Effect of Thermomechanical Coupling on Thermal Stress Problems in a Hollow Circular Cylinder

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SUMMARY

The problem of finding a complete solution for the transient thermal stress with considerations of the thermomechanical coupling effect and the inertia effect is one of considerable analytical difficulty. In our recent study, however, it can be seen that it is sufficient to consider only the effect of the coupling term for realistic cases. From the present paper, it may be stated that it is important to consider the coupling term, because of the coupling effect which acts visible affect on the temperature and stress distribution. This paper presents a general treatment of the axisymmetric coupled transient thermal stresses of a hollow circular cylinder with a finite length using a new technique, and numerical results are given.

The method used is developed by Love's strain function and the thermoelastic displacement potential function apart from the former method deviced by the one of authors. The value of thermomechanical coupling parameter adopted in our calculation is almost for a realistic material. In spite of the reduction of the temperature rise for the whole time interval in transient state, the maximum thermal stresses are increased by the thermomechanical coupling effect.

Moreover the solution rigourously meets traction-free boundary conditions on both the lateral surfaces and plane ends of the hollow circular cylinder.

I. Introduction

It is well known that uncoupling of the temperature and stress problems is permissible whenever simultaneously inertia effects are small and the value of the coupling paremeter is small. However, in order to treat thermal stress problems rigourously by a sudden change in temperature such as thermal shock problems, two effective terms must be taken into account. In the arthor's reference, we found that effect which is the most powerful for finding the thermal stress distribution. From our previous result [1], it is more important to consider the coupling term than to consider the inertia term for common metals in pure thermal stress problems. Therefore, it may be stated that it is sufficient to consider only the effect of the coupling term for realistic cases.

In this paper, we show the effects of thermomechanical coupling for a transient thermal stress problems in an axisymmetric temperature distribution in a hollow circular cylinder by means of the another new technique apart from our previous paper[2,3]. The numerical results show that the coupling term has a considerable affect on the temperature and stress distribution.

II. Basic Formulation

Let us consider a finite hollow circular cylinder with outer redius a, inner radius b and length $2L$ is heated partially in the axial direction, as shown in Fig. 1. Assuming the surface heat generation to be axisymmetric, the fundamental corrected heat conduction equation for the axisymmetric problems with the thermomechanical coupling term can be written in the non-dimensional form, as

$$\Delta_1 \bar{T} = \bar{T},_\tau + \delta m^{-1} \bar{e},_\tau , \qquad \Delta_1 = \partial^2/\partial\rho^2 + \rho^{-1}\partial/\partial\rho + \partial^2/\partial\zeta^2. \tag{1}$$

where, δ is so-called thermomechanical coupling parameter and comma(,) denotes the differntiation with respect to the variable.

If the body is initially at the uniform temperature(say at zero temperature) and assuming the surface heat generation on the cylindrical surfaces and the heat transfer on the both end surfaces to be prescribed, eq.(1) must be solved under the initial condition

$$at \quad \tau = 0 \quad ; \quad \bar{T} = 0 \tag{2}$$

and with the thermal boundary conditions

$$on \quad \rho = 1 \quad ; \quad \partial\bar{T}/\partial\rho + H_1\bar{T} = H_1 f(\zeta) \tag{3}$$

$$on \quad \rho = \rho_1 \quad ; \quad \partial\bar{T}/\partial\rho - H_2\bar{T} = -H_2 g(\zeta) \tag{4}$$

$$on \quad \zeta = \pm\zeta_0 \;; \quad \partial\bar{T}/\partial\zeta \pm H_3\bar{T} = 0 \tag{5}$$

where $f(\zeta)$ and $g(\zeta)$ are the arbitrary functions which are difined the region $-\zeta_1<\zeta<\zeta_1$ and $-\zeta_2<\zeta<\zeta_2$ respectively. They describe the temperature distribution on the cylindrical surfaces.

