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Dynamic Design of Piping Systems

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Abstract

A methodology for design of piping systems and their supports/restraints to satisfactorily perform under normal, expected, and postulated dynamic loads is presented. The methodology relies on a synthesis process utilizing preanalyzed equivalent piping elements such as straight spans, spans with concentrated weights, cantilever spans and elbow spans, and a set of procedural rules to construct the piping system. The application of the methodology is illustrated for the case of seismic excitations, so chosen because the present methodology can be directly compared with previously proposed and used simplified dynamic analyses methods. Comparison of this methodology with those methods indicates that it more closely predicts the dynamic behavior of piping systems as determined by detailed computer analyses.

1. Introduction

A typical nuclear power plant piping exceeds several miles in length, most of which, by international practice is usually categorized as Seismic Category I, II or III, or falls in the category of high energy piping. Design and analyses of nuclear piping is a major time consuming task particularly for the above categories, and generally requires iterative design verified by rigorous dynamic analyses performed by especially trained personnel. The further requirement that the vibratory motions of piping systems in the above categories be verified by test during the preoperational phases of the plant (1) imposes considerations in the piping design which are generally not recognized in the present design methods; such as vibrations caused by flow or pressure pulsations, and effects of supports/restraints with stiffness and/or location different than that used in the design.

Simplfied, approximate yet conservative methods are clearly preferable to the iterative design-sophisticated dynamic analysis process normally used, and in particular such methods should be capable of applicability to those loads which are normally not analyzed in the design process, so that recognition of those loads at the design stage will minimize the need for design changes during preoperational and start-up tests.

Several simplified methods have been suggested and used (2, 3, 4, 5). To date they concentrate on seismic design. All of these methods have varying complexity, but they are similar in assuming that piping seismic supports are rigid, an assumption which is not justifiable in many applications. A method proposed by N. Pal (6) overcomes the deficiency of the previous methods by permitting recognition of the potentially substantial effect of support/restraint flexibility. The latter method, just as all of the others, proceeds from an existing piping routing (normally predicated on thermal expansion requirements) and spaces the supports/restraints so that for the assumed stiffness of the restraint, the piping nowhere exceeds the allowable stresses. All of these methods however suffer by their lack of consideration of interaction effects between spans in the piping system.

The present approach is intended to overcome this limitation and to extend the utilization of simplified methods to the investigation of dynamic loads different from seismic. The central idea of the method is derived from the electrical network synthesis work of Brune (7). The method is thus predicated on the thesis that a complex piping network can be synthesized from the assemblage of elements having prescribed mechanical dynamic behavior by application of suitable rules. The manner in which the mechanical elements can be assembled to produce a piping/support system having the desired properties is not unique. For a given piping routing, however, the synthesis method can yield the minimum number of elements satisfying the imposed constraints.

In the present approach, the designer considers the physical constraints of the spaces in which the piping system is routed and constructs the piping/support network by utilizing elements of known dynamic properties together with a prescribed set of rules which ensure that the overall network will meet the required dynamic responses. In this sense the engineer designs the piping system truly dynamically. This method is considered to be reasonably conservative and to be applicable for (a) final seismic design of field run piping or small piping, (b) preliminary dynamic design of piping for flow or pressure-induced vibrations, and (c) preliminary seismic design of main piping. The method is not limited to seismic design, however for purposes of illustration this paper presents its application to seismic design.

2. Theory of the Method

Successful use of synthesis requires that the elements developed for the synthesis process be compatible with the general approach followed for detailed dynamic analyses, so that the synthesis process will produce results as close as possible to those predicted by the detailed analyses. The development of the elements and the methodology employs straightforward modal analyses methods.

The response of any piping system to an arbitrary dynamic excitation, expressed by its flexural deflection, $y(x,t)$ at any point, x and at any time, t is given by

$$y(x,t) = \sum_n A_n(t)\, \phi_n(x) \tag{1}$$

where $\phi_n(x)$ is the characteristic shape of the n^{th} mode of vibration.

The modal amplitude of the response at any time t, $A_n(t)$ can be written as

$$A_n(t) = D_n(t)\, P_n \tag{2}$$

where $D_n(t)$ and P_n are the time-dependent dynamic load factor and the modal participation factor for the n^{th} vibratory mode, which are defined as

$$\frac{d^2 D_n(t)}{dt^2} + 2\beta \frac{dD_n(t)}{dt} + \omega_n^2 D_n(t) = \psi(t) \tag{3}$$

and

$$P_n = \frac{\int_o^\ell f(x)\, \phi_n(x)\, dx + \sum_k F_k\, \delta(x-x_k)\, \phi_n(x)}{\int_o^\ell m\phi_n^2(x)\, dx + \sum_k M_k\, \delta(x-x_k)\, \phi_n^2(x)} \tag{4}$$

