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Bayes-based methodology to update seismic response of RC nuclear buildings

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ABSTRACT

The numerical prediction of the nonlinear behavior of structures under a given seismic load remains a very complex task regarding many aspects: the choice of the modeling strategy, the description of the soil behavior, the description of the soil-structure-interaction, the induced structural effects, the description of the reinforced concrete behavior under cyclic loadings, etc. Such complexity, coupled with the inherent and intrinsic randomness of material properties, usually induces a gap between the blindly modeled quantities of interest and the ones measured or observed (when measurements are available). To reduce such gap, one might increase the accuracy of the used physical models (using different models, adding more physical phenomena, etc.) and/or improve this knowledge of the model's inputs leading to the observed responses. In this work, we assume that the used model is sufficiently complex to be considered physically representative (main physical phenomena are accounted for already). Therefore, we consider that the lack of knowledge concerns mainly the used inputs and that is the issue that needs to be addressed. Eventually, this work focuses mainly on available tools and methods to solve the issue of updating the inputs of seismic behaviour models based on the acquired data (experimental observations preferably or in situ data from post-earthquake survey if experimental evidence is not available) expressed in terms of displacements, strains, forces, etc. The Bayesian approach is one of the most relevant and widely used technique. In this paper, it is applied to adjust material properties in the model of SMART2013 mock-up (3-story reinforced concrete structure at the scale 1/4) leading to a better quantification of the median properties of concrete at the structural level and a clear reduction of epistemic uncertainties based on the feedback provided by experimental measurements during successive seismic loads.

INTRODUCTION

This paper focuses on the application of Bayesian updating techniques to reduce the uncertainties on fragility curves. The Bayesian technique is applied to update the inputs characterizing the mechanical behavior of structural elements made from reinforced concrete and subjected to seismic ground motion. The main sources of uncertainties come either from the so-called aleatory uncertainty (associated to the intrinsic random nature of some specific phenomena) or from epistemic uncertainties (associated to the limited knowledge we may have given a specific phenomenon). The objective here is to develop a methodology based on the Bayesian approach using the available measurements to limit the effect of such uncertainties on the model's predictive capacity. This includes updating the current state of parameters for the constitutive law of the material and of boundary conditions to get what one observes or at least approach it to the best. With these new inputs, one can launch new numerical simulations with a realistic resulting mechanical state with, hopefully, reduced epistemic uncertainties.

In fact, the Bayesian updating technique is popular and widely used in various domains (see for example the work of (Richard B. A., 2012; Rossat D. B.-P., 2022; Rossat D. B.-M., 2021; Tekeste, 2022; Wang, 2018). For the sake of illustration, the Bayesian approach is applied here to the SMART2013 mock-up (Belletti, 2017; Richard B. C.-E., 2016; Richard B. M., 2015) using the linear elastic equivalent method (Figure 1). The aim, through this work, is to update the effective properties of cracked structural elements (Young's modulus and damping ratio) based on the recorded and experimental eigenfrequencies; acceleration and observed cracking patterns.

Bayesian updating using nonlinear modelling is considered as beyond the present scope of the present work. However, we mention that the global methodology remains the same and that it might take more computational time as the number of random variables increases including for instance the tensile strength, the fracture energy and the hysteretic damping ratio, etc.