Since the abovementioned system for the coupled thermal stress problems is imcomplete, the following additional relations are to be considered. Takig into account that the present problem is axisymmetric, we introduce the thermoelastic displacement potential Φ and Love's displacement function $\bar{L}$. These function Φ and $\bar{L}$ should satisfy the following fundamental differential equations

$$\Delta_1 \Phi = m\bar{T}, \qquad \Delta_1 \Delta_1 \bar{L} = 0 \;. \tag{6}$$

Then, the corresponding displacement u_i, volumetric strain $\bar{e}$ and thermal stress components $\bar{\sigma}_{ij}$ in the non-dimensional form can be written as

$$\bar{u}_r = \Phi,_\rho - \bar{L},_{\rho\zeta} \;, \quad \bar{u}_z = \Phi,_\zeta + 2(1-\nu)\Delta_1 \bar{L} - \bar{L},_{\zeta\zeta} \;. \tag{7}$$

$$\bar{e} = \Delta_1 \Phi + (1-2\nu)\Delta_1 \bar{L},_\zeta \;. \tag{8}$$

$$\begin{aligned}
\bar{\sigma}_{rr} &= m^{-1}[\Phi,_{\rho\rho} - \Delta_1 \Phi + (\nu\Delta_1 \bar{L} - \bar{L},_{\rho\rho}),_\zeta],\\
\bar{\sigma}_{\theta\theta} &= m^{-1}[\rho^{-1}\Phi,_\rho - \Delta_1 \Phi + (\nu\Delta_1 \bar{L} - \rho^{-1}\bar{L},_\rho),_\zeta],\\
\bar{\sigma}_{zz} &= m^{-1}[\Phi,_{\zeta\zeta} - \Delta_1 \Phi + \{(2-\nu)\Delta_1 \bar{L} - \bar{L},_{\zeta\zeta}\},_\zeta],\\
\bar{\sigma}_{rz} &= m^{-1}[\Phi,_{\rho\zeta} + \{(1-\nu)\Delta_1 \bar{L} - \bar{L},_{\zeta\zeta}\},_\rho] \;.
\end{aligned} \tag{9}$$

Futhermore, the abovementioned relations are supplemented by the initial condition

$$at \quad \tau = 0 \quad ; \; \bar{u}_i = 0, \quad \bar{e} = 0, \quad \bar{\sigma}_{ij} = 0.(\; \bar{L} = 0, \quad \Phi = 0) \tag{10}$$

and the mechanical boundary conditions that the surfaces are traction free,

$$\begin{aligned}
&on \quad \rho = 1 \; and \; \rho_1 \quad ; \; \bar{\sigma}_{rr} = \bar{\sigma}_{rz} = 0,\\
&on \quad \zeta = \pm\zeta_0 \qquad ; \; \bar{\sigma}_{zz} = \bar{\sigma}_{rz} = 0.
\end{aligned} \tag{11}$$

The system, eqs.(1)-(11), constitutes the complete set for the coupled themo-elastic problems.

Next, in order to separate the second term in the right hand side of eq.(1), substituting eqs.(6) and (8) into eq.(1), we obtain

$$\Delta_1 \bar{T} = (1+\delta)\bar{T},_\tau + m^{-1}(1-2\nu)\delta\Delta_1 \bar{L},_{\zeta\tau} \tag{12}$$

Now, we introduce a new temperature function defined as

$$\bar{T}' = \bar{T} + m^{-1}\{(1-2\nu)/(1+\delta)\}\delta\Delta_1 \bar{L},_\zeta \tag{13}$$

Substituting eq.(13) into eq.(12) and taking into account the relation of the second of eq.(6), we obtain the following differential equation for $\bar{T}'$.

$$\Delta_1 \bar{T}' = (1+\delta)\bar{T}',_\tau \tag{14}$$

In the preceding system, the non-dimensional quantities are difined as

$$\begin{aligned}
&\bar{T} = T/T_0, \quad \tau = \kappa t/a^2, \; \rho = r/a, \quad \zeta = z/a, \quad \rho_1 = b/a, \quad \zeta_0 = L/a, \quad \zeta_1 = l_1/a,\\
&\zeta_2 = l_2/a, \quad (\bar{u}_r,\bar{u}_z) = (u_r,u_z)/\alpha_t T_0 a, \quad \bar{e} = e/\alpha_t T_0, \quad \bar{\sigma}_{ij} = (1-\nu)\sigma_{ij}/\alpha_t E T_0,\\
&\delta = m\alpha_t T^*\beta/C_v\tilde{\rho}, \quad m = (1+\nu)/(1-\nu), \quad H_i = \gamma_i a/\lambda_t \; (i=1,2,3).
\end{aligned} \tag{15}$$