The functions $\psi(t)$, $f(x)$ and F_k are defined by arbitrary distributed and local dynamic force $F(x,t)$

$$F(x,t) = \psi(t) \left[f(x) + \sum_k F_k\, \delta(x-x_k) \right] \tag{5}$$

In the particular case of seismic excitation it can be shown that P_n is reducible to the following form

$$P_n = g \frac{\int_o^\ell m\phi_n^2\, dx + \sum_k M_k\, \delta(x-x_k)\, \phi_n(x)}{\int_o^\ell m\phi_n^2\, dx + \sum_k M_k\, \delta(x-x_k)\, \phi_n^2(x)} \tag{6}$$

In equations (3) and (6), m is the piping distributed mass, M_k is any k^{th} localized (lumped) mass, δ is the Kroenecker delta function, ω_n is the frequency of the n^{th} mode, and β is the damping coefficient generally prescribed by regulation (8).

The design process requires that the largest value of $D_n(t)$, D_{max}, be known as a function of frequency and damping. For seismic design, for instance D_{max} is the response spectra curve for a given damping (8). The modal frequencies, chapes and participation factors have to be determined. In particular the participation factors are a function of the number of spans in the piping system, their relative lengths, the restraint parameters, the overall boundary conditions, and the forcing functions.

The limit in application of the proposed method is inherent in the definition of the modal participation factor. For instance, its form as gvein in equation (6) is valid only in the case of uniform seismic exictation of the piping segment; this means that it is assured that there are no phase shifts between the exciting functions at the supports within a multi-supported segment, an assumption which is generally valid within legs of piping systems. Within this limitation, different support stiffnesses and unequal lengths of the span within a leg can be considered. However, the synthesis process is considerably simplified by assuming that all elements in the system are equivalent; i.e., straight spans have identical length and are supported by the equal stiffness restraints. Dynamic equivalency denotes that the first natural frequency of the different elements are equal. The equivalent elements form the element library for the synthesis process.

It can be shown that for a multi-supported piping segment consisting of k equivalent elements, the seismic excitation will result in k non-negligible participation factors. Their value for all of the modes required to simulate the response of a piping segment with a given number of elements can be precalculated as a function of the support stiffness and the boundary conditions of the segment using detailed modal analyses techniques.

From the knowledge of the modal participation factor $P_n^{(k)}$ for the given number of spans, the normalized stress factor $\alpha_n^{(k)}$ is obtained from

$$\alpha_n^{(k)} = \frac{\sigma_{max,n}^{(k)} \; P_n^{(k)}}{\sigma_s} \tag{7}$$

$$\sigma_s = 0.064 \, \frac{w \, D\ell^2}{I} \tag{8}$$

$\sigma_{max,n}^{(k)}$ is the maximum stress for the k-span pipe due to the n^{th} mode which is obtained from the detailed analyses and σ_s is the maximum stress for a simply supported single span pipe computed for a unit seismic acceleration, w is the distributed weight, ℓ the single span length, D the pipe outer diameter and I its moment of inertia.

Similarly, a normalized restraint force factor, $\beta_n^{(k)}$ can be obtained from the precalculated maximum restraint motions $\Delta_{max,n}^{(k)}$ for the k-span pipe, the corresponding participation factors, and the maximum reaction for a simple-supported single span pipe V_s

$$\beta_n^{(k)} = \frac{K\Delta_{max,n}^{(k)} \; P_n^{(k)}}{V_s} \tag{9}$$

$$V_s = 0.4\, w\ell \qquad (10)$$

wherein K is the restraint axial stiffness.

Finally, normalized frequency factors $\gamma_n^{(k)}$ are computed from

$$\gamma_n^{(k)} = \frac{\omega_n^{(k)}}{\omega_s} \qquad (11)$$

$$\omega_s = \frac{\pi}{2\ell^2}\sqrt{\frac{EI}{m}} \qquad (12)$$

where $\omega_n^{(k)}$ is the n^{th} natural frequency for the k-span segment mode, ω_s is the fundamental frequency of a simply-supported single span beam and E is Young's modulus of elasticity. Tables 1, 2 and 3 show the values of the normalized stress, reaction and frequency factors for the required number of modes for piping segments of up to six spans, and two different sets of boundary conditions applied to the ends of the piping segment.

Different types of forcing functions, unequal restraints stiffness, and different segment boundary conditions engendered similar sets of tables. One can readily see from these example tables that the representation of multi-supported systems by one span model can lead to substantially incorrect representation of dynamic system behavior.