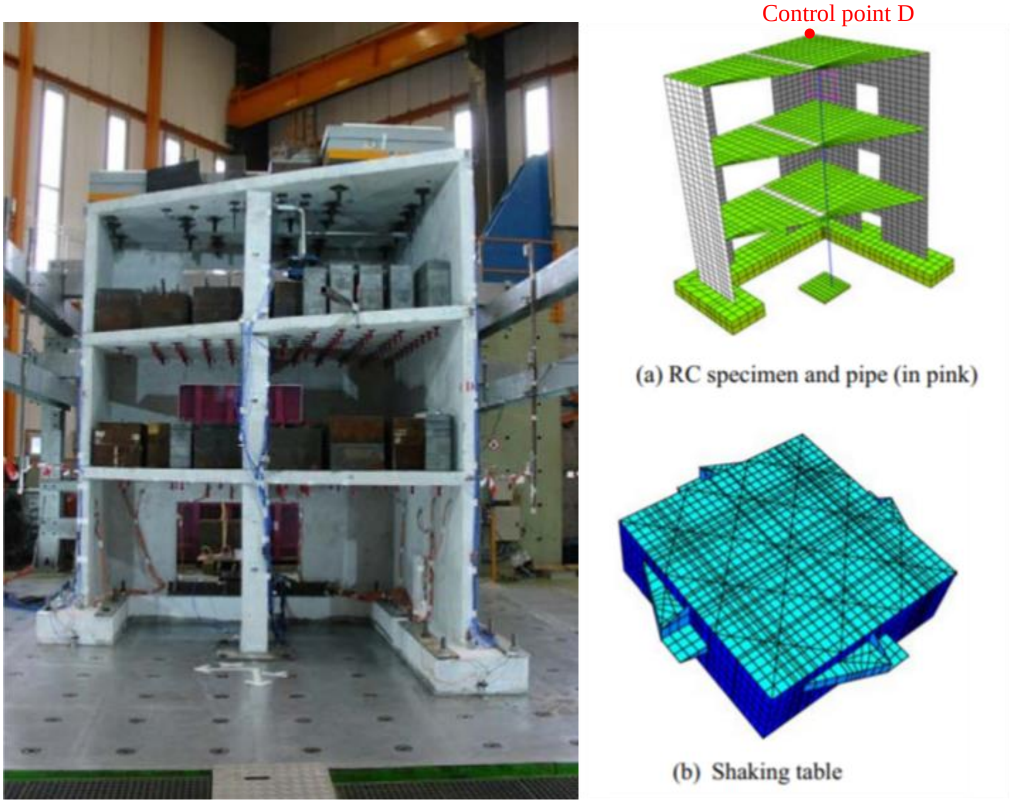

Figure 1: SMART mock-up: (left) Experimental set up and(right) FE mesh of the mock-up and of the shaking table (from Richard B. M., 2015)

OVERVIEW OF THE CASE STUDY SMART2013

Experimental campaign and available data

The SMART2013 experimental campaign is based on a ¼ scaled mock-up of a building that is attached to the reactor building with a strong sensitivity to the torsional behaviour under seismic loads. This mock-up is bolted to the shaking table through 34 thread stalks at the foundation level (Figure 2). The mock-up consists of walls (with a thickness of 10 cm); slabs (with a thickness of 10 cm); beams (with a section of15 cm x 32 cm); shallow foundation (with a section of 25 cm x 65 cm) and column (with a section of 20 cm x 20 cm) to assure the stability during the tests. Masses are added to each floor (around 11 tons) to meet the scaling effects criteria.

