Transient Stress Analysis of Fuel Cladding

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Abstract

A method of analysis of dynamic stresses in the cladding modelled as an elastic cylinder is presented. The elasticity modulus has been taken to be varying with time. This has been applied to evaluate the transient stresses due to varying external pressure and a sudden increase in internal pressure following a loss of coolant accident. These stresses are used for evaluation of stress intensity factors to verify the possibility of crack growth. Some sample numerical examples are presented in the paper.

1. Introduction

Integrity of fuel assembly to normal operating and various accident conditions is of paramount importance in the design of a nuclear reactor. Various core disruptive accidents viz. overpower transient, loss of coolant accident (LOCA) have been postulated. The mechanical fuel clad interaction due to differential expansion, clad loading due to fission gas release and consequent rupture of the clad and local contact with molten fuel are some of the postulated mechanisms of failure. In the text a study of brittle fracture of the cladding under LOCA conditions is presented. Apart from the contribution due to increasing fission gas pressure, the stress increases due to rapid depressurization and heating following the accident. The problem becomes more aggravated due to the loss of tensile strength at high temperature and the reduction of fracture toughness with irradiation.

While most of the previous works tackle the fuel failure problem from a static stand point the present work is concerned about the dynamic aspects of the same. Dynamic stresses in the cladding are evaluated for the transient external core pressure and temperature. The cladding is treated as an axi-symmetric shell loaded in plane-strain. The elastic modulus is taken to be gradually reducing. The equations of dynamic equilibrium have been solved for the boundary conditions of known internal and external radial stresses.

Free standing clad is treated as a cylindrical shell with a crack. After obtaining the dynamic stresses for the uncracked shell, the stress-intensity factors are evaluated for the cracked shell and a check is made to study the possibility of unstable crack growth.

Fracture of collapsible clad is treated as a 'peeling-off' problem. The fuel clad combination is treated as a double-cantilever specimen. A known force as given by the pressure differential across a ruptured section is assumed to be acting on this beam and causing the crack to propagate.

Parametric studies have been made for various values of initial crack length. Core depressurisation and clad temperature rise have been obtained. Numerical results for dynamic stresses and crack lengths are presented in this paper.

2. Theory

2.1 Dynamic Response of an Elastic Cylinder

The geometry and loading on the cylinder are assumed to be axi-symmetric. Under plane-strain conditions then the equation of equilibrium of an elastic cylinder can be written as[2]

$$\frac{\nu}{1-\nu}\frac{\rho}{\lambda}\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial r^2} - \frac{1}{r}\frac{\partial u}{\partial r} + \frac{u}{r^2} + \frac{1+\nu}{1-\nu}\beta\frac{\partial T}{\partial r} = 0 \qquad (1)$$

$$\lambda = \nu E/[(1+\nu)(1-2\nu)] \qquad (2)$$

r is the radius, t is the time, u is the radial displacement and T is the temperature at radius r and at time t. ν, E, ρ and β are Poisson's ratio, Young's modulus, density and coefficient of linear thermal expansion of the material respectively. The Young's modulus is assumed to be varying as

$$E = E_0 \exp(-t/\tau_0) \qquad (3)$$

The temperature T(r,t) is solved separately from the heat conduction equation with appropriate thermal boundary conditions.

The boundary conditions in terms of known pressure are

$$\sigma_r = -p_i(t) \text{ at } r = r_i \; ; \; \sigma_r = -p_o(t) \text{ at } r = r_o \tag{4}$$

Subscripts i and o refer to the inner and outer radii.

