Multigrid in Structure Assembling Problem

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ABSTRACT

The numerical treatment of problems involving unilateral contact with friction needs the use of iterative methods which could be expensive for large industrial structures. A convenient mathematical model and an associated numerical method are presented in an another paper (Raous and al. 1989). The present communication introduces the use of multigrid methods for this class of non linear problems. The objective is to reduce the computational time to allow the treatment of systems with a large number of degrees of freedom. This paper deals with structure assembling problems but the field of application of the model, and the numerical methods, is larger : metal forming processes, cracked solids...

INTRODUCTION

To describe the contact between several pieces occuring in structure assembling problems, we have to set several equilibrium problems for each part connected through inequality conditions on the contact boundaries. We know only the initial contact aeras and the real contact zones, after applying the loadings, are unknown and will be obtained by solving the whole problem. Due to the friction conditions, the contact forces are also unknown and depend on the whole solution. The model takes into account the eventual loss of contact as well as the new contacts which can occur in the zones. This is essential to control the tightness or the deformation of the joints for pressure vessels assembling. For structure assembling problems, elasticity, small deformation and quasistatic evolution will be assumed.

The friction is modelized using a Coulomb law. As shown in other works (Raous et al 1989 and Lebon et al 1987), this is a convenient model for metal/metal contact without lubrication : comparison with experimental measurements have been made.

The mathematical formulation leads to an implicit variational inequation including an undifferentiable term. Through a fixed point method on the sliding limit depending on the normal contact force, we get a variational inequation problem which can be set under the form of a minimization problem with constraints of a undifferentiable functional (see Raous et al, 1988). An incremental formulation is used to fit with the velocity formulation of the friction law. The numerical method is based on projection techniques associated to a relaxation process. The Gauss Seidel algorithm is a good smoother which will be used for the multigrid methods.

Multigrid methods, dating from the 1960's, have been strongly developped since the 1970's. Their main applications are in fluid mechanics and they are often associated to finite difference discretizations. This paper deals with the use of these methods in structure mechanics on a non linear problem with finite element approximation. They associate smoothing processes of the solution on a fine grid and total resolutions of a correcting error on a coarse grid.The processes of restriction and interpolation from one grid to an other is always delicate and more particulary for this non linear problem. They will be discussed. Applications are presented which show the efficiency of these methods.

NUMERICAL ALGORITHM FOR CONTACT PROBLEMS INCLUDING FRICTION.

Details about the formulation and the numerical method can be found in Raous et al, 1988, and Chabrand et al, 1989. Separating the displacement u and the contact force F into normal and tangential components ($u_N n$, u_T, $F_N n$, F_T) (n is the external unit normal to Γ_3) , we set the contact conditions under the following inequalities written on the part Γ_3 of the boundary Γ where the contact can occur.

The unilateral conditions are :

$$u_N \leqslant 0 \qquad F_N \leqslant 0 \qquad u_N F_N = 0 \tag{1}$$

The Coulomb's friction law is (μ is the friction coefficient):

$$|F_T| \leqslant \mu\ |F_N| \text{ with if } |F_T| < \mu\ |F_N| \text{ then } \dot{u}_T = 0 \tag{2}$$

$$\text{if } |F_T| = \mu\ |F_N| \text{ then } \dot{u}_T \text{ is equal and opposite to } F_T \tag{3}$$

Together with the kinematic relations and the equilibrium equations, these inequalities lead to an implicit varitional inequation including an undifferential term. As presented in Raous et al, 1988, and Chabrand et al, 1988, the problem can be transformed in the following manner. Let $(t_0,\ldots,t_M)$ be a time partition and $\dot{u}_k = \Delta u_k/\Delta t_k$ with $\Delta u_k = u(t_{k+1}) - u(t_k)$ and $\Delta t_k = t_{k+1} - t_k$, the final problem to be solved is :

<u>**Problem P1**</u> : Find the sliding limit function g_k fixed point of the application $g_k \rightarrow \mu\ |F_N(u_k)|$ where u_k is solution of the problem P2 (depending) on g_k .

<u>Problem P2</u> : Let $\varphi(t_k)$ be the load applied on a part Γ_1 of the boundary, g_k given, find $u_{k+1} = u_k + \Delta u_k$ with
$u_{k+1} \in \mathbb{K} = \left\{ v \in (H^1(\Omega))^3 ,\ v = 0 \text{ on } \Gamma_1 \text{ (boundary conditions) and } v_N \leqslant 0 \text{ on } \Gamma_3 \right\}$
such that :

$$J(\Delta u_k) \leqslant J(v) \quad \forall\ v \in \mathbb{K} \tag{4}$$

$$\text{with } J(v) = \tfrac{1}{2}\ a(v,v) - (\varphi,v) + j(v) \tag{5}$$

$$a(v,v) = \int_\Omega \text{grad}_s v\ K\ \text{grad}_s v\ dx \text{ (K is the elasticity matrix)} \tag{6}$$