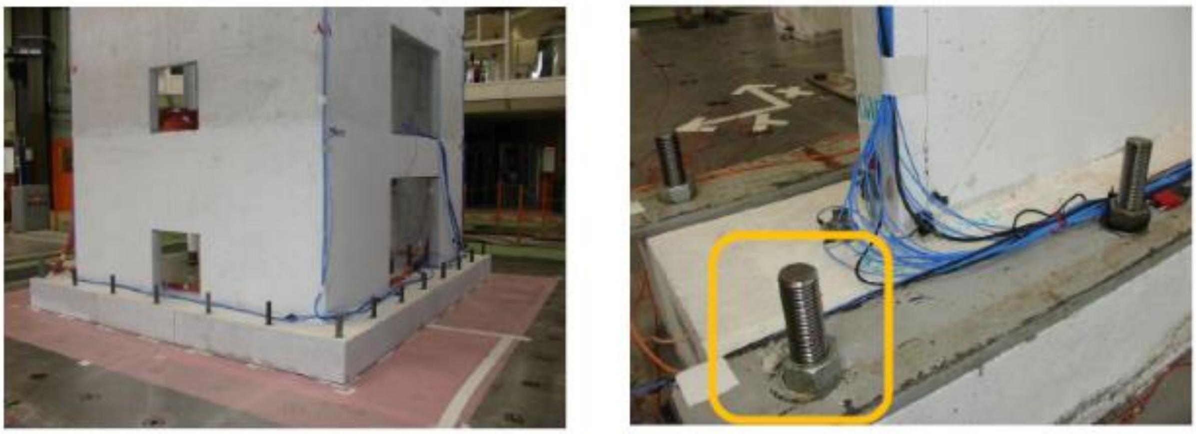

Figure 2: Fixation system of the mock-up to the shaking table

As for the seismic loads, three seismic sequences are considered: the design level (0.2g), Northridge main-shock (1.78g) and Northridge after-shock (0.37g). These loads are applied in an increasing way through several runs (Table 1). The seismic tests were realized through the shaking table AZALEE installed at CEA. The shaking table is a 6x6m aluminium plate with a maximum capacity load of 100 tons. It is equipped with eight hydraulic actuators, 4 horizontal actuators at the sides and 4 vertical actuators at the bottom of the table. Each actuator can provide at maximum of 1 MN. During each run, the mock-up is monitored to measure the eigenfrequencies and modal deformations, displacements, the accelerations, crack openings, strains within concrete, uplift at the foundation level (Figure 3).

Table 1: Peak ground acceleration measured at the shaking table level

# run	PGAx (g)	PGAy (g)	Description
7	0.13	0.13	Design level
9	0.21	0.24	
11	0.21	0.15	Northridge main shock
15	0.38	0.22	
17	0.56	0.39	
19	1.08	0.9	

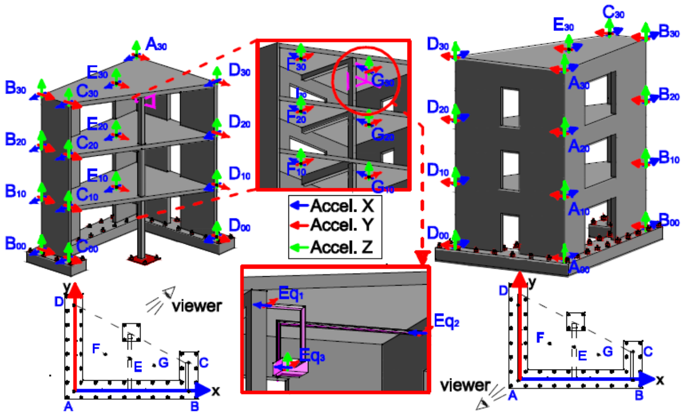

Figure 3: Position of measurement points

The material properties of concrete correspond to a mean Young's modulus of 27 GPa, a mean tensile strength of 3.7 MPa and a mean fracture energy of 129 N/m. Such properties show an intrinsic variation at the specimen scale around 10%. The used steel for rebars and stirrups has a mean Young's modulus of 205 GPa.

Numerical model

The numerical simulation is done using the Cast3m software (version 2022). The structural elements such as walls; floors and beams are modelled using multi-layered shell elements (5 layers for concrete). The foundation is modelled by the solid elements. The rebars are modelled using 4 additional layers (1 layer for each reinforcement direction). The mesh density is globally set at about 20 cm. The shaking table is also modelled by shell elements and is calibrated to obtain the relevant eigenfrequencies of the shaking table.

The damping of the structure is modelled through the Rayleigh model (Figure 4) where the damping matrix is a linear combination of the mass and stiffens matrix. Such model is well defined using two frequencies defining the range of interest (here we consider the first f1 and third f3 main eigenfrequencies) and a target damping value for each frequency. Based on experimental measurements, the damping ratios for these two frequencies are of 2.2% and 5.5% respectively (case of elastic behaviour – no damage). As for the connection between the mock-up and the shaking table, the model does not include any bolts and the nodes of the foundation and of the shaking table are fully merged (perfect kinematic bond). The interaction between the table and the mock-up is modelled by modifying the Young's modulus value of the massive foundations in the model.

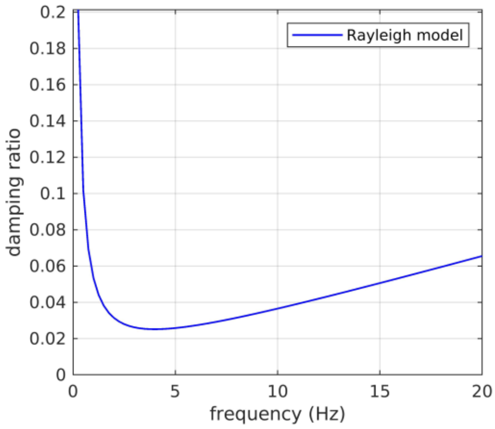

Figure 4: Damping based on Rayleigh model

The calibration of the Rayleigh model is not obvious since both the frequencies and the damping values evolve with the damage state. The boundary conditions (stiffness of the shaking table – mock-up interaction) are not easy to define precisely either in the model. So, the Bayesian updating is a good solution to better quantify the model's parameters based on available measurements.

