that we will be observing include: output, consumption, labor, capital, and government spending. To a lesser extent, we will also report results on the interest rate and the wage rate. First, we will compare the steady state values which result from different tax structures. Next, second moments generated by the simulation are reported and discussed. Finally, we examine these variables' behavior after a positive unit technology shock as they return to their steady states and use impulse response functions to illustrate these time paths. The parameters of our tax function and their computation are thoroughly described in Chapter 1. As only the tax parameters are altered, the remainder of the parameters are constant across experiments and are displayaed in Table 3.1.

We divide our experiments into two groups. Group One uses the entire set of tax parameters, as described in Table 3.2, from 2005 with the exception of letting α and β vary.

In this first group of experiments, we used several different values for both the α and β parameters. The 'small' value for each of these parameters is the smallest value historically seen, and the 'large' value is the largest. The 'average' quantity, not surprisingly,

is the average of all the values seen in the time series for these parameters. The specific values that correspond to these stylized descriptions are listed in Table 3.3. Due to the functional form of the income tax, when α equals zero, this tax is also equal to zero. Also because of the form, when β is equal to one, the income tax becomes a flat tax as the function loses all convexity. The combinations of α and β used are shown in Table 3.4. Also in this table, are the years when these particular combinations of α and β have been seen in history. These tests, in Group One, allow us to isolate the effects that α and β have on our economy.

The Group Two experiments use entire sets of tax parameters from certain years. The criterion that selected the particular years to be included was simply an evident and large movement in either the α or β parameters. These sets permit us to evaluate complete tax policies that have actually occurred in history. The specific years, and their parameters, included in this second group are displayed in Table 3.5.

3.7 Results

The implications of these different tax structures are measured by steady state values, second moments, and impulse responses. This section is arranged in that order with results of Group One experiments reported before those of Group Two. The steady state results for the Group One experiments are given in Table 3.6. We observe a number of expected relationships. Both α and β have a negative relationship with all of the variables except government spending; and with this variable, they both have a positive relationship. An increase in either α or β will increase the income tax. As this tax increases, our household

Table 3.1: Constant Parameter Values

Capital share in production, ω	0.36
Depreciation rate, δ	0.10
Coefficient of lagged value in shock process, ϱ	0.8145
Effective discount factor, Γ	0.961538
Weight on leisure in the utility function, μ	2.1092
Investment adjustment cost, ψ	0.50
Lump sum taxes, T	0.0118

Table 3.2: Group 1, Parameter Values from 2005

Aggregate deductions / AGI, θ	0.229378
AGI / GDP, ϕ	0.596956
Individual exemption amount, e /(GDP/pop)	0.055
Aggregate number of exemptions claimed, N / pop	0.9092
Proportional tax rate, τ	0.197

works less, has less income and consumes less. The substitution effect eclipses any wealth effect. Also, with less income, the household invests less; and, we therefore see capital falling as α and β increase. Because, in our model, government spending equals total tax revenue in every period, as income tax revenue increases so will government expenditure. Our results exhibit several other characteristics that are not as readily evident, and we discuss three. First, we see that the effect of a small α will dominate the effect of any sized β. A small α combined with: β equal to one, small β, and average β will return almost identical steady state values for each variable. Only when β is large does it have any effect on the steady states, and the effect is even then quite minor. Second, we see that β's effect increases as α grows. When we choose a fixed size for β then pair it with a small, average, and then large α we see a successively larger impact on all of the steady states. The change in steady states is nonlinear, and this holds for any size β. This result is consistent with what we would expect as β is the measure of convexity, or progressivity, in our tax function. Third, for most combinations of α and β, the steady states are not significantly different. We only see considerable changes in the steady state values from combinations where α is large and paired with any β and from an average α paired with a large β. Therefore, in general, and certainly for most combinations that we have seen historically, we do not observe large incentive effects from changes in α or β. Because the steady state values are

Table 3.3: Group 1, α and β Values

Size	α	β
Small	0.0000123	1.272363
Average	0.0080194	1.400436
Large	0.0192349	1.825267

Table 3.4: Group 1, Combinations of α and β

α	β	Description or Years Observed
0	-	no income tax
small	1	proportional income tax
large	1	proportional income tax
small	small	historically not seen
average	average	1948 to 1950
large	large	historically not seen
average	small	1993 to 2001
average	large	historically not seen
small	average	2002 to 2005
large	average	1943, 1952, and 1953
small	large	1924 to 1931
large	small	1964 to 1973, 1976, and 1977

not significantly different among our combinations of α and β - which include combinations where β equals one - we believe that for many experiments, using a proportional income tax will not return grossly inaccurate results. Therefore, unless specifically studying tax structure, using a proportional tax is sufficient.

Note, Table 3.6: Column Labels

The steady state groups in this table are labeled [x,x]. The (1,1) position corresponds to the α value, and the (1,2) position corresponds to the β value. This will hold for all following tables with Group One results.

However, when we look at our Group Two experiments, our conclusions are slightly different. We find that the different tax structures have some significant effects on the economy. In this set of experiments, we vary all six parameters in our tax function plus the proportional tax rate - as opposed to just two parameters, as in our Group One experiments. After inspection of the graphs that show the α and β parameters over time, we chose the years to include in our experiments. This determination was made by simply locating the years when large changes occured in either parameter. We will discuss the eight sets of years

Table 3.5: Group 2, Parameter Sets

Year	α	β	θ	ϕ	$\tilde{e}$	$\tilde{N}$	τ	Description
1916	0.000516	1.37818	0.245670	0.168345	7.376	0.0043	0.106	WWI
1919	0.000378	1.57455	0.114905	0.286560	1.125	0.1207	0.106	WWI
1923	0.000321	1.57189	0.152837	0.342478	1.701	0.1190	0.100	Mellon
1929	0.000012	1.82527	0.171844	0.289063	1.857	0.0620	0.080	Mellon
1936	0.000948	1.48496	0.120989	0.261198	2.803	0.0484	0.124	WWII
1941	0.012567	1.30559	0.091567	0.505960	1.294	0.2372	0.168	WWII
1940	0.001440	1.46718	0.111530	0.399578	1.983	0.1293	0.147	WWII
1943	0.019235	1.31038	0.073052	0.537689	0.623	0.4075	0.155	WWII
1949	0.009372	1.34530	0.317003	0.600725	0.334	0.8997	0.138	Korea
1952	0.016013	1.31481	0.266043	0.600865	0.262	0.9493	0.152	Korea
1963	0.010497	1.33997	0.220756	0.597018	1.183	0.9697	0.171	Kennedy
1964	0.018965	1.27439	0.213951	0.597739	1.173	0.9758	0.170	Kennedy
1987	0.010553	1.27688	0.218912	0.585257	0.097	0.8958	0.186	Reagan
1988	0.006252	1.32412	0.222494	0.604064	0.093	0.9056	0.188	Reagan
2001	0.007314	1.29630	0.221443	0.609262	0.064	0.8985	0.185	Bush
2002	0.001411	1.43454	0.230396	0.576296	0.066	0.8987	0.181	Bush

Table 3.6: Group 1, Steady State Values

Variable	[0,-]	[Small,Small]	[Average,Average]	[Large,Large]
Consumption	31.737	31.736	31.141	25.999
Labor	0.2901	0.2901	0.2889	0.2678
Wage	117.425	117.425	117.425	117.425
Output	53.224	53.224	52.997	49.127
Capital	136.861	136.860	136.279	126.326
Interest Rate	0.04	0.04	0.04	0.04
Govt Spending	7.801	7.801	8.229	10.495

Variable	[Small,1]	[Large,1]	[Small,Large]	[Large,Small]
Consumption	31.737	31.388	31.733	30.820
Labor	0.2901	0.2901	0.2901	0.2887
Wage	117.425	117.425	117.425	117.425
Output	53.224	53.223	53.221	52.966
Capital	136.861	136.858	136.855	136.197
Interest Rate	0.04	0.04	0.04	0.04
Govt Spending	7.801	8.150	7.803	8.526