where T_0 is reference temperature, t is time, κ is thermal diffusivity, α_t is coefficient of linear thermal expansion, E is Young's modulus, ν is Poisson's ratio, T^* is the absolute reference temperature, C_v is spcific heat at constant volume, $\tilde{\rho}$ is density, γ_i is the heat trasfer coefficient, λ_t is thermal conductivity, $\epsilon = u_{r,r} + u_r/r + u_{z,z}$ is volumetric strain and $\beta = \alpha_t E/(1-2\nu)$.

III. Analysis

To solve the system in the preceding section, we introduce the following expressions for $\bar{T}$ and $\bar{\sigma}_{ij}$.

$$\bar{T} = \bar{T}_0 + \bar{T}_1, \quad \bar{\sigma}_{ij} = \bar{\sigma}_{ij}^{\,0} + \bar{\sigma}_{ij}^{\,1}, \quad \bar{L} = \bar{L}_0 + \bar{L}_1. \tag{16}$$

in which $\bar{T}_0$ and $\bar{\sigma}_{ij}^{\,0}$ are the temperature and stress components for the steady state, while $\bar{T}_1$ and $\bar{\sigma}_{ij}^{\,1}$ are the additional terms for the transient state.

Let us now consider the steady state solutions. For the steady state, the right hand side of eq.(1) are all vanished, then,

$$\Delta_1 \bar{T}_0 = 0 \tag{17}$$

For convenience, assuming the arbitrary functions $f(\zeta)$ and $g(\zeta)$ to be symmetric functions with respect to $\zeta=0$, the solution for eq.(17) can be written as

$$\bar{T}_0 = \sum_{j=1}^{\infty} \cos\omega_j\zeta\{B_{1j}I_0(\omega_j\rho) + B_{2j}K_0(\omega_j\rho)\}. \tag{18}$$

where $I_n(x)$ and $K_n(x)$ are the n-th order modified Bessel functions of first and second kind of argument x, respectively,

$$\begin{aligned}
&B_{1j} = -[\{\omega_j K_1(\omega_j\rho_1) + H_2K_0(\omega_j\rho_1)\}F(\omega_j) + \{\omega_j K_1(\omega_j) - H_1K_0(\omega_j)\}G(\omega_j)]/\Delta_0,\\
&B_{2j} = -[\{\omega_j I_1(\omega_j) + H_1I_0(\omega_j)\}G(\omega_j) + \{\omega_j I_1(\omega_j\rho_1) - H_2I_0(\omega_j\rho_1)\}F(\omega_j)]/\Delta_0,\\
&[F(\omega_j), G(\omega_j)] = 2\int_0^{\zeta_0} [H_1 f(\zeta), H_2 g(\zeta)]\cos\omega_j\zeta d\zeta/I_z,\\
&I_z = \zeta_0 + H_3/(\omega_j^2 + H_3^2),\\
&\Delta_0 = \omega_j^2\{I_1(\omega_j\rho_1)K_1(\omega_j) - I_1(\omega_j)K_1(\omega_j\rho_1)\} - \omega_j H_1\{I_1(\omega_j\rho_1)K_0(\omega_j) + I_0(\omega_j)K_1(\omega_j\rho_1)\} -\\
&\quad -\omega_j H_2\{I_0(\omega_j\rho_1)K_1(\omega_j) + I_1(\omega_j)K_0(\omega_j\rho_1)\} + H_1H_2\{I_0(\omega_j\rho_1)K_0(\omega_j) - I_0(\omega_j)K_0(\omega_j\rho_1)\}.
\end{aligned} \tag{19}$$

and ω_j are the j-th positive roots of

$$H_3\cos\omega\zeta_0 - \omega\sin\omega\zeta_0 = 0 \tag{20}$$

Considering eq.(18), a suitable particular solution of the first relation of eq.(6) has the form

$$\Phi_0 = m\sum_{j=1}^{\infty} \cos\omega_j\zeta\{B_{1j}\omega_j\rho I_1(\omega_j\rho) - B_{2j}\omega_j\rho K_1(\omega_j\rho)\}/(2\omega_j^2) \tag{21}$$