Blind numerical analysis

In this part, runs #7 and #19 are experimentally considered as elastic (no cracking) and nonlinear (advanced cracks observed) runs. Before applying the Bayesian updating to the model, we run here blind calculations to quantify the gap between numerical and experimental results:

- for run #7, the blind simulation is exclusively based on the input parameters measured at the sample scale, including the concrete's Young's modulus and damping ratio as well.
- for run #19, the blind simulation is described by the hypothesis of equivalent elastic simulation where the Young's modulus of concrete's cracked elements are divided by 2 and the global damping ratio is set to 7%.

The comparison of first main eigenfrequencies between the blind numerical simulations and experimental results is provided in Table 2. Obviously, the blind results are rather good for the elastic run and did not match experimental results as the nonlinearities increased. The blind hypotheses tend to overestimate the observed eigenfrequencies which means that the Young's modulus reduction needs to be higher than the one considered beforehand.

Table 2: Main eigenfrequencies after runs #7 and #19: experimental vs. blind numerical results

	Run #7		Run #19	
	Exp.	Num.	Exp.	Num.
Freq. #1	6.1	6.2	2.8	5.1
Freq. #2	7.8	9.7	4.4	8.1
Freq. #3	16.5	20.6	9.0	17.2

As for acceleration response spectra at the control point D (Figure 1):

- for run#7, it is seen that the position of the peaks is rather well estimated (due to the good approximation of the main eigenfrequencies – the result is not that good for the third frequency). However, the amplitude of the peaks are overestimated in the numerical model meaning that the damping level needs to be increased in the model.

- for run #19, the same observations are made for the amplitude of the peak values. In addition, we see that the model fails at reproducing the shift to the left due to the damage and the associated frequency drop.

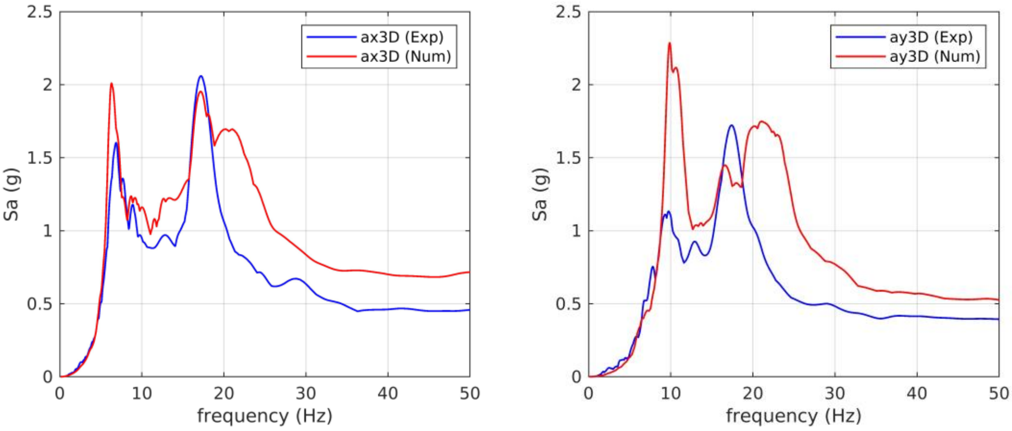

Figure 5: Acceleration response spectra (run #7)

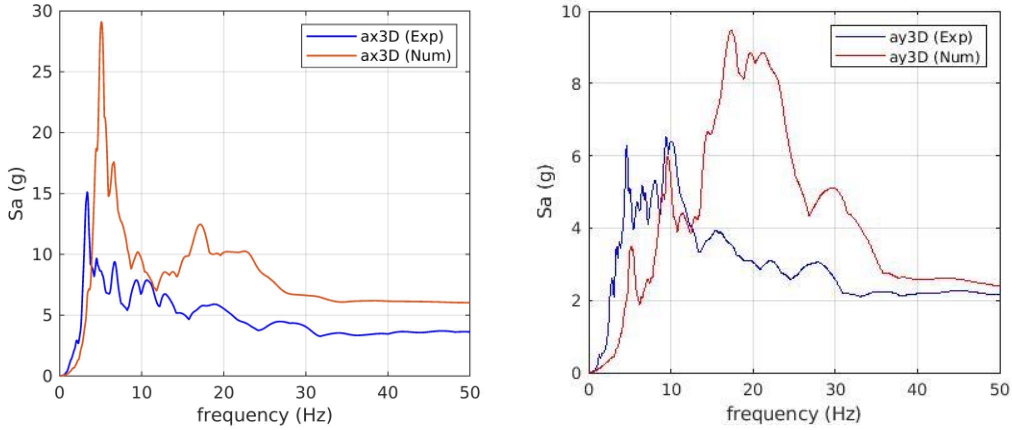

Figure 6: Acceleration response spectra (run #19)