Initial conditions are taken as

$$u(r,t) = \phi(r) \; ; \; \frac{\partial u}{\partial t}(r,t) = \psi(r) \quad \text{at } t = 0 \tag{5}$$

The solution of equations (1) and (4) are first analytically obtained assuming E to be constant. Natural frequencies (W_n) are obtained from the characteristic equation of (1)

$$\frac{C_0 Z Y_0(Z) - Y_1(Z)}{C_0 Z J_0(Z) - J_1(Z)} = -\mu = \frac{C_0 \gamma Z Y_0(\gamma Z) - Y_1(\gamma Z)}{C_0 \gamma Z J_0(\gamma Z) - J_1(\gamma Z)} \tag{6}$$

where,

$$C_0 = \frac{1-\nu}{1-2\nu} \; ; \; Z = \alpha r_i \; ; \; \gamma = r_o / r_i \tag{7}$$

$$\omega = \alpha \left[(1-\nu) \lambda / (\rho) \right]^{0.5} \tag{8}$$

$J_n(x)$ and $Y_n(x)$ are Bessel functions of the first and second kind respectively. Total dynamic response satisfying (4) and (5) are obtained by modal synthesis.

The final solution may be written as

$$u(r,t) = g(r,t) + \sum_{n=1}^{\infty} \{ C_n \cos \omega_n t + D_n \sin \omega_n t - \omega_n / \alpha_n^2 \int_0^t P_n(\tau) \sin \omega_n (t-\tau) d\tau \} * \tag{9}$$

$$[J_1(\alpha_n r) + \mu_n Y_1(\alpha_n r)] \tag{10}$$

$g(r,t)$ is an auxiliary function to render the boundary conditions homogeneous. Constants C_n and D_n are calculable from the initial conditions.

Variation of E is accounted for in a quasi-static manner by restarting the solution i.e. equations (6) to (9) for each time step with new initial values and new frequencies due to the change in Young's modulus.

2.2 Fracture of a Cylindrical Shell

The stress field in the vicinity of a through the thickness crack of length 2C is given by[3]

$$\sigma = P^{(e)} (C/2r)^{0.5} f_1(\theta) + P^{(b)} z/h (C/2r)^{0.5} f_2(\theta) + O(r^0) \tag{11}$$

where r is the radial distance measured from the crack tip, h is the cylinder thickness and f_1 and f_2 are two functions of O, the crack tip angle. $P^{(e)}$ and $P^{(b)}$ are the stress factors for extensional and bending stresses. Through the presence of cracks accentuate the bending even in axi-symmetrically loaded shells this effect has been ignored for the present study. Considering the stresses responsible for crack-opening for an axial crack we have

$$P^{(e)} = \sigma_\theta \left(1 + 0.317 \lambda_0^2\right)^{0.5} \tag{12}$$

and for a circumferential crack

$$P^{(e)} = \sigma_a \left(1 + 0.05 \lambda_0^2\right)^{0.5} \tag{13}$$

$$\lambda_0^4 = 12(1-\nu^2) C^4 / (R_m^2 h^2) \tag{14}$$

where R_m is the mean radius of the cylinder.

The onset of unstable crack growth is given by

$$2\gamma = \pi C/E * \{[P^{(e)}]^2 + C P^{(e)} dP^{(e)}/dC\} \quad (15)$$

where r is the surface energy per unit area.

$$2\gamma = K_{IC}^2 (1-\nu^2)/E \quad (16)$$

2.3 Fracture of a Double Cantilever Beam

For the collapsible clad, the clad is treated as a beam 'peeling off' the fuel pin. A section of the tube of circumferenced is assumed to be ruptured. This section is assumed to be acted upon by a force governed by the differential pressure. For conservatism this force is assumed to be concentrated at the tip of the ruptured section. The crack length at any instant and the radial deflection are taken as l and y respectively.

The equation of motion is [4]

$$\frac{\partial^4 y}{\partial x^4} + \frac{1}{a^4}\frac{\partial^2 y}{\partial t^2} = 0 \quad (17)$$

where,

$$a^4 = EI/(\rho dw) \quad (18)$$

I is the moment of inertia of the section and x is the axial coordinate.

Boundary conditions are taken as

$$y(\ell,t) = \frac{\partial y(\ell,t)}{\partial x} = 0 \quad (19)$$

$$\frac{\partial^2 y(o,t)}{\partial x} = 0 \; ; \; \frac{\partial^3 y(\ell,t)}{\partial x^3} = \frac{P}{EI} \quad (20)$$

The fracture criterion is

$$EI \frac{\partial^2 y(\ell,t)}{\partial x^2} = (2\gamma d EI)^{1/2} \quad (21)$$

The crack propagates as a flexural wave and the rate is obtained by solving the above equations. It was observed that the result is rather insensitive to the values of P and r.