For our present problem, a general solution for the function $\bar{L}_0$ is given by

$$\begin{aligned}
\bar{L}_0 = m[&\sum_{j=1}^{\infty} \sin\omega_j\zeta\{C_{j0}\omega_j\rho I_1(\omega_j\rho) + D_{j0}I_0(\omega_j\rho) + E_{j0}\omega_j\rho K_1(\omega_j\rho) + F_{j0}K_0(\omega_j\rho)\}/\omega_j^3 +\\
&+ \sum_{s=1}^{\infty} \sin k_s\zeta\{C_{s0}k_s\rho I_1(k_s\rho) + D_{s0}I_0(k_s\rho) + E_{s0}k_s\rho K_1(k_s\rho) + F_{s0}K_0(k_s\rho)\}/k_s^3 +\\
&+ \sum_{l=1}^{\infty} R_0(\beta_l\rho)(C_{l0}\beta_l\zeta\cosh\beta_l\zeta + D_{l0}\sinh\beta_l\zeta)/\beta_l^3 + C_0'\rho^2\zeta/2 + D_0'\zeta^3/3 + E_0'\zeta\ln\rho]
\end{aligned} \tag{22}$$

where $R_n(\beta_l\rho)$ is defined as

$$R_n(\beta_l\rho) = J_n(\beta_l\rho) - J_1(\beta_l\rho_1)Y_n(\beta_l\rho)/Y_1(\beta_l\rho_1), \quad n = 0,1\ . \tag{23}$$

$J_n(x)$ and $Y_n(x)$ are the n-th order Bessel functions of first and second kind of argument x, respectively,

$k_s=\pi s/\zeta_0 (s=1,2,\ldots\ldots)$ and β_l are the l-th positive roots of

$$R_1(\beta) = J_1(\beta) - J_1(\beta\rho_1)Y_1(\beta)/Y_1(\beta\rho_1) = 0 \tag{24}$$

Substituting eqs.(21) and (22) into eq.(9), we obtain the steady state stres stress components $\bar{\sigma}_{ij}^{0}$. It follows then that

$$\begin{aligned}\bar{\sigma}_{zz}^{0}&=2(2-\nu)C_0'+2(1-\nu)D_0'+\\&+\sum_{j=1}^{\infty}\cos\omega_j\zeta[C_{j0}\{2(2-\nu)I_0(\omega_j\rho)+\omega_j\rho I_1(\omega_j\rho)\}+D_{j0}I_0(\omega_j\rho)-\\&\quad -E_{j0}\{2(2-\nu)K_0(\omega_j\rho)-\omega_j\rho K_1(\omega_j\rho)\}+F_{j0}K_0(\omega_j\rho)] +\\&+\sum_{s=1}^{\infty}\cos k_s\zeta[C_{s0}\{2(2-\nu)I_0(k_s\rho)+k_s\rho I_1(k_s\rho)\}+D_{s0}I_0(k_s\rho)-\\&\quad -E_{s0}\{2(2-\nu)K_0(k_s\rho)-k_s\rho K_1(k_s\rho)\}+F_{s0}K_0(k_s\rho)] +\\&+\sum_{l=1}^{\infty}R_0(\beta_l\rho)[C_{l0}\{(1-2\nu)\cosh\beta_l\zeta-\beta_l\zeta\sinh\beta_l\zeta\}-D_{l0}\cosh\beta_l\zeta]-\\&-\frac{1}{2}\sum_{j=1}^{\infty}\cos\omega_j\zeta[B_{1j}\{2I_0(\omega_j\rho)+\omega_j\rho I_1(\omega_j\rho)\}+B_{2j}\{2K_0(\omega_j\rho)-\omega_j\rho K_1(\omega_j\rho)\}]\end{aligned} \tag{25}$$

For the sake of brevity, the other expressions for thermal stress components are omitted here.

The unknown constants in the above expression can be determined from the mechanical boundary conditions eq.(11). The procedure, however, becomes to be complicated, then the precise discriptions are omitted.