Variable	[Average,Small]	[Average,Large]	[Small,Average]	[Large,Average]
Consumption	31.354	29.263	31.736	30.309
Labor	0.2895	0.2812	0.2901	0.2871
Wage	117.425	117.425	117.425	117.425
Output	53.118	51.595	53.224	52.669
Capital	136.588	132.672	136.860	135.435
Interest Rate	0.04	0.04	0.04	0.04
Govt Spending	8.105	9.065	7.801	8.817

that include the largest adjustments. Although these years were not chosen for historical meaning and only by studying the graphs, they include three wars: World War I, World War II, and Korea and four tax cuts: the Mellon tax cuts that occurred in the 1920's, and those of Kennedy, Reagan and Bush. The steady states for these years are reported in Table 3.7. Three interesting results are immediately evident. First, we see that the α and β parameters move in opposite directions during large changes in the tax structure. Although they both have a positive relationship with the amount of income tax, they have different effects on it. The α can be seen as a general measure of the level of the tax while the β parameter describes the progressivity of the tax. Therefore, policy makers may make

decisions based on trading off one parameter for another according to what their priorities are. Second, during the years before the 1940's, which include those covering WWI and the Mellon tax cuts, we do not observe much change in α. At first glance, this is unexpected given the role of α in our tax function. Because tax revenue is needed to fund war efforts and a tax cut would have the opposite effect on tax revenue, we would presume that α would incur a noticable change during these years around WWI and around the Mellon tax cuts. This, however is not the case, and can be explained by the fact that only a small percentage of the population was paying income tax at that time. To create a large change in tax revenues during those years, the β parameter was a more effective tool. The number of people paying income tax rose dramatically in the early 1940's (four million taxpayers in 1939 increases to forty-three million in 1945). In the years since the 1940's α has played a more dramatic role in tax policy changes. Third, it is interesting to note that not all tax cuts were created equally, in fact, one was not actually a reduction in taxes at all. Because we simply use the names of tax changes as given to them by the Treasury Department and other lawmakers, they might not actually agree with the data. A reduction in income tax on the whole, is signalled by a decrease in the α parameter, and a reduction in the progressivity of this tax is signalled by a decrease in the β. During the years of the Mellon tax cuts we see a barely perceptible decrease in α and a sizeable increase in β. Taxes in general may have been decreased, but their progressivity was greatly increased. Through the Reagan and Bush tax cut years, α drops (There is a significant drop here, much different from the Mellon tax cuts.) and β increases. Again, we see the overall tax decreased, but with an increase in the progressivity. Therefore, the result of these tax policies is that taxes are reduced, but they are reduced proportionally more for lower income households than for higher income households. This type of tax policy would have been unexpected from Reagan and Bush. However, Kennedy's tax cut is even more surprising. With this cut, we see an increase in α and a decrease in β. The parameters now move in opposite directions from the previous three tax cuts. This policy, in fact, was not a reduction in taxes at all. Not only did taxes increase, but they increased proportionately more for lower income tax payers.

From the specific steady state numbers in these years, again - in Table 3.7, we observe that α's effect eclipses that of β. This dominant α holds for all years other than the WWI years. These years are anomalous because only an exceptionally small percentage

Table 3.7: Group 2, Steady State Values

Variable	1916	1919	% Change	1923	1929	% Change
Consumption	35.328	35.314	-0.040	35.551	36.361	2.278
Labor	0.2901	0.2900	-0.034	0.2900	0.2901	0.034
Wage	117.425	117.425	0.000	117.425	117.425	0.000
Output	53.220	53.214	-0.011	53.213	53.221	0.015
Capital	136.852	136.834	-0.013	136.833	136.855	0.016
Interest Rate	0.04	0.04	0.00	0.04	0.04	0.00
Govt Spending	4.207	4.216	0.214	3.979	3.175	-20.206

Variable	1936	1941	% Change	1940	1943	% Change
Consumption	34.589	32.216	-6.861	33.622	32.241	-4.107
Labor	0.2900	0.2890	-0.345	0.2899	0.2881	-0.6209
Wage	117.425	117.425	0.000	117.425	117.425	0.000
Output	53.209	53.016	-0.363	53.186	52.865	-0.604
Capital	136.823	136.326	-0.363	136.762	135.938	-0.603
Interest Rate	0.04	0.04	0.00	0.04	0.04	0.00
Govt Spending	4.938	7.167	45.140	5.887	7.030	19.416

Variable	1949	1952	% Change	1963	1964	% Change
Consumption	33.580	32.683	-2.671	32.124	31.871	-0.788
Labor	0.2892	0.2887	-0.1729	0.2889	0.2887	-0.069
Wage	117.425	117.425	0.000	117.425	117.425	0.000
Output	53.062	52.965	-0.183	53.012	52.962	-0.094
Capital	136.444	136.194	-0.183	136.315	136.187	-0.094
Interest Rate	0.04	0.04	0.00	0.04	0.04	0.00
Govt Spending	5.838	6.662	14.114	7.256	7.472	2.977

Variable	1987	1988	% Change	2001	2002	% Change
Consumption	31.665	31.726	0.193	31.817	32.257	1.383
Labor	0.2893	0.2894	0.035	0.2895	0.2898	0.104
Wage	117.425	117.425	0.000	117.425	117.425	0.000
Output	53.082	53.107	0.047	53.108	53.180	0.136
Capital	136.495	136.561	0.048	136.563	136.747	0.135
Interest Rate	0.04	0.04	0.00	0.04	0.04	0.00
Govt Spending	7.767	7.725	-0.541	7.635	7.248	-5.069

of the population paid income tax at that time. In all of the other years examined, the variables move in directions which are the reverse of the actual direction that the changes in β would stipulate. If β increases (decreases) we expect all variables to decrease (increase) except for government spending which increases (decreases) because an increase in β raises the income tax. However, this does not occur. Because α and β move in opposite directions, they create opposing forces on the direction of the variables' change. The variables follow the direction as set by the change in α. As α increases, the steady state values of all our variables decrease, except government expenditure which increases. This result informs us that the α is the principle determinant of the variables' movements. We also observe the largest changes occuring in the consumption and government spending variables. Notably, during the war periods: consumption falls as government spending rises, and during the tax cuts (with one exception): consumption rises and government spending falls. This exception is the Kennedy tax cut in which consumption decreases and government spending actually increases.

Second moments also offer insight into the effects that the different tax structures have on our economy. The volatility of our variables will be described by standard deviations. These standard deviations are calculated over each variable's value in the 40 periods of the simulation which include 10,000 runs each. Tables 3.8, 3.9, 3.10, and 3.11 summarize the statistics. We see the usual volatility ranking with output's volatility greater than consumption's in each case. Also, as expected, consumption is procyclical. Hours are procyclical in every case except one. We, however, are not trying to match the actual moments of U.S. data but simply want to compare the consequences of different tax structures.

In the Group One experiments, an obvious pattern emerges from the volatility and correlation results. For each variable, three combinations of α and β produce numbers which stand apart from the others. The large α paired with the large β, the average α paired with the large β, and the large α paired with the average β, in that order and ascending, have the lowest volatilites for the consumption, labor, capital, output, and interest rate variables. They have the highest volatilities in that same, but descending order for the wage and government spending variables.

The volatility ranking is not as straight forward in the Group Two experiments, which is expected as more parameters values are being adjusted. There is one year, however, that is notable: 1943. The α and β for this year correspond respectively to the large and

Table 3.8: Group 1, Volatilities in 1000's

Variable	[0,-]	[Small,Small]	[Average,Average]	[Large,Large]
Consumption	0.3128	0.3128	0.3111	0.2763
Labor	0.0227	0.0227	0.0227	0.0251
Wage	0.3138	0.3138	0.3140	0.3250
Output	0.3272	0.3273	0.3262	0.3066
Capital	0.1368	0.1361	0.1361	0.1290
Interest Rate	0.6171	0.6171	0.6154	0.5765
Govt Spending	0.3122	0.3122	0.3133	0.3229

Variable	[Small,1]	[Large,1]	[Small,Large]	[Large,Small]
Consumption	0.3127	0.3130	0.3128	0.3111
Labor	0.0227	0.0227	0.0227	0.0226
Wage	0.3138	0.3138	0.3138	0.3146
Output	0.3273	0.3270	0.3273	0.3262
Capital	0.1371	0.1364	0.1363	0.1363
Interest Rate	0.6171	0.6172	0.6171	0.6153
Govt Spending	0.3122	0.3099	0.3122	0.3120

Variable	[Average,Small]	[Average,Large]	[Small,Average]	[Large,Average]
Consumption	0.3121	0.2980	0.3129	0.3087
Labor	0.0227	0.0229	0.0227	0.0226
Wage	0.3136	0.3183	0.3138	0.3148
Output	0.3269	0.3192	0.3273	0.3250
Capital	0.1363	0.1332	0.1368	0.1358
Interest Rate	0.6164	0.6013	0.6171	0.6128
Govt Spending	0.3122	0.3218	0.3122	0.3143

slightly less than average values from the Group One experiments. This year's capital and output variables have the lowest volatilities of their group. The picture becomes a bit more clear when looking at the percent change in standard deviation among the year pairings. The largest changes are seen between the following pairs of years: 1940 and 1943, 1949 and 1952, 1936 and 1941, and 1963 and 1964, with the most considerable changes occuring in the first two sets. As seen in Table 3.9, the labor, output, and government spending variables exhibit the largest changes in the year pairs ending with 1943 and 1952, while labor and government spending display large changes in the pair ending with year 1941, and government spending alone experiences a large change in the group ending with 1964. These results are anticipated as the parameter sets of 1943 and 1952 have α's and β's that would correspond respectively to the large and slightly less than average values of Group One. The 1964 α and β are equivalent to the large and small values of those parameters in the first group, and 1941's values correspond to the greater than average α and slightly less than average β.

We see similar responses in the correlations detailed in Tables 3.10 and 3.11. In the Group One experiments, the large α paired with large β returns the lowest correlations between consumption and output and between labor and output, with the latter correlation being unexpectedly negative. The next lowest correlations are produced by the average α and large β combination. The correlation between capital and output does not significantly fluctuate across the experiments. In Group Two, the only correlation to show considerable variance is that between labor and output. In ascending order, the lowest correlations occur in 1943, 1952, and 1964 which is consistent with the pattern that is emerging.