3. Illustrative Seismic Design Procedure

The following illustrates how the method is used to permit simplified seismic design. It is presumed that the following previously defined parameters are known: D, w, I, E, and M_k. Also the design response spectrum for a given damping, minimum allowable frequency, ω_m, and allowable stress, σ_a must be known. The procedure then requires the following number of steps.

(a) Establish the preliminary equivalent span length, ℓ from

$$\ell^2 = \frac{\pi}{2\omega_m}\sqrt{\frac{EI}{m}} \qquad (13)$$

(b) Obtain the number of spans for a given segment of pipe, at length L, representing a portion of the piping system which can be isolated from the remainder by suitable boundary conditions

$$n = \text{number of spans} = \frac{L}{\ell} \qquad (14)$$

(c) Select the boundary conditions most representative of the segment, L.

(d) Obtain $\alpha_n^{(k)}$ from a suitably expanded (for more than six spans), Table 1.

(e) Compute the natural frequencies of the modes with non-zero participation factor using Table 2 and

$$\omega_n^{(k)} = \gamma_n^{(k)}\, \omega_m \qquad (15)$$

(f) Determine $D_n^{(k)}$ from the design response spectrum curve, D_n

$$D_n = D_n\ (\omega_n^{(k)}) \tag{16}$$

(g) Calculate the maximum stress σ_{max} from

$$\sigma_{max} = \sqrt{\left[\sum_n (\alpha_n^{(k)}\ D_n)^2\right]}\ \sigma_s = \bar{\sigma}\ell^2 \tag{17}$$

If $\sigma_{max} > \sigma_a$, then a new value of ℓ is computed from

$$\ell = \sqrt{\frac{\sigma_a}{\bar{\sigma}}}$$

and steps (b) through (g) are repeated. If $\sigma_{max} < \sigma_a$, then the procedure continues through the following steps.

(h) Obtain the normalized restraint force factor $\beta_n^{(k)}$ from Table 3, suitably expanded to cover more spans and modes, and compute the maximum restraint force, V_{max} from

$$V_{max} = \sqrt{\sum_n (\gamma_n^{(k)}\ D_n^{(k)})^2}\ \ V_s \tag{18}$$

(i) Compute the required restraint/support stiffness, K from

$$K > \Xi\ \frac{EI}{\ell^3} \tag{19}$$

where Ξ is a dimensionless restraint stiffness which in Tables 1, 2, and 3 is taken to be equal to or higher than 10^3, meaning that the restraints are rigid in the practical sense. The values of the tables would of course change for non-rigid restraints.

(j) Calculate the maximum displacement, δ_m if it is suspected that the displacement may be limiting from

$$\delta_m = \sqrt{\sum_n (\xi_n^{(k)}\ D_n)^2}\ \ .013\ \frac{w\ell^4}{EI} \tag{20}$$

where $\xi_n^{(k)}$ would be a normalized displacement table similar to the preceding tables, derived in an equal manner.

The procedure and tables are used together with an element library comprising typical piping elements consisting of straight spans, elbows, spans with concentrated masses, cantilevers, Ts and Ys. Figures 1 and 3 permit determination of the equivalent span for straight and cantilever spans with maximum bending stress and deflection. Figure 2 for instance illustrates the effect of concentrated mass on the maximum bending stress, deflection and restraint force.

Results of these figures are normalized relative to the results of the span without concentrated mass. One of the unexpected results of this development is that the span length normalized in such fashion is virtually independent of whether the span is assumed to be fixed at both ends of simply supported.

4. Comparison with Other Methods

To demonstrate the advantages of the proposed methodology, the method is compared to that of N. Pal (6) and to accurate computer generated solution of the problem analysed in Reference (6). The problem consists of a five span uniaxial beam of constant span length.

Table 4 compares the results of the proposed methodology with those obtained by Pal (6). Results are presented for both maximum bending moments and support reactions as a function of span length and rigid restraints. The regid restraint case is chosen since the simplified technique gives less reliable results for this case. Results from both methodologies are normalized to the maxima as computed by accurate computer solution of the same problem; the value given in the table being the ratio of the accurate solution and the approximate solution. Both methodologies conservatively predict bending moments higher than computer by accurate analysis. However, the proposed methodology is considerably closer to the accurate solution. The single span method of Pal, however, considerably overpredicts the correct value of the maxmium reaction force; whereas, the present method is successful in predicting values which closely approximate the correct value.

5. Conclusions

The methodology developed herein is general and can be applied to the preliminary design of piping systems subject to arbitrary dynamic forces, and in particular to seismic design. For the latter, the behavior of multi-supported piping, as modelled by multi-supported beams, is substantially different from that predictable from the modelling as a one-span beam, and this must be recognized at the preliminary design stage. The present form of the methodology permits a designer to synthesize the piping system from a library of piping elements whose dynamic properties are given in a set of charts by application of a set of procedural rules.