For eq.(14), there also exists the steady state solution $\bar{T}_0'$. Function $\bar{T}_0'$ can be derived from eq.(13) as follow

$$\begin{aligned}\bar{T}_0'=&\sum_{j=1}^{\infty}\cos\omega_j\zeta\{B_{1j}I_0(\omega_j\rho)+B_{2j}K_0(\omega_j\rho)\}+2\delta\delta_1[\sum_{j=1}^{\infty}\cos\omega_j\zeta\{C_{j0}I_0(\omega_j\rho)-E_{j0}K_0(\omega_j\rho)\}+\\&+\sum_{s=1}^{\infty}\cos k_s\zeta\{C_{s0}I_0(k_s\rho)-E_{s0}K_0(k_s\rho)\}+\sum_{l=1}^{\infty}\cosh\beta_l\zeta C_{l0}R_0(\beta_l\rho)+C_0]\end{aligned} \tag{26}$$

where $\delta_1=(1-2\nu)/(1+\delta)$, $C_0=C_0'+D_0'$.

Obviously, function $\bar{T}_0'$ satisfies the following equation

$$\Delta_1\bar{T}_0' = 0 \tag{27}$$

Next step is to find the transient solution. Applying the method of separation of variables, we have

$$\bar{T}_1' = \sum_{j=1}^{\infty}\cos\omega_j\zeta\sum_{m=1}^{\infty}C_mG_0(\alpha_m\rho)P_{jm}\exp\{-(\alpha_m^2+\omega_j^2)\tau/(1+\delta)\} \tag{28}$$

where $G_n(\alpha\rho)$ is defined as

$$G_n(\alpha\rho)=J_n(\alpha\rho)-\{H_2J_0(\alpha\rho_1)+\alpha J_1(\alpha\rho_1)\}/\{H_2Y_0(\alpha\rho_1)+\alpha Y_1(\alpha\rho_1)\}Y_n(\alpha\rho) \tag{29}$$

$$C_m = 2\alpha_m^2/\{(\alpha_m^2+H_1^2)G_0^2(\alpha_m)-\rho_1^2(\alpha_m^2+H_2^2)G_0^2(\alpha_m)\} \tag{30}$$

and α_m are the m-th positive roots of

$$\begin{aligned}&\alpha^2\{J_1(\alpha)Y_1(\alpha\rho_1)-J_1(\alpha\rho_1)Y_1(\alpha)\}+H_2\alpha\{J_1(\alpha)Y_0(\alpha\rho_1)-J_0(\alpha\rho_1)Y_1(\alpha)\}-\\&-H_1\alpha\{J_0(\alpha)Y_1(\alpha\rho_1)-J_1(\alpha\rho_1)Y_0(\alpha)\}-H_1H_2\{J_0(\alpha)Y_0(\alpha\rho_1)-J_0(\alpha\rho_1)Y_0(\alpha)\}=0\end{aligned} \tag{31}$$

The above transcendental equation for the eigenvalue α is derived from the following thermal boundary conditions

$$\begin{aligned}&on\quad \rho = 1 \quad ; \quad \partial\bar{T}_1'/\partial\rho + H_1\bar{T}_1' = 0,\\&on\quad \rho = \rho_1 \quad ; \quad \partial\bar{T}_1'/\partial\rho - H_2\bar{T}_1' = 0.\end{aligned} \tag{32}$$

The unknown constants P_{jm} are to be determined from the initial condition eq.(2). In order to obtain P_{jm}, we rewrite the initial condition. From eq.(13), $\bar{T}_0$ and $\bar{T}_1$ can be written as

$$\bar{T}_0 = \bar{T}_0' - m^{-1}\delta\delta_1\Delta_1\bar{L}_{0,\zeta}\ , \qquad \bar{T}_1 = \bar{T}_1' - m^{-1}\delta\delta_1\Delta_1\bar{L}_{1,\zeta}\ . \tag{33}$$

Substitution eq.(33) into (2), we obtain the following condition

$$at\ \ \tau = 0 \quad ; \quad (\bar{T}_0'+\bar{T}_1')-m^{-1}\delta\delta_1\Delta_1(\bar{L}_0+\bar{L}_1)_{,\zeta} = 0 \tag{34}$$