We now look at the effects that the different tax structures have on our economy as illustrated by impulse response functions. We again divide the discussion by the Group One and Group Two tests. Because the tax structures return very similar response functions we report results for the Group One experiments in one set of graphs and the Group Two results in a second set of graphs. We present the results from each group, as a whole, for each variable. This arrangement reveals exactly how similar, with few exceptions, the outcomes actually are. Figures 3.1 and 3.2 show the responses of each variable from Group One to a positive unit technology shock. Figures 3.3 and 3.4 show the responses of the variables from Group Two, again to a positive unit technology shock. The y-axis measures the percent deviation from each variables' steady state and the x-axis measures time in years. Because

Table 3.9: Group 2, Volatilities in 1000's

Variable	1916	1919	% Change	1923	1929	% Change
Consumption	0.3128	0.3127	-0.01232	0.3128	0.3127	-0.05666
Labor	0.0227	0.0227	0.0000	0.0227	0.0227	0.0000
Wage	0.3138	0.3138	0.0000	0.3138	0.3138	0.0000
Output	0.3273	0.3272	-0.02836	0.3273	0.3273	0.0000
Capital	0.1368	0.1365	-0.20579	0.1362	0.1361	-0.08657
Interest Rate	0.6171	0.6171	0.0000	0.6172	0.6171	-0.01756
Govt Spending	0.3117	0.3109	-0.20579	0.3106	0.3115	0.28898

Variable	1936	1941	% Change	1940	1943	% Change
Consumption	0.3128	0.3146	0.58971	0.3132	0.3139	0.22059
Labor	0.0227	0.0232	2.00189	0.0229	0.0227	-1.11717
Wage	0.3138	0.3155	0.54681	0.3142	0.3164	0.70795
Output	0.3273	0.3253	-0.58791	0.3271	0.3233	-1.16402
Capital	0.1361	0.1371	0.70341	0.1370	0.1357	-0.91776
Interest Rate	0.6172	0.6198	0.42586	0.6180	0.6176	-0.06648
Govt Spending	0.3104	0.2869	-7.56104	0.3082	0.2774	-10.00055

Variable	1949	1952	% Change	1963	1964	% Change
Consumption	0.3159	0.3139	-0.65404	0.3127	0.3128	0.04498
Labor	0.0250	0.0228	-8.93396	0.0226	0.0225	-0.33674
Wage	0.3165	0.3152	-0.42658	0.3150	0.3151	0.02412
Output	0.3284	0.3244	-1.22557	0.3253	0.3247	-0.19836
Capital	0.1373	0.1367	-0.47883	0.1363	0.1362	-0.11476
Interest Rate	0.6262	0.6180	-1.30217	0.6166	0.6164	-0.02651
Govt Spending	0.2908	0.2830	-2.67419	0.2988	0.2942	-1.55240

Variable	1987	1988	% Change	2001	2002	% Change
Consumption	0.3121	0.3121	0.0000	0.3120	0.3124	0.12282
Labor	0.0226	0.0227	0.11656	0.0227	0.0227	0.0000
Wage	0.3136	0.3136	0.0000	0.3136	0.3136	0.0000
Output	0.3264	0.3266	0.06999	0.3268	0.3269	0.05558
Capital	0.1362	0.1368	0.43749	0.1363	0.1368	0.38838
Interest Rate	0.6163	0.6164	0.01607	0.6163	0.6168	0.07761
Govt Spending	0.3089	0.3104	0.46341	0.3119	0.3122	0.09028

Table 3.10: Group 1, Correlations

	[0,-]	[Small,Small]	[Avg,Avg]	[Large,Large]
corr(c,y)	0.9764	0.9764	0.9764	0.9745
corr(m,y)	0.3082	0.3084	0.2849	-0.1702
corr(k,y)	0.7661	0.7687	0.7677	0.7682

	[1,Small]	[1,Large]	[Small,Large]	[Large,Small]
corr(c,y)	0.9764	0.9764	0.9764	0.9764
corr(m,y)	0.3080	0.3025	0.3078	0.2805
corr(k,y)	0.7637	0.7717	0.7665	0.7654

	[Avg,Small]	[Avg,Large]	[Small,Avg]	[Large,Avg]
corr(c,y)	0.9765	0.9756	0.9763	0.9764
corr(m,y)	0.2967	0.1108	0.3082	0.2506
corr(k,y)	0.7662	0.7667	0.7660	0.7674

Table 3.11: Group 2, Correlations

	1916	1919	1923	1929	1936	1941	1940	1943
corr(c,y)	0.9764	0.9764	0.9765	0.9764	0.9764	0.9763	0.9764	0.9765
corr(m,y)	0.3081	0.3067	0.3065	0.3082	0.3053	0.2461	0.3016	0.2071
corr(k,y)	0.7660	0.7663	0.7677	0.7660	0.7659	0.7687	0.7632	0.7648

	1949	1952	1963	1964	1987	1988	2001	2002
corr(c,y)	0.9759	0.9765	0.9764	0.9766	0.9764	0.9765	0.9765	0.9764
corr(m,y)	0.2800	0.2325	0.2600	0.2434	0.2859	0.2917	0.2951	0.3033
corr(k,y)	0.7663	0.7690	0.7642	0.7687	0.7692	0.7655	0.7654	0.7647

the paths of the variables are not greatly affected by the different tax structures, we discuss their shapes without differentiation. As labor is more productive after the shock, demand for it is increased. As the demand rises for labor, so does its price, the wage. We also see an initial increase in the marginal productivity of capital. But, as capital comes online and as the shock disipates, the productivity of capital decreases. Therefore we see capital's initial value higher, then it further increases, and finally the capital stock falls. The interest rate also responds to the shock with an opening jump, but then falls and overshoots its steady state. As the interest rate falls the household has less incentive to postpone consumption. Also, with a higher income, the household is able to consume more and pays higher taxes which lead to higher government expenditures. Labor, however begins to fall immediately after its initial jump. This decline overshoots the steady state; and therefore, labor must rise as it returns to steady state. Output's (income's) initial increase occured because of greater marginal productivity of inputs and greater use of inputs. But, it follows a steadily decreasing path back to its steady state as the shock dies out, productivity is decreased, and inputs are reduced. After their initial increases, wages, consumption, and government spending fall consistently as they return to their steady states.

In the Group One experiments, the varying values of α and β have no perceptible effect on the wage and government spending variables. There are only slight differences in the impulse responses of the other variables. The paths from the experiments with the combinations of: large α paired with large β and average α paired with large β are slightly different from the other combinations which move collectively. These two exceptions have paths for the: capital, consumption, and output variables, below the others. And, their responses for the interest rate variable are just above the others. Because these paths measure the deviations from steady state, these two combinations do not move as far from their steady states as the others do, with the large-large combination moving from its steady states the least. However, these two experiments have different effects on the labor variable. Again, their initial jumps are not as far from their steady states. But, as labor overshoots its steady state, these two experiments overshoot farther than the others. They, however, do converge on the other groups' paths well before they reach their steady states.

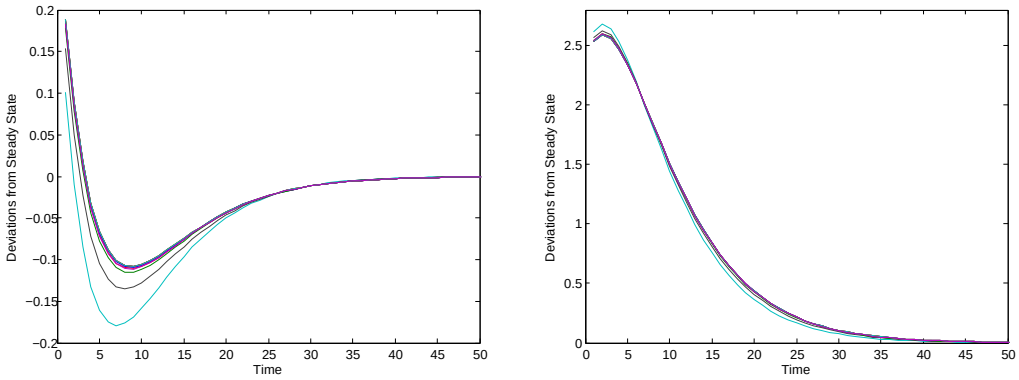
Only two variables in the Group Two experiments have impulse response functions that show differences according to the parameter sets used. All of the other variables: capital, consumption, output, wage, and the interest rate show no distinguishable changes

in the time paths generated from different parameter sets. Government spending and labor show only barely perceptible changes. The parameter sets that deviate from the balance of the group are from years: 1943, 1952, 1941, 1949, 1964, and 1963 and are described in Tabel 3.5. The α's in these years are either large or above average. They are paired with a slightly less than average β's in every case except one which includes a small β. The impulse responses for these specific years fall below the group, for the government spending variable, which means that they deviate less from their steady states. As in the Group One experiments, the labor variable in these exceptional years initially deviates less from its steady state, then exceeds the group in the amount of overshooting, and finally return to the same level of deviation as the other years before reaching its steady state.