$$(\varphi,v) = \int_{\Gamma_1} \varphi\ v\ dl \tag{7}$$

$$j(v) = \int_{\Gamma_3} g\ |v_T|\ dl \tag{8}$$

For solving the minimization problem (4) with constraint, we use a Successive Over-Relaxation method with Projection. For the special form of the undifferentiable term j(v), we have a theorem of convergenve for the algorithm (see Glowinski et al, 1976). We obtain an acceleration of the procedure using a diagonal process on the fixed point method, a condensation of the problem to the contact variables alone, and a sparse matrix storage (Morse storage). In structure assembling, the model and the algorithm have to be generalized to the contact of several deformable solids (instead of the contact with a rigid obstacle here described). This is presented in Lebon et al, 1987.

MULTIGRID METHODS :

The basical idea of the multigrid method can be explained in terms of spacial frequencies of the error between the solution and the approximation computed by the use of an iterative method such as Gauss Seidel. This type of method reduces rapidly the high frequency components of the error but needs a large number of iterations to reduce the low frequencies. A two grid method solves completely a problem involving a correcting error on a coarse grid and then interpolates this correction on the fine grid so to ameliorate the solution u on this grid where we want the final solution to be. So, the high frequencies of the error are reduced by the correction on the fine grid (with eventually a few iterations of smoothing) and the low frequencies by a complete resolution on a coarse grid concerning the defects of equilibrium and the error correction. We consider here a full multigrid method and the sequence will be the following one in this case :

a - We first solve the problem totally on the coarse grid and we get an initial solution U_0 (the capitals denote the variables on the coarse grid and the small letters the ones on the fine grid).

b - U_0 is interpolated to obtain $u_0 = \mathfrak{J} U_0$ on the fine grid ($\mathfrak{J}$ is the interpolation mapping) (l=0)

c - Two smoothing iterations are done $u^{l+1/2} = GS(u^l)$. The smoother is a mapping Gauss Seidel method which is considered to be a good one. Simpler ones could be used. This smoothing is not necessary but the numerical experience shows that it gives a good acceleration of the procedure. The choice of two iterations has been done after several tests : it is a good compromise between the acceleration and the computational expenses.

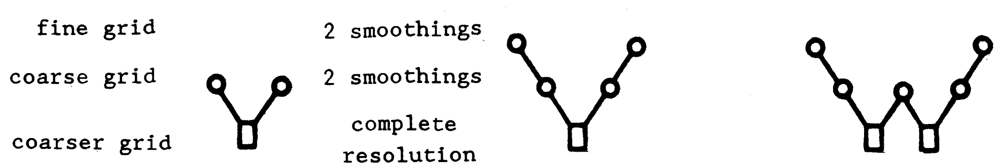
d - We compute the defects of the equilibrium r on the fine grid and we restrict it to the coarse grid $R = \mathfrak{R}(r)$ ($\mathfrak{R}$ is a restriction mapping).

e - We do a complete resolution of the problem involving the error E (the loads are replaced by the defects R in the second member). For a linear problem it is natural to use a direct method (Choleski) ; for the contact problem, we use the SOR method with projection

f - The solution E is interpolated on the fine grid e=J(E).

g - The previous solution u^l is corrected : $u^{l+1} = u^{l+1/2} + e$ and we return to step (c) untill a given precision is reached, the test being made at the end of the step g (with l=l+1).

This two grid process can be generalized to a multigrid one following the diagram of Fig. 1.



(a) two grid (b) multigrid V cycle (c) multigrid W cycle

Fig. 1 : diagram of various multigrid algorithms.

Three particular difficulties occur in treating the non linearities of contact problems with friction.

Transportation of the non penetration condition from the fine grid to the coarse one.

This condition assumes that the solution stays in the convex $\mathbb{K}$. The solution on the coarse grid where E is the variable has to be such that $(U+E)_N \leqslant 0$ on Γ_3, but U is never defined on the coarse grid. So we have to choose an estimation which has to be convenient for the projection. After testing different choices, we keep as value of U on a contact mode of the coarse grid the smallest value of u among the value on the same node and on the two adjacent ones on the fine grid. Other choices are presented in Lebon, 1989.

Treatment of the absolute value of j(v) on the coarse grid.

The problem is analogue to the previous one : the variable is the error E and we need to treat a term of displacement $|U+E|$ by trying both alternatives successively.

Several choice for U are given in Lebon, 1989. We use finally the first value U_0 computed totally at the beginning which has been chosen as an initial condition for the algorithm. This choice implics a full-multigrid, which we recommend for this class of problems.

Transportation of the sliding limit g on the thin grid to the coarse grid.

Once again, our choice for G is the one obtain with the first resolution on the coarse grid at the first step of the full-multigrid scheme.

Example 1 :

The first test has been done on a benchmark test defined in a group of validation of computer codes of the GRECO "Grandes déformations et endommagement". It is a long bar (plane strain hypothesis) squeezed on a plane surface where it is submitted to an unilateral contact with friction (Fig. 2). Details about the mechanical solution can found in Raous et al, 1988. We focus in this presentation only on effeciency of the multigrid method.