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TABLE 1
NORMALIZED STRESS FACTORS - $\alpha_n^{(k)}$

MODES / SPANS	HINGED-HINGED BOUNDARY						FIXED-HINGED BOUNDARY					
	1	2	3	4	5	6	1	2	3	4	5	6
1	1.	–	–	–	–	–	.77	–	–	–	–	–
2	0.0	0.77	–	–	–	–	.41	.77	–	–	–	–
3	0.34	0	0.6	–	–	–	.306	.29	.76	–	–	–
4	0.0	0.41	0.0	0.77	–	–	.24	.167	.27	.76	–	–
5	0.2	0	0.28	0	0.7	–	.19	.15	.17	.26	.76	–
6	0.0	0.306	0.0	0.29	0.0	0.76	.166	.136	.131	.164	.258	.76

TABLE 2
NORMALIZED RESTRAINT FORCE FACTORS - $\beta_n^{(k)}$

MODES / SPANS	HINGED-HINGED BOUNDARY						FIXED-HINGED BOUNDARY					
	1	2	3	4	5	6	1	2	3	4	5	6
1	1.	–	–	–	–	–	1.83	–	–	–	–	–
2	0.	1.83	–	–	–	–	0.435	1.44	–	–	–	–
3	0.28	0	1.54	–	–	–	.27	.69	1.79	–	–	–
4	0.0	0.41	0.0	1.44	–	–	.21	.376	.63	1.93	–	–
5	0.2	0.0	0.63	0.0	1.88	–	.17	.27	.41	.65	1.69	–
6	0.	0.306	0	0.29	0.0	1.79	.144	.204	.255	.393	.683	2.03

TABLE 3
NORMALIZED NATURAL FREQUENCIES - $\gamma_n^{(k)}$

MODES / SPANS	HINGED-HINGED BOUNDARY						FIXED-HINGED BOUNDARY					
	1	2	3	4	5	6	1	2	3	4	5	6
1	1.0	–	–	–	–	–	1.56	–	–	–	–	–
2	1.0	1.56	–	–	–	–	1.17	2.02	–	–	–	–
3	1.0	1.28	1.87	–	–	–	1.08	1.56	2.13	–	–	–
4	1.0	1.17	1.56	2.01	–	–	1.04	1.34	1.8	2.19	–	–
5	1.0	1.1	1.39	1.74	2.1	–	1.04	1.23	1.56	1.93	2.22	–
6	1.0	1.08	1.27	1.56	1.91	2.13	1.02	1.17	1.42	1.71	2.01	2.22

TABLE 4
NORMALIZED SEISMIC RESPONSE OF CONTINUOUS FIVE SPAN UNIAXIAL PIPE

SPAN LENGTH (FT)	MAXIMUM SUPPORT REACTION		MAXIMUM SEISMIC MOMENT	
	a)	b)	a)	b)
20	0.47	0.96	0.39	0.89
30	0.32	0.88	0.29	0.88
40	0.87	1.01	0.73	0.86

a) DETAILED ANALYSIS/ONE SPAN METHOD OF PAL (6)
b) DETAILED ANALYSIS/PRESENT METHOD

FIGURE 1
RELATIVE SINGLE SPAN LENGTH FOR SPAN WITH CONCENTRATED MASS

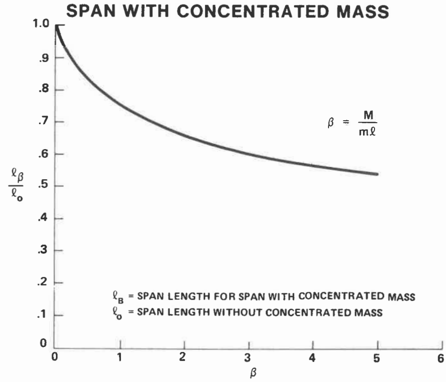

FIGURE 2
EFFECT OF CONCENTRATED MASS ON MAX MOMENT, M max, RESTRAINT FORCE, Vmax, AND DEFLECTION δ max

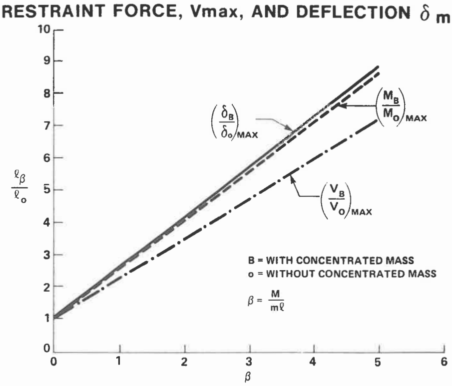

FIGURE 3
RELATIVE SPAN LENGTH FOR CANTILEVER WITH END MASS