3.8 Conclusion

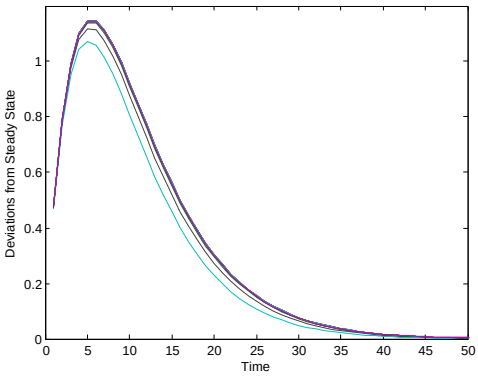
This chapter details the construction of our model, how we solved this model, and finally the experiments that we performed using it. The model is standard with the exception of the addition of our income tax function which was introduced in Chapter 1 and further described in Chapter 2. The experiments include changing, in certain combinations, the values of the parameters in this tax function. The results from this chapter support the results found in Chapter 2. Several noteworthy findings come from this chapter. We observe that α is the dominant parameter in our income tax function. It is this parameter's value, almost regardless of the other parameters' values, that governs the behavior of this tax. However, we do see β's effect increase as the α, that it is paired with, grows. Next, we find that the steady state values are not significantly different for most combinations of the tax parameter values. We only see large incentive effects when α is large and paired with a large or average β and when α is average and paired with a large β. These same three combinations of α and β produce volatilites different, and lower, from the other parameter value combinations. Also, the impulse response functions demonstrate how little the incentive effects are that emerge from these changes in the parameters. The only visible differences in these graphs come from the three combinations that stood out in the steady state and volatility experiments. Although these combinations have rarely been seen in history thus far, it is quite possible that our current government may enact changes in our tax system such as these in the near future. We believe that this type of model, with it's

more appropriate income tax function, would offer valuable insight into the effects of the structure of the income tax on the economy. Perhaps it could cause the government to think before it imposes a potentially harmful tax structure. With the data that details the history of our income tax from Chapter 2 and the tools for forecasting the effects of future changes in Chapter 3, there is no need for a change in our tax system to be made blindly.

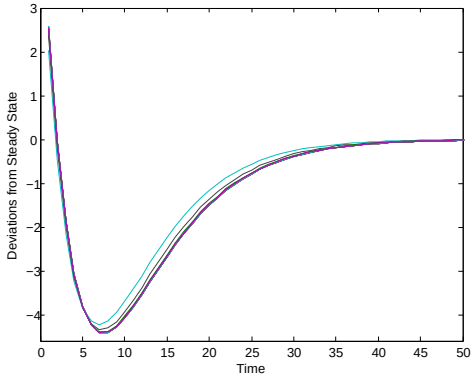


(a) Labor

(b) Wage

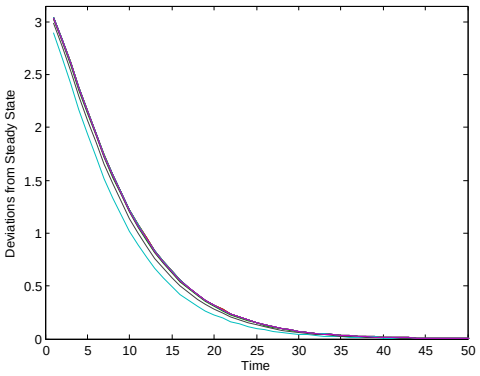


(c) Capital

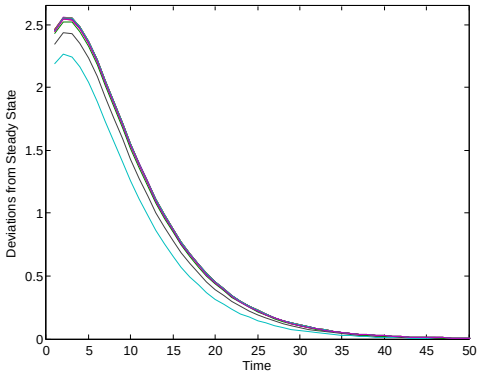


(d) Interest Rate

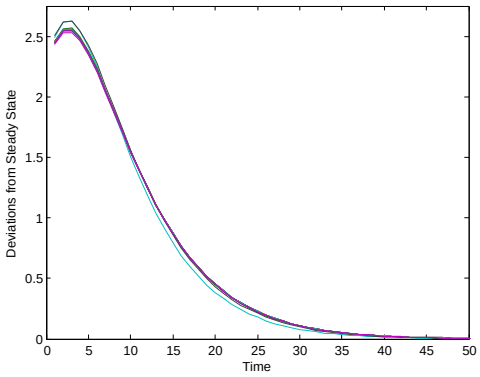
Figure 3.1: Impulse Response Functions, Group 1