However, the results of the simulation demonstrate the reasonable representativeness (yet not perfect) of the numerical model. Indeed, the uncertainties of the input parameters and the boundary conditions are considered to have effects on the numerical results. For this reason, the uncertainty propagation is proposed, and the Bayesian updating approach is applied to update the uncertain parameters based on the available measurements of the experiment.

THEORY OF BAYESIAN UPDATING AND METAMODELING

Even though numerical calculations are, in this work, based on equivalent linear analysis, the computational time is still not compatible with the use of Monte Carlo methods. Each calculation of eigenfrequencies requires about 2 minutes and each computation of the full transient elastic analysis requires about 6 minutes. Instead, we base our development on techniques of metamodeling to limit the computational cost and be able to achieve Bayesian updating within a reasonable time.

Polynomial Chaos Expansion

From a conceptual point of view, any continuous input–output map can be approximated by a Polynomial Chaos Expansions (PCE) surrogate model, which consists of a truncated series expansion formed by orthonormal polynomials (Ghanem, 1991):

$$Y(X) \approx \check{Y}(X) = \sum_{\alpha \epsilon A} a_\alpha \, \Psi_\alpha(X)$$

where $A \subset N^d$ is a set of multi-indices, α the PCE coefficients and Ψ_α the associated tensorized polynomial term defined by: $\Psi_\alpha(X) = \prod_{i=1}^{d} \Psi_{\alpha_i}(X_i)$ with Ψ_{α_i} is the univariate polynomial of degree α_i. In this work, the PCE coefficients are computed with the procedure based on the Least Angle Regression algorithm (LARS). The maximal polynomial order is set at 8. However, a lower order can be obtained if it is sufficient to get a good fit based on the so-called Leave One Out validation error method.

MCMC Bayesian updating

Let be $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space. One considers a mathematical model M (X, t) (analytical or numerical), representing the temporal evolution of a given system taking values in the space $\mathbb{R}^N$, and depending of M input parameters gathered in a random vector $X : \Omega \longrightarrow D_X \subset \mathbb{R}^M$.

The uncertainties on input parameters are represented by a prior probability density function π_X. Then, the model response Y is also a random vector: Y(t) = M (X, t). This setup defines a probabilistic model.

One considers a set of N_{obs} observations $Y_{obs} = (y_1,\dots, y_{Nobs})$ of the model response obtained at the corresponding instants $(t_1,\dots, t_{Nobs})$. Then, model predictions M (X) differ from observations through a discrepancy ε_i: $y_i = M(X, t_i) + \varepsilon_i$. This term ε_i comes not only from measurement error, but also from the model error since models are always simplified descriptions of real systems.

For the sake of simplicity, the model/measurement discrepancy is supposed to follow a zero mean Gaussian distribution $\mathcal{N}$ hereafter. That means that the conditional distribution of observations with respect to input parameters reads: $Y_i|X = x \sim \mathcal{N}(M(x, t_i), C)$ with C the covariance matric of the model/measurement discrepancy.

This reformulates in terms of PDF as follows: $\pi_{Y_i|X}(y, x, t_i) = \varphi_C\big(y - M(x, t_i)\big)$ with $\pi_{Y_i|X}$ is the conditional PDF of observations, and φ_C is the density of the distribution $\mathcal{N}(0, C)$.

Using Bayes' theorem, the posterior probability density function π_X^* of input parameters could be written as a function of the prior PDF (Berveiller et al., 2012; Tarantola, 1987):

$$\pi_X^*(x) = c\pi_X(x)\mathcal{L}(x; Y_{obs})$$

where $c = \int_{D_X} \pi_X(x)\mathcal{L}(x; Y_{obs})dx$ is a normalization constant also known as Bayesian integral, and $\mathcal{L}(x, Y_{obs})$is the likelihood of the observations equal to: $\prod_{i=1}^{N_{obs}} \pi_{Y_i|X}(y, x, t_i)$.

