Eigenoscillations for PWR-Core Barrel — Comparison Between Various Theoretical Results for the HDR Geometry

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Abstract

In this paper the results of three approaches to the eigensolutions of a cylindrical shell are compared. The first method, code CYLDY3, is based on analytical solutions of the full partial differential equations for thin-walled cylindrical shells derived by Flügge. This method gives exact results for eigenfrequencies and mode shapes in the framework of the theory. The given boundary conditions are matched exactly. On the other hand, as the differential equations as well as the boundary conditions enter the computer program, it is difficult to take into account any irregularities with respect to the geometry or the material of the shell.

The code CYLDYFE, representing the second approach, is confined to cylindrical shells, too. It yields the eigensolutions by means of the finite element method, using ring type elements where the mode shape functions in azimuthal direction are given analytically. The elastic law for each element is based on Timoshenko's theory for cylindrical shells or, optionally, on Flügge's simplified equations. Since geometrical and material properties may be defined separately for each element, it is easy to treat irregularities of the shell like changes in wall thickness, for example, as long as they remain axisymmetrically.

In the third approach the shell is modelled using the standard finite element program STRUDL/DYNAL. Its advantage is the adaptability to various geometrical configurations. Its disadvantage arises from the computational effort and from limitations in spatial resolution.

The comparison is performed for the core barrel of the HDR-test facility. It is clamped rigidly at the top and stiffened by a heavy ring at the lower end. The wall thickness is constant all over the length. The numerical results show very good agreement between CYLDY3 and CYLDYFE and reasonable, however, less good agreement between CYLDY3 and STRUDL/DYNAL, although a high number of degrees of freedom has been used in the latter.

It is not possible to recommend one of these methods in generality. Rather the proper approach depends on the peculiarities of the problem. But if one of the finite element methods is used, the adequate discretization effort may be estimated by comparison of the eigensolutions with those obtained by the analytical approach in CYLDY3.

1. Introduction

Safety standards for pressurized water reactors (PWR) require that the reactor vessel internals must withstand a sudden break of the primary coolant circuit. The most severe loadings are expected to occur in the first period of the subsequent blowdown phase where depressurization waves travel from the point of break into the vessel and induce considerable pressure differences across the internal structures.

One of the most essential components inside the vessel is the core barrel which supports and confines the reactor core. Therefore the dynamical response of the core barrel under blowdown loading must be investigated.

Besides this accidental loading the core barrel may be submitted to vibrations induced under service conditions. It must be guaranteed that these vibrations do not cause material fatigue with subsequent failure. Therefore the frequencies and amplitudes of the vibrations are of some interest.

For analysis the core barrel is modeled as a circular cylindrical shell, at the upper end clamped to the vessel and at the lower end stiffened by a ring (but not supported). The dynamical response of such a shell to time dependent loadings may be estimated, in principle, by two different approaches:

- direct numerical integration of the appropriate differential equations (e.g. [1, 2]). Here the differential forms have to be replaced by finite difference formulations. The accuracy and especially the resolution of the results is highly affected by the degree of discretisation.
- modal synthesis of pre-calculated eigenvibrations. In this case the integration is performed with respect to generalized coordinates. The resolution depends mainly on the resolution of the modes of the particular eigenvibrations, whereas the accuracy additionally is a matter of the number of modes involved in the analysis as well as of the degree of approximation used for determination of the mode shapes.

In any case the question arises whether the discretization is fine enough and/or whether the number of modes taken into account is high enough in order to meet the required accuracy conditions. Some confidence may be got from comparisons with experimental findings which, however, are available only for a limited number of cases. In addition, possible discrepancies may be hard to interprete. Therefore it is desirable to find solutions which do not involve errors due to the numerical procedures.

Recently analytical eigensolutions [3] of Flügge's full equations for thin-walled circular cylindrical shells [4, 5] have been presented. They provide a quasi-exact determination of eigenfrequencies and modes of the eigenvibrations. To date these solutions are restricted to two sets of boundary conditions, but they may easily be expanded to some other types of boundary conditions.

In applications where the first approach of direct integration is used, the analytical solutions for eigenvibrations may be used as test cases. In applications where the second approach of modal superposition is used, the pre-calculations of the eigenvibrations can be replaced or directly verified by corresponding analytical solutions.

In this paper some eigenvibrations obtained by the above analytical solution are presented for the core barrel of the HDR, a former reactor in Germany which is now used as an experimental facility. They are used to check eigenvibrations determined by two different numerical methods, namely a special program for the analysis of cylindrical shells which employs a ring-type of finite elements, and a standard finite element program, where the core barrel is modeled by plane triangular elements.

Finally the results will be compared with the outcomings from experimental investigations at the HDR. At the time of writing this paper, however, the evaluation of the experimental data is not finished. If possible, these information will be provided during the conference. The geometrical and material properties of the HDR-core barrel are listed in Table I.