3. Numerical Results

Coolant pressure and clad surface temperature following LOCA are shown in figures 1 and 2. The internal pressure rise due to fission gas release is taken as a step function with respect to time. Dynamic hoop stress of the clad is presented in figure 3. Figure 4 shows the variation of the expression on the right hand side of equation (14) with initial crack length when the peak hoop stress is of concern. Figure (5) shows the propagation of crack for the beam model.

References

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[2] Liu, C.K., Chang, C.H., "Thermal and Dynamic Response of an Infinite Hollow Cylinder", Developments in Theoretical and Applied Mechanics, Vol.2, pp. 487-501 (1964).

[3] Folias, E.S., "On the Fracture of Nuclear Reactor Tubes", paper C 4/5, 3rd SMiRT, London (1975).

[4] Bilek, Z.J., Burns, S.J., "The Dynamics of Crack Propagation in Double Cantilever Beam Specimens", Dynamic Crack Propagation - G.C. Sihed, Noordhoff International Publishing (1973).

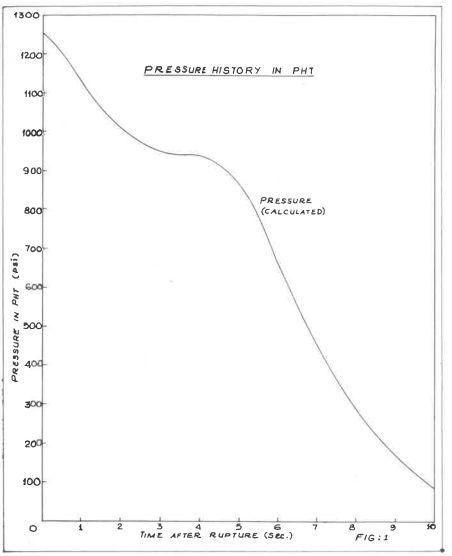

FIG: 1

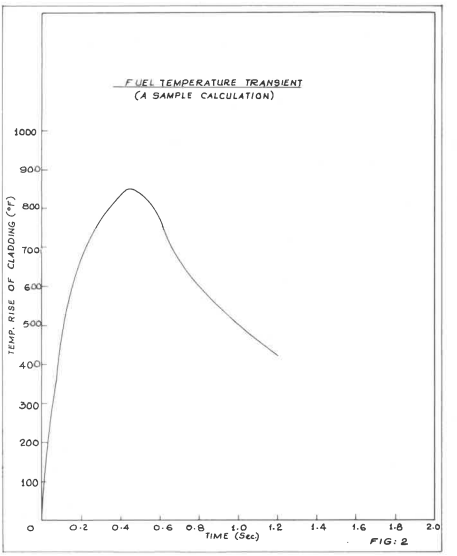

FIG: 2

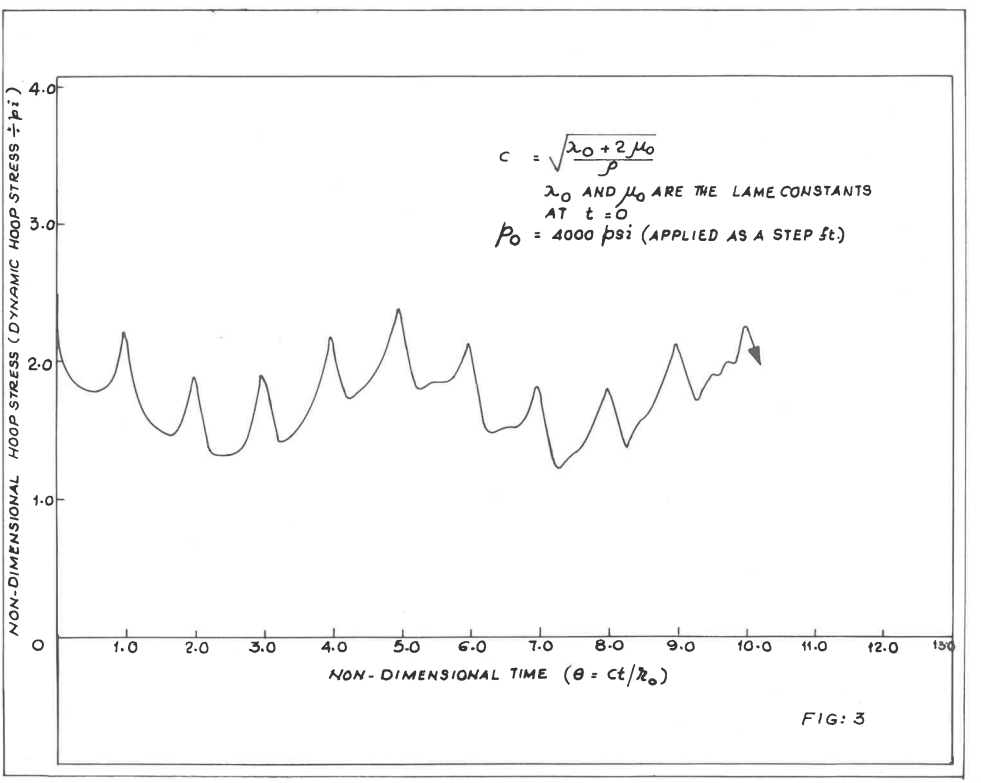

FIG: 3

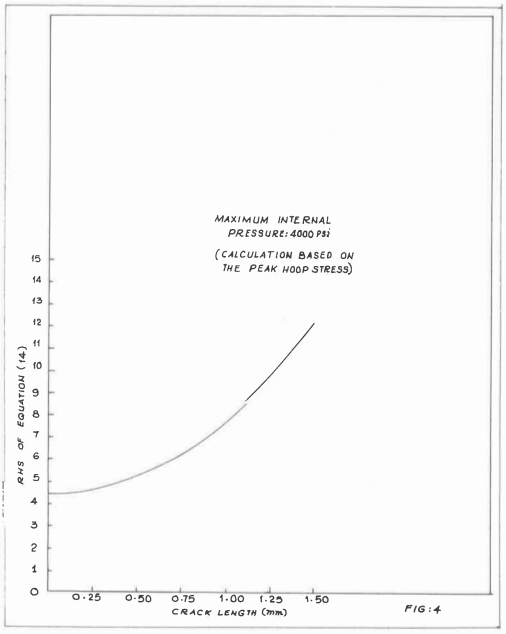
MAXIMUM INTERNAL
PRESSURE: 4000 psi
(CALCULATION BASED ON
THE PEAK HOOP STRESS)
RHS OF EQUATION (14)
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
0·25
0·50
0·75
1·00
1·25
1·50
CRACK LENGTH (mm)

FIG: 4

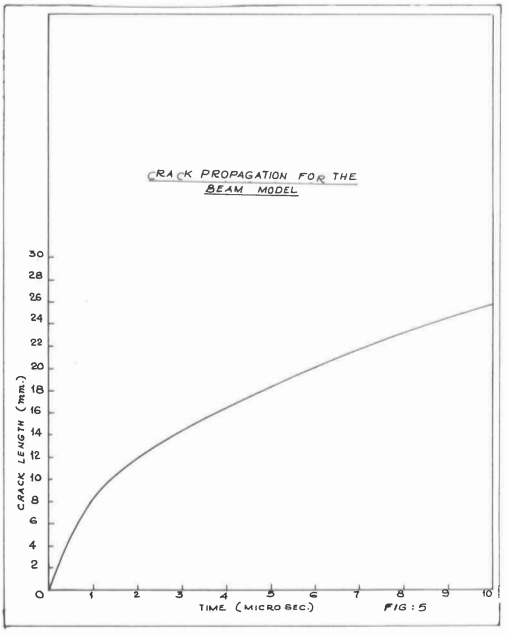
CRACK PROPAGATION FOR THE
BEAM MODEL
CRACK LENGTH (mm.)
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
1
2
3
4
5
6
7
8
9
10
TIME (MICRO SEC.)

FIG: 5