(a) Output

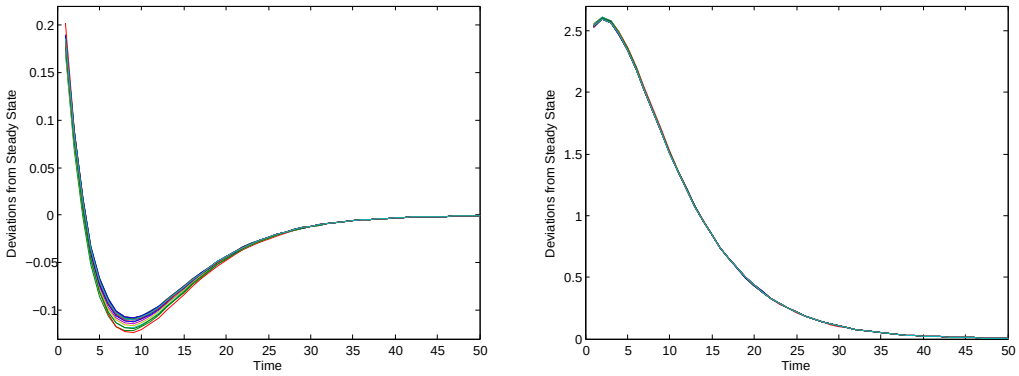


(b) Consumption



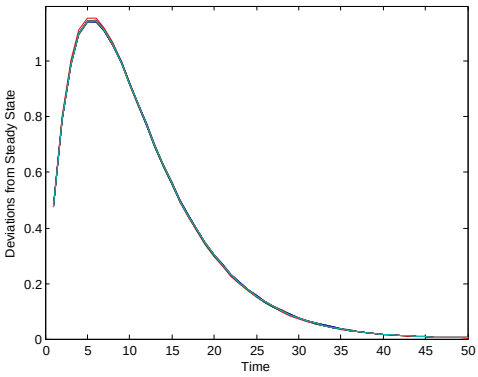
(c) Government Spending

Figure 3.2: Impulse Response Functions Continued, Group 1

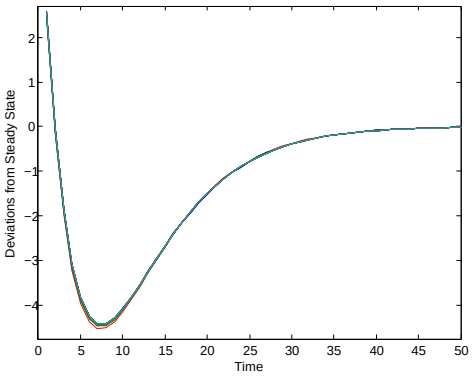


(a) Labor

(b) Wage

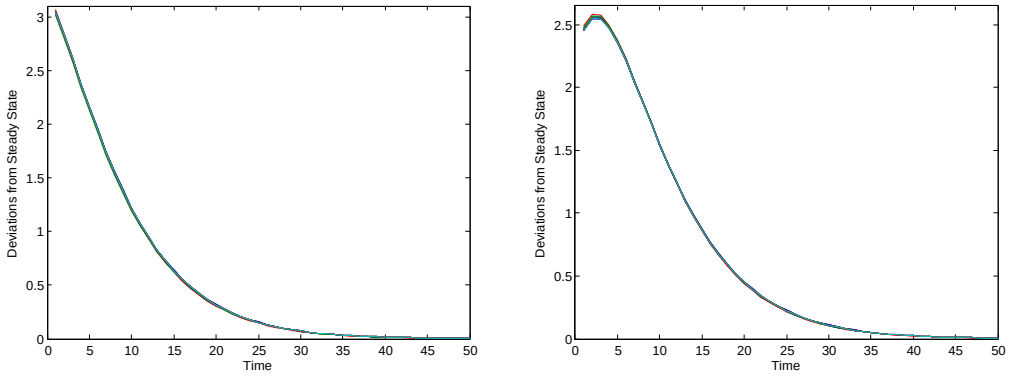


(c) Capital



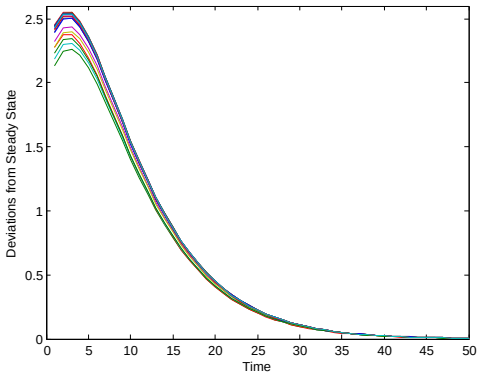
(d) Interest Rate

Figure 3.3: Impulse Response Functions, Group 2



(a) Output

(b) Consumption



(c) Government Spending

Figure 3.4: Impulse Response Functions Continued, Group 2

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APPENDIX

A Percentages of AMT Returns per Measure

Year	Number of Returns Using AMT	Amount of AMT Included in AGI	Amount of AMT Included in Total Tax Liability
1979	0.17%	0.06%	0.40%
1980	0.13%	0.05%	0.34%
1981	0.14%	0.07%	0.44%
1982	0.14%	0.06%	0.39%
1983	0.27%	0.13%	0.92%
1984	0.37%	0.21%	1.49%
1985	0.42%	0.16%	1.16%
1986	0.59%	0.27%	1.83%
1987	0.13%	0.06%	0.45%
1988	0.10%	0.03%	0.25%
1989	0.10%	0.03%	0.19%
1990	0.12%	0.02%	0.19%
1991	0.21%	0.04%	0.27%
1992	0.25%	0.04%	0.28%
1993	0.29%	0.06%	0.39%
1994	0.32%	0.06%	0.41%
1995	0.35%	0.05%	0.39%
1996	0.40%	0.06%	0.43%
1997	0.50%	0.08%	0.55%
1998	0.68%	0.09%	0.64%
1999	0.80%	0.11%	0.74%
2000	1.01%	0.15%	0.98%
2001	0.86%	0.11%	0.76%
2002	1.47%	0.11%	0.86%
2003	1.81%	0.15%	1.27%
2004	2.34%	0.19%	1.57%
2005	16.93%	0.57%	4.54%

B Values of Tax Parameters

Note for Appendix B: θ and ϕ for Years 1913-1915

As described in Chapter 1, data was unavalable for the calculation of θ and ϕ during the years 1913-1915. The structure of the function and the available data did, however, allow us to calculate these parameters as one singular quantity. The values for $[(1\text{-}\theta)\phi]$ are: 1913, .045845; 1914, .053040; 1915, .054806.

Year	α	β	θ	ϕ	e/(GDP/pop)	N/pop	τ
1913	0.000063	1.51483	-	-	8.922	0.0037	0.106
1914	0.000063	1.51483	-	-	9.743	0.0036	0.106
1915	0.000063	1.51483	-	-	9.323	0.0033	0.106
1916	0.000516	1.37818	0.245670	0.168345	7.376	0.0043	0.106
1917	0.000509	1.49228	0.093636	0.202295	1.358	0.0796	0.106
1918	0.001135	1.49107	0.102623	0.234113	1.069	0.1014	0.106
1919	0.000378	1.57455	0.114905	0.286560	1.125	0.1207	0.106
1920	0.000378	1.57455	0.110701	0.301926	0.945	0.1613	0.106
1921	0.000378	1.57455	0.160813	0.316967	1.328	0.1452	0.106
1922	0.000162	1.64896	0.142156	0.338854	1.325	0.1511	0.110
1923	0.000321	1.57189	0.152837	0.342478	1.701	0.1190	0.100
1924	0.000051	1.71995	0.132623	0.339988	1.595	0.1176	0.104
1925	0.000021	1.78589	0.133644	0.278941	2.078	0.0650	0.106
1926	0.000021	1.78589	0.137103	0.262345	1.909	0.0655	0.104
1927	0.000021	1.78589	0.139781	0.274435	2.067	0.0608	0.111
1928	0.000021	1.78589	0.129756	0.297614	1.999	0.0609	0.106
1929	0.000012	1.82527	0.171844	0.289063	1.857	0.0620	0.080
1930	0.000021	1.78589	0.191582	0.245751	2.193	0.0550	0.087
1931	0.000021	1.78589	0.212147	0.225731	3.667	0.0345	0.097
1932	0.000450	1.55167	0.190127	0.245180	2.568	0.0565	0.124
1933	0.000450	1.55167	0.178081	0.237479	2.660	0.0554	0.135
1934	0.000582	1.52926	0.152134	0.228681	3.608	0.0373	0.129
1935	0.000582	1.52926	0.138983	0.236242	3.227	0.0413	0.126
1936	0.000948	1.48496	0.120989	0.261198	2.803	0.0484	0.124
1937	0.000948	1.48496	0.131492	0.266095	2.559	0.0558	0.131
1938	0.000948	1.48496	0.162853	0.253984	2.792	0.0537	0.137
1939	0.000948	1.48496	0.130170	0.282482	2.584	0.0652	0.137
1940	0.001440	1.46718	0.111530	0.399578	1.983	0.1293	0.147
1941	0.012567	1.30559	0.091567	0.505960	1.294	0.2372	0.168
1942	0.011027	1.35260	0.089085	0.531548	0.782	0.3497	0.162
1943	0.019235	1.31038	0.073052	0.537689	0.623	0.4075	0.155
1944	0.015415	1.32929	0.207616	0.529868	0.302	0.8044	0.142

Table is Continued from Appendix B

Year	α	β	θ	ϕ	e/(GDP/pop)	N/pop	τ
1945	0.015415	1.32929	0.218864	0.537914	0.297	0.8324	0.141
1946	0.010388	1.35550	0.211223	0.603161	0.315	0.8700	0.148
1947	0.010388	1.35550	0.201438	0.613170	0.294	0.8859	0.146
1948	0.009403	1.34534	0.312704	0.607415	0.326	0.9098	0.139
1949	0.009372	1.34530	0.317003	0.600725	0.334	0.8997	0.138
1950	0.009879	1.34337	0.294017	0.609763	0.310	0.9106	0.161
1951	0.012539	1.33143	0.275522	0.596335	0.272	0.9367	0.162
1952	0.016013	1.31481	0.266043	0.600865	0.262	0.9493	0.152
1953	0.015991	1.31483	0.258086	0.602816	0.251	0.9669	0.152
1954	0.010688	1.33844	0.255158	0.602580	0.255	0.9538	0.148
1955	0.010602	1.33924	0.246878	0.599157	0.239	0.9693	0.155
1956	0.010661	1.33855	0.236628	0.611941	0.231	0.9737	0.156
1957	0.010632	1.33884	0.233166	0.607939	0.223	0.9768	0.155
1958	0.010664	1.33851	0.233935	0.601785	0.224	0.9618	0.151
1959	0.010685	1.33825	0.228710	0.602240	0.210	0.9673	0.160
1960	0.010663	1.33842	0.229051	0.599290	0.205	0.9703	0.165
1961	0.010533	1.33958	0.224934	0.605583	0.202	0.9664	0.166
1962	0.010491	1.34002	0.222254	0.595460	0.190	0.9641	0.166
1963	0.010497	1.33997	0.220756	0.597018	1.183	0.9697	0.171
1964	0.018965	1.27439	0.213951	0.597739	1.173	0.9758	0.170
1965	0.016591	1.27998	0.209845	0.596859	0.161	0.9817	0.167
1966	0.016661	1.27960	0.204487	0.594631	0.149	0.9919	0.170
1967	0.017140	1.28677	0.200335	0.606305	0.142	0.9966	0.171
1968	0.018971	1.27242	0.199296	0.609253	0.131	1.0029	0.179
1969	0.017727	1.28233	0.199935	0.612986	0.123	1.0174	0.181
1970	0.018405	1.27236	0.200431	0.608274	0.123	0.9955	0.177
1971	0.016978	1.27682	0.233161	0.597657	0.124	0.9820	0.179
1972	0.017010	1.27672	0.264043	0.602418	0.127	0.9850	0.183
1973	0.017128	1.27610	0.255623	0.598212	0.115	0.9898	0.189
1974	0.015808	1.28901	0.249432	0.603682	0.107	1.0056	0.189
1975	0.012025	1.30907	0.235477	0.578517	0.099	0.9825	0.181
1976	0.017324	1.27524	0.234980	0.577382	0.089	0.9896	0.186
1977	0.017376	1.27501	0.238388	0.570433	0.081	0.9770	0.185
1978	0.015784	1.28947	0.233623	0.567589	0.073	0.9878	0.183
1979	0.013694	1.29115	0.227214	0.571683	0.088	0.9984	0.179
1980	0.011882	1.30925	0.225564	0.578502	0.081	1.0009	0.177
1981	0.011880	1.30924	0.226316	0.566617	0.073	1.0055	0.179
1982	0.012927	1.28541	0.229317	0.569012	0.071	1.0000	0.173
1983	0.011888	1.28356	0.230963	0.549266	0.066	1.0004	0.176
1984	0.011469	1.28289	0.233461	0.544062	0.060	1.0192	0.179
1985	0.011500	1.28287	0.240566	0.546395	0.059	1.0240	0.179