Table I: Geometrical and material properties of the HDR-core barrel (Input data for calculations)

Radius of the shell middle surface	1.3185 m
Wall thickness	0.023 m
Shell length	7.57 m
Young's modulus	1.7×10^{11} N/m^2
Poisson's ratio	0.3
density of the shell material	7.8×10^3 kg/m^3
Mass of the ring at the lower end	1.2054×10^4 kg
Moment of inertia of the ring about an horizontal axis in the plane of lower shell end	6.096×10^3 kg m^2
Moment of inertia of the ring about the shell axis	1.219×10^4 kg m^2

2. Analytical Solution (computer code CYLDY3)

The analytical solution of Flügge's shell equations as described by Ludwig and Krieg /3/ is realized in the computer code CYLDY3.

First the homogeneous partial differential equations are reduced to ordinary differential equations by splitting up the displacement field into a set of products, where the factors are shape functions of the displacement distributions in axial and azimuthal direction and a time function. The azimuthal shape functions as well as the time functions are a priori known to be harmonics; under these conditions Flügge's shell equations become ordinary differential equations with respect to the axial coordinate. These equations can be integrated analytically separately for each azimuthal mode shape. The solutions, however, contain characteristic numbers which depend on the yet unknown eigenfrequencies, as well as eight integration constants which are also yet undetermined.

Now introducing eight appropriate boundary conditions (four for each edge) yields an homogeneous set of eight linear algebraic equations for the eight integration constants; the elements of the coefficient matrix consist of transcendental functions of the eigenfrequency. Thus finding the zero of the determinant of this coefficient matrix is equivalent to finding the eigenfrequencies. With this result also the integration constants can be calculated, except for a common factor.

This solution is essentially of analytical nature. The only approximative procedure involved is the root-searching algorithm for the zeros of the determinant which, however, is much simpler than the numerical solution of a differential equation. Practically it does not limit the accuracy of the solution.

3. Ring-type of Finite Elements (computer code CYLDYFE)

As a numerical method a finite element description of circular cylindrical shells has been developed based on ring-type of elements. For the calculations the computer program CYLDYFE has been written.

In a first step the shell is divided axially into a number of "ring elements". The axial boundaries of these elements are introduced as nodes.

Now in each element the unkown displacement components are represented by a set of products of harmonic functions describing the azimuthal distributions and of other functions describing the axial distributions. The latter are approximated by Hermite type interpolation polynomials. These polynomials - of third degree for radial displacements, of first degree for axial and azimuthal displacements - contain the unknown nodal displacement components as coefficients. At boundaries of the shell some of them are determined by prescribed boundary conditions.

The above representation is introduced into expressions for the potential and the kinetic energy of a ring element. Due to the orthogonality of the harmonic functions these expressions split up into separate expressions for each azimuthal mode. Now applying a variational principle with respect to the nodal displacements - as far as they are not prescribed - yields the mass- and stiffness matrices for each element. They are summed up in a proper manner in order to get the overall mass- and stiffness-matrices for each azimuthal mode. These overall matrices define algebraic eigenvalue problems which may finally be solved using standard procedures. Then the eigenvalues represent the square of the angular eigenfrequencies, and the eigenvectors describe the corresponding axial mode shapes.

Shell theory enters this procedure where the potential energy is used. In CYLDYFE three options are available: Timoshenko's shell theory [6] with and without azimuthal extension of the middle surface, as well as Flügge's simplified relations [4].

For application to the HDR-core barrel two different discretizations, one with 35 and one with 70 elements, have been used. In each case the distancies between neighbouring nodes, i.e. the element lengths, were constant all over the shell length.

4. Standard Finite Elements (computer code STRUDL/DYNAL)

As a second numerical method the standard finite element code STRUDL/DYNAL has been used. The core barrel was modeled by plane triangular elements of type SBCT. This type represents a superposition of a plane stress and a bending element. It has nodes at the three corners only. The in-plane displacements are described by linear distributions; the bending displacements are described by polynomials of third degree. The discretization with about 1500 elements is shown in Fig. 1. At the boundaries where the core barrel is clamped or ring-stiffened, the axial length of the elements has been reduced in order to describe the expected bendings as well as possible. On the other hand, symmetry with respect to a plane containing the shell axis is assumed in order to reduce the computational effort. Therefore torsional modes are excluded. Furthermore, the number of degrees of freedom has been reduced

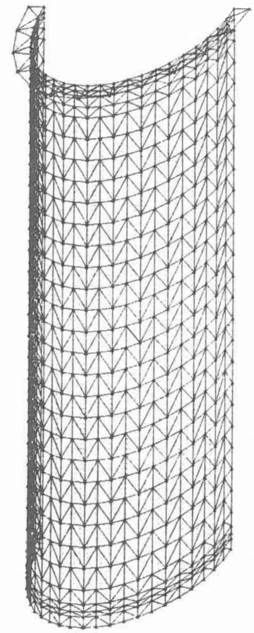


Fig. 1: Finite element model of the HDR-core barrel, used in STRUDL/DYNAL analysis

for the dynamic analysis to 195 using standard condensation techniques.