Taking into account eqs.(10) and (16), we can rewrite as

$$at\ \ \tau = 0 \quad ; \quad \bar{T}_1' = -\bar{T}_0' \tag{35}$$

Substituting eqs.(26) and (28) into eq.(35) and applying the method of series expansion over the whole region by using the function $\cos\omega_j\zeta G_0(\alpha_m\rho)$ as a kernel, we can find the unknown constants P_{jm} an each j and m.

$$\begin{aligned}P_{jm}=&-\{(B_{1j}+2\delta\delta_1C_{j0})a_{jm}+(B_{2j}-2\delta\delta_1E_{j0})b_{jm}\}/(\alpha_m^2+\omega_j^2) -\\&-4\delta\delta_1\cos\omega_j\zeta_0[H_3\sum_{s=1}^{\infty}(-1)^s(C_{s0}a_{sm}-E_{s0}b_{sm})/\{(\alpha_m^2+k_s^2)(\omega_j^2-k_s^2)\}+\\&+\sum_{l=1}^{\infty}C_{l0}(\beta_l\sinh\beta_l\zeta_0+H_3\cosh\beta_l\zeta_0)c_{lm}/\{(\omega_j^2+\beta_l^2)(\alpha_m^2-\beta_l^2)\}+\\&+H_3\{H_1G_0(\alpha_m)+H_2\rho_1G_0(\alpha_m\rho_1)\}C_0/(\omega_j^2\alpha_m^2)]/I_z\end{aligned} \tag{36}$$

where

$$\begin{aligned}a_{jm}&=\{H_1I_0(\omega_j)+\omega_jI_1(\omega_j)\}G_0(\alpha_m)+\rho_1\{H_2I_0(\omega_j\rho_1)-\omega_jI_1(\omega_j\rho_1)\}G_0(\alpha_m\rho_1)\ ,\\b_{jm}&=\{H_1K_0(\omega_j)-\omega_jK_1(\omega_j)\}G_0(\alpha_m)+\rho_1\{H_2K_0(\omega_j\rho_1)+\omega_jK_1(\omega_j\rho_1)\}G_0(\alpha_m\rho_1)\ ,\\c_{lm}&= H_1R_0(\beta_l)G_0(\alpha_m)+H_2\rho_1R_0(\beta_l\rho_1)G_0(\alpha_m\rho_1)\ .\end{aligned} \tag{37}$$

and a_{sm} and b_{sm} have the similar form to eq.(37) which is obtained by changing eigenvalues ω_j and subscript j for k_s and s, respectively.
For the second of eq.(33), we obtain the following expression for $\bar{T}_1$

$$\begin{aligned}\bar{T}_1=&\sum_{j=1}^{\infty}\cos\omega_j\zeta\sum_{m=1}^{\infty}C_mG_0(\alpha_m\rho)\Theta_{jm}(\tau)-2\delta\delta_1[\sum_{j=1}^{\infty}\cos\omega_j\zeta\{C_{j1}I_0(\omega_j\rho)-E_{j1}K_0(\omega_j\rho)\}+\\&+\sum_{s=1}^{\infty}\cos k_s\zeta\{C_{s1}I_0(k_s\rho)-E_{s0}K_0(k_s\rho)\}+\sum_{l=1}^{\infty}\cosh\beta_l\zeta C_{l1}R_0(\beta_l\rho)+C_1\ ]\end{aligned} \tag{38}$$

where

$$\Theta_{jm}(\tau) = P_{jm}\exp\{-(\alpha_m^2+\omega_j^2)\tau/(1+\delta)\} \tag{39}$$

Taking into account eq.(38), the corresponding particular solution for Φ_1 has the form

$$\begin{aligned}\Phi_1=&-m\sum_{j=1}^{\infty}\cos\omega_j\zeta\sum_{m=1}^{\infty}C_mG_0(\alpha_m\rho)\Theta_{jm}(\tau)/(\alpha_m^2+\omega_j^2) -\\&-m\delta\delta_1[\sum_{j=1}^{\infty}\cos\omega_j\zeta\{C_{j1}\omega_j\rho I_1(\omega_j\rho)+E_{j1}\omega_j\rho K_1(\omega_j\rho)\}/\omega_j^2 +\\&+\sum_{s=1}^{\infty}\cos k_s\zeta\{C_{s1}k_s\rho I_1(k_s\rho)+E_{s1}k_s\rho K_1(k_s\rho)\}/k_s^2 +\\&+\sum_{l=1}^{\infty}C_{l1}R_0(\beta_l\rho)(\cosh\beta_l\zeta+\beta_l\zeta\sinh\beta_l\zeta)/\beta_l^2 +C_1'\rho^2/2 +D_1'\zeta^2]\end{aligned} \tag{40}$$