In practice, there are several ways to calculate the posterior density of input parameters. Markov Chain Monte Carlo (MCMC) simulations are often used to determine this density (Tarantola, 1987). They consist of constructing a Markov Chain over the input parameters space, which behaves asymptotically as the posterior density of interest (Perrin, 2008). In this context, the Random Walk Metropolis–Hastings (RW-MH) algorithm (Hastings, 1970) constitutes the basis of MCMC algorithms. Given a Markov chain length N_{MCMC}, this algorithm could be described as follows. First, an initial realisation x_0 of the input X is randomly chosen. At iteration k from a realisation x_k, a candidate point $\tilde{x}$ is drawn by making a translation from x_k with a zero-mean random vector, usually Gaussian or uniform. Then, the candidate is accepted with a probability equal to: $\alpha(\tilde{x}, \tilde{x}_k) = min(1, \frac{\mathcal{L}(\tilde{x}; Y_{obs})}{\mathcal{L}(\tilde{x_k}; Y_{obs})})$. The iterations are pursued until reaching the Markov chain length N_{MCMC}.

APPLICATION OF BAYESIAN UPDATING TO SMART2013

The use of Markov Chain Monte Carlo (MCMC) during the Bayesian updating is expected to be a time-consuming process. To overcome this issue, the construction of Metamodels is proposed. Thus, the Metamodel is a mathematical function, and is constructed through the polynomial chaos expansion (PCE). Afterward, the Bayesian approach calls directly to the Metamodel to update the uncertainties parameters with a comparable output to those obtained experimentally. In this work, we focus only on the updating of inputs' parameters based on measured eigne frequencies and the accelerations response spectra at the control point D (top of the structure). Also, to optimize the updating process further, the Bayesian updating is realized in two steps:

- first we update Young's modulus to get the right global eigenfrequency of the structure.
- second we update, after fixing the mean values of the Young's modulus, the damping ratios to get the peaks of acceleration response spectra at the main eigenfrequencies of the structure (control point D).