Table is Continued from Appendix B

Year	α	β	θ	ϕ	e/(GDP/pop)	N/pop	τ
1986	0.010900	1.28675	0.246322	0.556082	0.058	1.0212	0.181
1987	0.010553	1.27688	0.218912	0.585257	0.097	0.8958	0.186
1988	0.006252	1.32412	0.222494	0.604064	0.093	0.9056	0.188
1989	0.006121	1.32508	0.227373	0.593749	0.090	0.9046	0.187
1990	0.006007	1.32540	0.231965	0.586829	0.088	0.9104	0.186
1991	0.005941	1.32482	0.236327	0.577816	0.078	0.9155	0.188
1992	0.005790	1.32569	0.233797	0.572626	0.079	0.9028	0.189
1993	0.007321	1.30210	0.234343	0.559278	0.078	0.9025	0.190
1994	0.007051	1.30485	0.227961	0.552518	0.077	0.8930	0.193
1995	0.007070	1.30384	0.224607	0.566305	0.075	0.9016	0.195
1996	0.006968	1.30451	0.220161	0.580278	0.072	0.8989	0.193
1997	0.006964	1.30382	0.213786	0.598479	0.069	0.9006	0.193
1998	0.006882	1.30418	0.209735	0.619181	0.066	0.8903	0.193
1999	0.006896	1.30359	0.205848	0.631767	0.063	0.8911	0.192
2000	0.006871	1.30351	0.203159	0.648403	0.059	0.8944	0.192
2001	0.007314	1.29630	0.221443	0.609262	0.064	0.8985	0.185
2002	0.001411	1.43454	0.230396	0.576296	0.066	0.8987	0.181
2003	0.001744	1.40885	0.234835	0.566301	0.064	0.8990	0.185
2004	0.001673	1.41166	0.229668	0.580940	0.059	0.9001	0.189
2005	0.001743	1.40738	0.229378	0.596956	0.055	0.9092	0.197

C Statistics from α and β Regressions

	logα		β	
Year	Standard Error	T-Value	Standard Error	T-Value
1913	0.29827	-32.42	0.02582	58.66
1914	0.29827	-32.42	0.02582	58.66
1915	0.29827	-32.42	0.02582	58.66
1916	0.26449	-28.62	0.02105	65.46
1917	0.18851	-40.23	0.01613	92.53
1918	0.09776	-69.37	0.00903	165.21
1919	0.12457	-63.26	0.01150	136.91
1920	0.12457	-63.26	0.01150	136.91
1921	0.12457	-63.26	0.01150	136.91
1922	0.09696	-90.03	0.00901	182.93
1923	0.08632	-93.19	0.00802	195.88
1924	0.19643	-50.31	0.01816	94.69
1925	0.10205	-105.57	0.00988	180.70
1926	0.10205	-105.57	0.00988	180.70
1927	0.10205	-105.57	0.00988	180.70
1928	0.10205	-105.57	0.00988	180.70
1929	0.22864	-49.45	0.02214	82.43
1930	0.10205	-105.57	0.00988	180.70
1931	0.10205	-105.57	0.00988	180.70
1932	0.17859	-43.15	0.01623	95.62
1933	0.17859	-43.15	0.01623	95.62
1934	0.22080	-33.74	0.02017	75.81
1935	0.22080	-33.74	0.02017	75.81
1936	0.28752	-24.21	0.02547	58.30
1937	0.28752	-24.21	0.02547	58.30
1938	0.28752	-24.21	0.02547	58.30
1939	0.28752	-24.21	0.02547	58.30
1940	0.37703	-17.35	0.03336	43.98
1941	0.24111	-18.15	0.02152	60.65
1942	0.11148	-40.43	0.01088	124.34
1943	0.10222	-38.65	0.00998	131.36
1944	0.11894	-35.08	0.01161	114.53
1945	0.11894	-35.08	0.01161	114.53
1946	0.12524	-36.47	0.01222	110.91
1947	0.12524	-36.47	0.01222	110.91
1948	0.09320	-50.07	0.00866	155.33
1949	0.09385	-49.76	0.00872	154.25
1950	0.09631	-47.94	0.00895	150.10

Table is Continued from Appendix C

	logα		β	
Year	Standard Error	T-Value	Standard Error	T-Value
1951	0.10548	-41.52	0.00986	135.02
1952	0.09937	-41.60	0.00929	141.52
1953	0.09958	-41.53	0.00931	141.23
1954	0.14114	-32.15	0.01320	101.44
1955	0.14281	-31.83	0.01335	100.31
1956	0.14237	-31.90	0.01331	100.58
1957	0.14274	-31.83	0.01335	100.33
1958	0.14257	-31.85	0.01333	100.43
1959	0.14207	-31.95	0.01328	100.76
1960	0.14288	-31.78	0.01336	100.21
1961	0.14372	-31.68	0.01344	99.71
1962	0.14430	-31.58	0.01349	99.34
1963	0.14466	-31.50	0.01352	99.09
1964	0.15445	-25.68	0.01500	84.98
1965	0.14893	-27.53	0.01457	87.86
1966	0.14859	-27.56	0.01454	88.03
1967	0.13085	-31.12	0.01280	100.56
1968	0.04103	-96.61	0.00481	264.51
1969	0.09476	-42.61	0.00968	132.54
1970	0.10642	-37.54	0.01083	117.44
1971	0.15102	-26.99	0.01478	86.41
1972	0.15101	-26.98	0.01478	86.41
1973	0.15054	-27.02	0.01473	86.64
1974	0.13956	-29.72	0.01365	94.40
1975	0.10664	-41.45	0.01043	125.46
1976	0.14859	-27.29	0.01454	87.72
1977	0.14666	-27.63	0.01435	88.86
1978	0.13928	-29.79	0.01363	94.62
1979	0.16508	-25.99	0.01674	77.11
1980	0.16687	-26.56	0.01693	77.35
1981	0.16697	-26.55	0.01694	77.31
1982	0.18517	-23.48	0.01949	65.95
1983	0.16999	-26.07	0.01754	73.16
1984	0.17482	-25.56	0.01789	71.71
1985	0.15342	-29.11	0.01569	81.74
1986	0.15890	-28.44	0.01606	80.12
1987	0.25055	-18.17	0.02563	49.82
1988	0.47891	-10.60	0.04602	28.78
1989	0.47915	-10.64	0.04586	28.89
1990	0.48359	-10.58	0.04608	28.76

Table is Continued from Appendix C

	logα		β	
Year	Standard Error	T-Value	Standard Error	T-Value
1991	0.50872	-10.08	0.04826	27.45
1992	0.51590	-9.99	0.04871	27.22
1993	0.20958	-23.46	0.01883	69.14
1994	0.21175	-23.40	0.01900	68.67
1995	0.21087	-23.48	0.01888	69.06
1996	0.21038	-23.61	0.01879	69.43
1997	0.21188	-23.44	0.01888	69.07
1998	0.21025	-23.68	0.01869	69.80
1999	0.20922	-23.79	0.01857	70.21
2000	0.20780	-23.97	0.01841	70.80
2001	0.20344	-24.17	0.01798	72.11
2002	0.32322	-20.31	0.02942	48.76
2003	0.31569	-20.12	0.02849	49.46
2004	0.32437	-19.71	0.02921	48.33
2005	0.31570	-19.27	0.02962	47.51

D Fit Statistic for α and β Regressions

Year	Adj R^2	Year	Adj R^2	Year	Adj R^2
1913	0.9985	1944	0.9983	1975	0.9977
1914	0.9985	1945	0.9983	1976	0.9952
1915	0.9985	1946	0.9982	1977	0.9953
1916	0.9970	1947	0.9982	1978	0.9959
1917	0.9977	1948	0.9984	1979	0.9948
1918	0.9980	1949	0.9984	1980	0.9948
1919	0.9971	1950	0.9983	1981	0.9948
1920	0.9971	1951	0.9980	1982	0.9941
1921	0.9971	1952	0.9982	1983	0.9946
1922	0.9986	1953	0.9982	1984	0.9942
1923	0.9988	1954	0.9965	1985	0.9943
1924	0.9954	1955	0.9964	1986	0.9944
1925	0.9994	1956	0.9965	1987	0.9944
1926	0.9994	1957	0.9964	1988	0.9916
1927	0.9994	1958	0.9964	1989	0.9917
1928	0.9994	1959	0.9965	1990	0.9916
1929	0.9969	1960	0.9964	1991	0.9908
1930	0.9994	1961	0.9964	1992	0.9906
1931	0.9994	1962	0.9964	1993	0.9973
1932	0.9942	1963	0.9963	1994	0.9973
1933	0.9942	1964	0.9948	1995	0.9973
1934	0.9952	1965	0.9952	1996	0.9973
1935	0.9952	1966	0.9952	1997	0.9973
1936	0.9910	1967	0.9964	1998	0.9973
1937	0.9910	1968	0.9978	1999	0.9974
1938	0.9910	1969	0.9974	2000	0.9974
1939	0.9910	1970	0.9967	2001	0.9975
1940	0.9852	1971	0.9951	2002	0.9933
1941	0.9919	1972	0.9951	2003	0.9939
1942	0.9986	1973	0.9951	2004	0.9936
1943	0.9987	1974	0.9959	2005	0.9934

E Data Construction from Chapter 2

A. Federal Government Expenditures

For years before the advent of national income accounting in 1929, data are available only in nominal units. Those are converted to real values using appropriate price indices.