Thus, by eq.(9), we can obtain the required solution for $\bar{\sigma}_{ij}^1$ for the transient state. For example,

$$\bar{\sigma}_{zz}^{1}=2(\delta\delta_1+2-\nu)C_1'+2(1-\nu)D_1'+$$
$$+\sum_{j=1}^{\infty}\cos\omega_j\zeta[C_{j1}\{2(\delta\delta_1+2-\nu)I_0(\omega_j\rho)+(\delta\delta_1+1)\omega_j\rho I_1(\omega_j\rho)\}+D_{j1}I_0(\omega_j\rho)-$$
$$-E_{j1}\{2(\delta\delta_1+2-\nu)K_0(\omega_j\rho)-(\delta\delta_1+1)\omega_j\rho K_1(\omega_j\rho)\}+F_{j1}K_0(\omega_j\rho)]+$$
$$+\sum_{s=1}^{\infty}\cos k_s\zeta[C_{s1}\{2(\delta\delta_1+2-\nu)I_0(k_s\rho)+(\delta\delta_1+1)k_s\rho I_1(k_s\rho)\}+D_{s1}I_0(k_s\rho)-$$
$$-E_{s1}\{2(\delta\delta_1+2-\nu)K_0(k_s\rho)-(\delta\delta_1+1)k_s\rho K_1(k_s\rho)\}+F_{s1}K_0(k_s\rho)]+$$
$$+\sum_{l=1}^{\infty}R_0(\beta_l\rho)[C_{l1}\{\delta_1\cosh\beta_l\zeta-(\delta\delta_1+1)\beta_l\zeta\sinh\beta_l\zeta\}-D_{l1}\cosh\beta_l\zeta]-$$
$$-\sum_{j=1}^{\infty}\cos\omega_j\zeta\sum_{m=1}^{\infty}\alpha_m^2C_mG_0(\alpha_m\rho)\Theta_{jm}(\tau)/(\alpha_m^2+\omega_j^2) \qquad (41)$$

For the sake of brevity, the other expressions for thermal stress components are omitted here. The unknown coefficients in the preceding expressions can be determined from eq.(11). Hence, using eq.(16), we find the final expressions for $\bar{T}$ and $\bar{\sigma}_{ij}$.

IV. Numerical Results

The numerical calculations are performed for the following characteristic case

$$f(\zeta) = 1 \ (\ -\zeta_1<\zeta<\zeta_1\ ),\ = 0\ (\ \text{for other } \zeta\ ).$$
$$g(\zeta) = 0\ (\ \text{for all } \zeta\ ).$$
$$\nu = 0.3,\quad \rho_1 = 0.5,\quad \zeta_0 = 3.0,\quad \zeta_1 = 1.0,$$
$$H_1 = 1.0,\quad H_2 = 0.5,\quad H_3 = 0.1,\quad \delta = 0.08\ (\text{Coupled}),\ = 0.0\ (\text{Uncoupled}).$$

In Figs.2-6, we show the numerical calculation results. In thses figures the coupled solutions are plotted by the dotted line and also the corresponding numerical results for the uncoupled solutions are shown by the solid lines for comparison.

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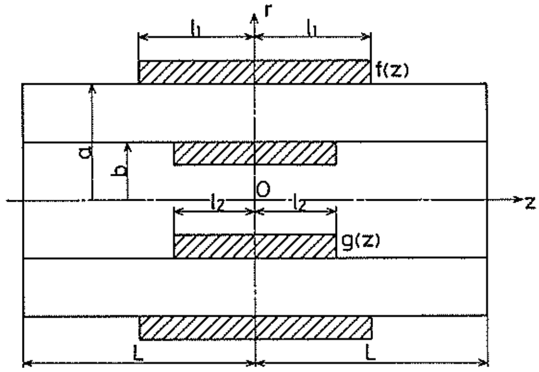