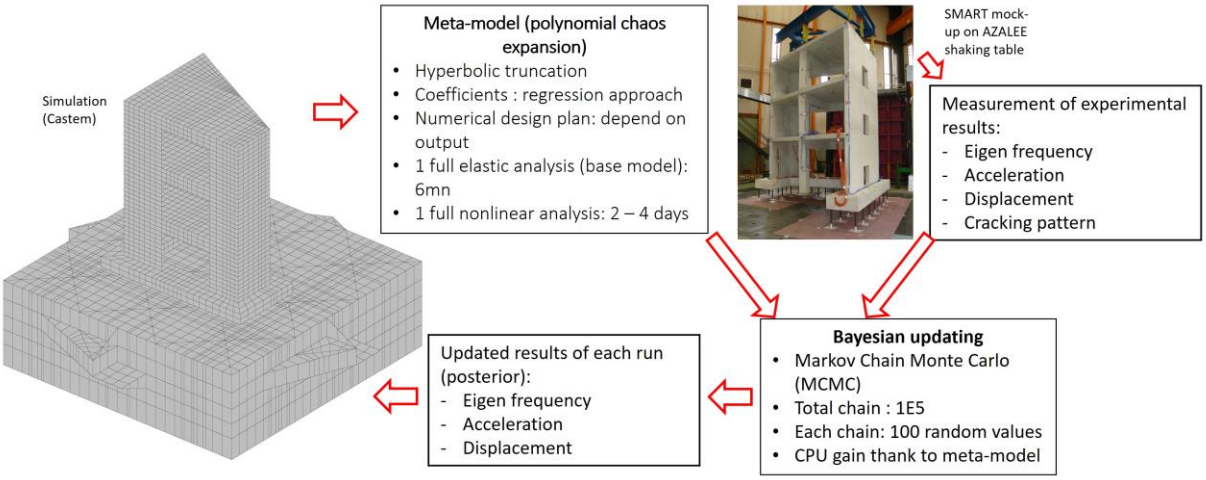

Figure 7: Global methodology for Bayesian updating of SMART2013 behaviour

Eigenfrequencies

For the 1st Bayesian updating step, a numerical design plan is proposed to cover all possibilities of most probable Young's modulus distributions (both for prior and posterior estimations). One should note that there are 15 Young's modulus values in the model, one value for each structural element (walls, floors, foundations). The values of these Young's modulus are updated depending on the cracking state- and observed damage. For metamodeling purposes, the numerical design plan covers 8000 random sampling of 15 young's modulus values between 0.2 GPa and 50 GPa using a uniform law. Each random set of Young's modulus values provides us with a set of three main eigenfrequencies. Then, a metamodel is constructed relating those eigenfrequencies with the Young's modulus values. In Figure 8-left**Erreur ! Source du renvoi introuvable.**, Y(PCE) represent the results computed from the fitted metamodel whereas Y(true) are obtained directly by means of numerical simulations. It is observed that metamodel reproduces well the estimated eigenfrequencies. With this metamodel, the use of MCMC during updating becomes accessible with a reasonable computational time.

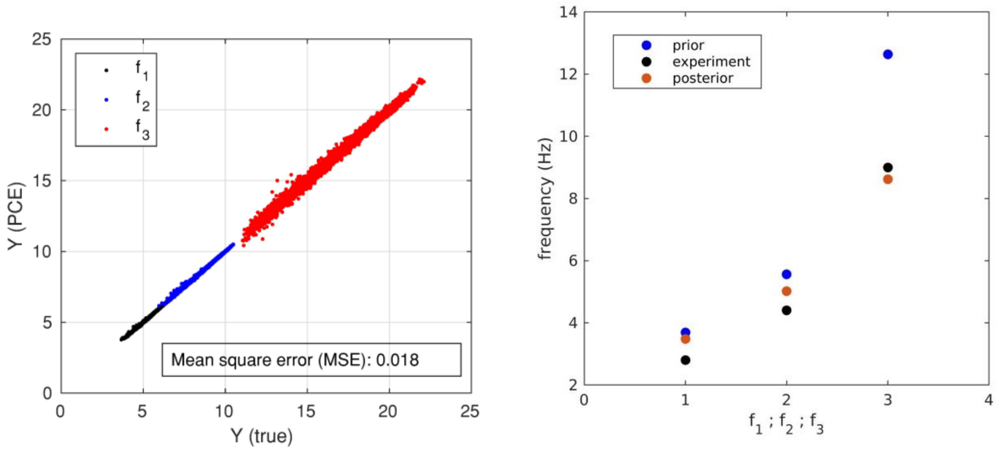

Figure 8: (left) Comparison of metamodel vs. numerical results (right) Prior and posterior eigenfrequencies vs. experimental results (run #19)

The application of the Bayesian process allows for a better estimation of the eigenfrequencies (Figure 8 right) and, more importantly, allows for an accurate estimation of the overall stiffness of each structural element without having to model in a refined and precise way crack initiation and propagation. For instance, for run #19 (strongly nonlinear), Young's modulus reduction is of a factor 6 for cracked structural elements whilst the prior estimation was of 2.

Spectral accelerations

In this 2nd Bayesian updating step, metamodels are constructed for each run. The proposed numerical design plan covers damping ratios at the frequencies of interest. The damping ratios, ξ1 and ξ2, are generated according to a uniform distribution with boundaries between 1% to 30%. Each metamodel is based on 100 simulations. The outputs of interest are the spectral accelerations at the control point D. In Figure 9-left, we see that the estimations from the metamodel provides good agreement with those of the exact numerical simulation. It is seen that the mean square error is relatively small, and all the dot points are slightly scattered along the diagonal. For run #19, the prior hypothesis was of a uniform distribution between 1% and 30% for the two damping ratios. Provided the acceleration spectra at point D, and the updated Young's modulus values, the posterior distributions are rather lognormal with mean values around 12% and 28% respectively for ξ1 and ξ2. The posterior coefficient of variation is limited to 2% (Figure 10). So, the posterior results are closer to the experimental accelerations with a reduced confidence interval (Figure 9-right).