5. Comparison between Results of the Different Methods

The eigenfrequencies of the shell under investigation up to about 600 Hz, calculated by the analytical method of CYLDY3, are shown in Fig. 2. More than 250 eigenfrequencies are found in this range. Some of them are listed in Table II.

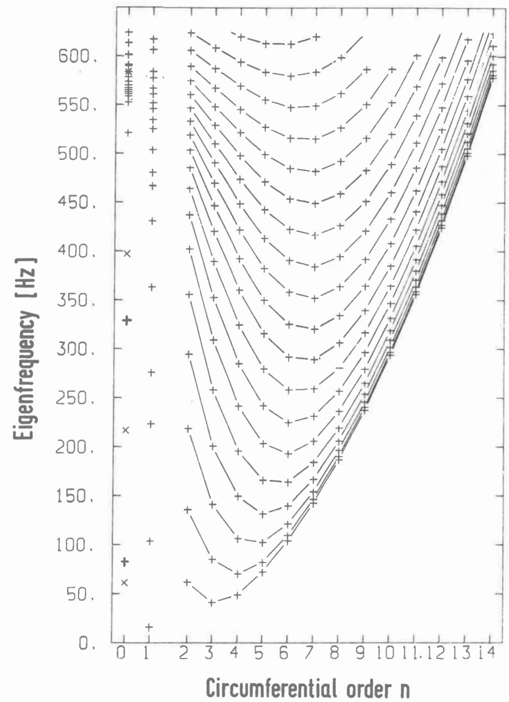


Fig. 2: Eigenfrequencies of the HDR-core barrel calulated with CYLDY3

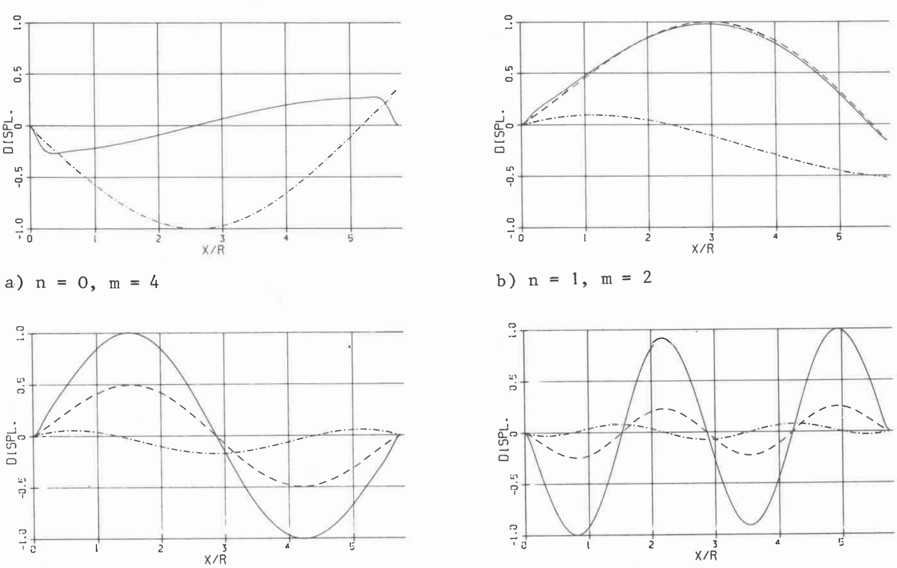
The circumferential order n describes the azimuthal mode shape: i.e., the azimuthal mode shape functions read cos $n\theta$ or sin $n\theta$ with θ the angular coordinate. The axial order m is an integer number which primarily indicates the ascending values of the corresponding eigenfrequencies. Besides this it may serve as a rough descriptor of the axial mode shapes: the number of extrema of the axial modal functions generally coincides with m. Some deviations from this rule occur with n = 0 and n = 1, as well as for high frequencies ($\gtrsim$ 700 Hz).

In Fig. 3 some axial mode shapes of several circumferential orders are plotted. Almost all mode shapes exhibit strong curvatures close to the boundaries. This corresponds to strong bending deformations of the shell.