Fig. 1 Circular cylinder with axisymmetric heat supplies

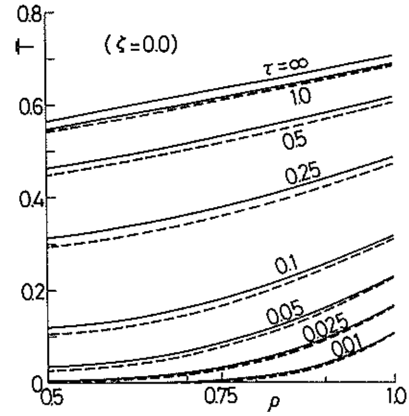

Fig. 2 Temperature distribution on the middle cross section of the cylinder

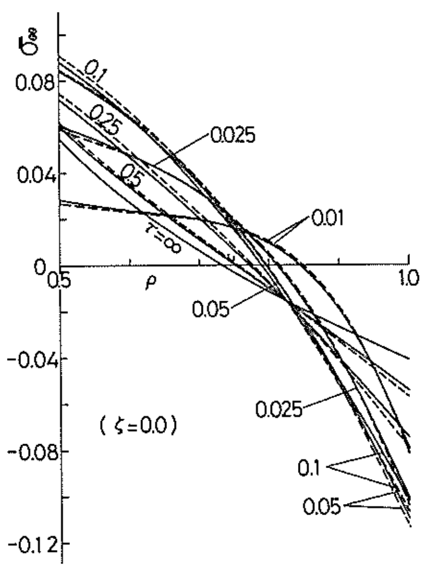

Fig. 3 Hoop stress distribution on the middle cross section of the cylinder

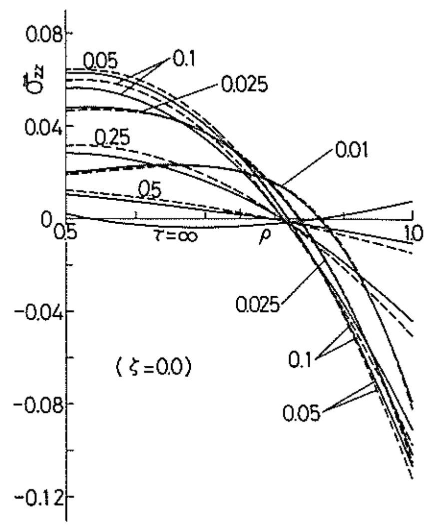

Fig. 4 Axial stress distribution on the middle cross section of the cylinder

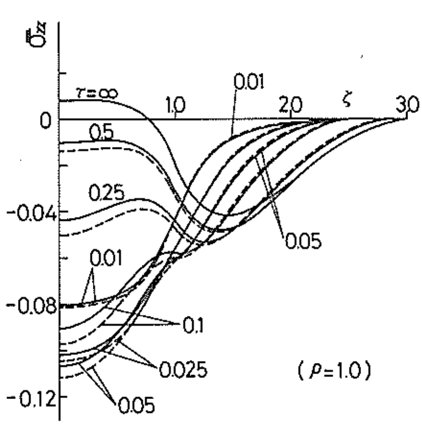

Fig. 5 Axial stress distribution on the lateral surface of the cylinder

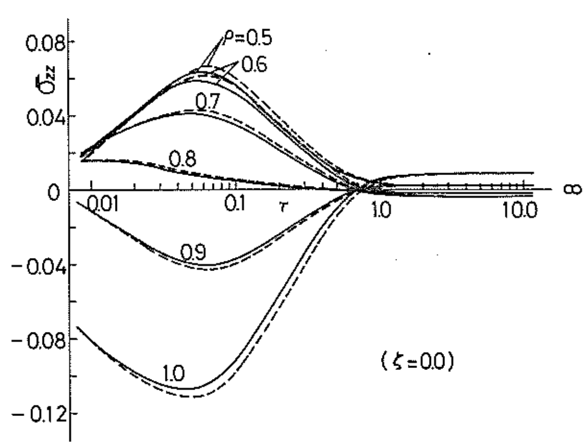

Fig. 6 Axial stress variation with time on the middle cross section of the cylinder