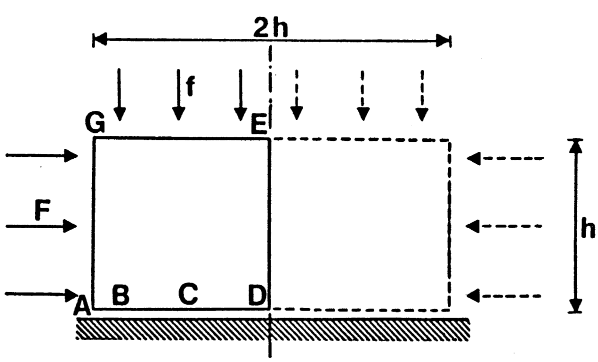


Fig. 2 : the geometry (h = 40 mm) and the loadings

Example 2 :

This is a dovetail assembling problem treated by the same group (see Raous et al, 1988). This is a more general problem of a two body contact contact which can be generalized to multibody contact for larger classes of assembling (see Fig. 3).

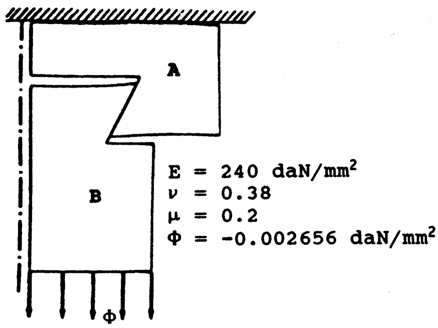


Fig. 3 : dove tail assembling displacement (amplified twenty times)

Example 3 :

It concerns the behaviour of a bolted junction of vessels submitted to internal pressure loadings. The numerical results have been discussed in Lebon et al, 1988 (see Fig. 4).

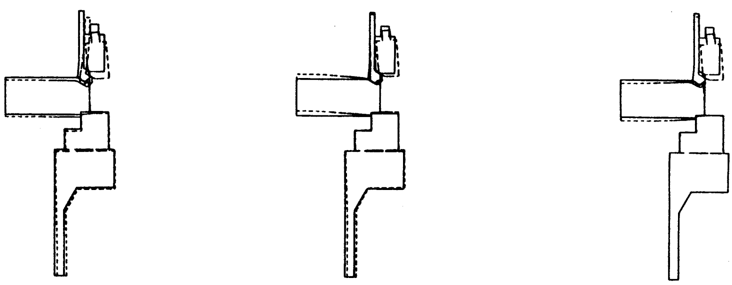


Fig. 4 : bolted junction displacement amplified 100 times for the total loadings.

Efficiency of multigrid method.

The discussion is done on example 1 in the paper and results on exemple 2 and 3 are presented in the talk.

We test up to a four grid method with a full-multigrid scheme two smoothing iterations on the fine grid with a Gauss Seidel method (the over relaxation is not necessary). Grid 1 has 1089 nodes, grid 2 has 289, the grid 3 has 81, the grid 4 has 25. Table 1 gives the computational time on a Vax 730.

Method	Node number	CPU time
Regular resolution	1089	12h
Two grid method	1089/289	1h40'
Three grid method	1089/289/81	33'12"
Four grid method	1089/289/81/25	26'25"

Table 1 : Computational times

Usually, it is not necessary to use more than 3 grids and it can be observed that a two grid method is already very efficient. This example is treated with P1 triangles and the mesh refinement is made in parting each triangle in three smaller ones. This uniform refinement penalizes the computation on the fine grid because for contact problems the non linearities are localized and we do not need a fine mesh on all the structure. We are now developping adaptive meshing to construct the fine grid from a coarse one : in this case, the different grids are not imbricated and the restriction and the interpolation mappings $\mathfrak{R}$ and $\mathfrak{I}$ are more complicated. The real efficiency of multigrid methods goes through this adaptative mesh technique.

For this class of non linear problem the coarse grid has to be enough fine because of the use of the initial solution computed on this grid along the next iterations (full multigrid method).

We do two smoothing iterations on each fine grid. We use a Gauss Seidel method with projection. It is a good smoother which is rapidly efficient on the high frequencies on the error. Over relaxation is not necessary because its acceleration concerns the low frequencies. We give on table 2, the influence of the smoothing number on the computational time. Table 2 concerns example 1 for a frictionless case and a two grid method (289 / 81 nodes).

smoothing number	iteration number two grid process	cpu time
1	12	8'
2	8	5'10"
3	8	5'15"
4	10	7'

Table 2 : influence of the smoothing iteration number.

Table 3 gives a comparaison between a regular two grid method and a full two grid method on the same example.

Method	CPU time
Regular resolution	24'
Two grid method	11'
Full two grid method	5'10"

Table 3 : efficiency of the full two grid method.

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