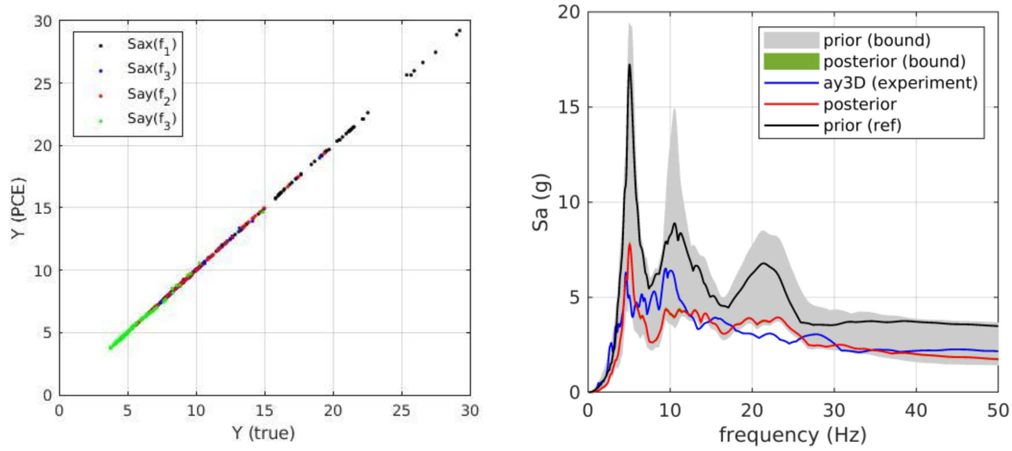

Figure 9: (left) Comparison of metamodel vs. numerical results (right) Prior and posterior acceleration vs. experimental spectra along the Y direction at point D (run #19)

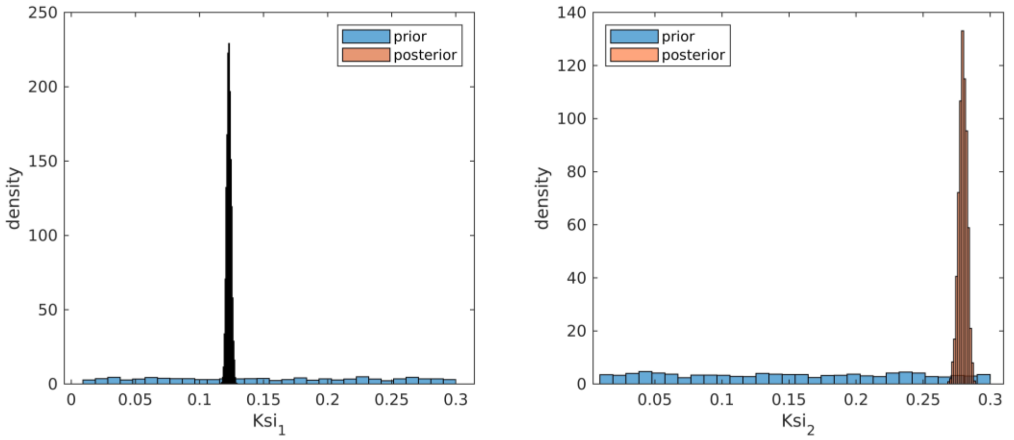

Figure 10: prior and posterior distributions of the damping ratio (left) ξ1 (right) ξ2 (run #19)

CONCLUSIONS AND PERSPECTIVES

In this paper, the methodology of Bayesian updating is presented and applied to better assess the seismic behaviour of structures through the case of SMART2013 mock-up. Even with the use of linear or linear equivalent models, it is shown that the coupling of metamodeling and Bayesian updating methods allows for a considerable computational time reduction. In this work, updating was geared towards the Young's modulus and damping ratio values. However, one should understand that this can be applied to other inputs as well. Also, in this work, as we directly updated the inputs of our model, all EDPs of interest were calculated using standard uncertainty propagation methods such as basic Monte Carlo method (with no optimization) or Markov Chain Monte Carlo approaches.

In the obtained results, the improvement of the estimations of eigenfrequencies and spectral accelerations was obvious coping with the difficulty of knowing blindly what values should be considered for each input. The physical trends were also met during the process such as the Young's modulus reduction and damping ratio increase with the resulting damage due to the seismic loads.

Finally, one should note that the current updating methodology was applied within an equivalent linear framework. Future work would be geared towards the use of direct nonlinear models. The main challenge is, as often, related to the limitation of the computational cost considering adapted numerical design plan. Another possible improvement would be the application of the updating process to the fragility curves directly without going through the updating of the model's inputs. These points will be considered for future developments.

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