Table II: List of some eigenfrequencies of the HDR-core barrel (results from CYLDY3), in Hz

n \ m	1	2	3	4	5
0	61.4 (T)	82.4	217.0 (T)	328.6	397.1 (T)
1	16.2	103.7	223.4	275.9	363.3
2	62.0	135.8	218.7	294.6	355.9
3	41.4	85.2	141.3	200.9	258.1
4	49.0	70.5	106.3	149.6	195.9
5	72.4	82.2	102.4	131.4	166.1
6	104.1	109.7	121.3	139.7	164.2
7	142.4	146.4	154.3	166.8	184.2

n : circumferential order, m : axial order, (T) denotes torsional modes



a) n = 0, m = 4

b) n = 1, m = 2

c) n = 2, m = 2

d) n = 4, m = 4

Fig. 3: Axial distributions of displacements for some eigenoscillation modes of the HDR-core barrel calulated with CYLDY3.—— radial, --- azimuthal, -·-·- axial displacement.

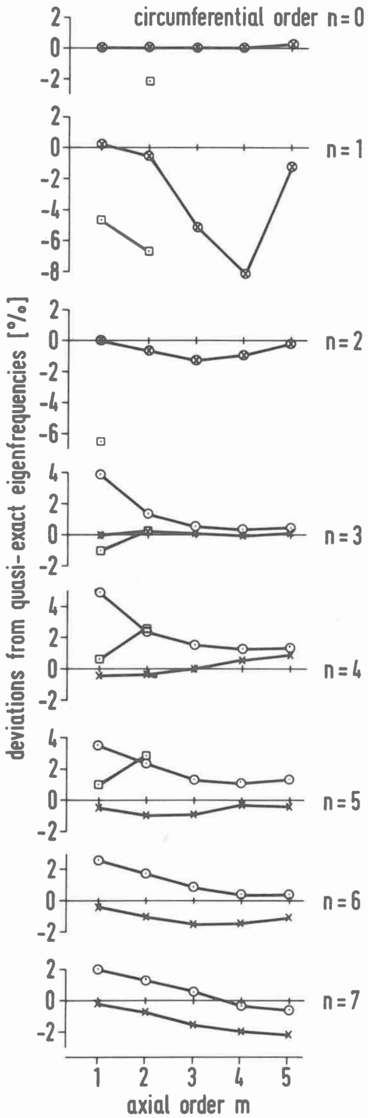


Fig. 4: Deviations of eigenfrequencies calculated with CYLDYFE (x Timoshenko, o Flügge) and STRUDL/DYNAL (□) from quasi-exact values calculated with CYLDY3

The eigenfrequencies obtained by the numerical methods are presented in Fig. 4. Strictly speaking their relative deviations from the corresponding results of CYLDY3 are plotted. Applying 35 as well as 70 finite elements in CYLDYFE showed that the calculated eigenfrequencies were almost the same; therefore only the first discretization was used for the representation in the diagrams. The influence of the two different formulations of Timoshenko's theory was even smaller. This means that extensional deformations are unimportant in this case.

Generally the results of CYLDYFE using Timoshenko's equations agree very well with the quasi-exact eigenfrequencies. Only for some higher axial orders of the "beam mode" ($n = 1$) relatively large deviations occur. With increasing circumferential and axial orders the deviations seem to increase, too. For these discrepancies the linear representation of the in-plane deformations used in CYLDYFE may be of some influence.

Flügge's simplified and Timoshenko's equations differ with regard to the bending forces which are partially neglected by Flügge. This simplification implies additional constraints for the shell model. Therefore somewhat higher eigenfrequencies are computed if bending in azimuthal direction becomes important, which is true for $n \geq 3$.

Although a high number of degrees of freedom has been taken into account in the calculations with STRUDL/DYNAL, only a few modes are included into the comparison. The main reason for this arises from the difficulty of unique identification of the modes, which has to be done by inspecting plots of the modal deformation patterns. The comparison of the frequencies of the identified modes with the results of CYLDY3 shows relatively large deviations for the circumferential orders $n = 1$ and $n = 2$, but smaller deviations, comparable to those of CYLDYFE, for $3 \leq n \leq 5$.

6. Conclusions

Finite element models as well as other numerical methods provide information only for the finite number of nodes or meshpoints. Besides this limited resolution even these results may be object to uncertainties due to their approximative nature. Moreover, the analysis of a transient process often requires a very high number of oscillation modes to be included in order to meet convergence conditions (in precalculations of the HDR-core barrel response due to blowdown loading more than 400 modes have been taken into account, for instance). For these reasons, the use of analytical methods like that of CYLDY3 is preferable, whenever possible.

In many cases, however, such an approach will be impassable due to irregularities of the structure or due to peculiar boundary conditions. Then approximative methods can well substitute the exact methods if properly applied. The appropriate discretization effort may then be estimated by the following procedure: modify the problem such that quasi-exact methods can be applied; carry out computations with both the numerical and the quasi-exact methods; if the results are in good agreement, one may conclude that the discretization is suitable to serve also as a basis for the original problem.

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