1. G$ = nominal expenditures on goods and services. For 1913-1928, Table #185, Statistical Abstract 1930. Calendar year data obtained by averaging fiscal year data. Before 1919, the fiscal year value equals ordinary expenditures minus the sum of pensions, Veterans' Bureau expenditures (largely veterans' benefits), and interest on the debt. After 1928, data are from NIPA, Table 1.1.5, line 21.

2. GMil$ = nominal defense purchases. For 1913-1928, from Statistical Abstract's Historical Statistics, Table HS-51, "Defense Outlays, Current Dollars. Calendar year data obtained by averaging fiscal year data. For years after 1928, data are from NIPA, Table 1.1.5. line 22.

3. GCiv$ = nominal federal non-defense purchases. For 1913-1928, calculated as G$ minus GMil$. For years after 1928, data are from NIPA, Table 1.1.5, line 23.

4. TrP$ = nominal federal transfers to persons. Before 1919, TrP$ is taken to be Veterans' Bureau expenditures, cited above. In 1929, veterans programs were 5/6 of all public welfare programs, which also include Social Insurance, Health and Medical Programs, Education, and Other Social Welfare Programs. See Historical Statistics, Tables Bf197-203. Calendar year data obtained by averaging. After 1928, TrP$ is from NIPA, Table 3.2, line 23.

5. TrSL$ = nominal federal grants-in-aid to state and local governments. Before 1919, data are intermittent, necessitating interpolation. Instead of interpolating the available TrSL$ data directly, we interpolated the ratio TrSL$/G$ whenever possible and then multiplied the result by G$. This approach extracts some of the information in the correlation of TrSL$ and G$. Observations on TrSL$ are available for 1907, 1913, 1922, and 1927 in the Census Bureau's Historical Statistics of the United States, Colonial Times to 1970, Series Y654. For 1907-1912, TrSL$ was estimated by the ratio interpolation method just described. The effects of the First World War on TrSL$ are difficult to sort out, and we were forced to resort to somewhat arbitrary procedures. The TrSL$/G$ ratio changes its behavior between 1913 and 1922, but only partly because of increased military expenditures. When the U.S. actually entered the War in 1917, G$ rose abruptly. Clearly the pre-War relationship between G$ and TrSL$ was broken. However, some other change occurred by 1922, for the 1922 and subsequent values reported for TrSL$/G$ are much higher than before the War. On the assumption that no change in TrSL$ occurred before 1917, we simply used the 1913 ratio of TrSL$/G$ for 1914-16. For the years 1917-21, we abandoned the ratio altogether and directly interpolated between the estimated 1916 TrSL$ value and that for 1922. For 1923-26, we used ratio interpolation, except for 1925, for which we used actual values because of anomalous behavior in G$ in 1925 arising from liquidation of the War Finance Corporation and repayment of loans by the railroads. After 1927, we could find no fiscal year basis figures for TrSL$. We estimated a fiscal year value for 1930 by averaging the calendar year values for 1929 and 1930 in NIPA. This value provides the necessary endpoint to use ratio interpolation for 1928, giving a complete fiscal year series for 1913-28, which was converted to a calendar year basis by averaging. After 1928, data are from NIPA, 3.2, line 26.

6. Int$ = nominal federal interest payments to persons and business, $billions. For 1913-1928, data are from Historical Statistics, Table Ea648. The data for 1913-1928 are for fiscal years and include interest payments to other government agencies as well as to persons and business. Fiscal year data were converted to calendar years by averaging. To correct for the inclusion of irrelevant interest payments, we reduced the resulting series by multiplying it by the ratio of interest payments to persons and business in 1929, reported

in NIPA, to total interest in 1929, reported in Historical Statistics. After 1928, data are from NIPA, Table 3.2, line 28.

7. TrP = TrP$ deflated by PPCE (see below).

8. TrSL = TrSL$ deflated by PGSL (see below).

9. GMil = real military expenditures. For 1913-1928, data are from Kendrick (1961), Table A-I, column 5, converted from a 1919 base to a 2000 base using the overlap with NIPA data for 1929. After 1928, data are from obtained by deflating GMil$ by PGMil.

10. GCiv = GCiv$ deflated by PGCiv.

11. Int = Int$ deflated by PGNP.

B. Market Value of Government Debt

GDR = real market value of federal government debt outstanding. Construction is in two steps: construction of the nominal market value and then deflation by a price index.

1. Nominal market value.

For 1919-1975, the nominal market values are Seater's MVPRIV3 series (Journal of Monetary Economics, July 1981, Table 1), which is the market value of outstanding federal debt not held by any federal entity. The series was updated to 2005 and also extended back to 1913. The methods differ for the forward and backward extensions because of data availability.

For 1976-2005, the series was updated using the method presented by Butkiewicz (Journal of Monetary Economics, May 1983), which uses the formula MV = PV[(1+nc)/(1+nr)], where MV is the market value, PV is the par value, n is the average maturity of debt outstanding, c is the average coupon rate on debt outstanding, and r is the market yield for an instrument of maturity equal to the average maturity of federal debt. All data are for December of each year.

a. Par value PV is measured as total interest-bearing debt held by private investors, taken from the Treasury Bulletin's Table OFS1, Distribution of Federal Securities by Class of Investors and Type of Issues.

b. The average term to maturity n is measured as the Average Length series reported in the Treasury Bulletin's Table, Maturity Distribution and Average Length of Marketable Interest-Bearing Public Debt. The table's number varies over the years: FD4 for 1976-1978, FD7 for 1978-1982, and FD5 for 1983-2005.

c. For 1976-1982, the coupon rate c is the computed interest rate on total interest-bearing public debt, taken from the Treasury Bulletin's Table FD2, Computed Interest Charge and Computed Interest Rate on Interest-Bearing Public Debt. It is the weighted average of the coupon rates that would have been paid if all the issues outstanding at the end of a year were to remain outstanding for one more year. After 1982, the computed interest rate series was dropped from the Treasury Bulletin, so an approximation to it was constructed by dividing the total interest paid on the federal debt by the amount of debt outstanding. The total interest payment was obtained by summing the monthly payments reported for year in the Treasury Bulletin's Table FFO3, Budget Outlays by Agency. (It is not possible to base the computed rate on just the percentage December interest payment converted to an annual rate because the payments in June and December are always much larger than for other months, indicating a monthly seasonality.) The reported interest payment is the gross payment on all outstanding debt, including debt held by government entities. Therefore, it was divided by December value of the total outstanding interest-bearing federal debt rather than just debt held by private investors. The total outstanding interest-bearing debt is taken from the Treasury Bulletin's Table OFS1.

d. For 1976-1982, the market yield r is taken from the Treasury Bulletin's Tables MQ1, MQ2, and MQ3, which report market quotations for bills, notes, and bonds, respectively. The yield was taken to be that of the security with maturity closest to the average term to maturity of the outstanding debt. After 1982, tables MQ1, MQ2, and MQ3 were replaced by Table MBY1 (for 1983-1988) and MB1 (for 1983-2005). The new tables do not report the quotations for each outstanding security but instead report only

bid yields at constant maturities. The maturities are discrete, and usually there is not one that matches the average maturity of the outstanding debt. Consequently, an approximate yield for the average maturity was constructed by linearly interpolating between the two constant maturity yields that bracket the average maturity for that year.

For 1913-1918, the Treasury Bulletin did not exist, and the available data are not as detailed as for later years. Market values were approximated as follows.

a. The par value PV was taken from Historical Statistics of the United States, Earliest Times to the Present: Millennial Edition, series Ea654, for total outstanding interest-bearing debt. For these years, there seem to be no data on the amount of debt held by federal entities. However, Seater's Table 3 shows that there was virtually no such debt for 1919 and the years immediately after, suggesting the same for the period 1913-1918. Consequently there is very little inaccuracy introduced by not correcting for debt held by federal entities.

b. The average term to maturity n is estimated from data available in Table No.147 of the Federal Reserve System's Banking and Monetary Statistics, 1914-1941. The table reports the amounts of outstanding securities maturing within 5 years, 5 to 10 years, 10 to 20 years, and more than 20 years. In the "Within 5 years" category, it lists bills separately. The data start in 1916 and are for the end of June. Estimates for December of 1916-1918 were obtained by averaging the data for the two years straddling the December in question. The average maturity then was obtained by the formula

$$n = 0.5*\text{Bills} + 2.5*\text{LT5} + 7.5*\text{LT10} + 15*\text{LT20} + 25*\text{GT20}$$

where Bills, LT5, LT10, LT20, and GT20 are the amounts outstanding of bills and securities maturing in less than 5 years (excluding bills), more than 5 but less than 10 years, more than 10 years but less than 20 years, and more than 20 years, respectively. The numbers multiplying each amount of debt outstanding are the mid-points of the interval of maturity, taken as an approximation to the average term to maturity of each class of securities. For 1913-1915, the average maturity was assumed to be the same as in 1916. Almost the only debt outstanding was bonds and postal savings issues (92% of all debt in 1916), both long-term securities. Their amounts changed very little over 1913-1916. It was only with the onset of America's involvement in the First World War in 1917 that the volume and maturity structure of the debt began to change substantially. That suggests an approximate steady state in government financing before 1917, so that the value for 1916 should be a good approximation for the earlier years.

c. The average coupon rate c for the years 1916-1918 is the computed interest rate reported in series Ea661 of Historical Statistics of the United States, Earliest Times to the Present: Millennial Edition, the same type of series reported in the Treasury Bulletin before 1983 and used above. For 1913-1915, the coupon rate c was taken to be the value reported in Ea661 for 1916. That choice seems reasonable in light of the maturity structure of the federal debt for 1913-1915 and the history of interest rates for the preceding 30 years. Judging from the data for 1916, the outstanding debt before the First World War was almost entirely long-term bonds. There are no data on interest rates for federal debt before 1919, but series Cj1193 and Cj1194 of Historical Statistics of the United States, Earliest Times to the Present: Millennial Edition report municipal interest rates back to 1798. Comparison of the municipal rates with the federal bond rates in series Cj1192 of Historical Statistics shows the two to be nearly identical for the first five years after the federal series starts in 1919, so the municipal rates seem a good proxy for the federal rates. The municipal rates hardly changed at all over the 30 years before 1916. Thus it seems very likely that the coupon rates on federal bonds outstanding in 1913-1915 were essentially the same for all issues, which means they probably also were essentially the same as in 1916, justifying the use of the 1916 coupon rate as a proxy for the 1913-1915 rate.

d. The market yield r for 1913-1918 is measured as the rate on high-grade municipal bonds, reported in series Cj1193 of Historical Statistics.

2. Real market value

Finally, real values were obtained by deflating the nominal values by PG, the price index for federal expenditures (see below).

C. Price Indices

1. PGDP = GDP deflator, 2000 = 100. For 1913-28, data are Balke and Gordon's series, reported in Historical Statistics, Table Ca215, converted to a 2000 base using the overlap with NIPA data for 1929. After 1928, data are from NIPA, Table 1.1.4, line 1.

2. PGMil = deflator for federal defense purchases, 2000=100. For 1913-1928, the data are constructed by taking the ratio of nominal defense purchases from NIPA to Kendrick's real purchases and converting from a 1919 base to a 2000 base using the overlap with the PGMil series from NIPA in 1929. For years after 1928, data are from NIPA, Table 1.1.4, line 22.

3. PGCiv = deflator for federal non-defense purchases, 2000=100. For 1913-1928, set equal to PGMil because the two series are almost identical over the subsequent period 1929-2005, for which data are available on both. After 1928, data are from NIPA, Table 1.1.4, line 23.

4. PG = deflator for federal purchases, 2000=100. For 1913-1928, set equal to PGMil for same reason as in constructing PGCiv. After 1928, data are from NIPA, Table 3.9.4, line 6.

5. PGSL = deflator for state and local government purchases, 2000=100. For 1913-1928, the values of PGSL are obtained by regressing the growth rate of PGSL for 1929-2005 (available from NIPA, explained momentarily) on the growth rate of PGNP for the same years and using the result to backcast the levels of PGSL. Estimates obtained by regressing the level of PGSL on the levels of PGNP and PGMil and using the estimated coefficients for backcasting gave similar values to the growth rate method. After 1928, data are from NIPA, Table 1.1.4, line 24.

6. PPCE = deflator for personal consumption expenditures, 2000=100. For 1913-1928, PPCE is set equal to PGNP because the two series are almost identical over the subsequent period 1929-2005, for which data are available for both. For years after 1928, data are from NIPA, Table 1.1.4, line 2.

7. PGDP_HP = HP-filtered PGDP.

D. Other Aggregate Variables

1. GDP$ = nominal GDP. For years before 1929, data are from Balke and Gordon's estimates, reported in Historical Statistics, Table Ca214. For years after 1928, data are from NIPA, Table 1.1.5, line 1.

2. GDPR = real GDP, calculated as GDP$ deflated by PGDP.

3. GDPR_HP = HP-filtered GDPR.

E. Normal and Transitory Values

1. For all variables except military expenditure, normal variables are taken the be the Hodrick-Prescott filtered series.

2. The Hodrick-Prescott filter is inappropriate for military expenditure because it has large one-time shocks arising from unexpected wars that would induce artificial trends in the HP filtered data. Consequently, a composite approach is used to construct normal military expenditure, based on US military history before the Korean War and on the HP filter after that, when military expenditure ceases to have large one-time shocks.

A brief outline of US military history since 1913 will provide the background for our method of constructing normal military spending. The two World Wars clearly were unanticipated shocks. World War I was clearly viewed as a transitory affair. The US entered in early 1917, and the various armistices were signed in the late summer and fall of 1918. All fighting stopped by November 1918, but the peace treaty was not signed until the following year. When the war ended , US military spending almost immediately returned to its pre-war level. Military spending behaved differently after World War II. Although it dropped precipitously, it did not reach its pre-war level. Harry Truman saw the implications of Stalin's behavior much more clearly than Roosevelt had, and Cold War thinking first appeared among US policy makers with the death of Franklin Roosevelt and the ascension of Harry Truman to the presidency. Roosevelt gave little or no indication that he saw any need for substantial US military spending to continue after World War II ended, whereas Truman clearly saw the need for it. The Potsdam conference, at which Truman

represented the United States for the first time, marked a sharp change in the US attitude and posture toward the Soviet Union. The events in post-war Europe further clarified the situation for Truman, and he embarked on an active involvement of the US in world affairs. US military spending did fall greatly after the war, but that was because the war was a huge effort that did not have to be matched during the Cold War. Many soldiers and sailors were discharged, and a large fraction of the huge stock of military hardware was reduced to scrap. The outbreak of the Korean War guaranteed that US military spending stayed high for several decades. After the Korean War ended, the ratio of US military spending to GDP declined slowly for 30 years (even during the Vietnam War), reaching a trough in 1979. It rose in the last year of Jimmy Carter's presidency and continued to rise through Ronald Reagan's two terms until Reagan's last year in office, 1988, when it turned down. The downward trend continued thereafter.

In light of this history, we constructed normal military expenditure as a spline function comprising the following pieces:

a. After 1947, military expenditure was much smoother, even including the Korean War, than it had been across the two World Wars. Consequently, the Hodrick-Prescott filter is valid for the data, and normal military expenditure was taken to be the HP filtered series. We used the HP-filtered series for the entire sample. The HP filter is forward-looking, so it constructs "normal" values that are much too big before 1945 to be consistent with the history of what people expected before 1945. After 1945, however, people quickly caught on to the implications of the Soviet Union's behavior and increased their forecasts of US military activity abruptly, as if they learned what they "should" have known earlier. It turns out, however, to make no perceptible difference if one uses the HP-filtered series constructed from the full sample 1913-2005 or just from the sub-sample 1945-2005. The values for 1947 for the two series are almost identical.

b. Before 1941, military spending was a tiny fraction of GDP except for the years surrounding World War I, when there was a hump in military spending. In 1913, the ratio of military purchases to GDP was 0.009. In 1914, the first year of the war but not of US involvement, it rose to 0.014 and stayed about there through 1916. In 1917, the ratio rose to 0.069, then to 0.246 in 1918. The ratio then began to fall, dropping to 0.147 in 1919 (even though the war had effectively ended the previous year), to 0.036 in 1920, and then gradually to 0.011 in 1926 and 1927. Thus it took 10 years for spending to return to its pre-war level, even though the US was involved in the war for only about 19 months. The average value for the years 1913-1916 plus 1923-1927 was 0.012, and that value was taken to be the normal ratio of military spending to GDP over the period 1913-1940. The actual ratio was somewhat higher than 0.012 in the 1930s, but that was mostly because the denominator (GDP) was unusually low because of the Great Depression, itself definitely a transitory event. The normal level of military spending for each year in the period 1913-1940 was computed simply as 0.012*GDP.

c. The US entered World War II in early December of 1941, but its military spending had increased markedly in 1940, when the ratio of military spending to GDP rose to 0.028 from 0.018 the previous year. It then rose dramatically, going to 0.130 in 1941, 0.401 in 1942, 0.562 in 1943, and peaking at 0.593 in 1944. In 1945, it fell slightly to 0.517 (hardly down from the previous year, even though the war ended in August), then substantially to 0.139 in 1946 (even though the war ended the previous year), and then reaching 0.095 in 1947, which is about where it stayed until the Korean War. As mentioned earlier, Roosevelt had a much more limited vision of what US involvement in world affairs would be after the war ended than Truman did. However, even Roosevelt clearly began to change his views of Stalin and Soviet intentions as the war progressed. In light of these data and historical facts, the normal value of military spending from 1941 to 1945 was generated with the equation

$$\mathrm{GMil}_t = \mathrm{W1exp[W2(t\text{-}1941)]}$$

where t is the year, W1=0.012*GDP1941, the value of normal military purchases as constructed in part (b) above, and W2=0.51953, the slope necessary to make GMil equal

to the HP filtered value of military expenditure in 1945.